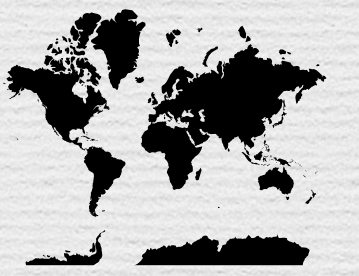


Synthetic gauge fields in trapped-ion crystals

From background $U(1)$ Peierls' phases to $\mathbb{Z}_2$ gauge fields

Collaborators



O. Bazavan, S. Saner, G. Araneda, R. Srinivas, C. Ballance, $\mathbb{Z}_2$ – LGTs
D. Lucas (Oxford University)



T. Schätz, U. Warring (Freiburg University) $U(1)$ Peierls



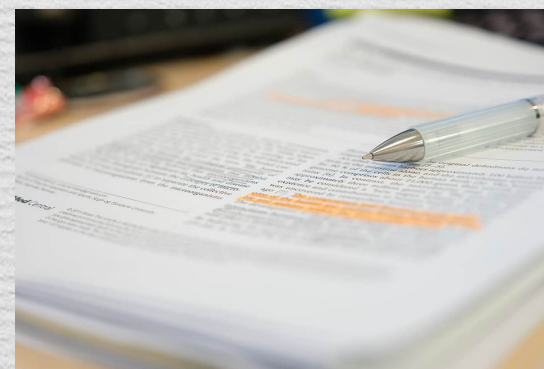
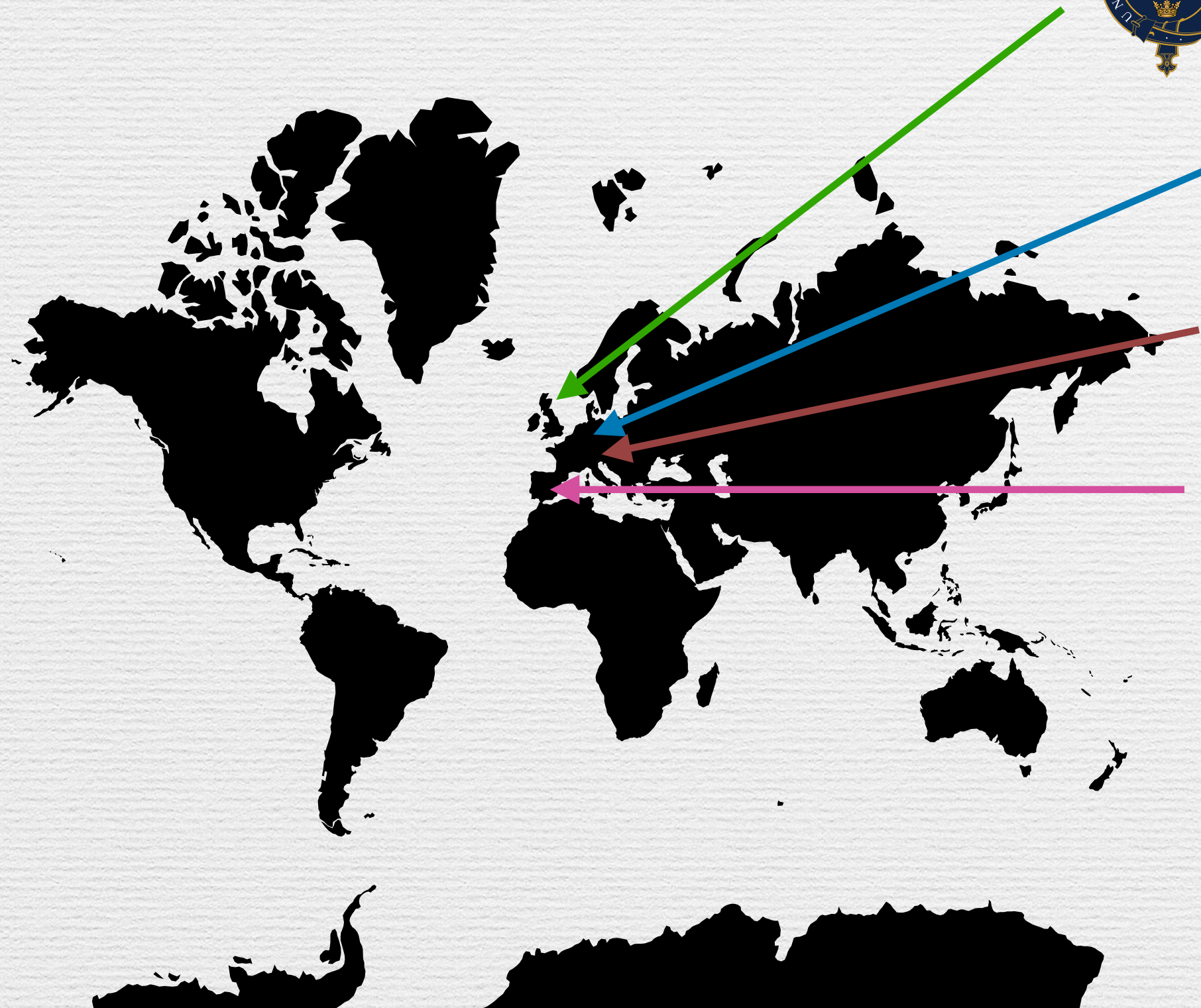
E. Tirrito (ICTP Trieste) $\mathbb{Z}_2$ – LGTs



D. Porras, (IFF, CSIC Madrid) $U(1)$ Peierls



E. Domanti, L. Amico (Catania & TTI Abu Dhabi) $\mathbb{Z}_2$ – LGTs



A.B, T. Schätz, D. Porras, *PRL* **107**, 150501 (2011)

P. Kiefer, ..., A.B, ..., T. Schätz, *PRL* **123**, 213605 (2019)

O. Bazavan, S. Saner, ..., A.B, *Comms. Phys.* **7**, 229 (2024)

S. Saner, O. Bazavan, ..., A.B. R. Srinivas (in prep.) , *arXiv:24xx.SOON*

E. Domanti, L. Amico, A.B, (in prep) *arXiv:24xx.SOON*

Contents

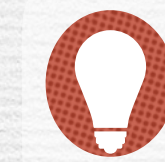
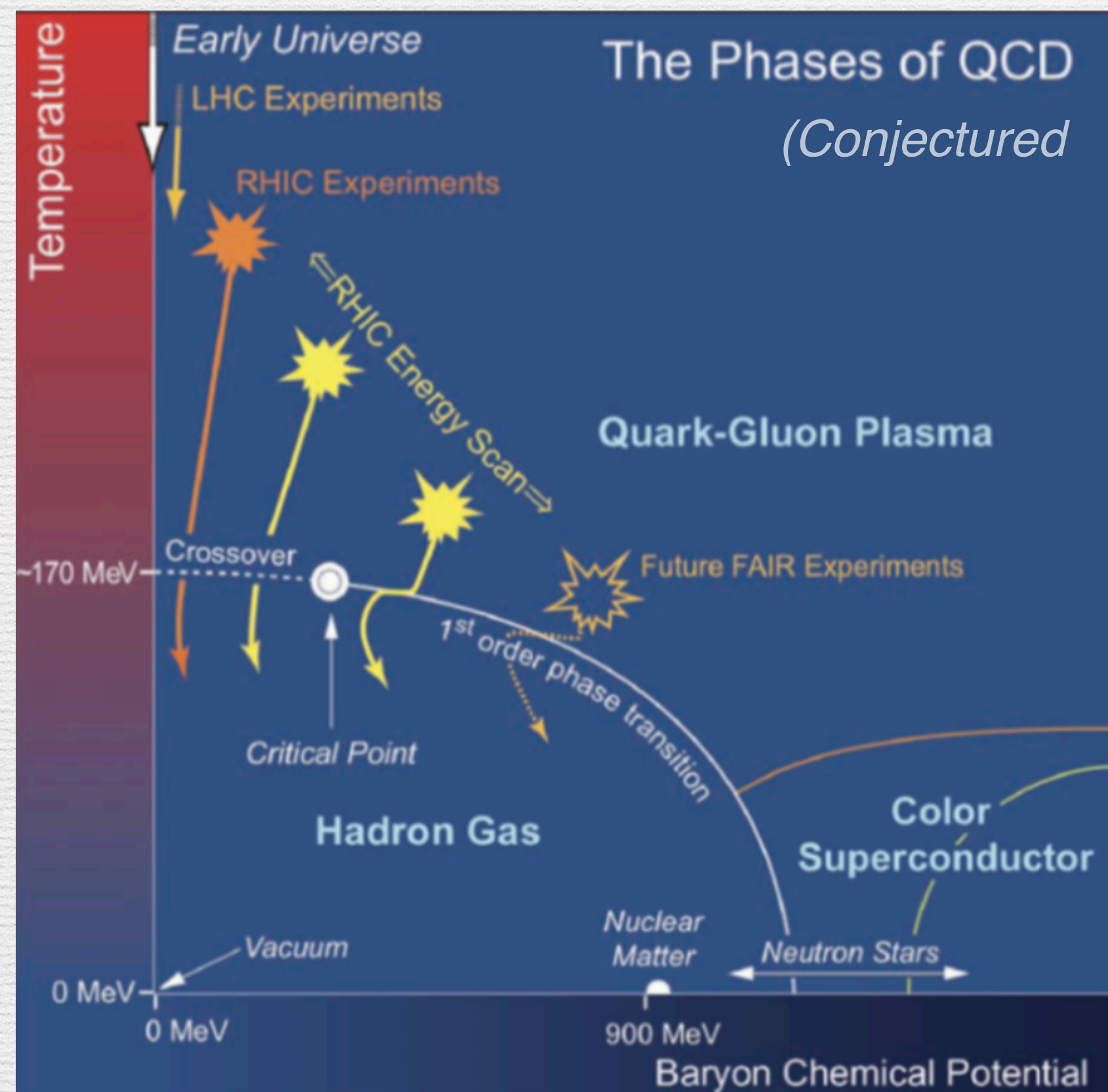


- ❧ 1. Motivation
- ❧ 2. Floquet engineering and $U(1)$ Peierls' phases for phonons
- ❧ 3. Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons
- ❧ 4. $\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression
- ❧ 5. Aharonov-Bohm instability for deconfined $\mathbb{Z}_2$ LGTs

Motivation



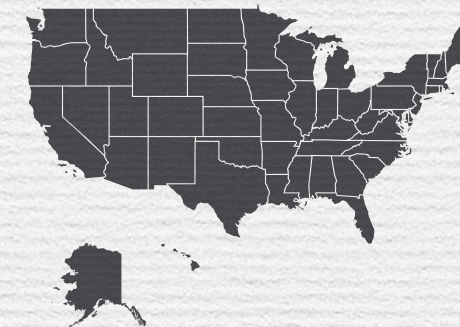
Can we exploit trapped-ions as quantum simulators for **high-energy physics**?



R. Feynman, *Int.J.Th.P.* **21**, 467 (1982).



S. P. Jordan, K. S. Lee,
and J. Preskill, *Science* **336**, 1130 (2012)



E. Zohar, et al., *PRL* **109**, 125302 (2012)
D. Banerjee, et al., *PRL* **109**, 175302 (2012)

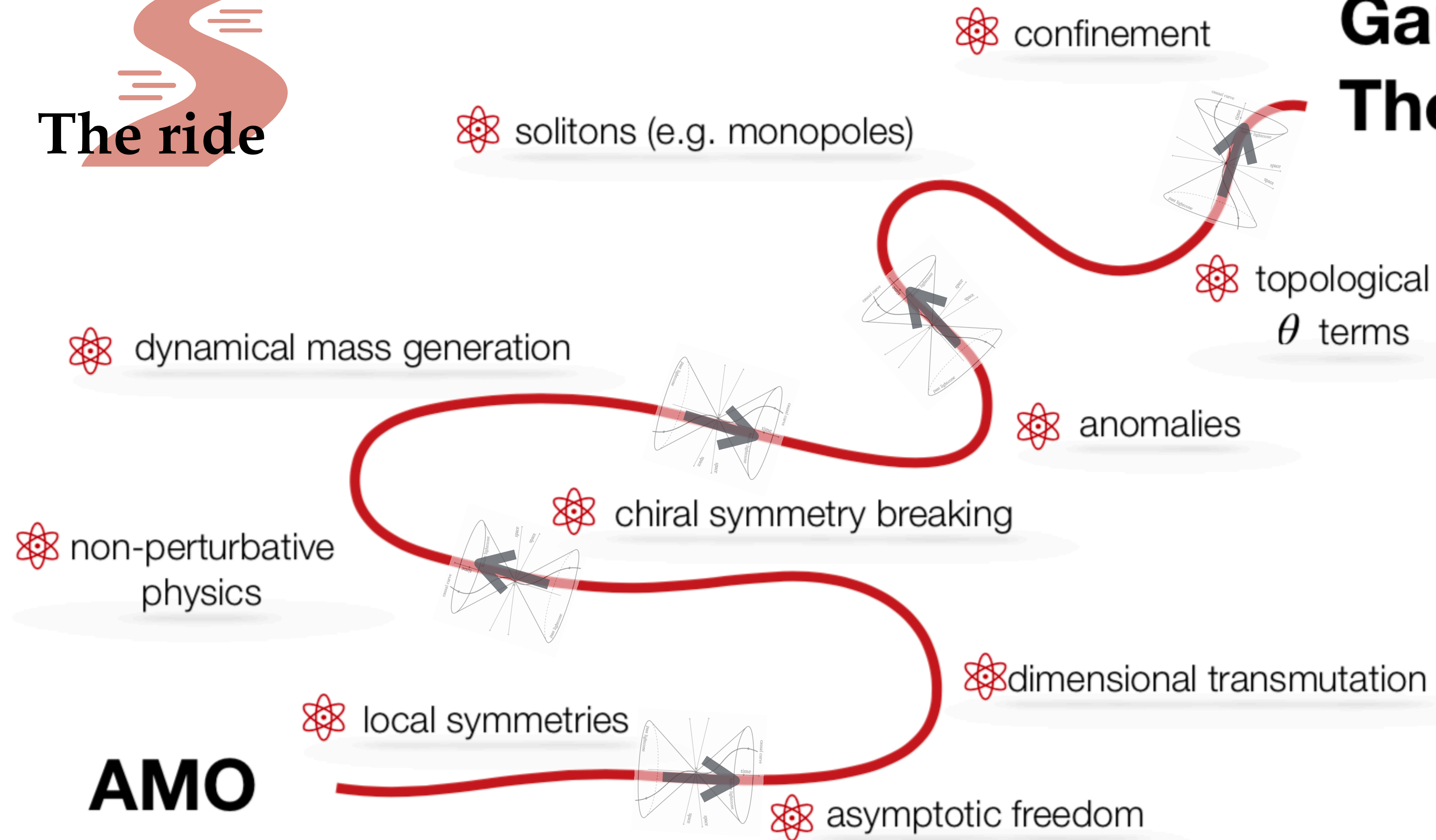


M. Bañuls et al., *Eur. Phys. J. D* **74**, 165 (2020).
C. Bauer et al., *PRX Quantum* **4**, 027001 (2023).



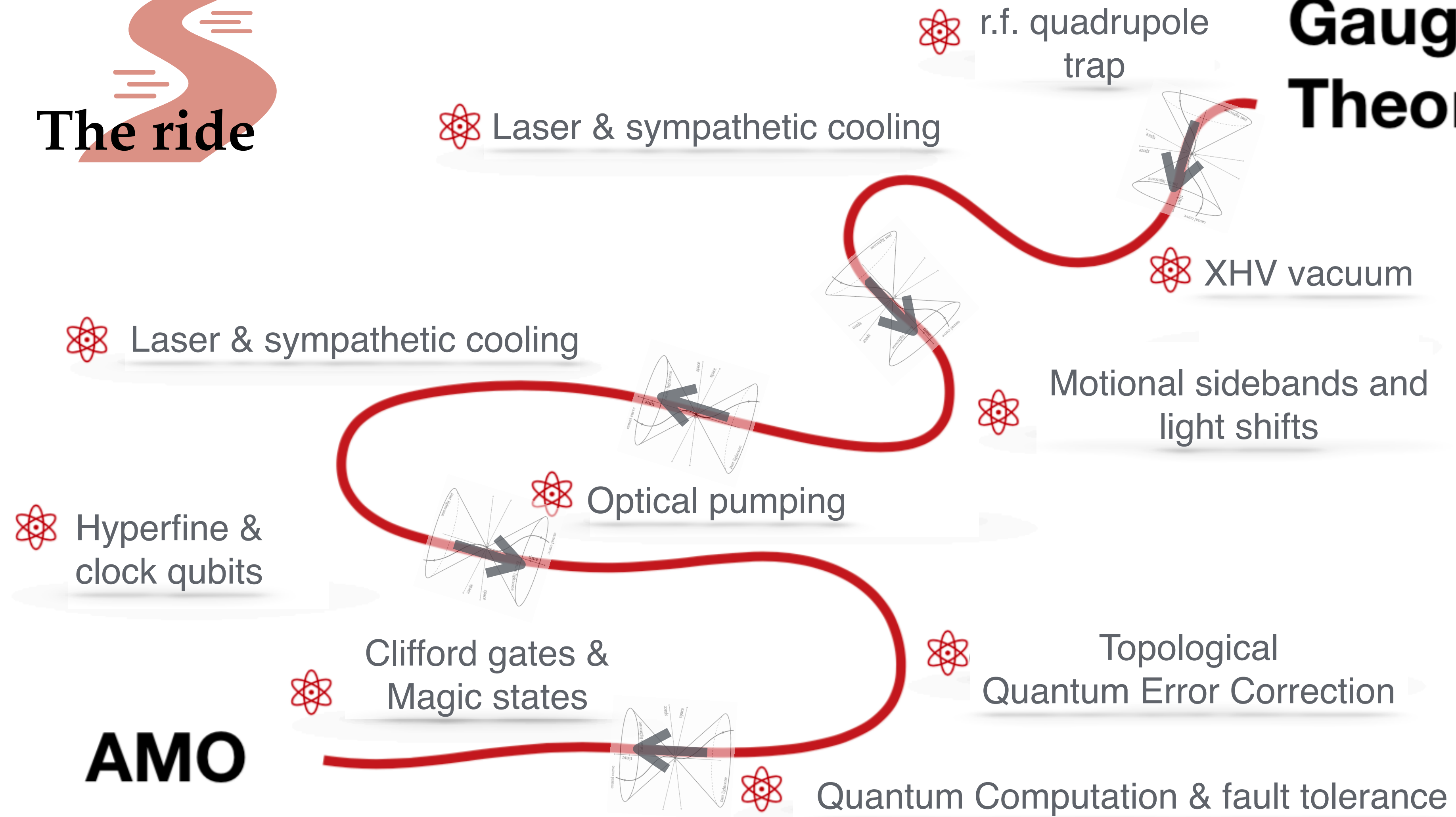
The ride

Gauge Theory

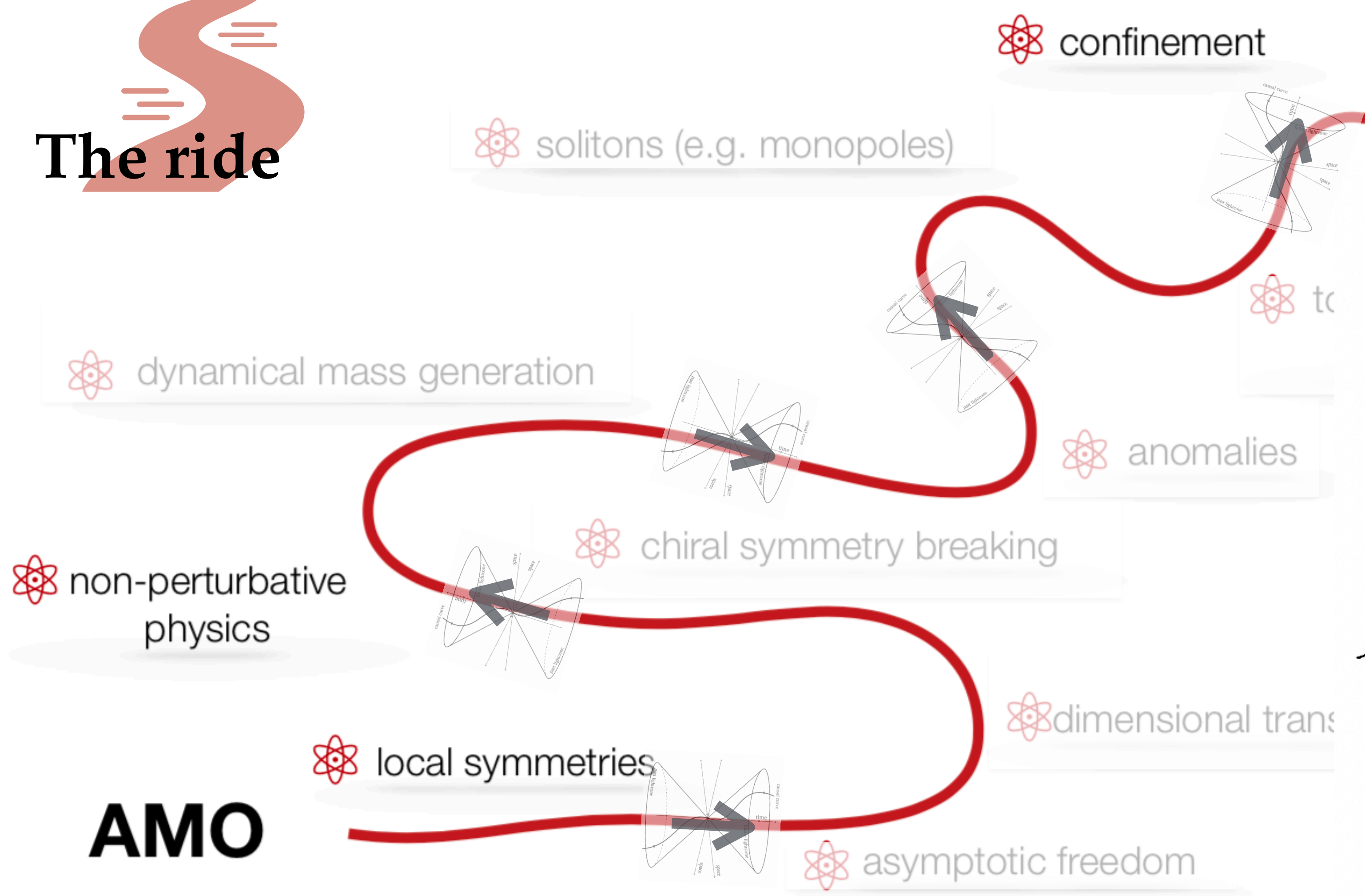


The ride

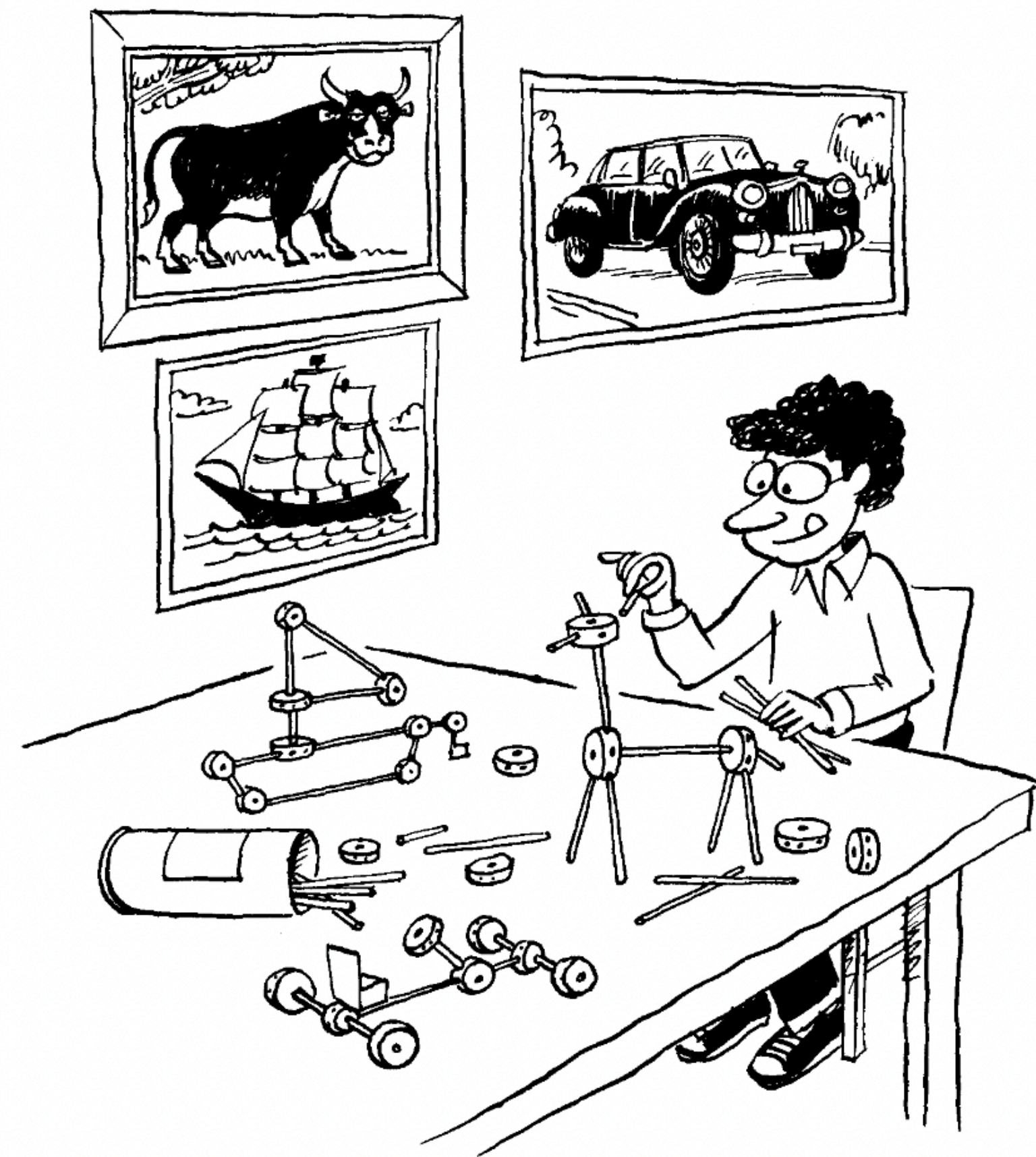
Gauge Theory



The ride



Gauge Theory



AMO



Motivation

Practical applications of quantum computers in general require **large resources** in *qubit #* and *T-gate counts*



