

THE STUDY OF NEUTRINO-NUCLEUS INTERACTION USING THE FERMI GAS AND THE MODIFIED FERMI GAS MODELS

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ABSTRACT: This study explores the Relativistic Fermi Gas Model (RFG) and the Modified Fermi Gas Model (MFG), which have been widely used to characterize quasi-elastic neutrino-nucleus interactions. The RFG considers nucleons as a gas of free particles within the nucleus; nevertheless, it is limited in its ability to account for nuclear correlations and binding energies. The MFG improves the RFG by integrating adjustments for these correlations and delivering a more precise spectral function. Comparisons between these models and experimental data are performed to enhance the understanding of neutrino interactions and the reconstruction of their energy.

Introduction

Experiments with (anti)neutrinos cover a wide range of energies, from a few MeV to hundreds of GeV, using nuclear targets. At lower energies, neutrinos scatter elastically or quasi-elastically through weak interactions (NC or CC), leading to the emission of particles such as photons, electrons, or neutrinos. As energy increases, hadrons are produced along with new particles, such as pions and kaons, accompanied by the residual nucleus. To adequately describe these processes, modeling of the initial and final nuclear states is necessary, which becomes increasingly complex as the energy increases, leading to the use of approximations such as the Fermi gas model, simplifying the description of these states. These models are essential components in simulators like Geant4, where they are employed to simulate particle interactions within the nucleus.

Theoretical Description

RELATIVISTIC FERMI GAS MODEL (RFG):

The **RFG model** considers the nucleus as a gas of free nucleons (protons and neutrons). These nucleons obey the Pauli Exclusion Principle, meaning they occupy quantum states up to a certain limit known as the Fermi energy [1].

Principal assumptions of the RFG model:

1. Within the nucleus, there are no nuclear correlations and nucleons can travel freely;
 2. Up to the Fermi momentum, nucleons have a uniform momentum distribution;
- The **Fermi momentum** and **Fermi energy** [1] can be expressed as follows:

$$p_F = [3\pi^2\rho]^{1/3} \quad E_F = \sqrt{p_F^2 + M^2}$$

Where ρ is the density of the nucleon inside the nucleus and M is the mass of the nucleon.

Smith and Moniz [2] obtained the formula for the **double differential cross section** by applying the **SM of weak interactions** and the **Fermi gas model's** considerations:

$$\frac{d^2\sigma}{dk'd\Omega_l} = \frac{G_F^2 k'^2 \cos^2(\frac{1}{2}\chi)}{2\pi^2 M} \left\{ W_2 + [2W_1 + \frac{m_l^2}{M^2} W_3] \tan^2(\frac{1}{2}\chi) + (W_5 + W_8) m_l^2 / (ME_l \cos^2(\frac{1}{2}\chi)) \right. \\ \left. - 2W_8/M \tan(\frac{1}{2}\chi) \sec(\frac{1}{2}\chi) [-Q^2 \cos^2(\frac{1}{2}\chi) + |\vec{q}|^2 \sin^2(\frac{1}{2}\chi) + m_l^2]^{1/2} \right\},$$

The **Spectral Function** [1], which provides the energy and momentum distribution, can be utilized to take **relativistic particles** and **binding energies** into account:

$$S(\vec{p}, E) \propto \Theta(p_F - p) \delta(E - \sqrt{|\vec{p}|^2 + M^2} + \epsilon)$$

Leading to a **relativistic response function** calculated by Gaisser and O'Connell [3]:

$$R(\vec{q}, q_0) = \frac{1}{\frac{4}{3}\pi p_F^3} \int \frac{d^3p_N M^2}{E_N E_{N'}} \delta(E_N + q_0 - E_B - E_{N'}) \theta(p_{F_N} - |\vec{p}_N|) \theta(|\vec{p}_N + \vec{q}| - p_{F_{N'}})$$

That **adjusted** the expression for the cross section [1]:

$$\frac{d^2\sigma}{d\Omega_l dE_l} = C \frac{d\sigma_{free}}{d\Omega_l} R(\vec{q}, q_0)$$