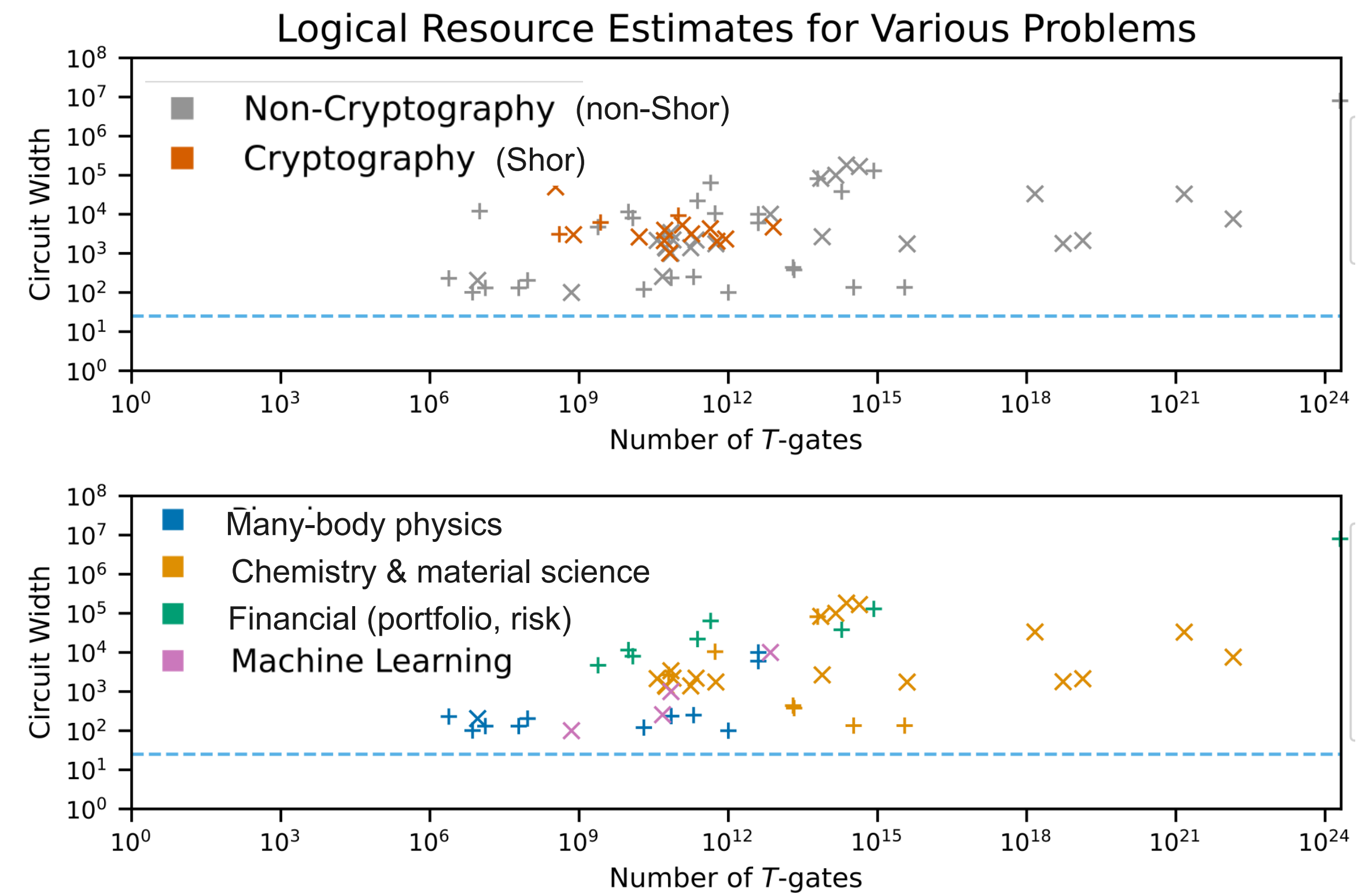
Goal of digital QS

$$U(t) = e^{-iHt}$$

$$u(n) \in \{U_i^{(n)}, U_{ij}^{(n)}\}_{n=1}^{N_U}$$

$$\prod_{n \in \mathbf{n}} u(n) \xrightarrow{N_U \gg 1} U_{\text{eff}} \approx U(t)$$

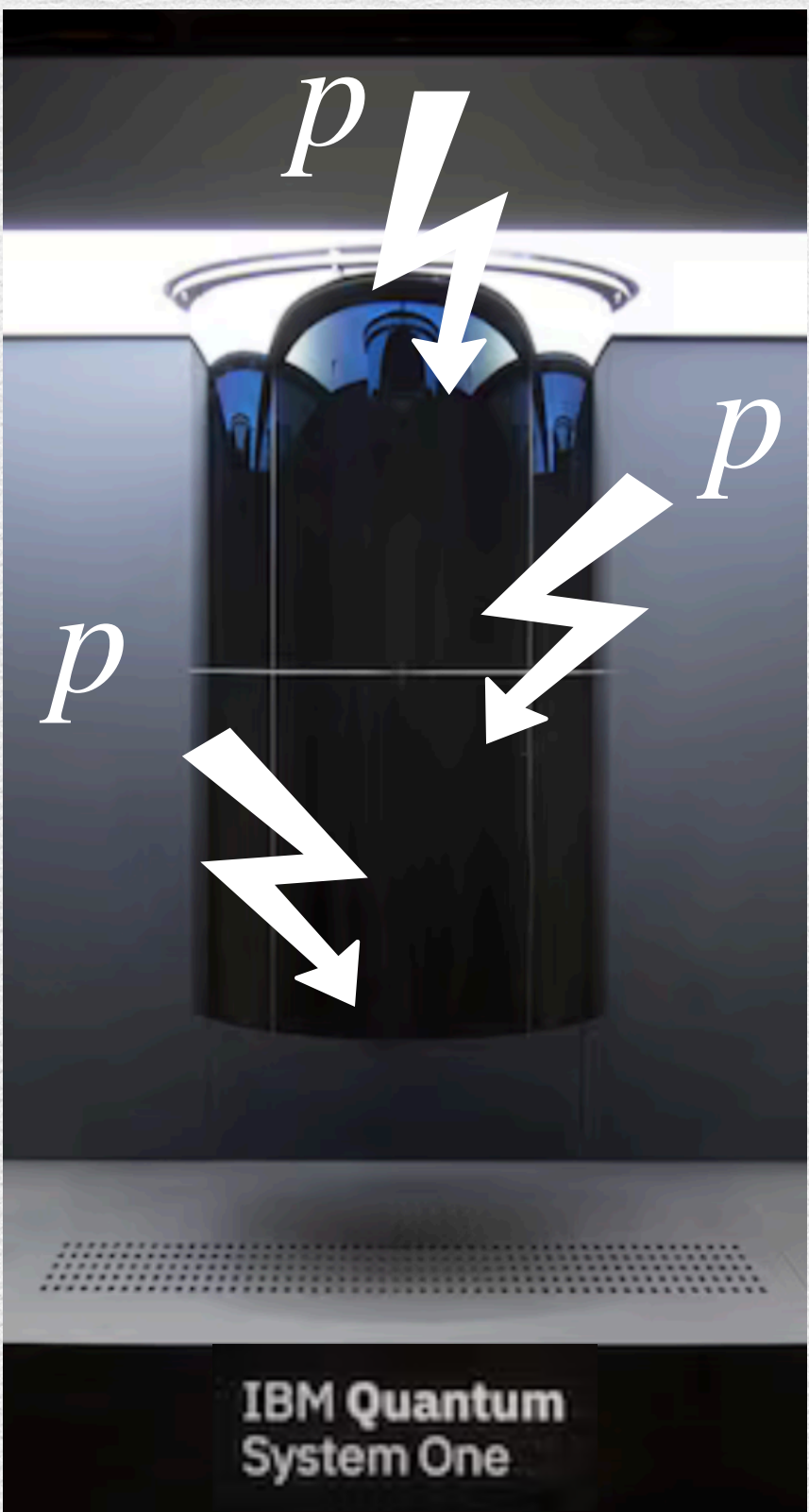
Eq. Effective evolution (Trotter-Suzuki)





Motivation

Gates are **noisy**, & current **quantum error correction** approaches **increase** the **resources** even further



Goal of digital QS

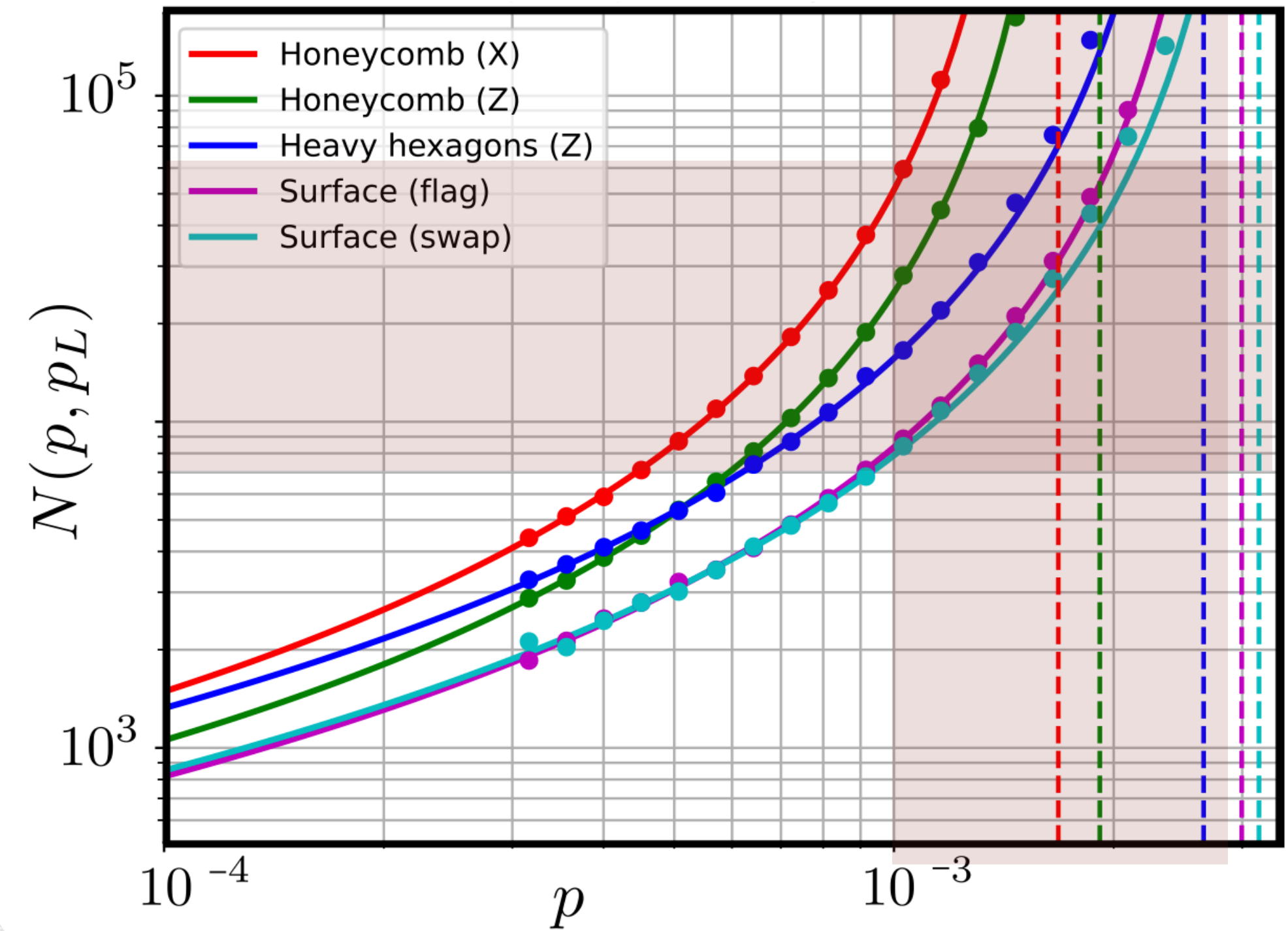
$$U(t) = e^{-iHt}$$

$$u(n) \approx \{U_i^{(n)}, U_{ij}^{(n)}\}_{n=1}^{N_U}$$

$$\prod_{n \in \mathbf{n}} u(n) \xrightarrow{N_U \gg 1} U_{\text{eff}} \approx U(t)$$

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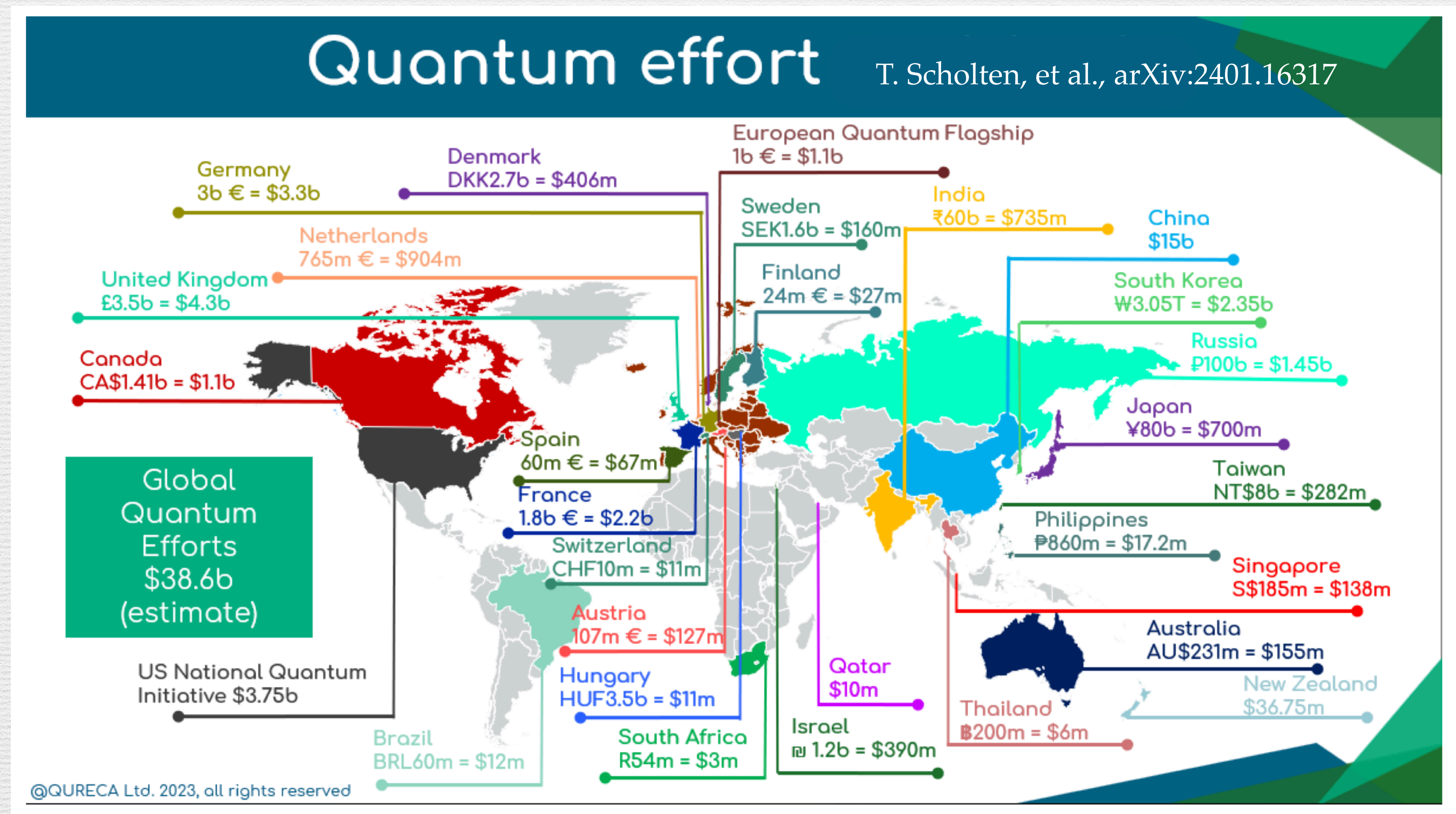
QEC teraquop footprint $p_L = 10^{-12}$





Motivation

And yet ...





Motivation



We may benefit from “opening the black box” & exploiting the native d.o.f.



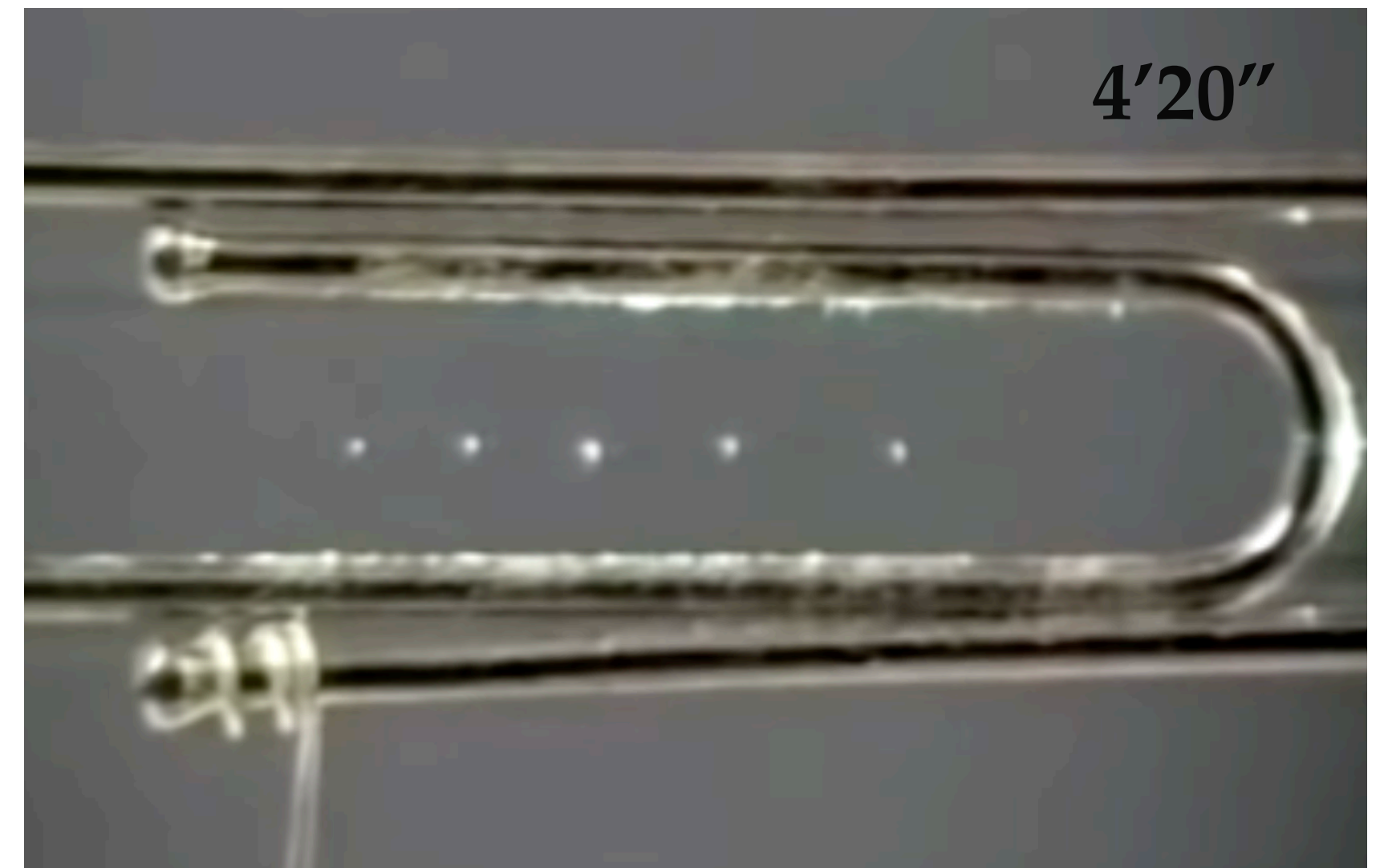
Goal of analog QS

$$H$$

$$H_{\text{sys}}(g_1, g_2 \dots)$$

$$H_{\text{sys}} \xrightarrow{\{g_i\} \rightarrow \{\tilde{g}_i\}} H_{\text{eff}} \approx H$$

Eq. Effective Hamiltonian



T. W. Hansch, Nobel prize '05 YouTuber



Motivation

Trapped-ion “*Art brut*” or “made by theorists who are untrained and untutored in the traditional beauty of experimental physics”

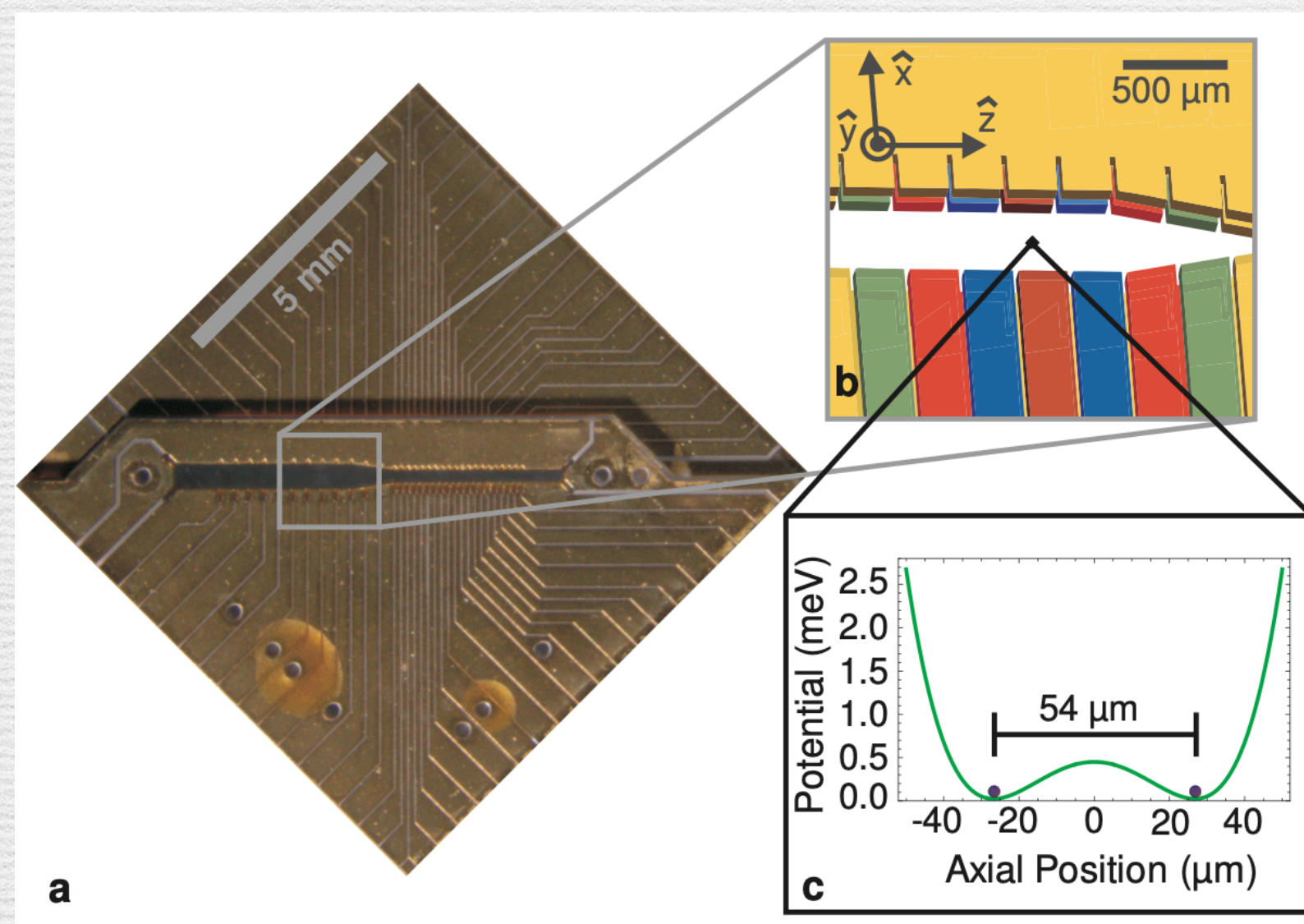


Motivation

H. Heizen and D. Wineland, *PRA* **42**, 2977 (1990)

K. R. Brown, *et al.*, *Nature* **471**, 196 (2011)

J. M. Harlander, *et al.*, *Nature* **471**, 200 (2011)

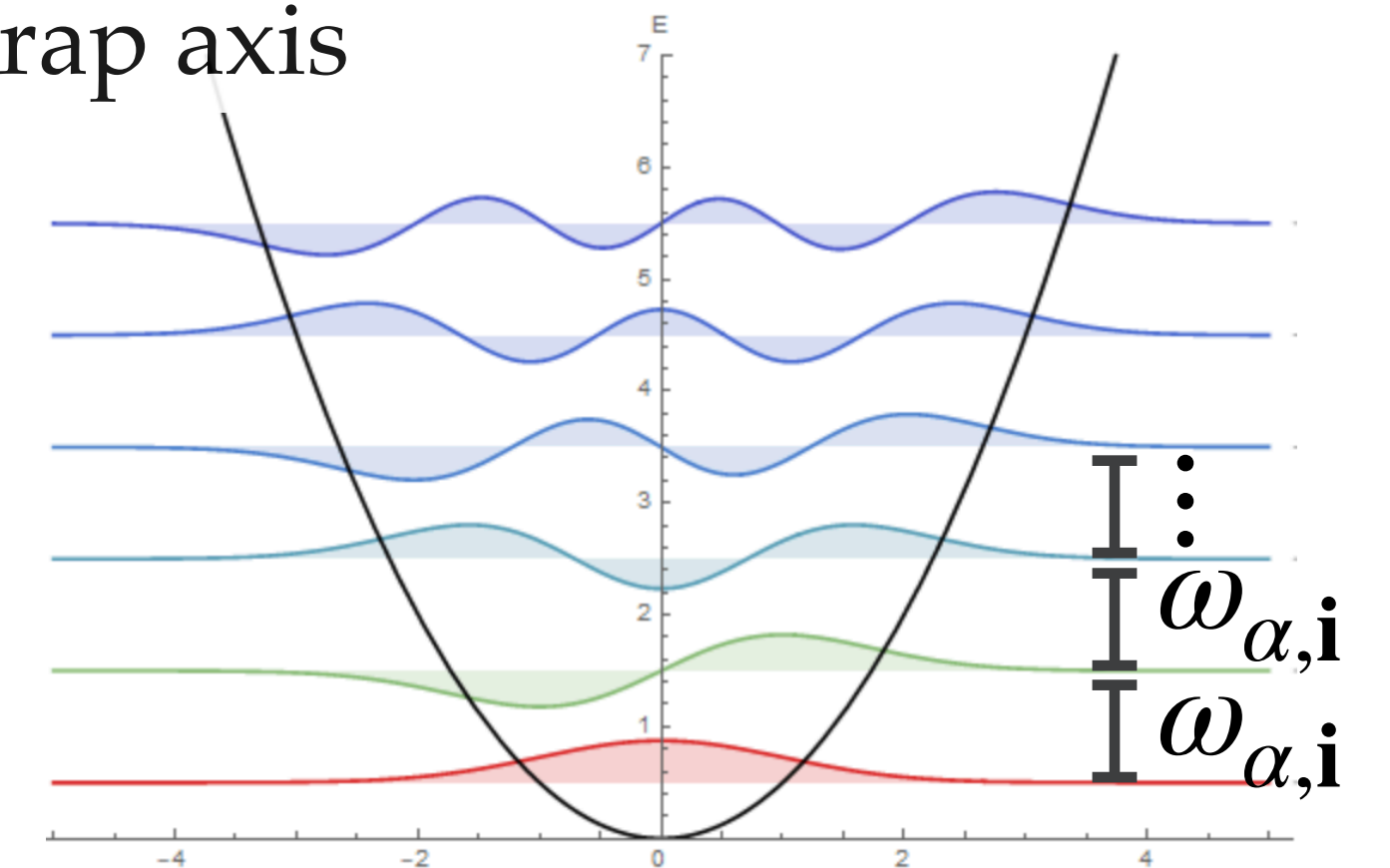


D. Porras and J.I. Cirac, *PRL* **93**, 263602 (2004)

One bosonic mode per ion and trap axis

$$\delta r_{\alpha,i} = \sqrt{\frac{1}{2m\omega_{\alpha,i}}} (b_{\alpha,i}^{\dagger} + b_{\alpha,i}),$$

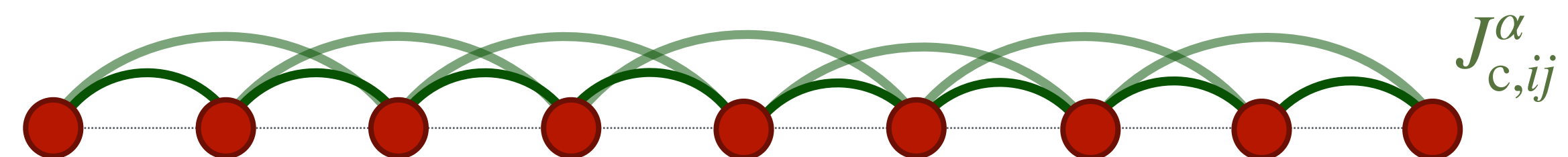
$$p_{\alpha,i} = i\sqrt{\frac{m\omega_{\alpha,i}}{2}} (b_{\alpha,i}^{\dagger} - b_{\alpha,i}),$$



get coupled by Coulomb interaction (2nd order *dipole-dipole*)

$$H = H_0 + H_c = \sum_{\mathbf{i},\alpha} \omega'_{\alpha,i} b_{\alpha,i}^{\dagger} b_{\alpha,i} + \sum_{\alpha} \sum_{\mathbf{i}>\mathbf{j}} (J_{c;\mathbf{ij}}^{\alpha} b_{\alpha,i}^{\dagger} b_{\alpha,j} + \text{H.c.}),$$

$U(1)$ symmetry $\rightarrow$ “Local” vibrations as bosonic particles”



Floquet engineering and $U(1)$ Peierls' phases for phonons

Dressing the microscopic parameters in **solids** by external fields is sometimes *possible*

Dynamic localization of a charged particle moving under the influence of an electric field

D. H. Dunlap and V. M. Kenkre

Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131

(Received 27 May 1986)

The motion of a charged particle on a discrete lattice under the action of an electric field is studied with the help of explicit calculations of probability propagators and mean-square displacements. Exact results are presented for arbitrary time dependence of the electric field on a one-dimensional lattice. Existing results for the limiting cases of zero frequency and zero field are recovered. A new phenomenon involving the dynamic localization of the moving particle is shown to result in the case of a sinusoidally varying field: The particle is generally delocalized except for the cases when the ratio of the field magnitude and the field frequency is a root of the ordinary Bessel function of order 0. For these special cases it is found to be localized. This localization could be used, in principle, for inducing anisotropy in the transport properties of an ordinarily isotropic material.



D.H. Dunlap and V.M. Kenkre., **PRB 34**, 3625 (1986).

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_i \Delta i \cos(\omega t) c_i^\dagger c_i \quad t \ll \Delta$$



$$H_{\text{dressed}} = -t J_0(\eta) \sum_{\langle i,j \rangle} c_i^\dagger c_j$$

$$\eta = \Delta/\omega$$

Floquet engineering and $U(1)$ Peierls' phases for phonons

Assisting the tunnelling *against a gradient* by absorbing photons from a drive introduces *recoil*

$$H = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_i \Delta i c_i^\dagger c_i + \sum_i \tilde{\Delta} \cos(\Delta \mathbf{k} \cdot \mathbf{x}_i + \omega t) c_i^\dagger c_i$$

$t \ll \Delta$

Plane wave

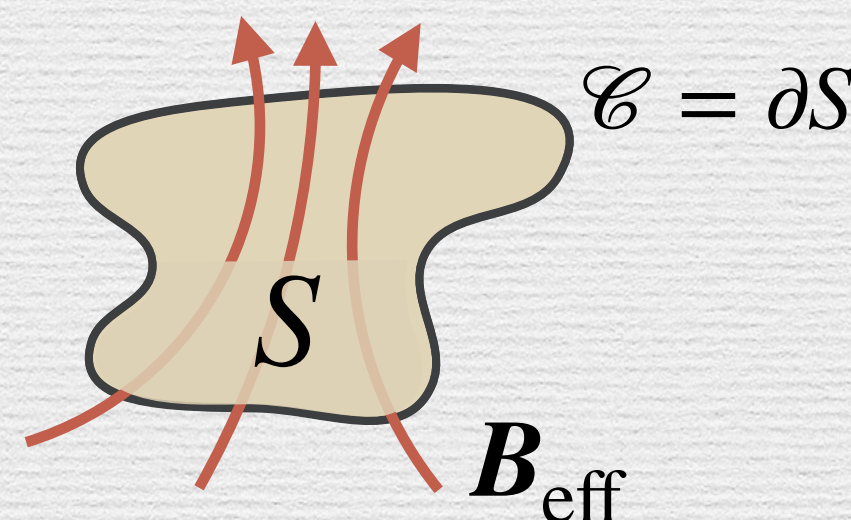
$$H_{\text{dressed}} = -t J_1(2\eta \sin \Delta k a) \sum_{\langle i,j \rangle} e^{-i\Delta \mathbf{k} \cdot \mathbf{x}_i} c_i^\dagger c_j$$

$\omega = \Delta$ Resonant recoil kick

We gain control of a **Peierls' phase** defined on the **links** to mimic a **background gauge field**

$$e^{i \frac{e}{\hbar} \int_{\mathcal{C}} \mathbf{d}\mathbf{x} \cdot \mathbf{A}_{\text{eff}}}$$

Stokes' th
→

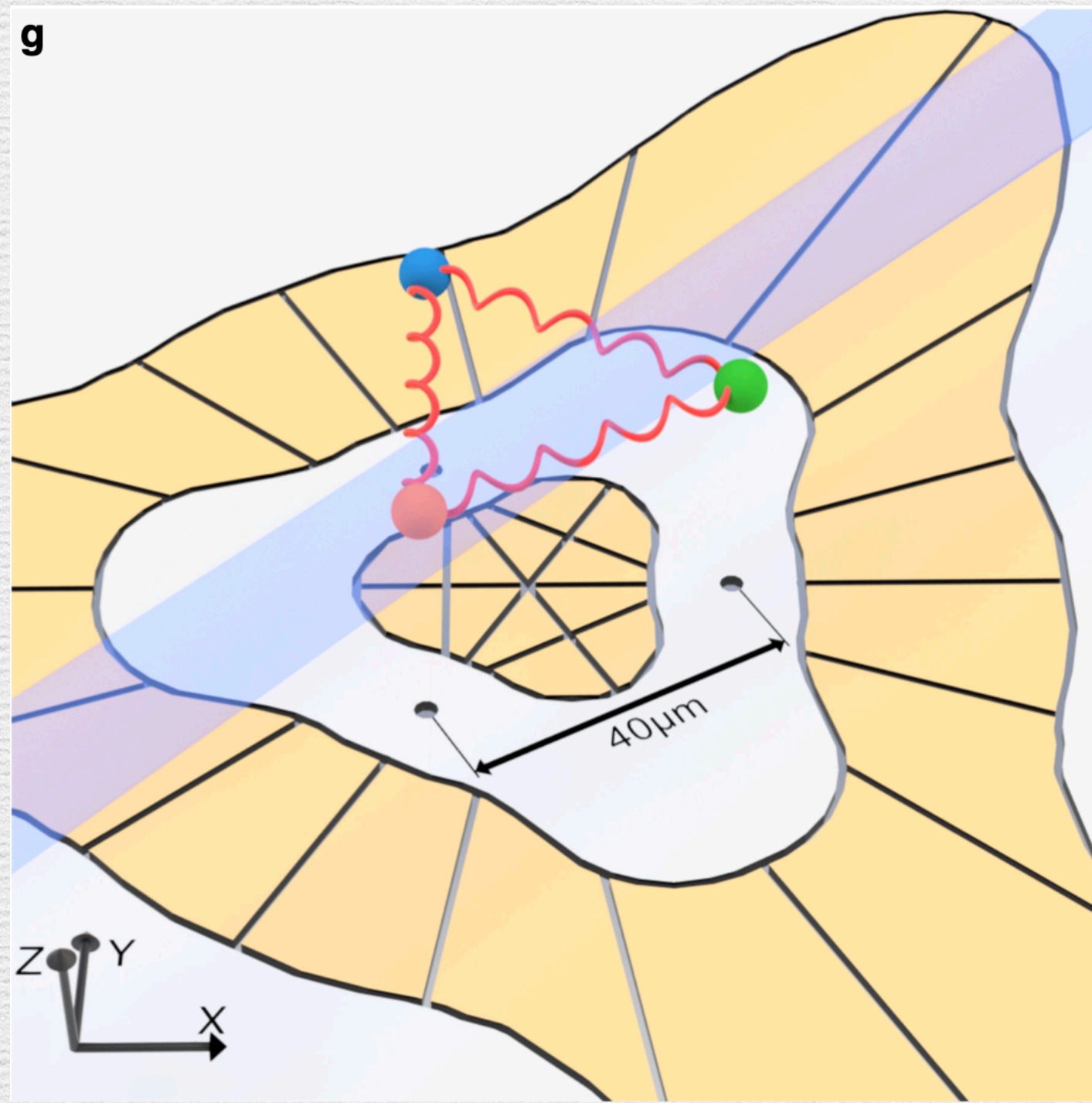


th AB, T. Schätz, and D. Porras, [PRL 107, 150501 \(2011\)](#)
(atoms) M. Aidelsburger, et al., [PRL 107, 255301 \(2011\)](#)

Floquet engineering and $U(1)$ Peierls' phases for phonons

M. Mielenz *et al.*, *Nat. Comm.* **7**, 11839 (2016)

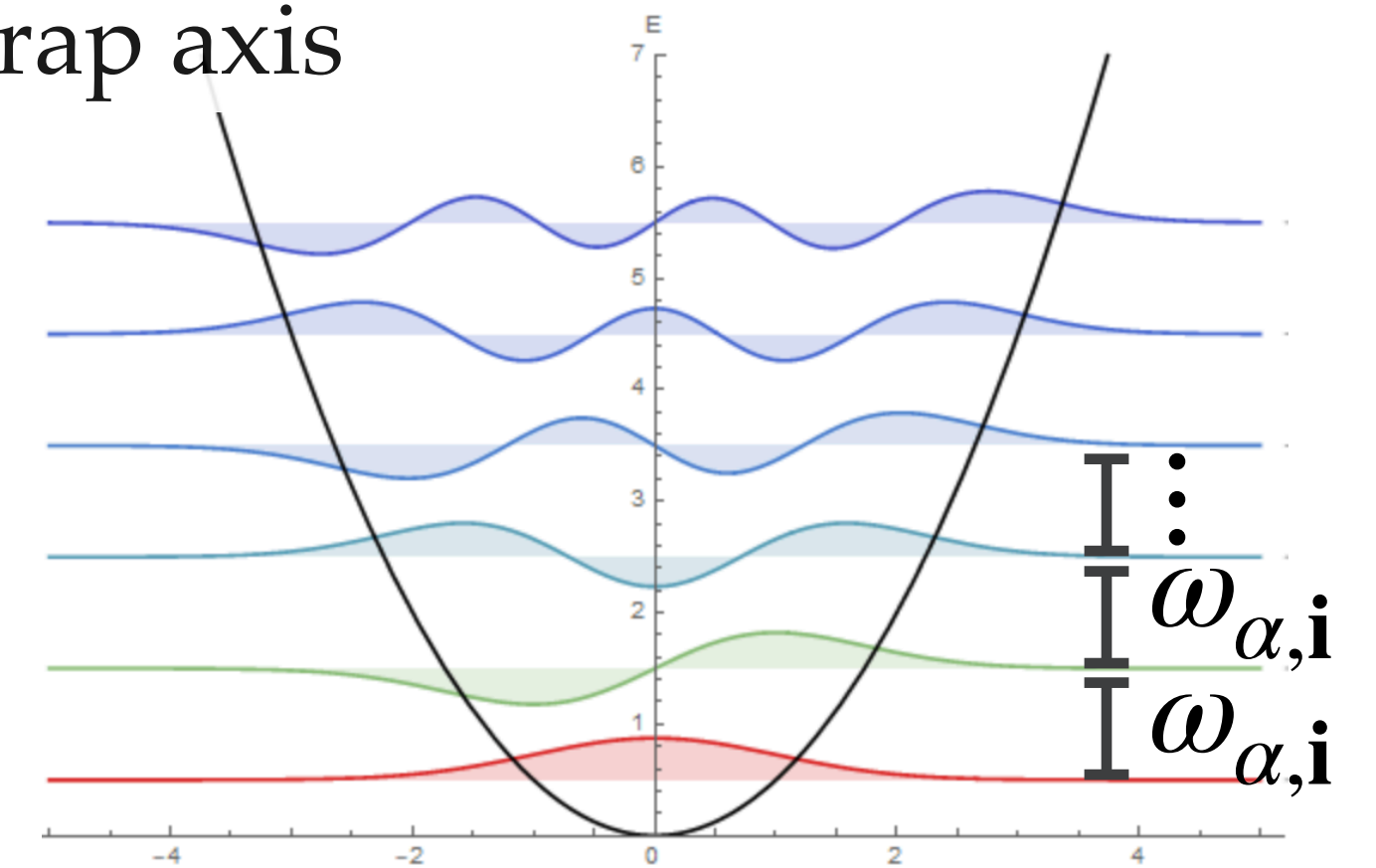
J. Niedermeyer, *et al.*, *Oxford seminar* (2024)



One bosonic mode per ion and trap axis

$$\delta r_{\alpha,i} = \sqrt{\frac{1}{2m\omega_{\alpha,i}}} (b_{\alpha,i}^\dagger + b_{\alpha,i}),$$

$$p_{\alpha,i} = i\sqrt{\frac{m\omega_{\alpha,i}}{2}} (b_{\alpha,i}^\dagger - b_{\alpha,i}),$$



get periodically modulated by **crossed-beam ac-Stark shifts**

$$H_L \approx -\Omega_L \sum_{\alpha,i} \eta_\alpha^2 \cos(\Delta \mathbf{k} \cdot \mathbf{r}_i^0 - \omega_L t) b_{\alpha,i}^\dagger b_{\alpha,i}.$$