So the final expression of the the **diferencial cross section for neutrino-nucleus scattering** [1] is:

$$\sigma_{\nu A}(E_l, \Omega_l) = \left(\frac{d^2\sigma}{dE_l d\Omega_l} \right)_{\nu A} = 2 \int d\vec{r} d\vec{p} \frac{1}{(2\pi)^3} n_n(\vec{p}, \vec{r}) \left(\frac{d^2\sigma}{dE_l d\Omega_l} \right)_{\nu n}$$

Comparison Between Models

The Figure 1 represents the results presented for the **ratio of scattering cross section** per interacting nucleon obtained using **LFG model** and **LFG model with RPA effect (LFG + RPA)** for (anti)neutrino induced processes in ¹²C, ⁴⁰Ar, ⁵⁶Fe and ²⁰⁸Pb to the **scattering cross section on free nucleon target** in the energy region from **threshold to 0.8 GeV**.

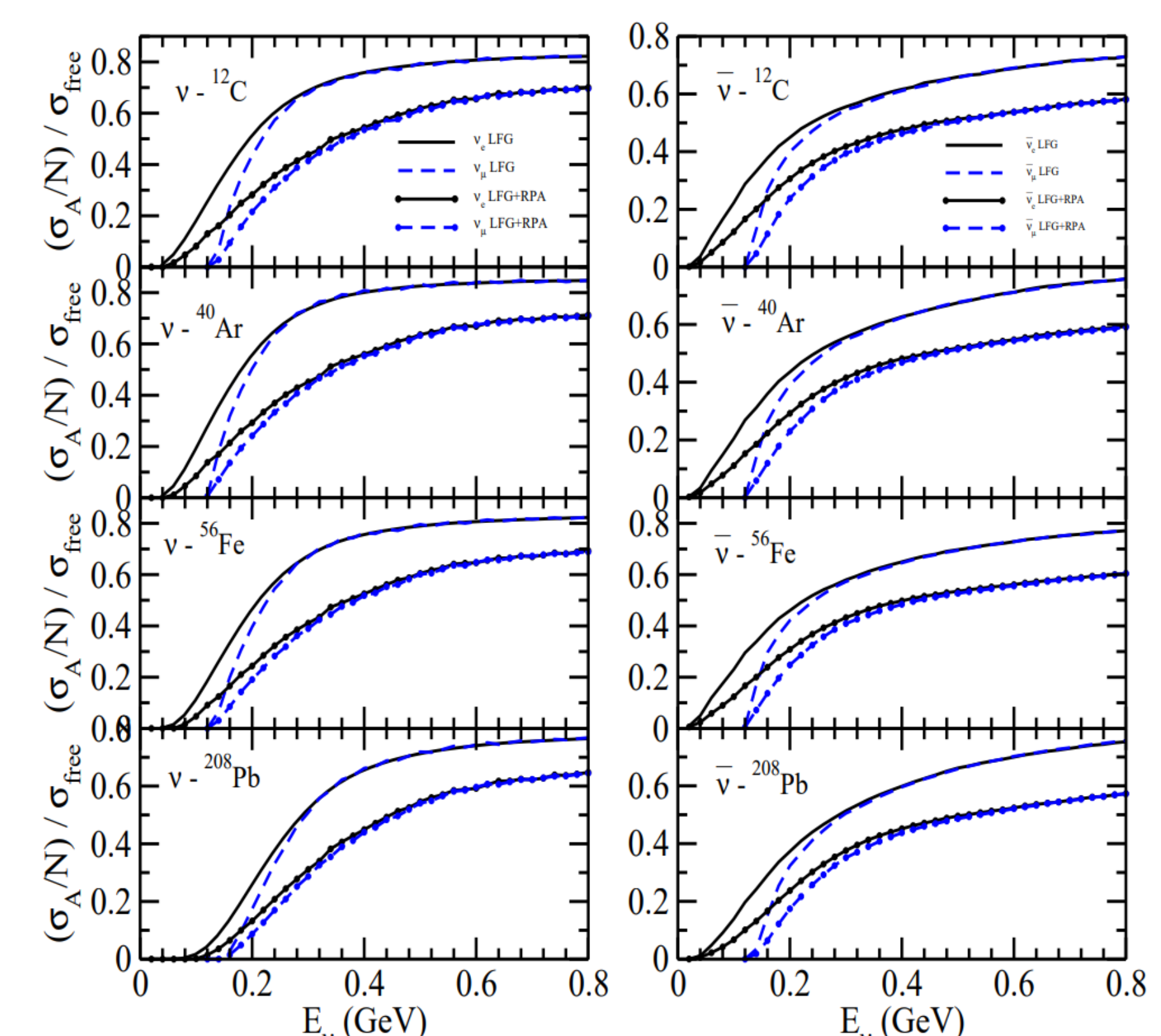
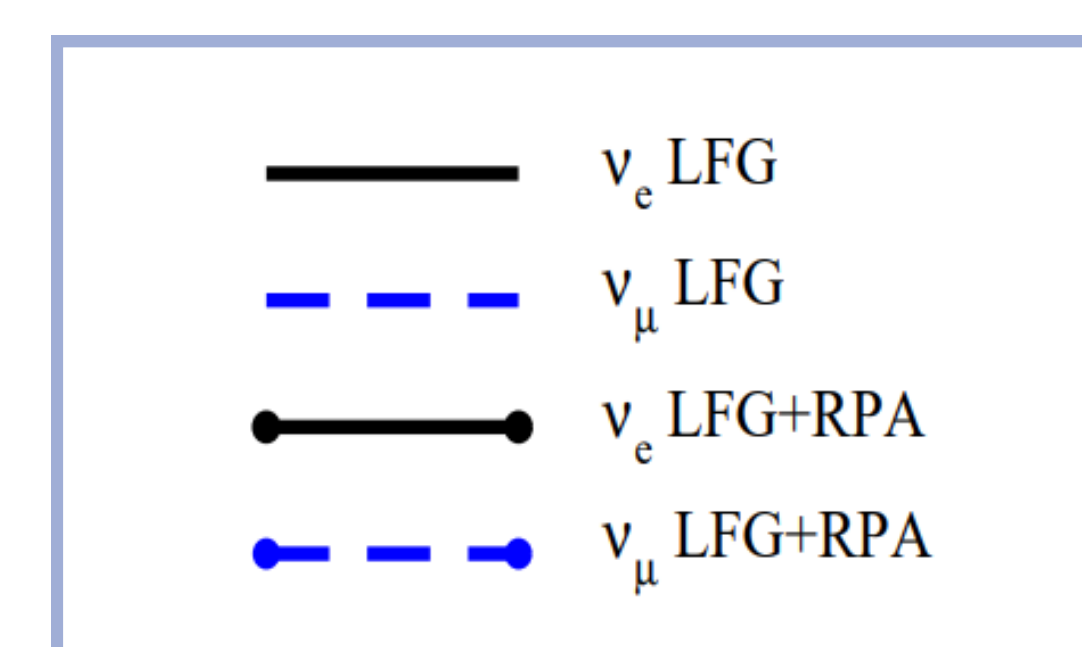


Figure 1: Ratio, for neutrino (left panel) and antineutrino (right panel) induced processes in ¹²C, ⁴⁰Ar, ⁵⁶Fe, and ²⁰⁸Pb. The solid (dashed) line represents cross section obtained from electron (muon) type neutrino and antineutrino. Ref. [1].

Conclusion

This work analyzed the Relativistic Fermi Gas (RFG) and Local Fermi Gas (LFG) models, emphasizing their significance in characterizing neutrino-nucleus interactions. While the RFG offers a simple approach, its limitations make it less accurate at higher energies. The LFG model provides a more realistic description. These studies provide the framework for future advances, aiming to refine these models for better accuracy in describing neutrino-nucleus interactions, especially in complex, high-energy regions.

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