Optical version of parametric drive

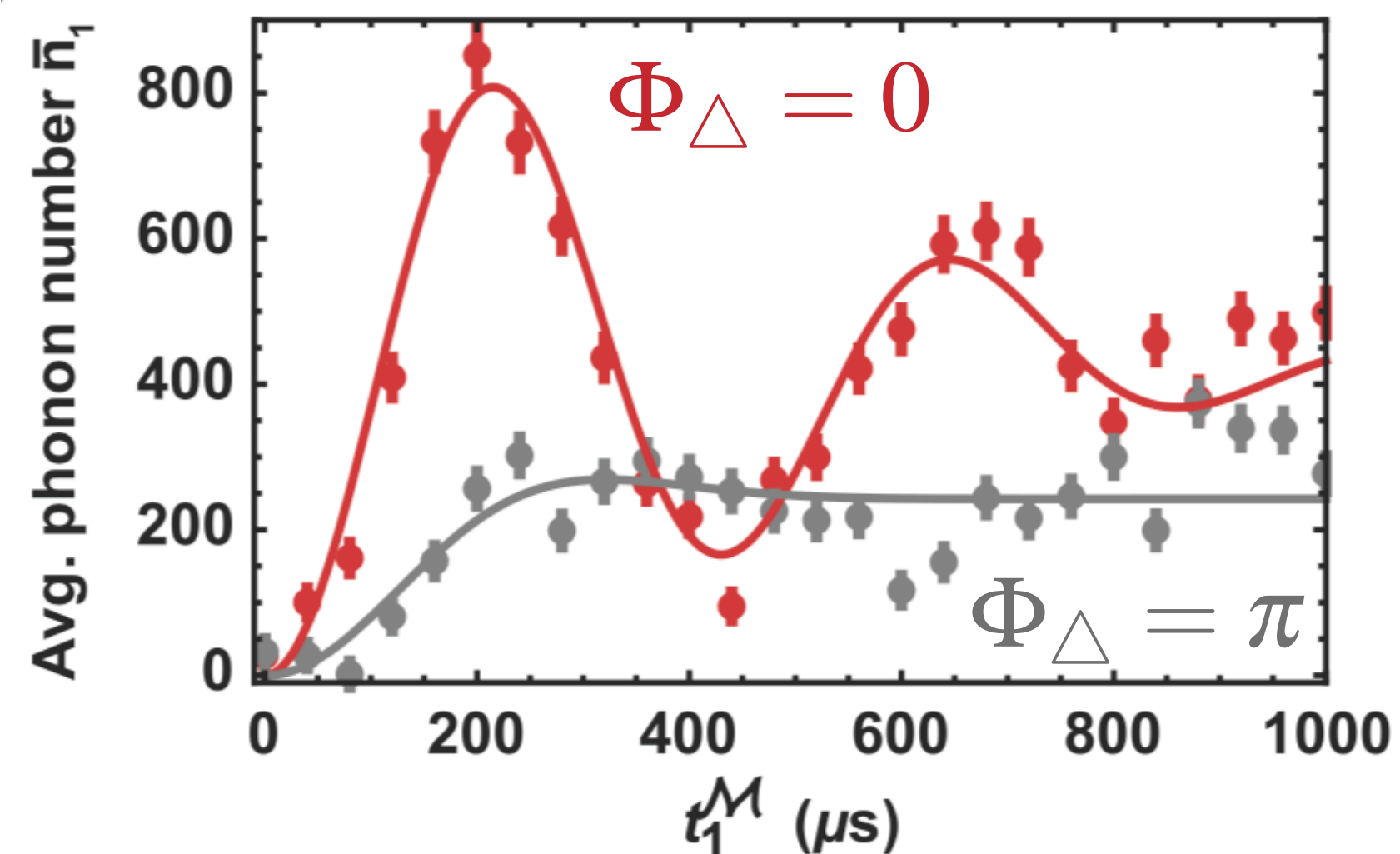
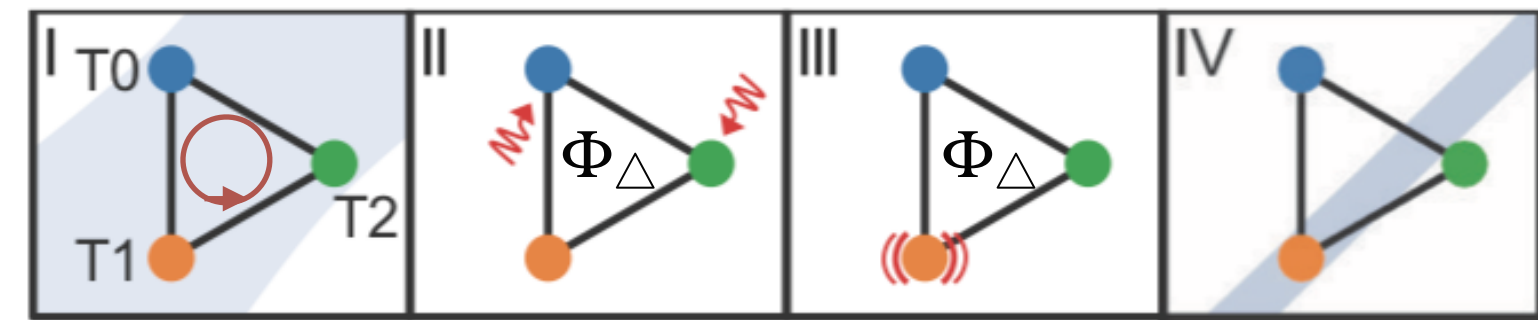
$U(1)$ symmetry $\rightarrow$ gets “gauged” by Floquet engineering

$$J_{d;ij}^\alpha = \tilde{J}_{d;ij}^\alpha e^{-i \frac{f_\alpha(\mathbf{i},\mathbf{j})}{2} (\phi_i + \phi_j)} \quad \phi_i = -\Delta \mathbf{k} \cdot \mathbf{r}_i^0.$$

AB, T. Schätz, and D. Porras, *PRL* **107**, 150501 (2011)

Floquet engineering and $U(1)$ Peierls' phases for phonons

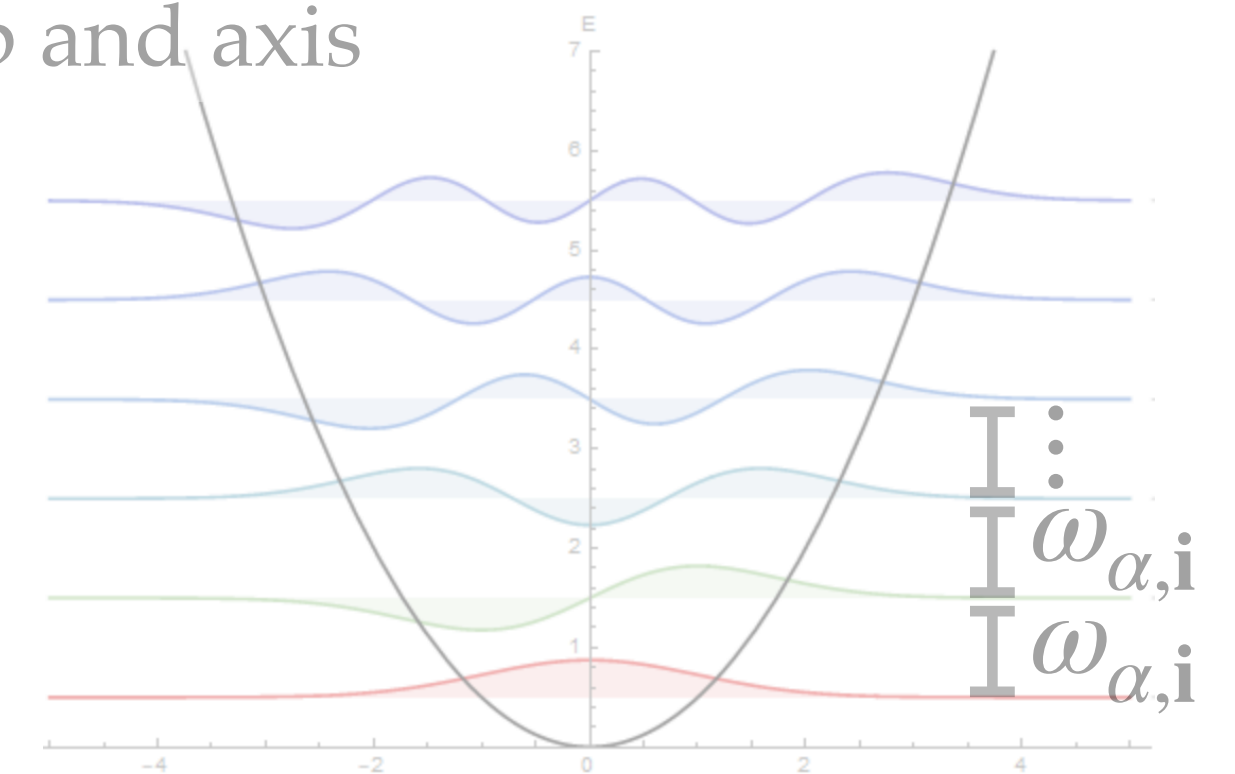
Real life is not that simple (noise, heating...)



One bosonic mode per micro trap and axis

$$\delta r_{\alpha,i} = \sqrt{\frac{1}{2m\omega_{\alpha,i}}} (b_{\alpha,i}^\dagger + b_{\alpha,i}),$$

$$p_{\alpha,i} = i\sqrt{\frac{m\omega_{\alpha,i}}{2}} (b_{\alpha,i}^\dagger - b_{\alpha,i}),$$



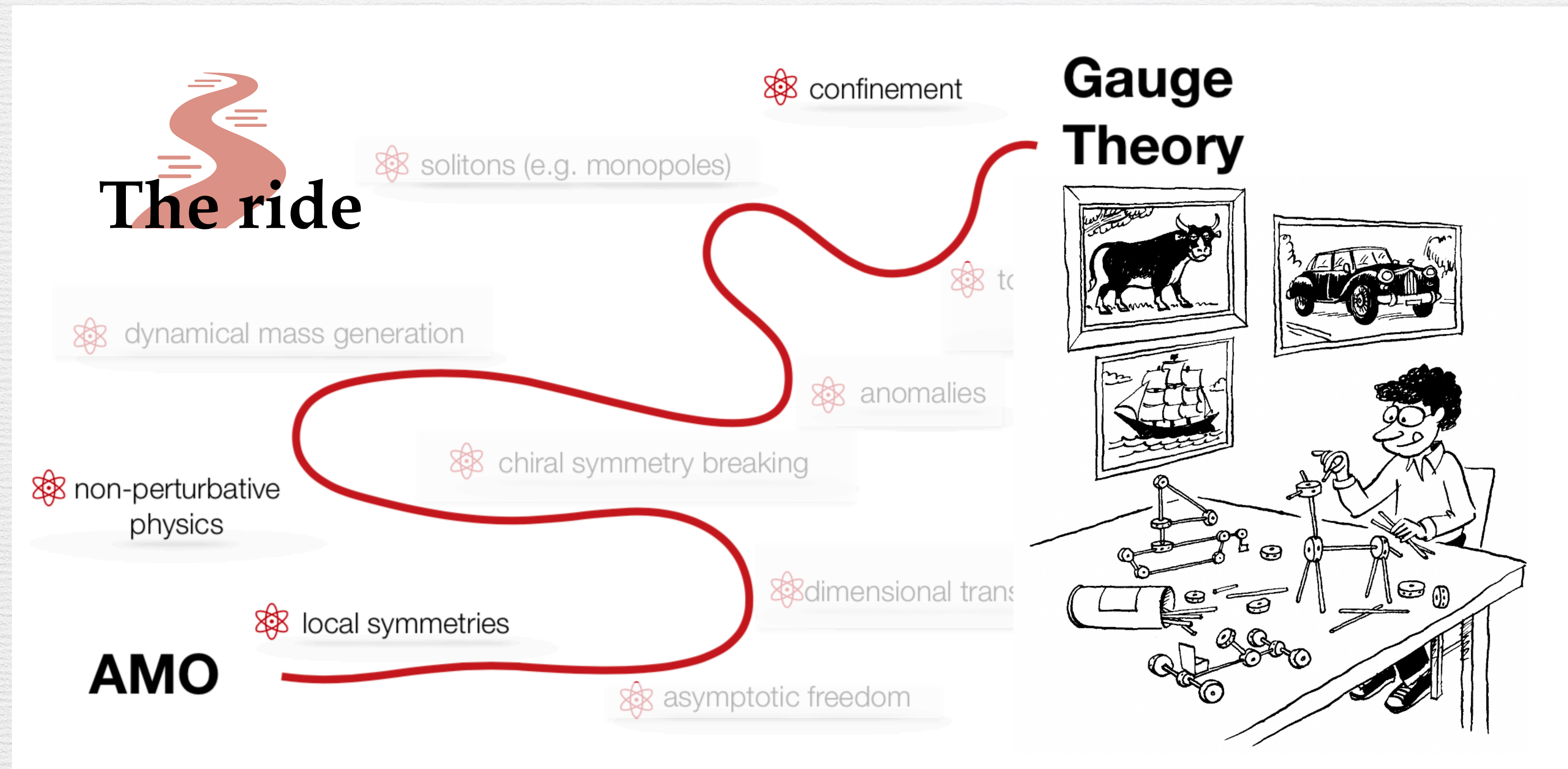
get periodically modulated by crossed-beam ac-Stark shifts

$$H_L \approx -\Omega_L \sum_{\alpha,i} \eta_\alpha^2 \cos(\Delta \mathbf{k} \cdot \mathbf{r}_i^0 - \omega_L t) b_{\alpha,i}^\dagger b_{\alpha,i}.$$

$U(1)$ symmetry $\rightarrow$ gets “gauged” by Floquet engineering

$$J_{d;ij}^\alpha = \tilde{J}_{d;ij}^\alpha e^{-i\frac{f_\alpha(i,j)}{2}(\phi_i + \phi_j)} \quad \phi_i = -\Delta \mathbf{k} \cdot \mathbf{r}_i^0.$$

But **hold on...** we were aiming for toy models that could address...

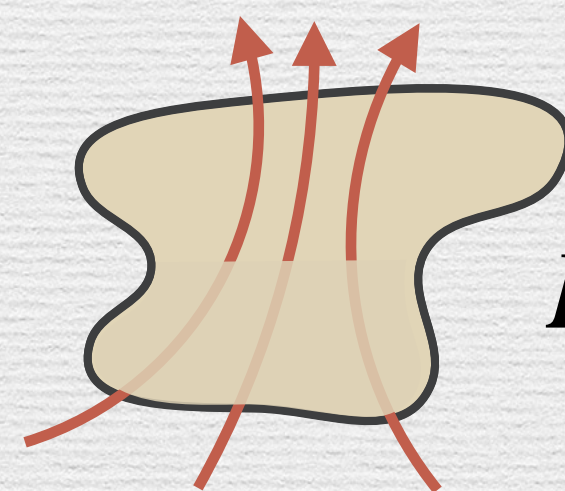


Quantum links

$$e^{i\frac{e}{\hbar} \int_{\mathcal{C}} d\mathbf{x} \cdot \hat{\mathbf{A}}_{\text{eff}}(t, \mathbf{x})}$$

parallel transporter

Stokes' th
→



$$\hat{\mathbf{B}}_{\text{eff}}(t, \mathbf{x}) ; \hat{\mathbf{E}}_{\text{eff}}(t, \mathbf{x})$$

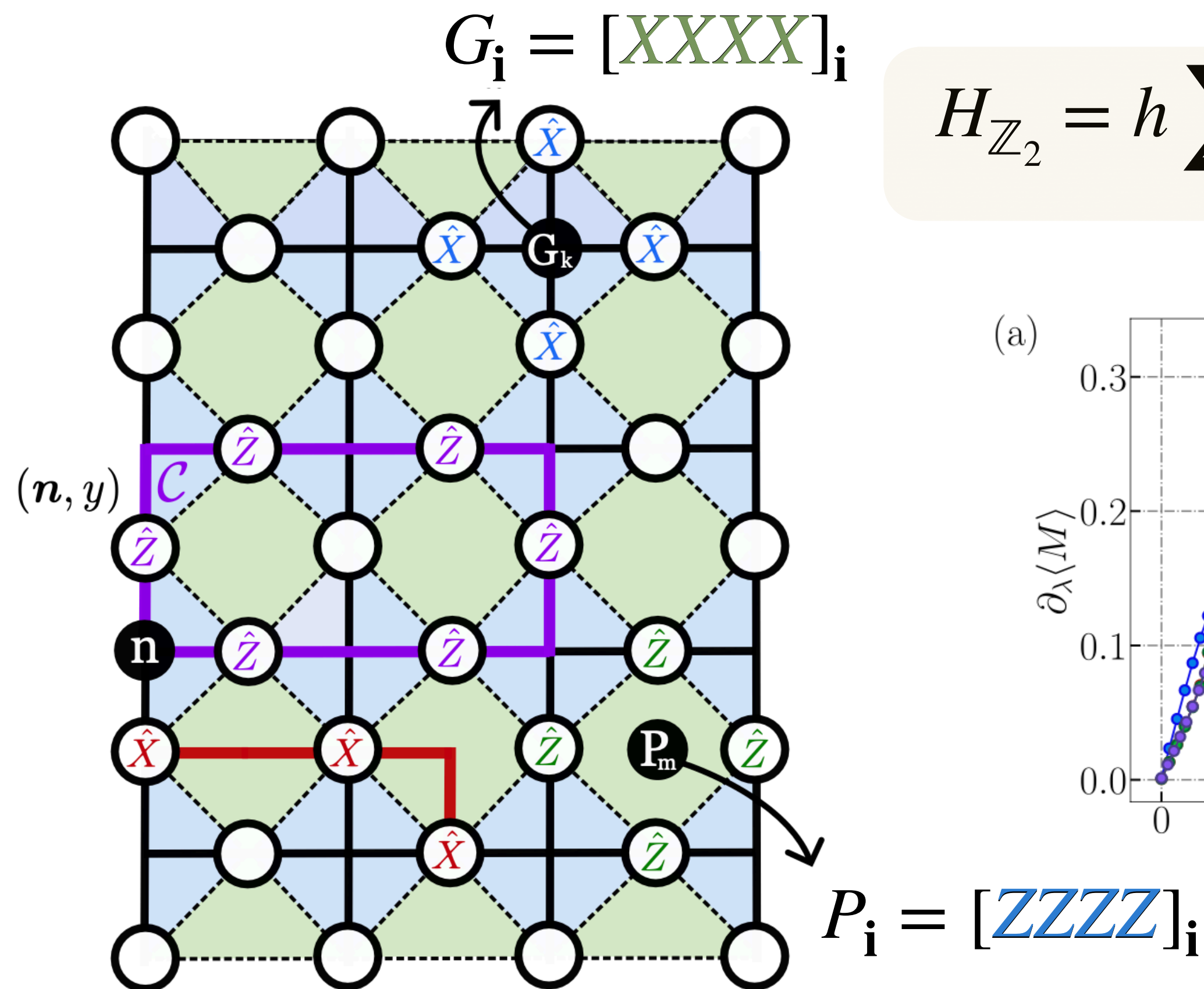
Maxwell
→

Gauge-invariant quantum dynamics

$$H = \frac{1}{2} \int d^D x \left(\epsilon_0 \hat{\mathbf{E}}_{\text{eff}}^2(t, \mathbf{x}) + \frac{1}{\mu_0} \hat{\mathbf{B}}_{\text{eff}}^2(t, \mathbf{x}) \right)$$

Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

“Ising gauge theory” is a paradigm of *confinement phase transitions, topological order & QEC*



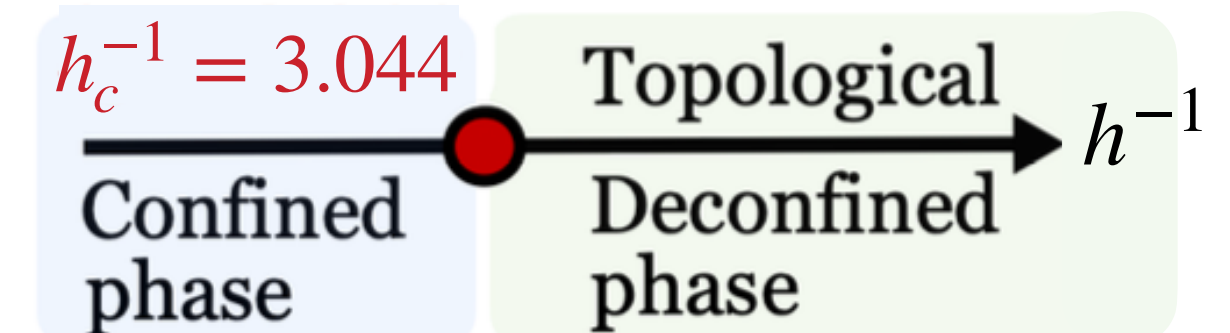
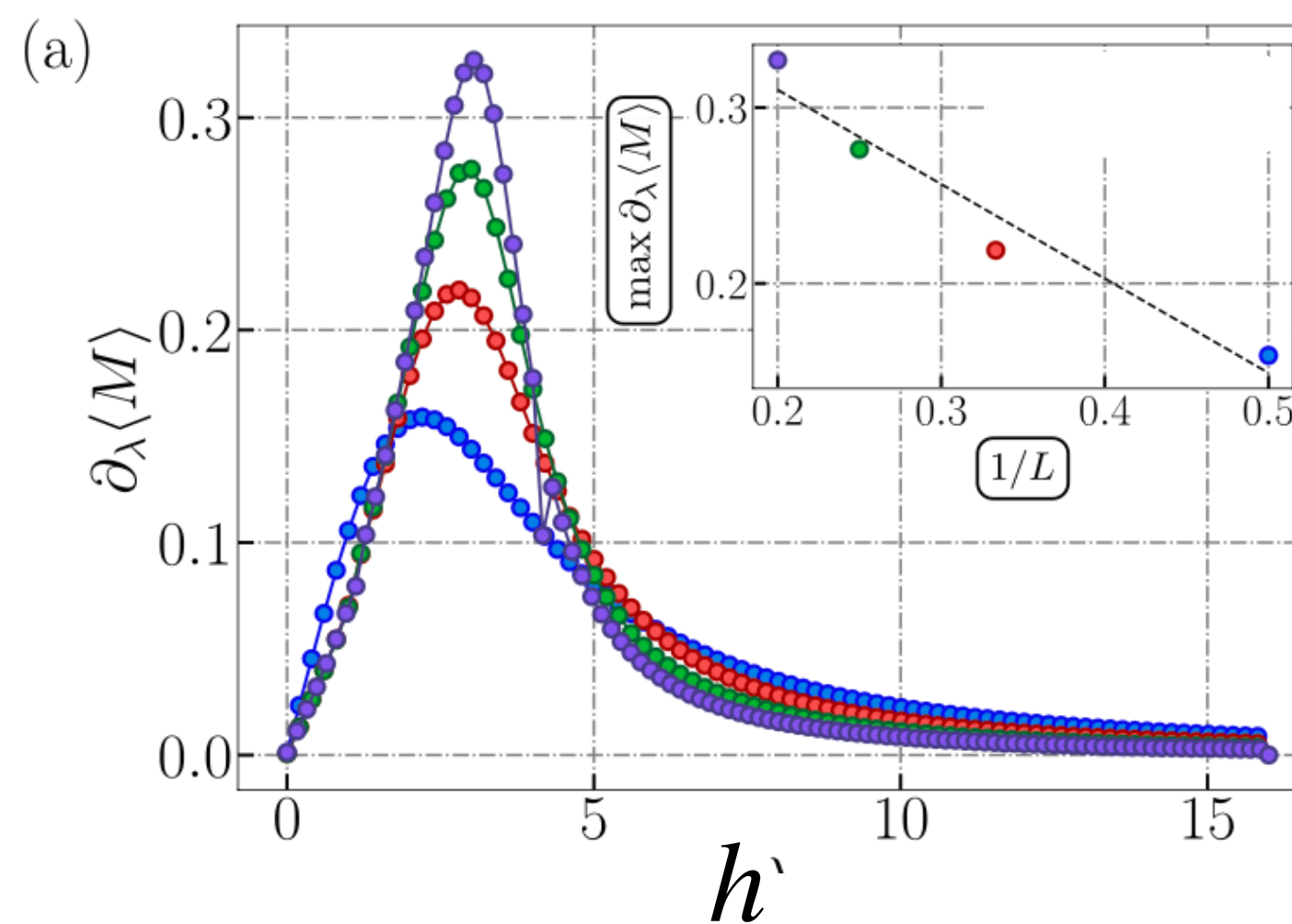
$$H_{\mathbb{Z}_2} = h \sum_i X_i + \sum_i [ZZZZ]_i$$

Local symmetry

$$[H_{\mathbb{Z}_2}, G_i] = 0, \forall i$$

Gauss' law

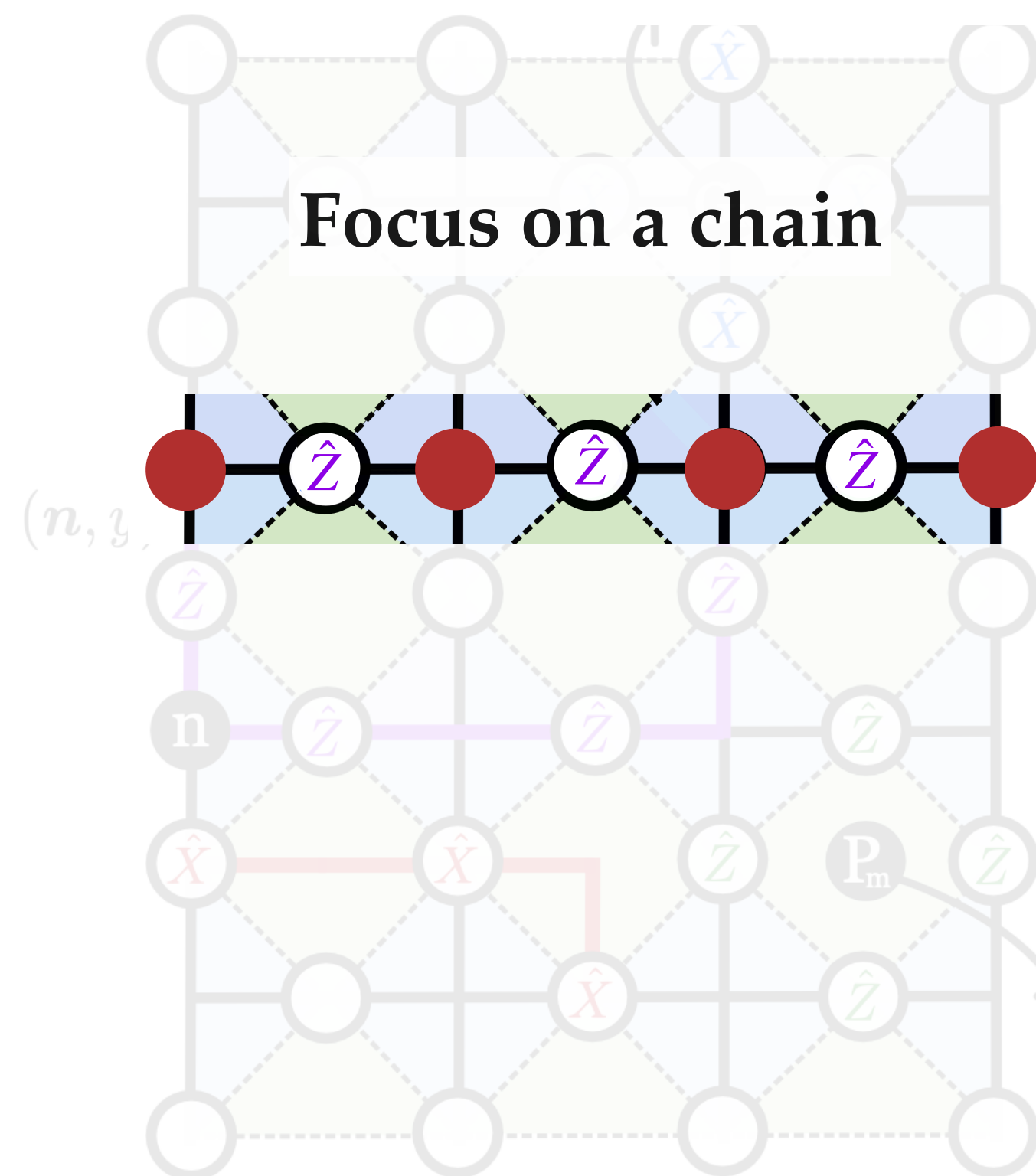
$$[XXXX]_i |\psi\rangle = + |\psi\rangle$$



 F. Wenger, *J. Math. Phys.* **12**, 2259 (1971).
 Fradkin & Kogut, *PRD* **17**, 2637 (1978).

Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

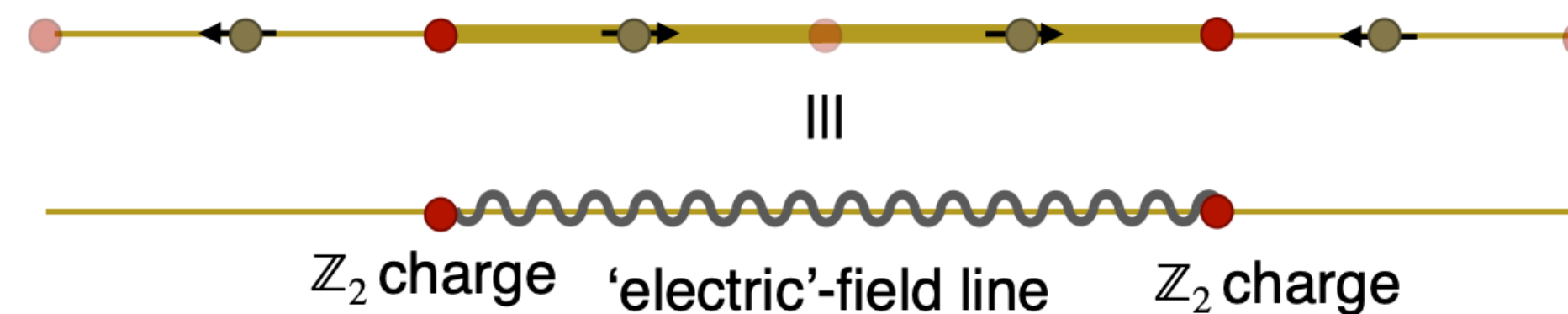
1+1 models become non-trivial by introducing **dynamical matter** (moving charges as sources / sinks)



$$H_{\mathbb{Z}_2} = \sum_i (a_i^\dagger Z_i a_{i+1} + \text{H. c.}) + h \sum_i X_i$$

Gauss' law $X_{i-1}(-1)^{a_i^\dagger a_i} X_i |\psi\rangle = |\psi\rangle$

$\mathbb{Z}_2$ charges connected electric strings

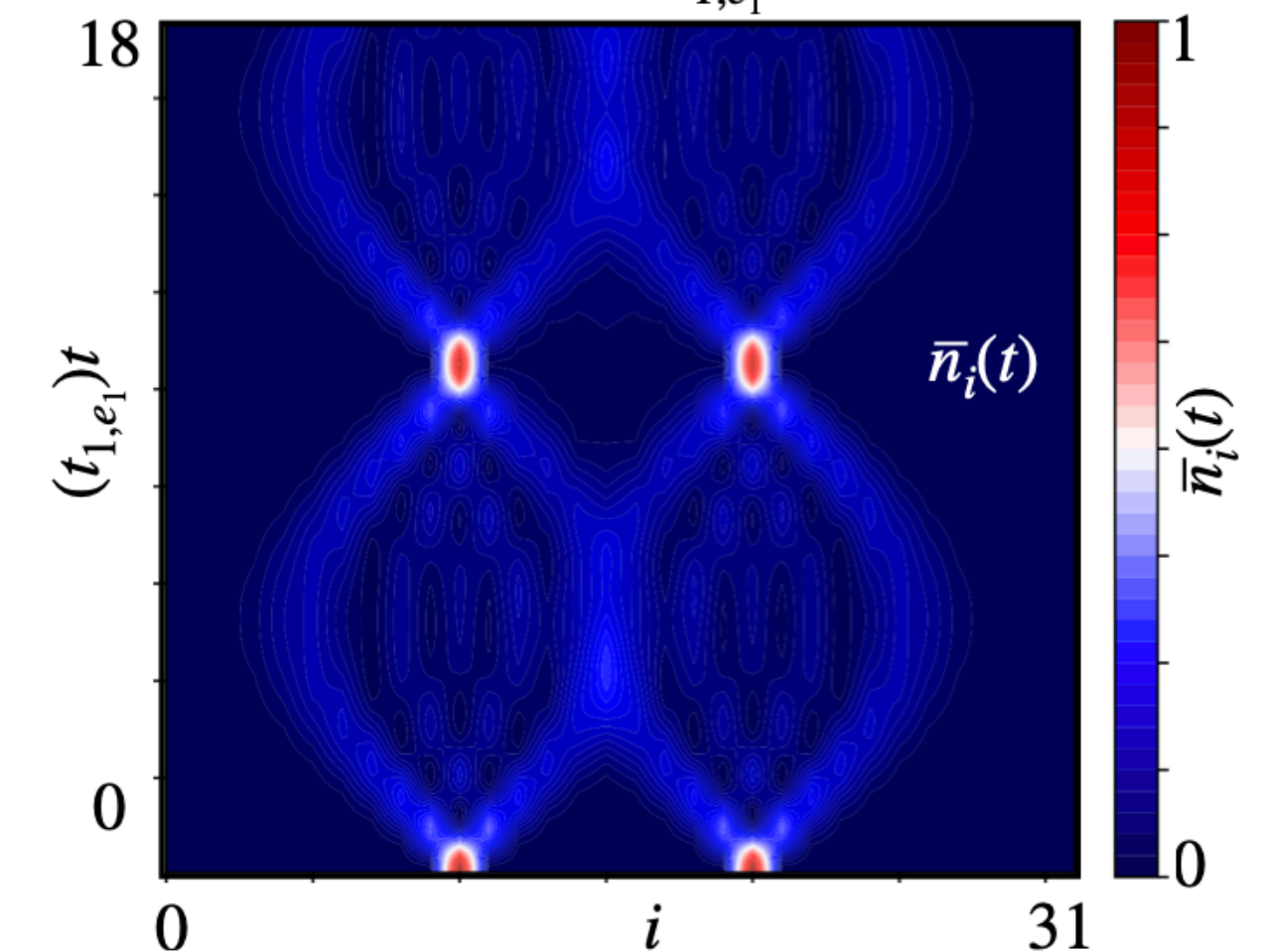


Local symmetry

$$[H_{\mathbb{Z}_2}, G_i] = 0, \forall i$$

Wannier-Stark confinement

$$h = 0.3t_{1,e_1}$$



GOING BEYOND...

Can one use the qubits not merely as sensors
but as dynamical degrees of freedom Q5?

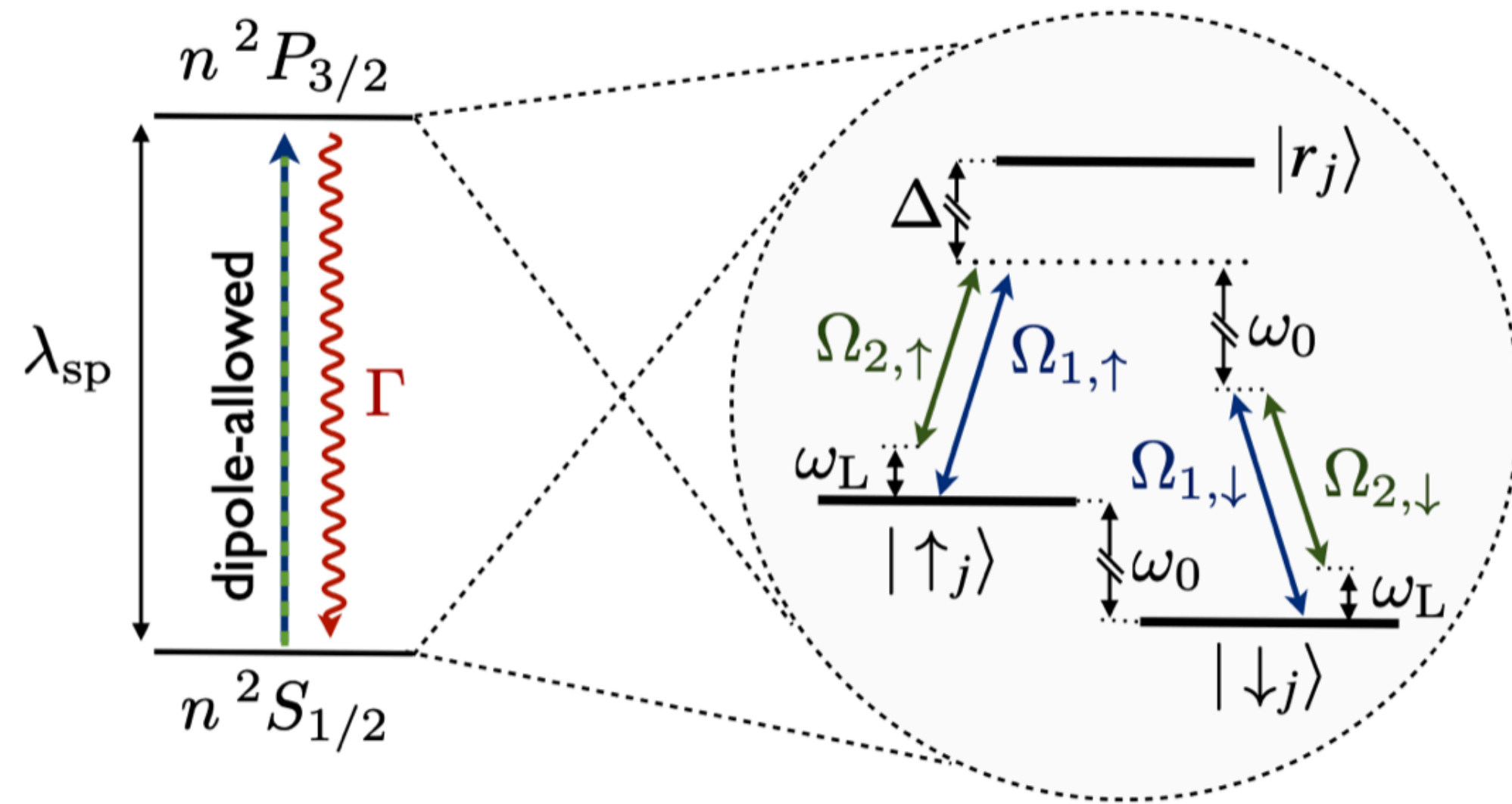
$\mathbb{Z}_2$ gauge theories {

- phonons play the role of dyn. $\mathbb{Z}_2$ charges
- qubits " dyn. $\mathbb{Z}_2$ fields

Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

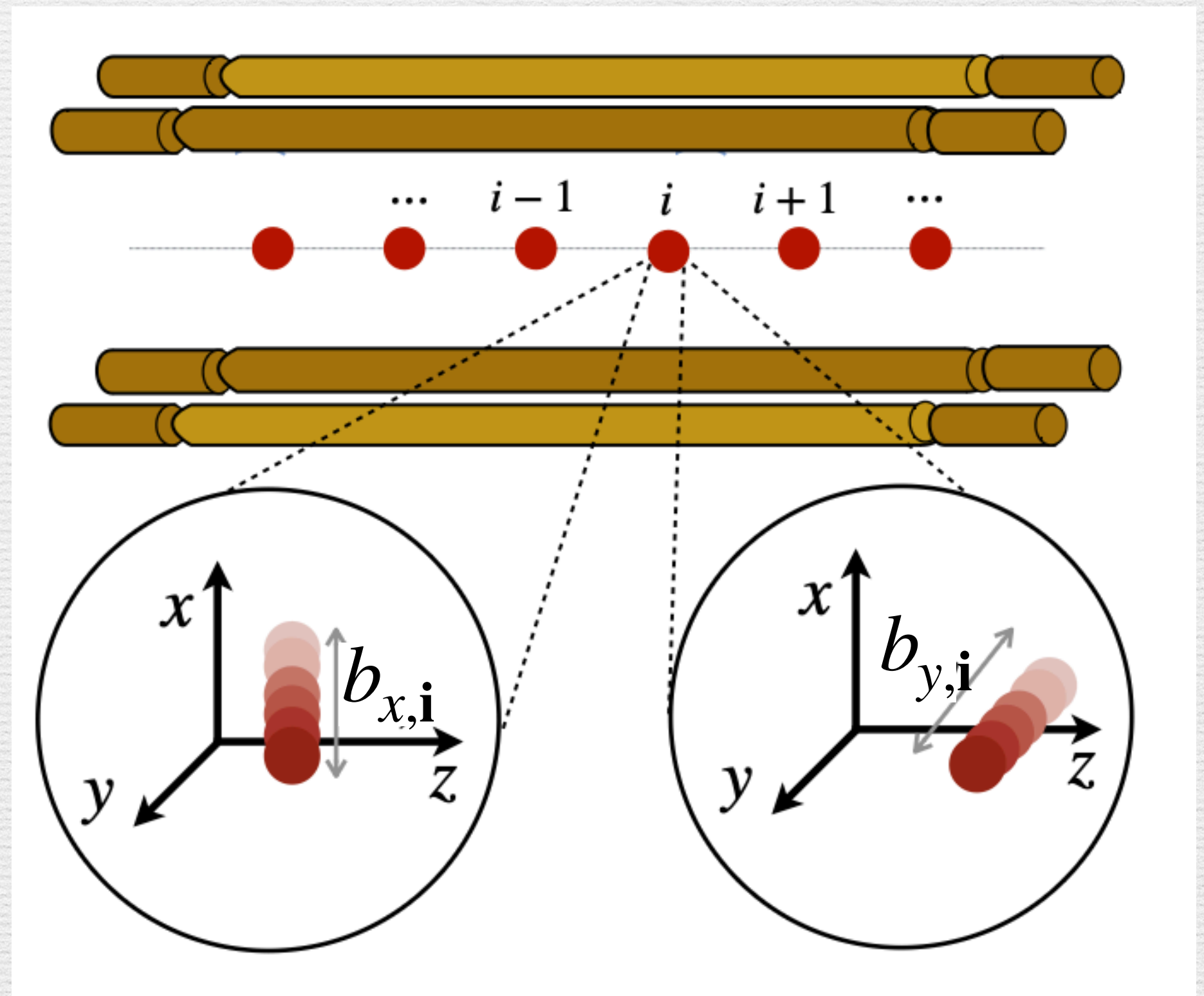
We assumed that all ions were in the same electronic state under the crossed-beam ac-Stark shifts, but...

1 Differential crossed-beam ac-Stark shifts



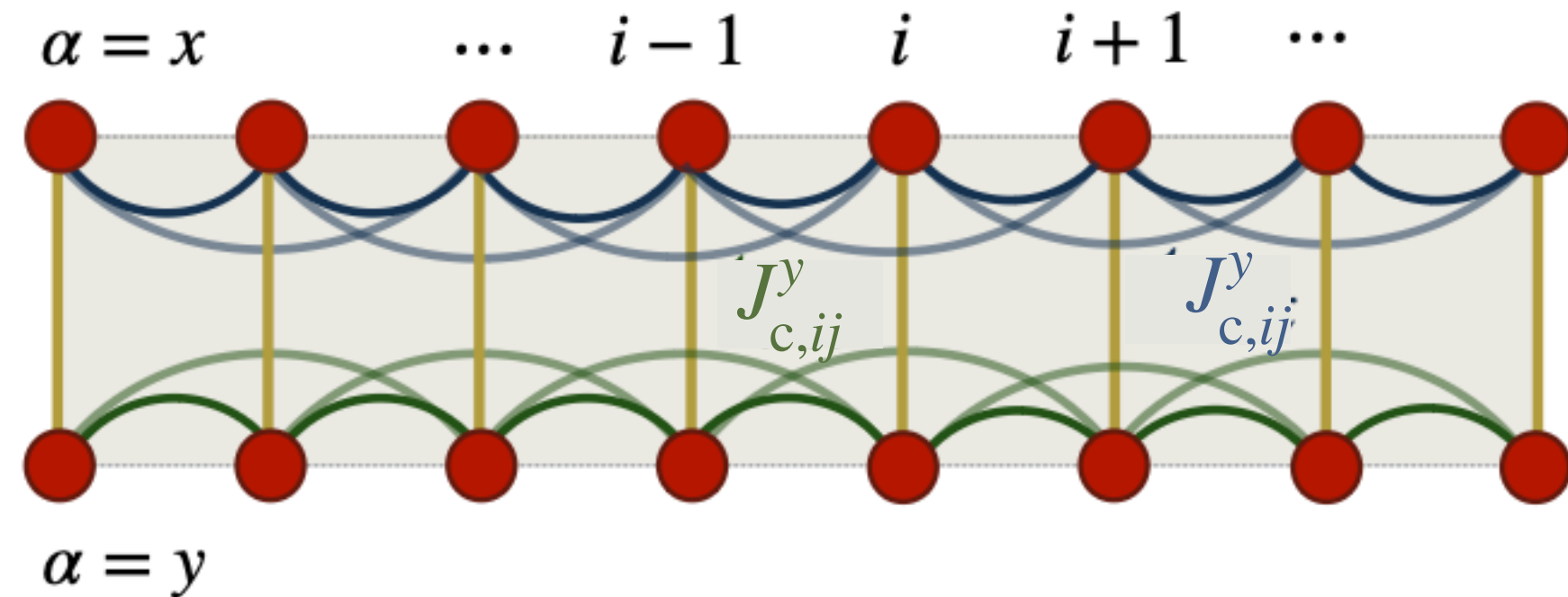
$$H_L \approx -\Omega_L \sum_{\mathbf{i}} \sum_{\alpha, \beta} \eta_{\alpha} \eta_{\beta} \cos(\Delta \mathbf{k} \cdot \mathbf{r}_i^0 - \omega_L t) b_{\alpha, \mathbf{i}}^{\dagger} b_{\beta, \mathbf{i}} Z_{\mathbf{i}}$$

2 Beatnote tilting w.r.t. trap symmetry axes

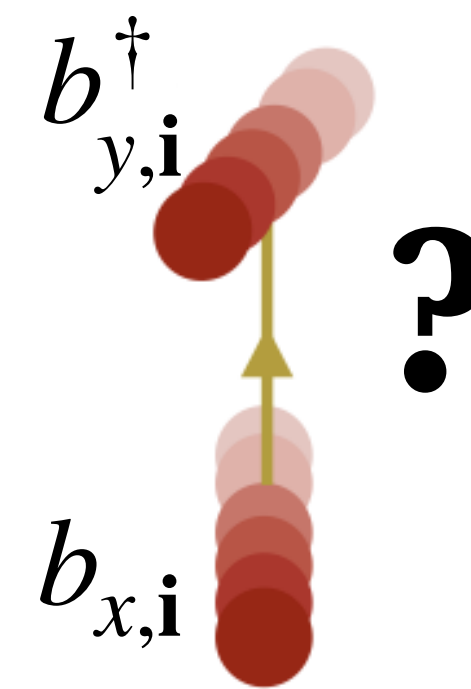


Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

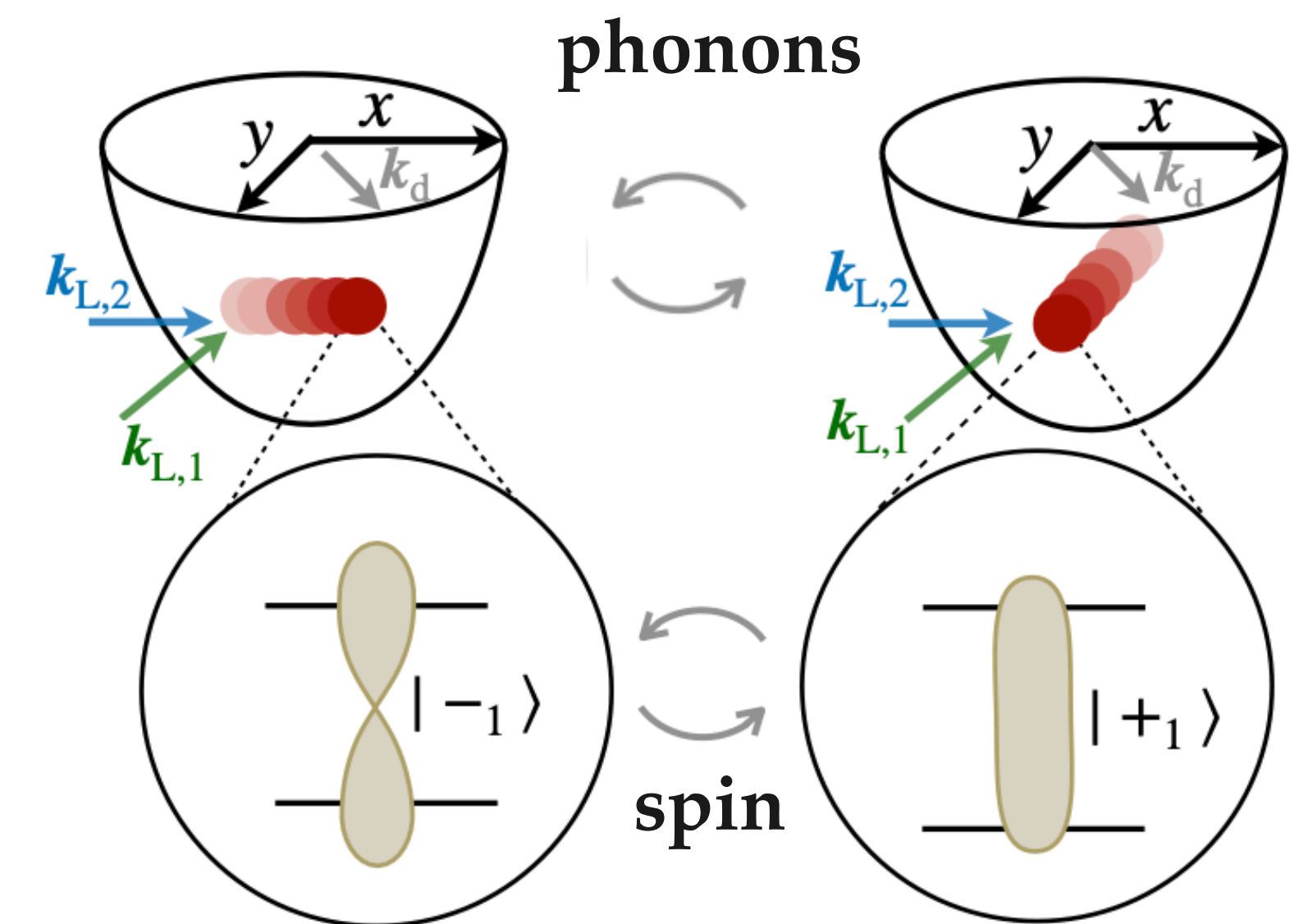
Synthetic “dimension” ladder



Synthetic link



Spin-dependent tunneling



Laser beat note for **parametric frequency conversion**

$$\Delta\omega_L \approx \omega_x - \omega_y$$

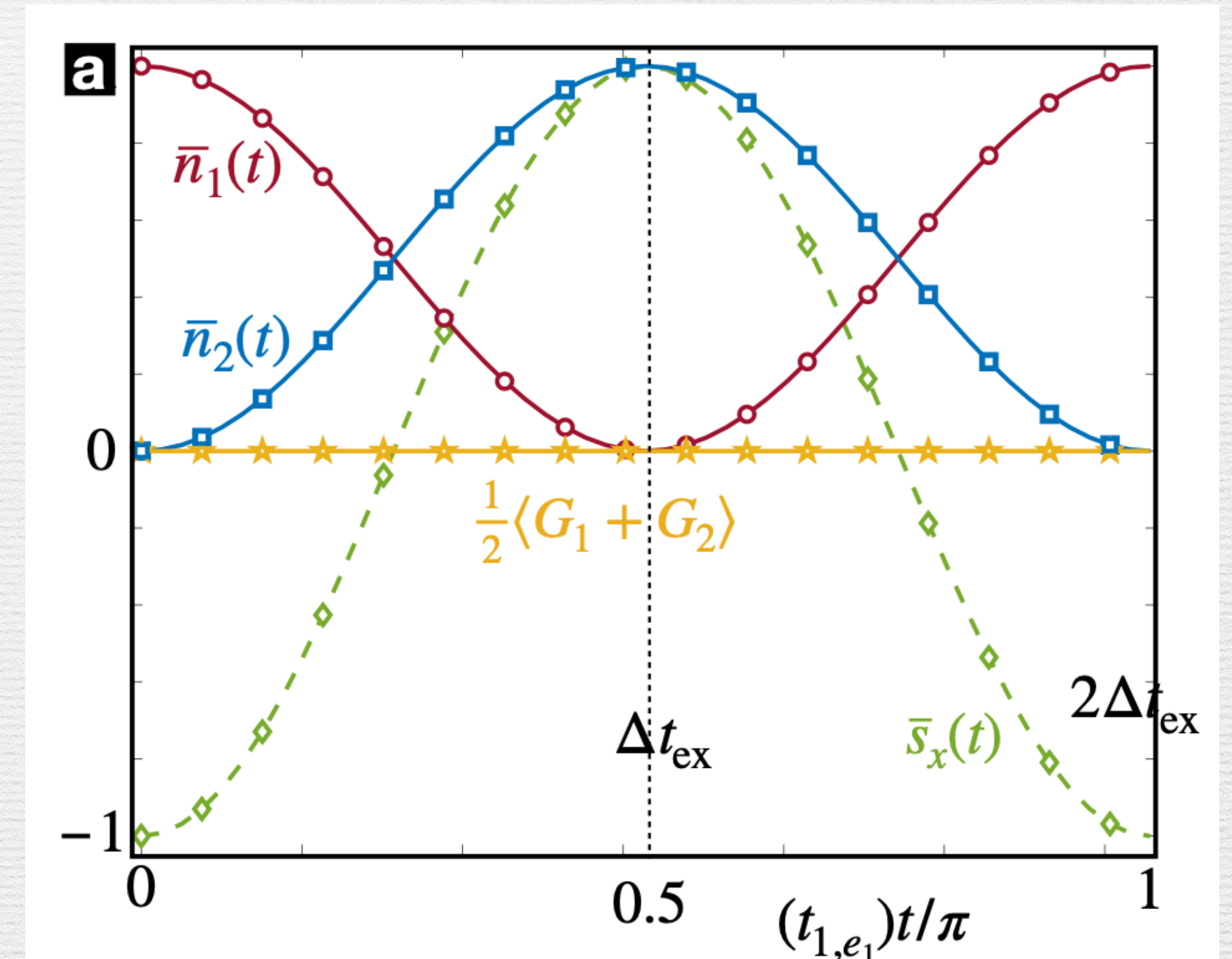
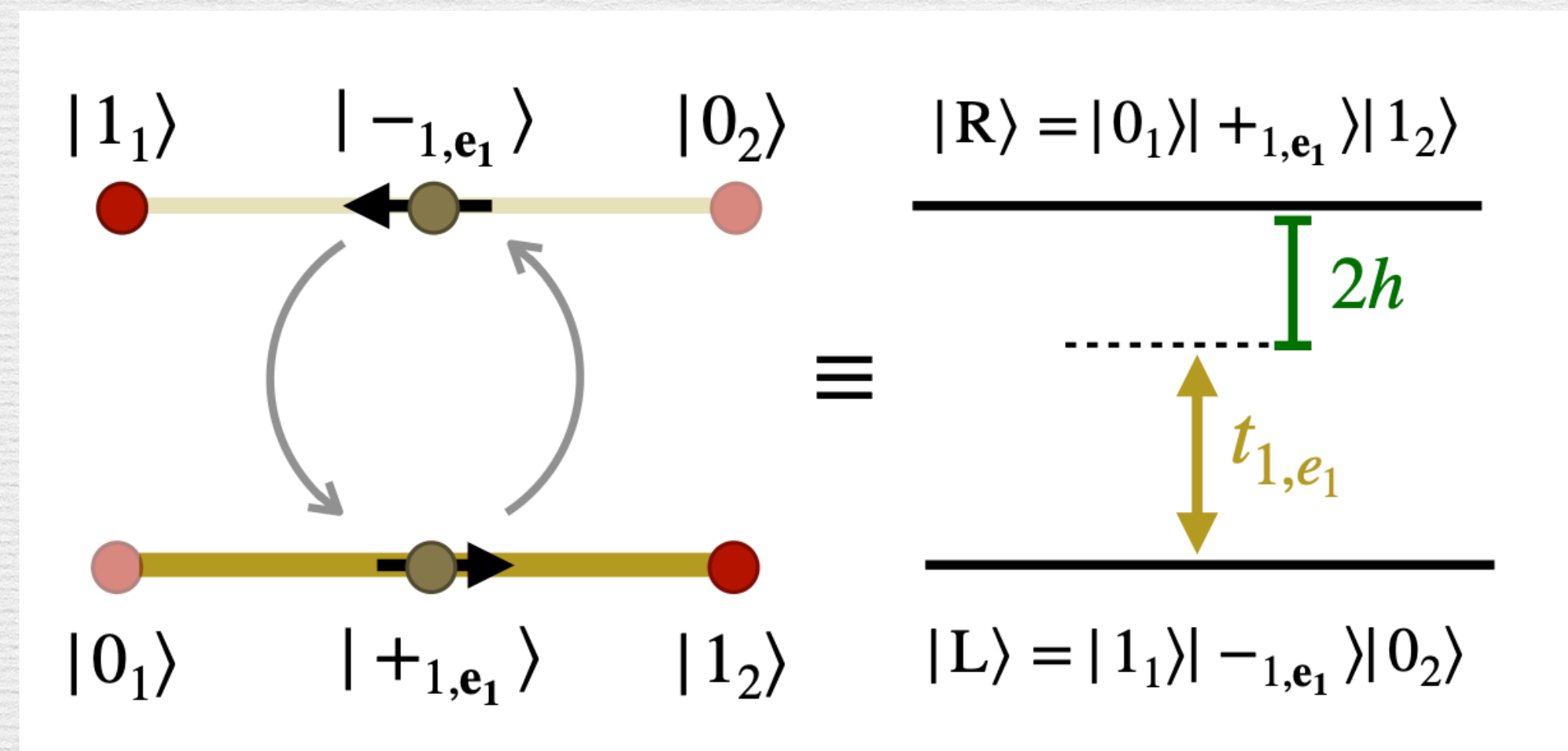
$$H_{\text{eff}} = \left(J_{\text{eff}} b_x^\dagger \mathbb{Z} b_y + \text{H.c.} \right) + hX$$



th O. Bazavan, S. Saner..., *Comm. Phys.* **7**, 229 (2024)

Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

The single-excitation maps to **correlated spin-phonon exchange** (detuned Rabi oscillations)



Other analogies to quantum optical schemes (Lambda drives, dark states, mater/ gauge entanglement, squeezed $\mathbb{Z}_2$ charges, phase-space dynamics* ...)

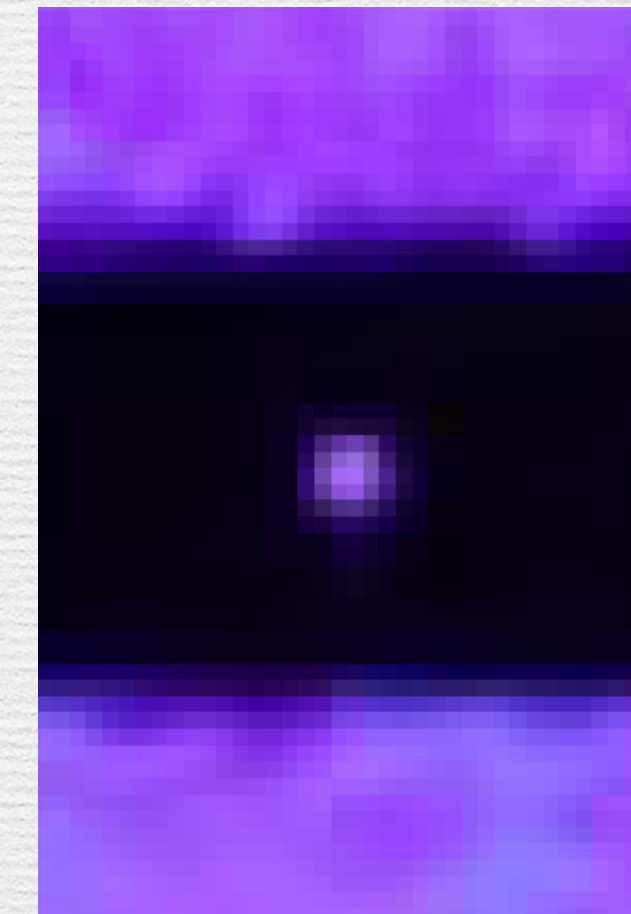
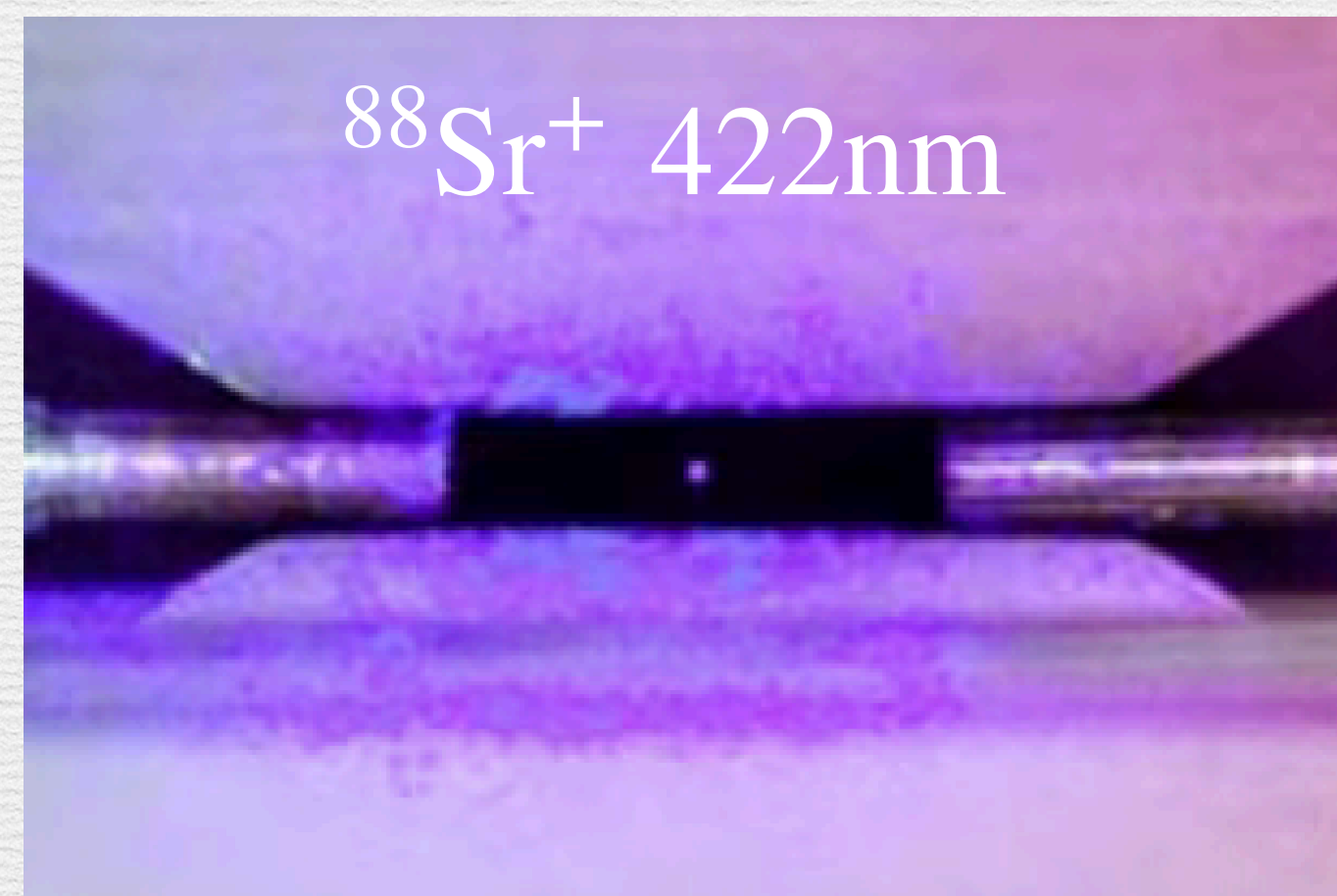
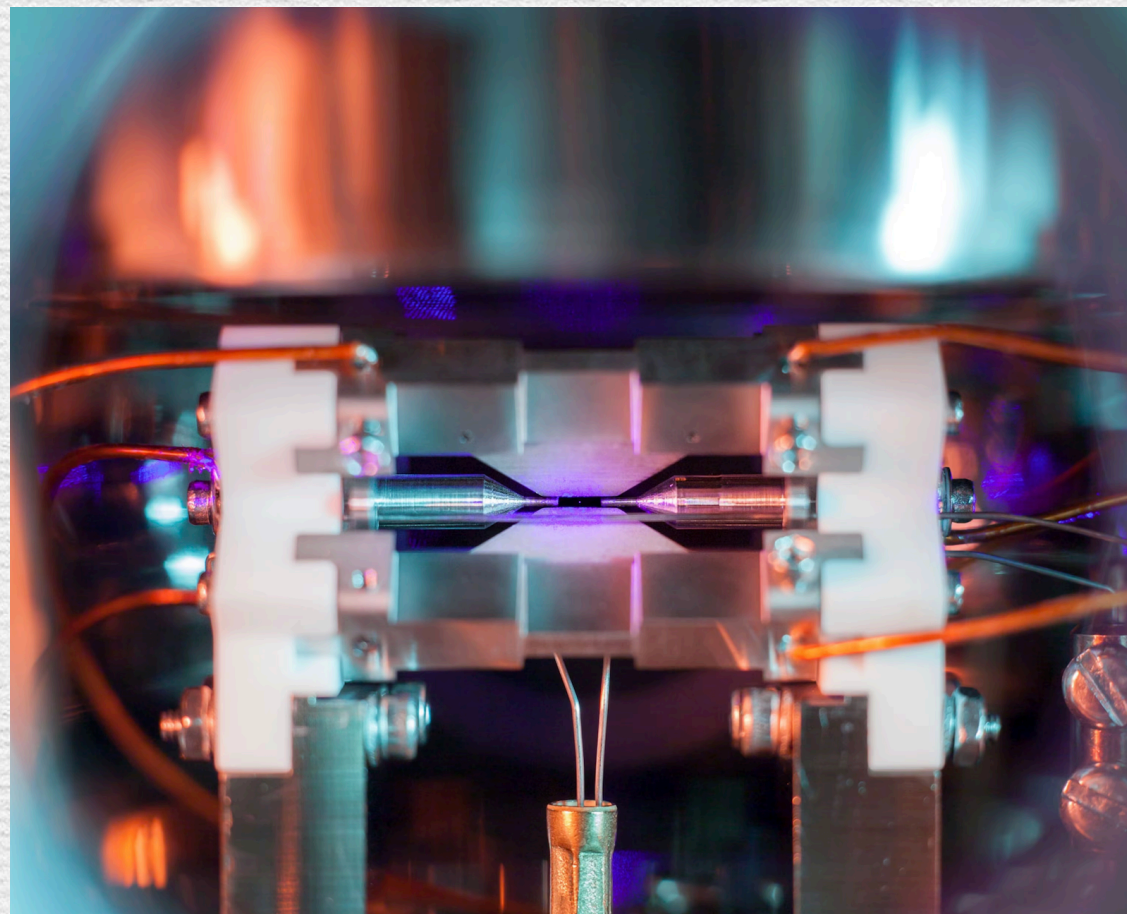
th O. Bazavan, S. Saner...A.B, *Comm. Phys.* **7**, 229 (2024)

ex S. Saner, O. Bazavan,..., A.B, R. Srinivas, *arXiv:24xx.SOON*

* cat states and $\mathbb{Z}_2$ LGTs, J. Del Pino *et al.*, *Phys. Rev. Lett.* **130**, 171901 (2023)

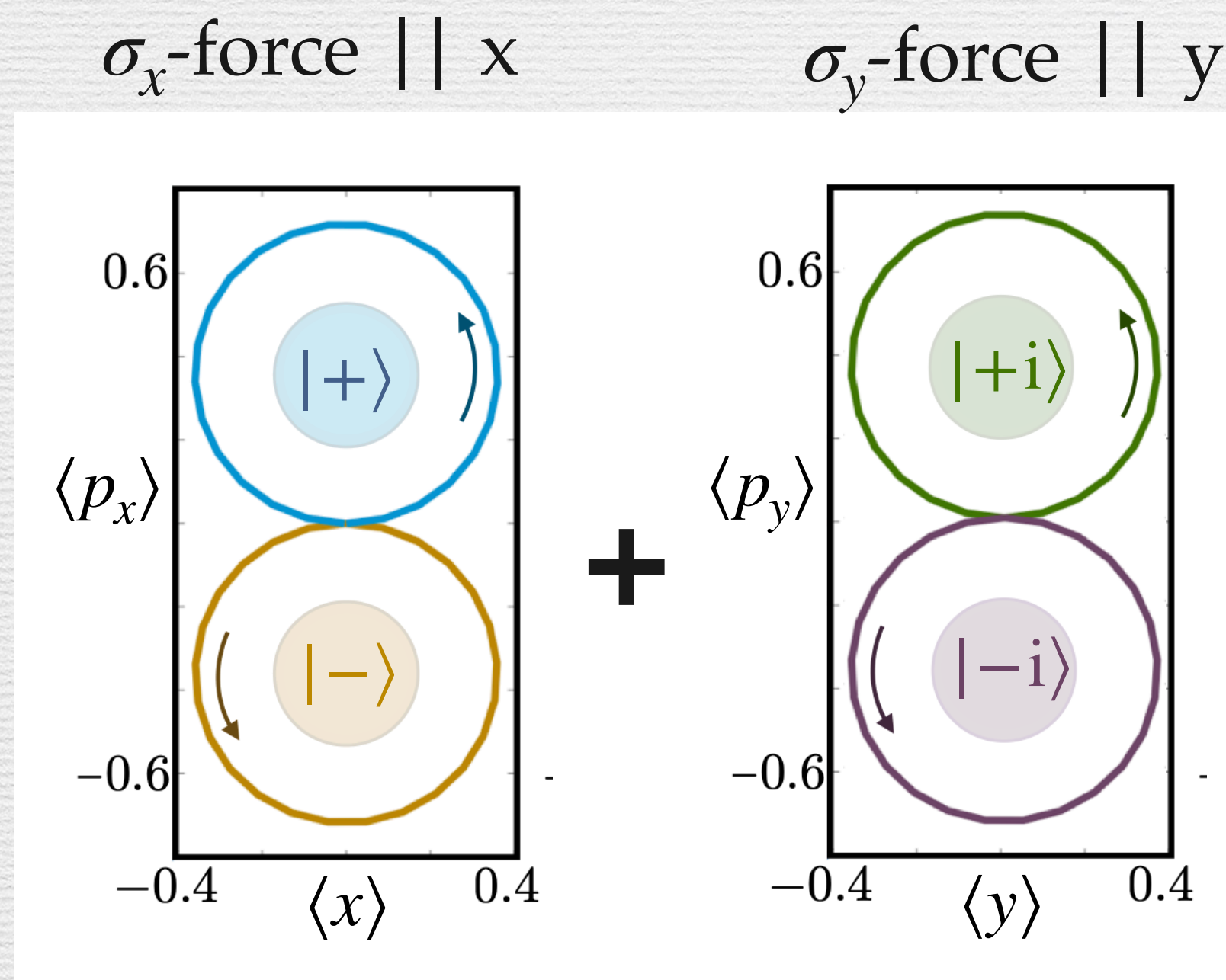
Parametric drivings and $\mathbb{Z}_2$ gauge theories for phonons

Trapped-ion “*Art brut*” or “made by theorists who are untrained and untutored in the traditional beauty of experimental physics”



$\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression

An alternative scheme is to use “**interference**” of two orthogonal and detuned **state-dependent forces**

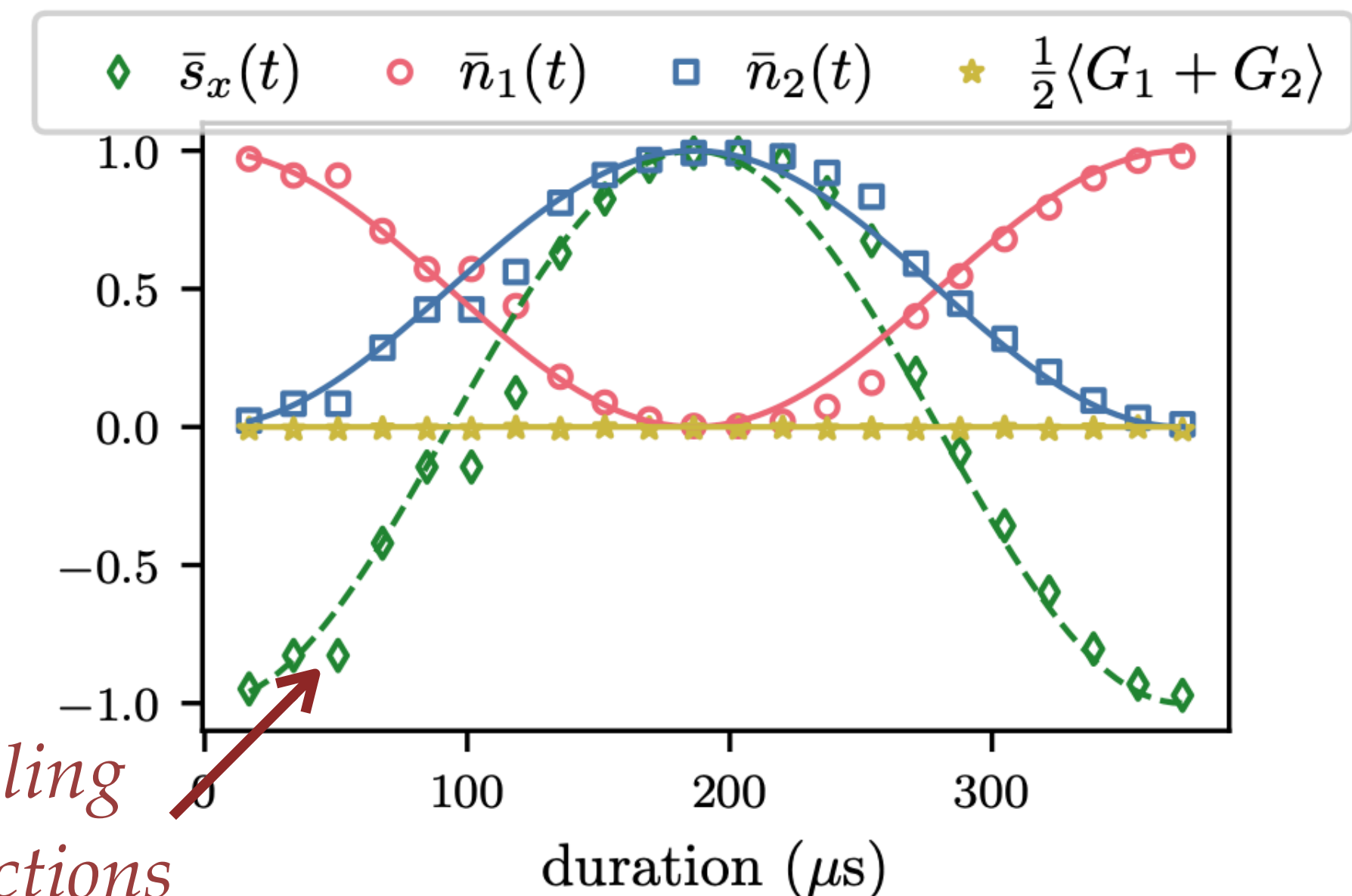


*R.T. Sutherland, R. Srinivas, **PRA** **104** 032609 (2024)*

new schemes w/ transverse field Bazavan & Saner

$$[F_x, F_y] \approx \textcolor{blue}{Z} \textcolor{red}{b}_x^\dagger \textcolor{red}{b}_y e^{i\Delta\varphi} e^{i(\omega_x - \omega_y - \delta)t} + \text{H.c.}$$

Forces for **parametric freq. conversion** $\delta \approx \omega_x - \omega_y$



*Modelling
imperfections*

$\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression

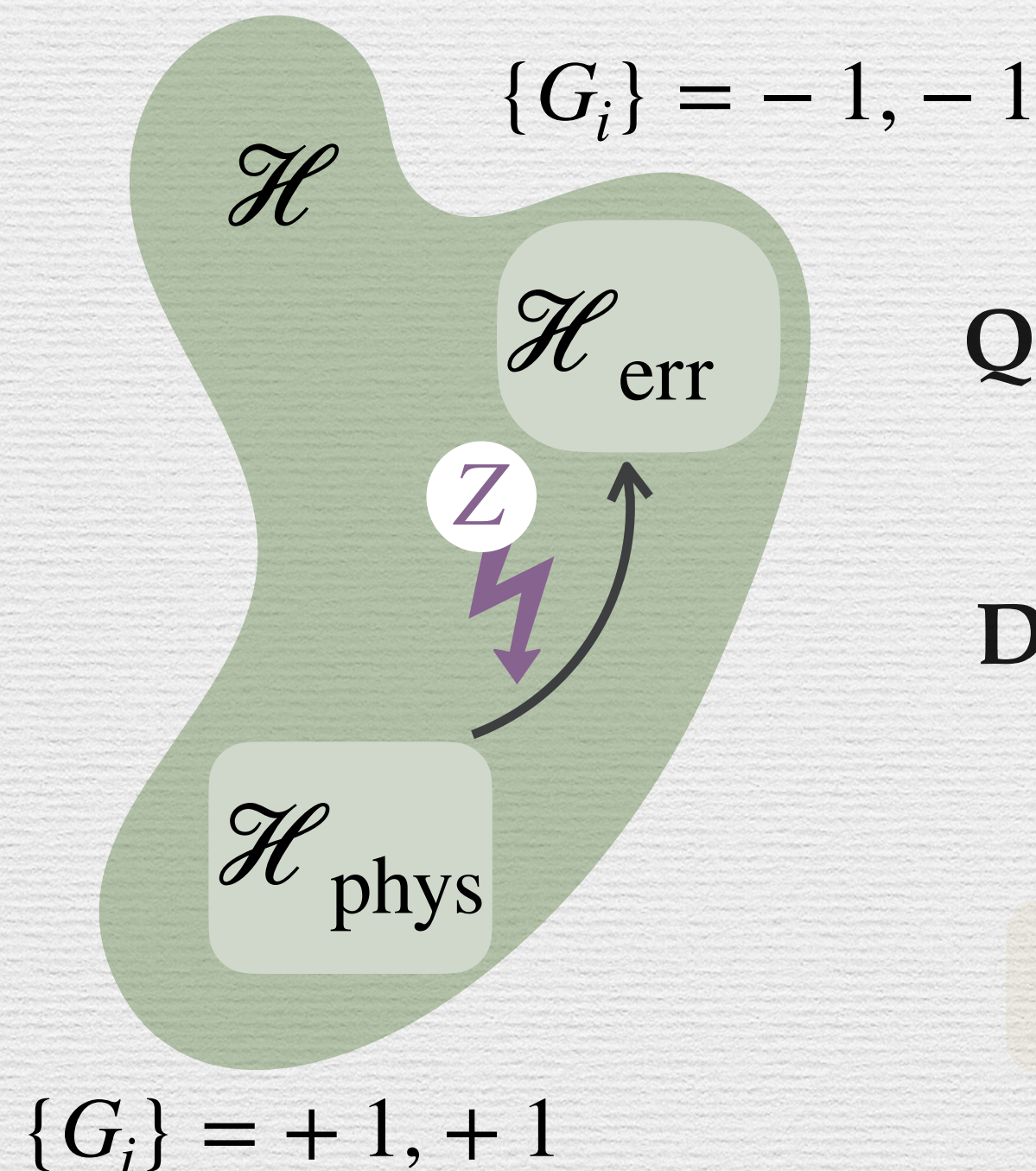
Phase noise is a typical source of errors that **break gauge invariance**

$$H_{\text{eff}}(t) = \left(J_{\text{eff}} b_x^\dagger Z b_y + \text{H.c.} \right) + hX + \delta\omega(t) Z$$

$$[H_{\text{eff}}, G_i] \neq 0, \quad G_1^2 = G_2^2 = \mathbb{I}$$

stabilizers (Gauss' law)

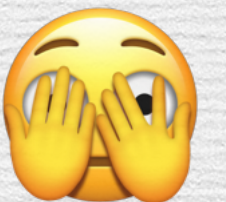
$$G_1 = (-1)^{b_x^\dagger b_x} X, \quad G_2 = X(-1)^{b_y^\dagger b_y}$$



QEC would require QND measurement of bosonic spin-dependent parity



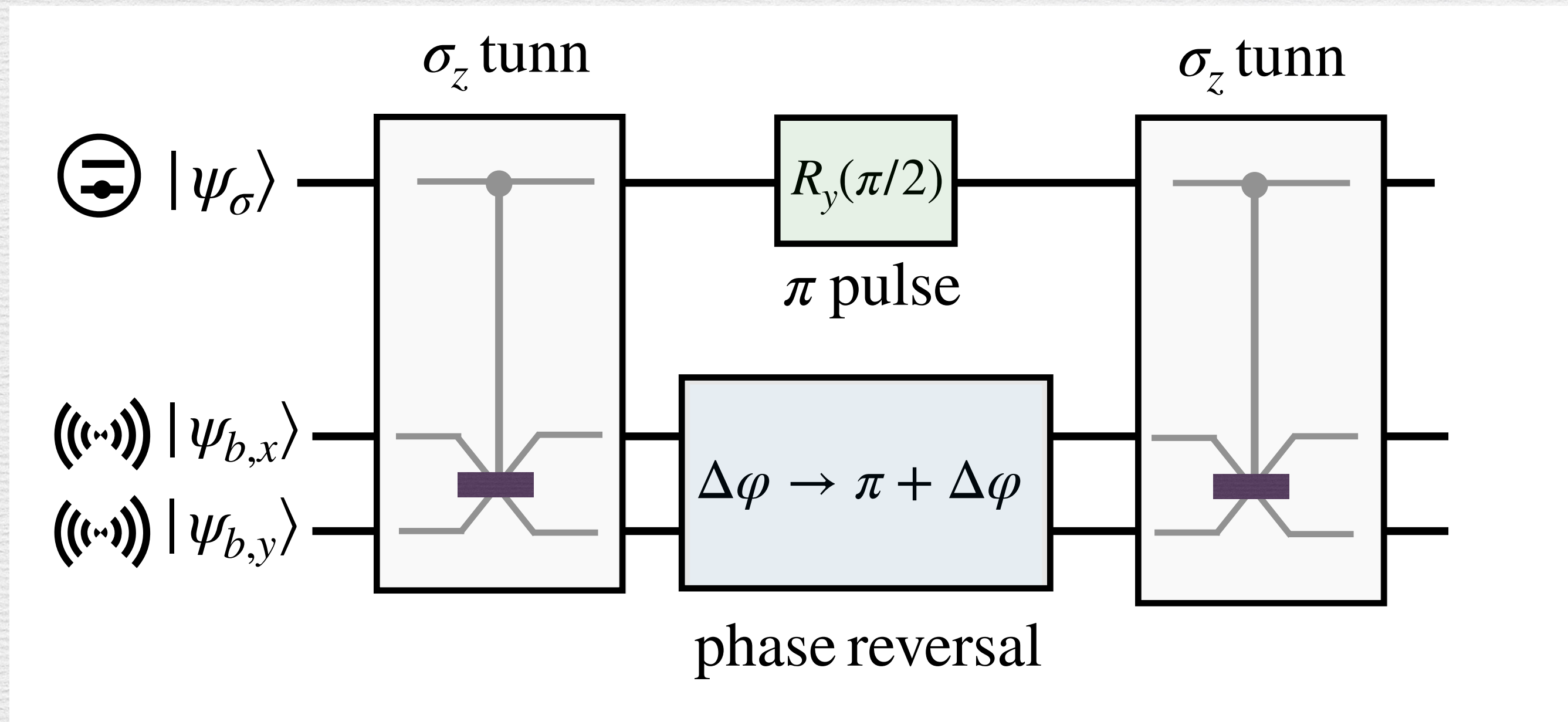
Dynamical decoupling require effectively implementing the gauge transformation



$$Z \mapsto -Z \quad X \mapsto X \quad b_x^\dagger \mapsto -b_x^\dagger \quad b_y \mapsto b_y \quad b_x^\dagger b_y \mapsto -b_x^\dagger b_y$$

$\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression

Phase noise can be refocused while preserving gauge invariance

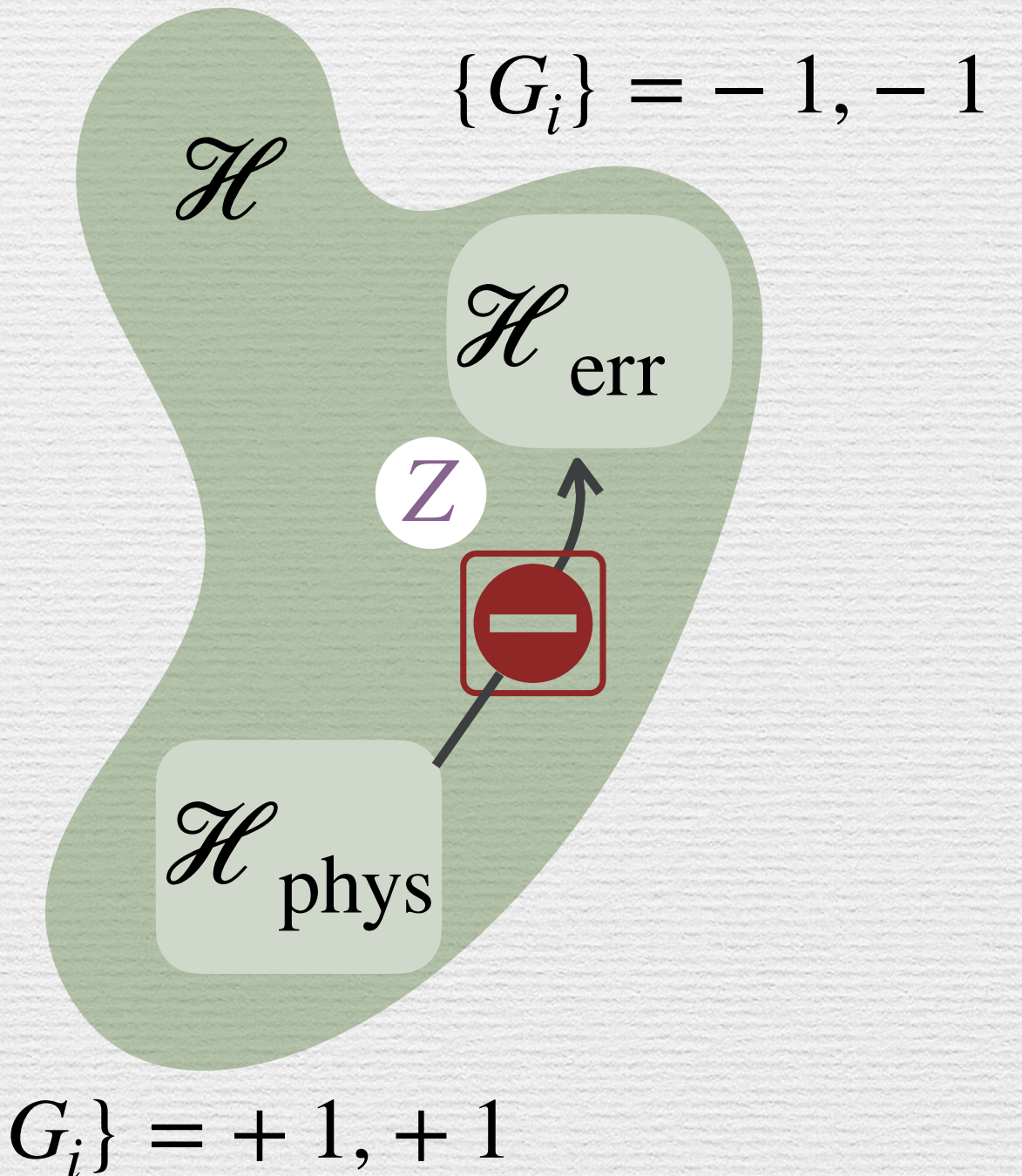


Combined spin-echo that refocuses slow components phase noise

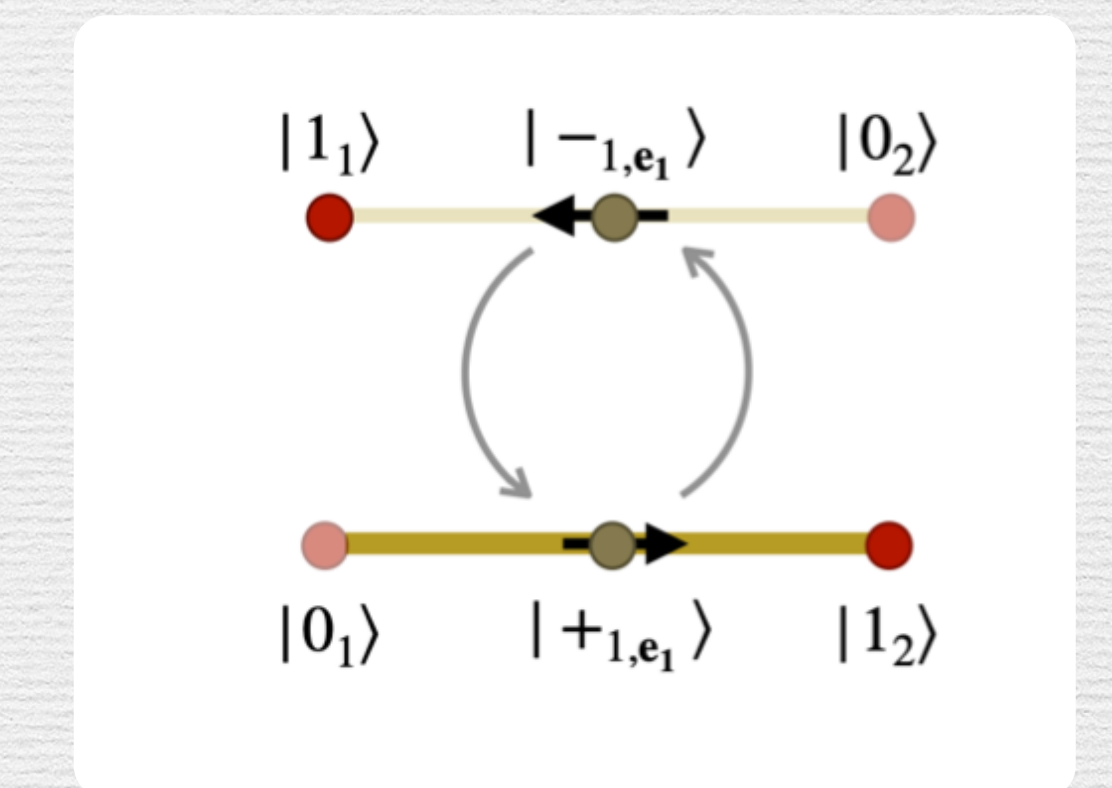
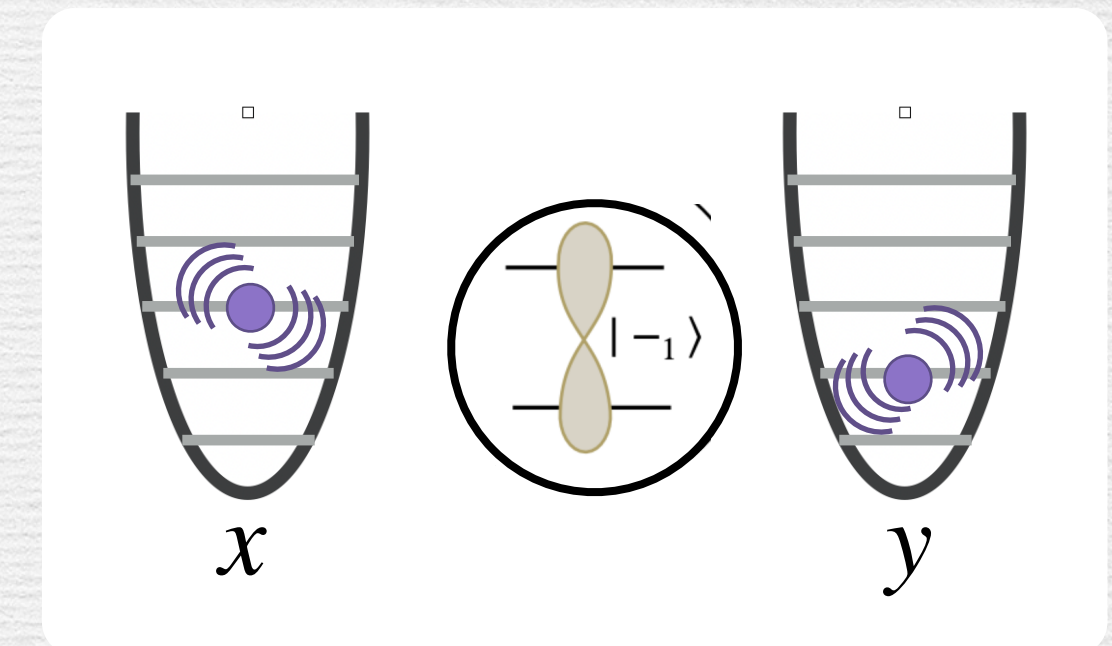
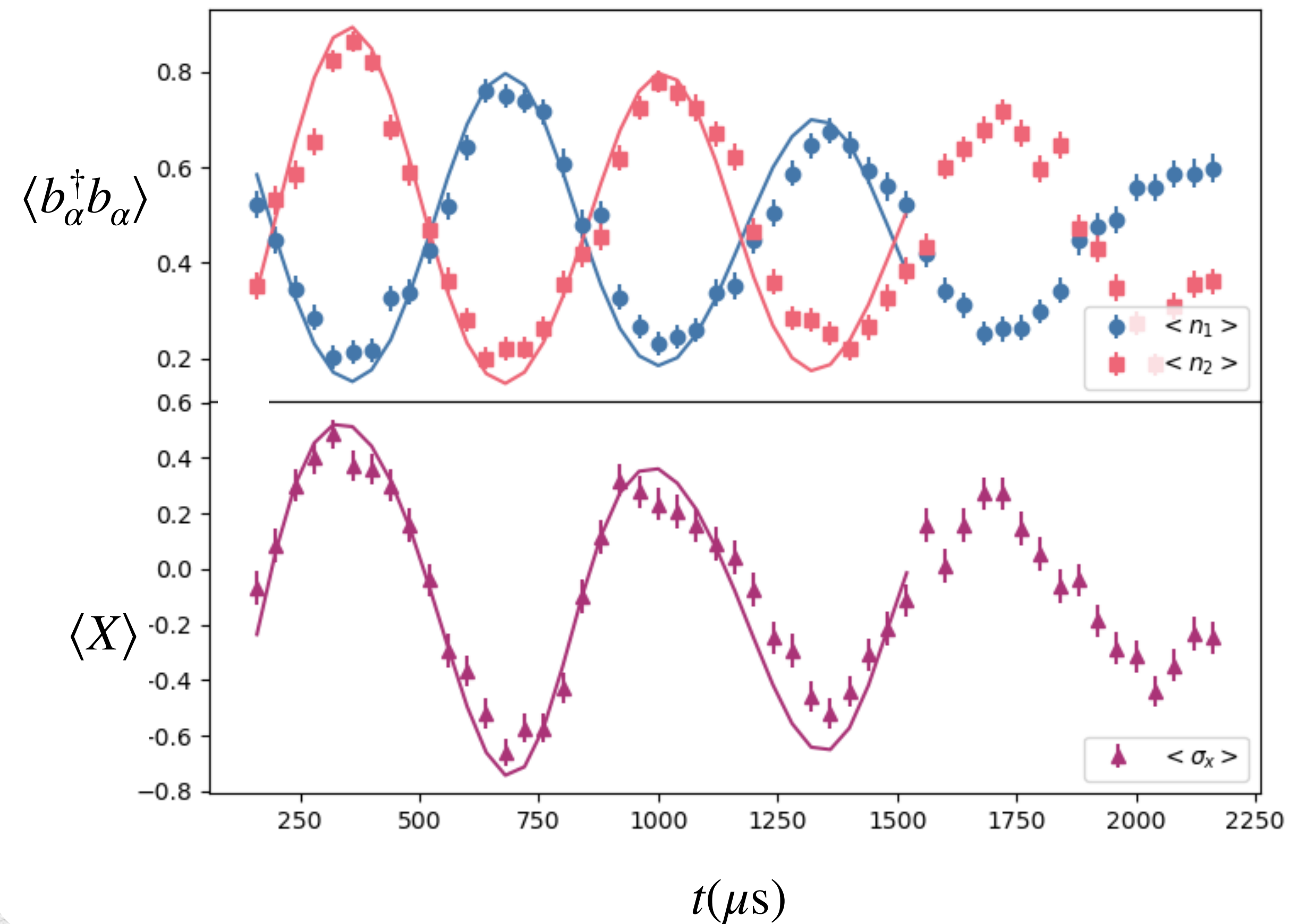
$$\delta\omega(t) \sim \delta\omega_0 \in N[0, \sigma^2]$$

Only valid for phase noise with $\tau_c \gg t$ (otherwise QEC required)

$$H_{\text{eff}} \approx \left(J_{\text{eff}} b_x^\dagger \mathbf{Z} b_y + \text{H.c.} \right) + hX$$



$\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression



No fitting parameters
(Independent characterization of noise heating, state-dep force imperfections)

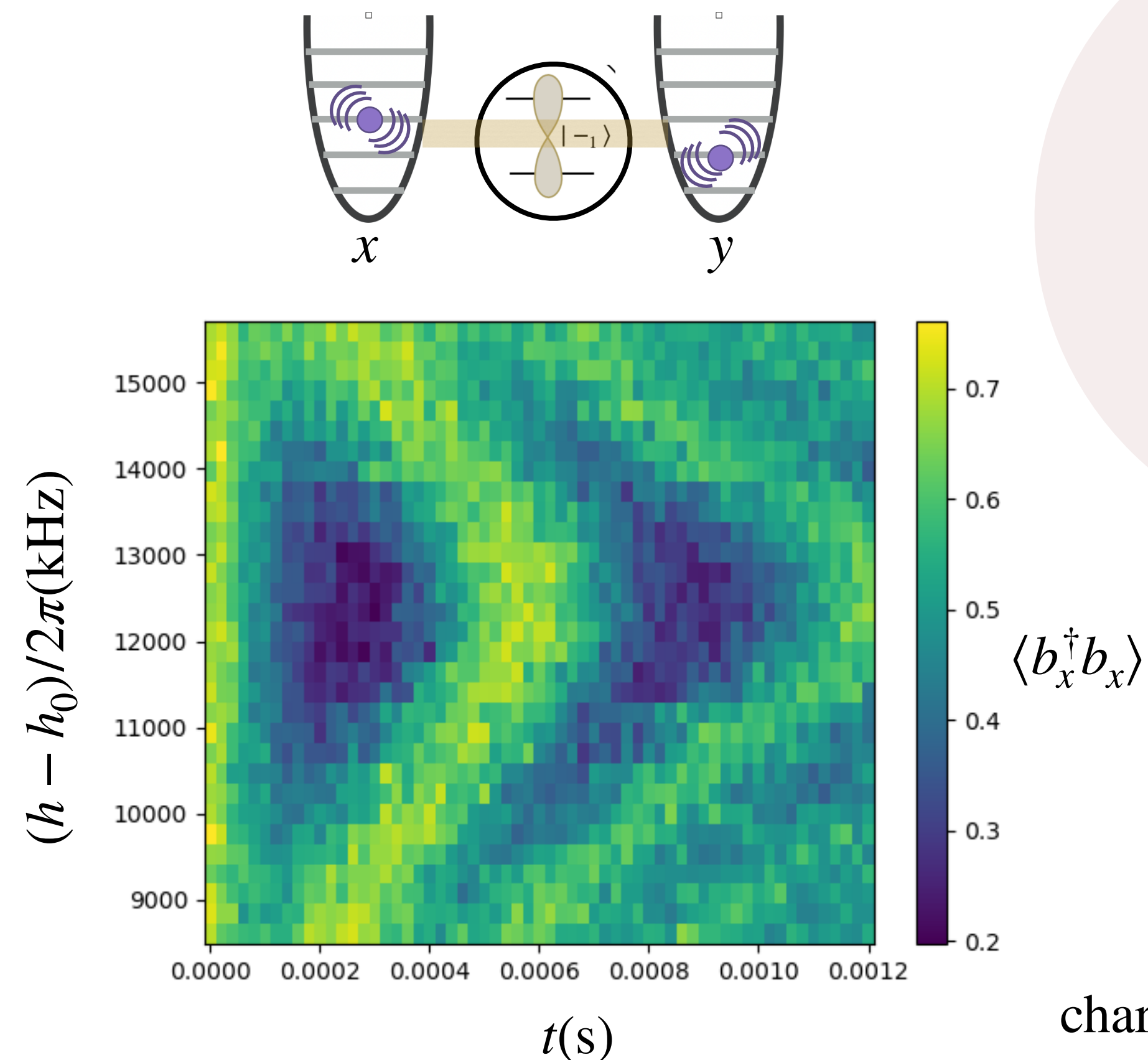
ex S. Saner, O. Bazavan *et al.*, [arXiv:24xx.SOON](#)

(Floquet-Hubbard atoms) C. Schweizer *et al.*, **Nat.Phys.** **15**, 1168 (2019)

$\mathbb{Z}_2$ link in the lab: gauge invariance and error suppression

Finite **electric field** inhibits the **dynamics** of charged particles be (**energetic penalty of stretching E -strings**)

ex S. Saner, O. Bazavan *et al.*, [arXiv:24xx.SOON](#)

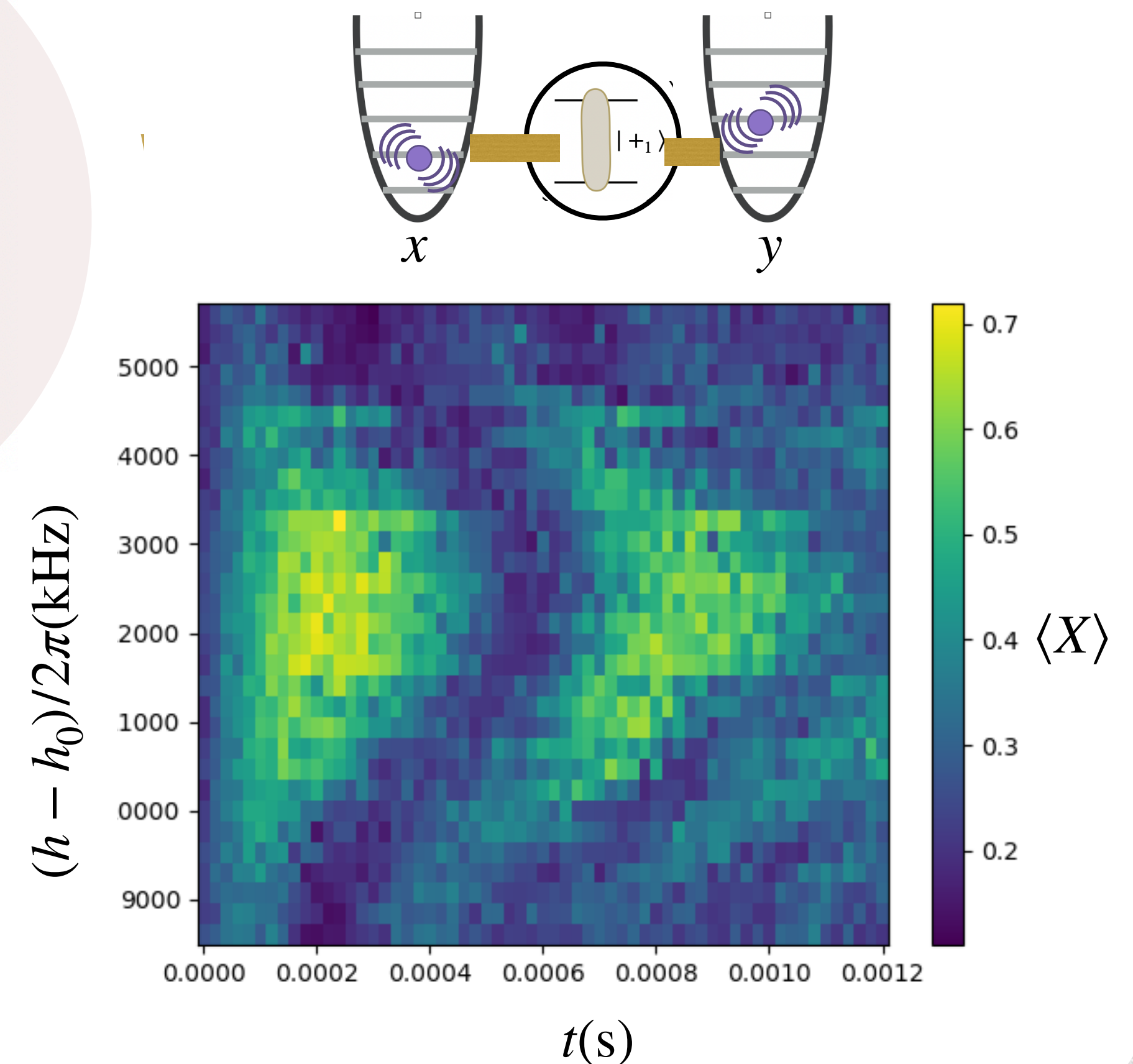


$$|R\rangle = |0_1\rangle |_{+1, e_1} |1_2\rangle$$

$$|L\rangle = |1_1\rangle |_{-1, e_1} |0_2\rangle$$

The energy gap between these states is indicated as $2h$. An effective time t_{eff} is shown between a dashed line and the lower state.

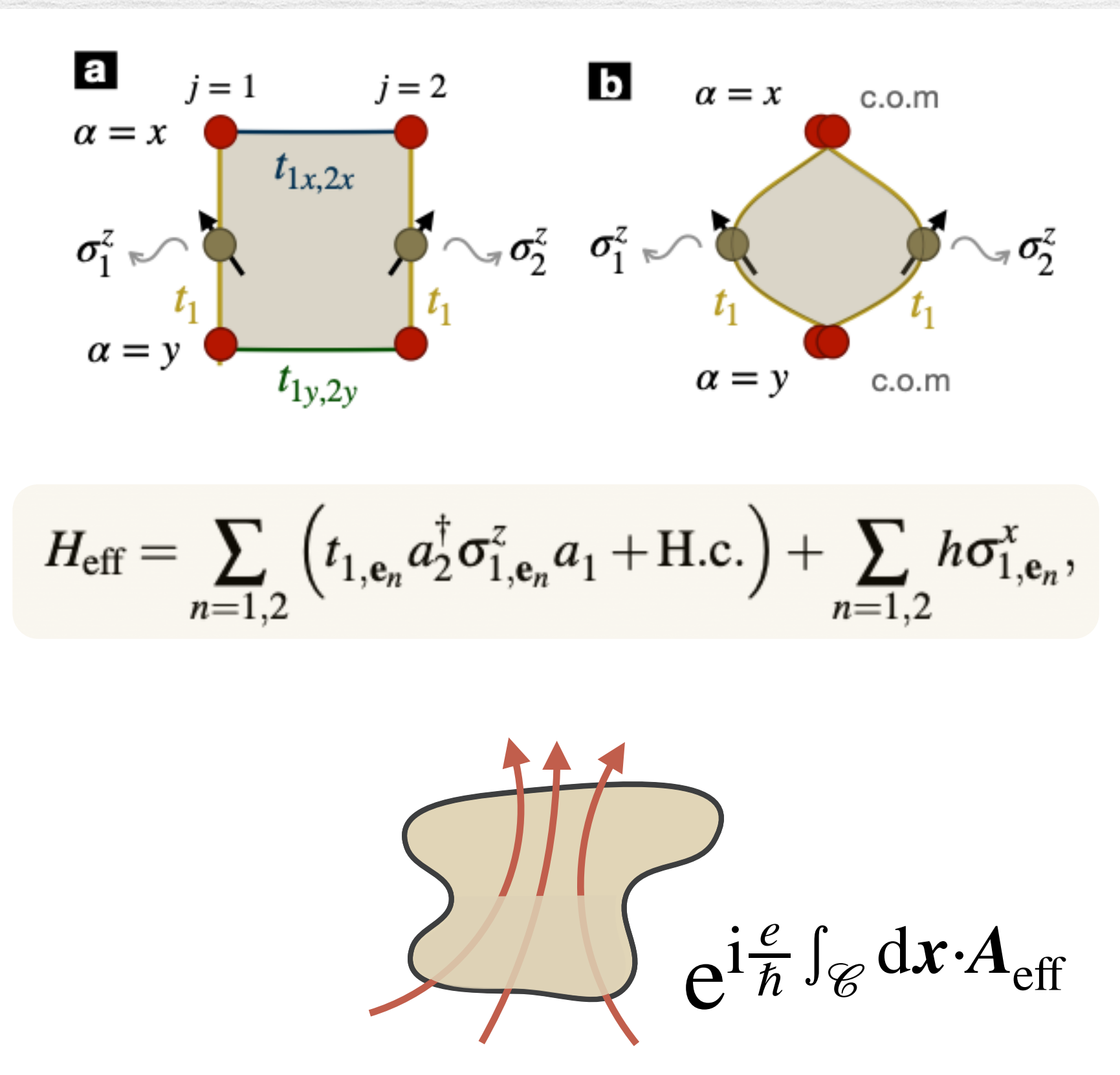
charge $\longleftrightarrow$ gauge



$\mathbb{Z}_2$ LGT in a chain and a chain of loops

More ions and normal modes

Gauge theory on a loop



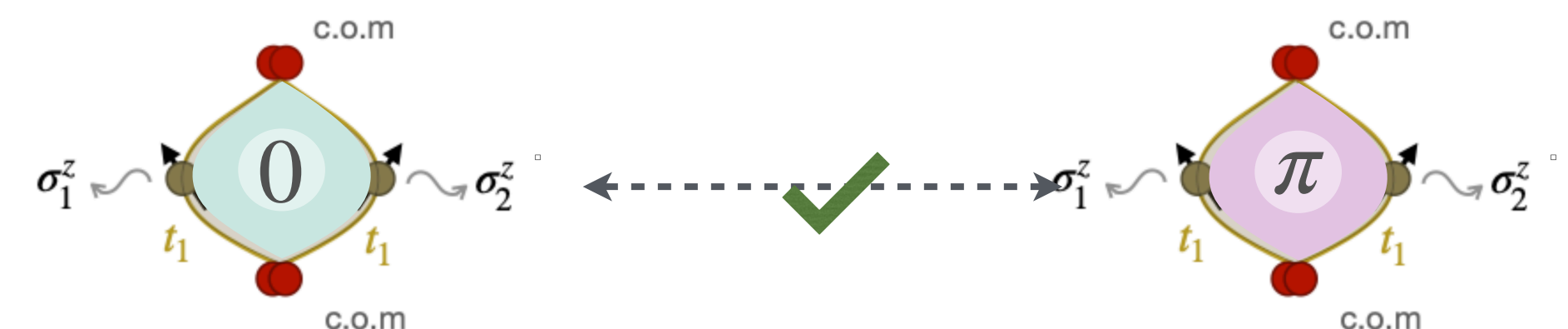
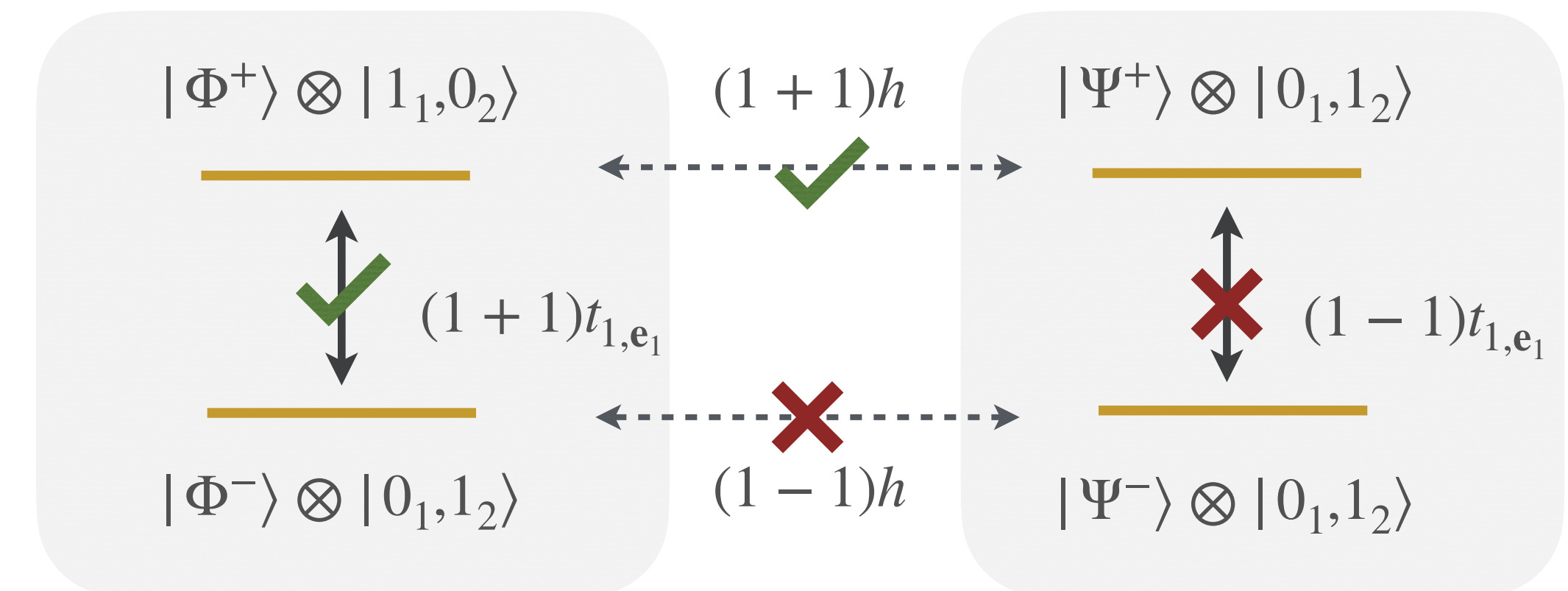
Dynamical Aharonov-Bohm effect

The gauge-invariant states in the **B-field** basis require **Bell pairs**

$$|\Phi^\pm\rangle = (|\uparrow, \uparrow\rangle \pm |\downarrow, \downarrow\rangle) / \sqrt{2} \quad |\Psi^\pm\rangle = (|\uparrow, \downarrow\rangle \pm |\downarrow, \uparrow\rangle) / \sqrt{2}.$$

Const. AB interference

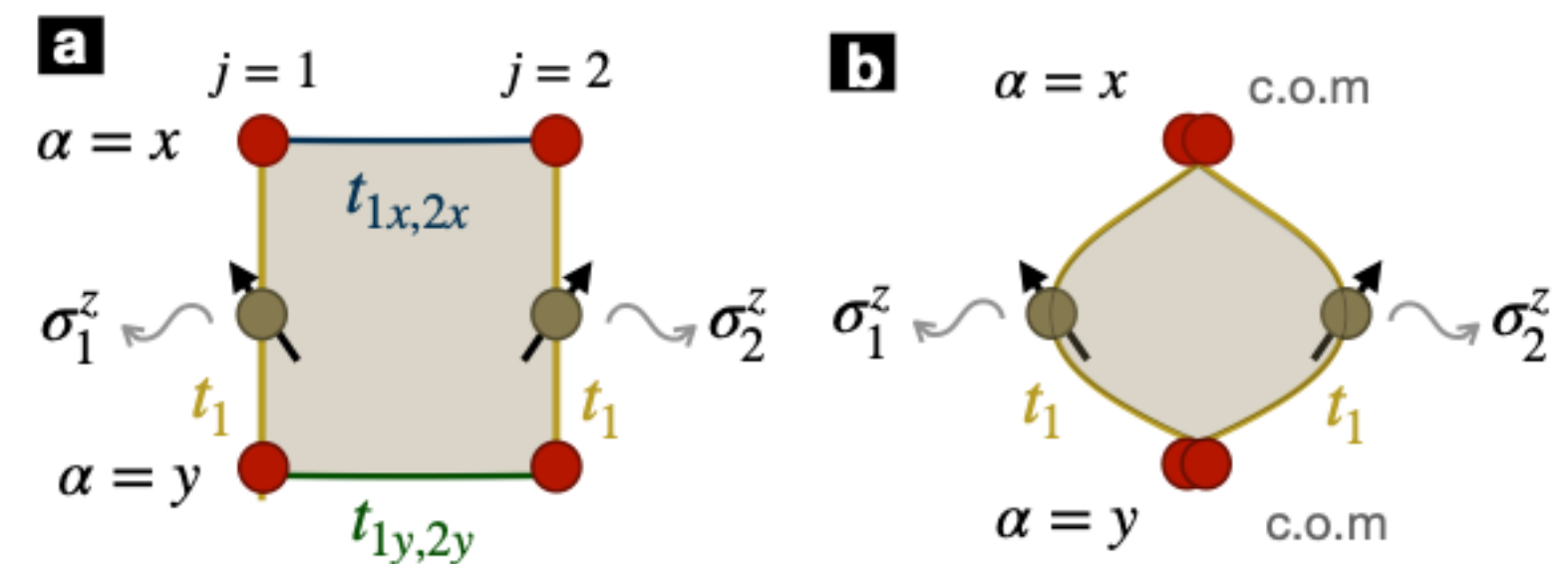
Dest. AB interference



$\mathbb{Z}_2$ LGT in a chain and a chain of loops

More ions and normal modes

Gauge theory on a loop

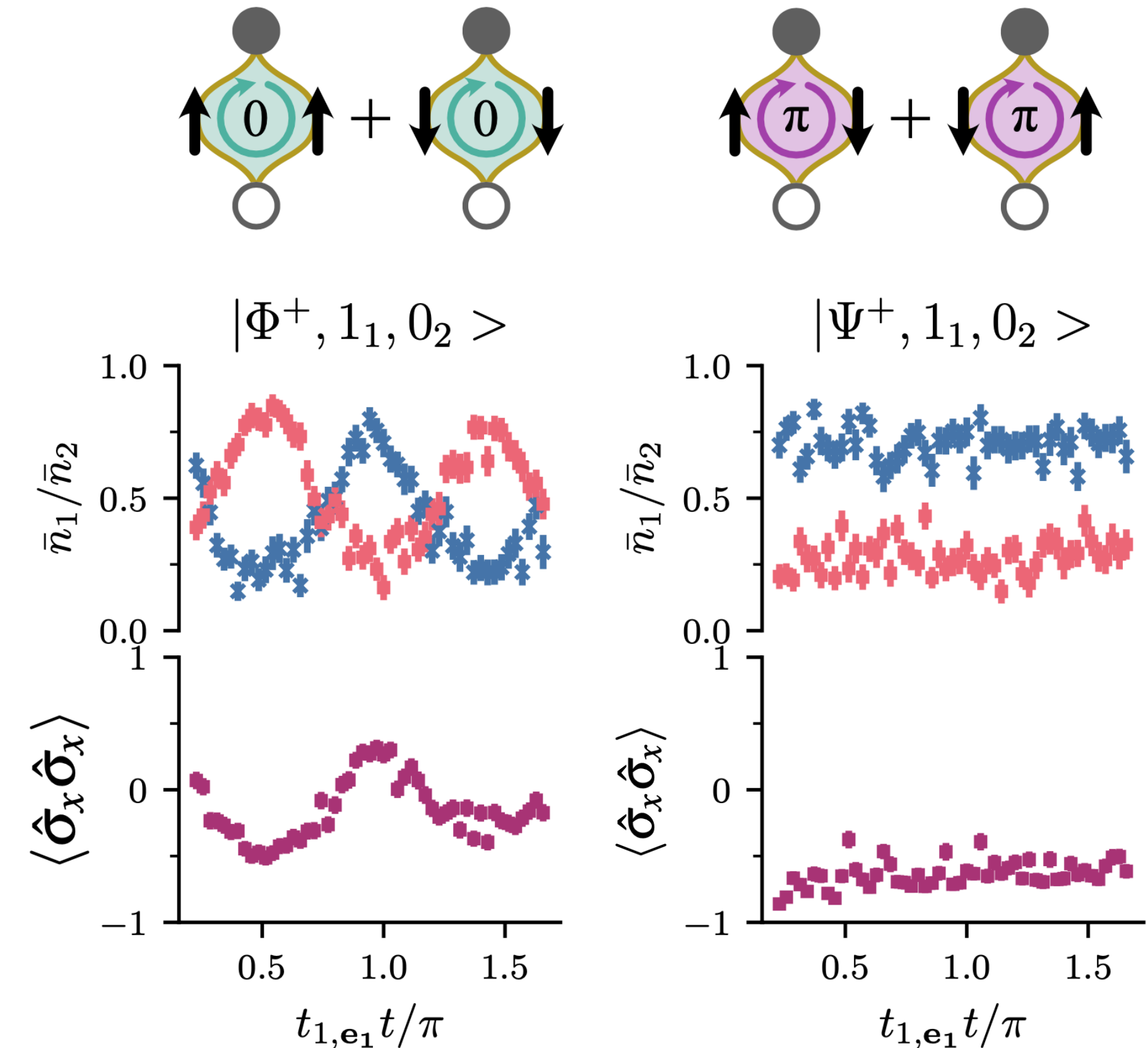


$$H_{\text{eff}} = \sum_{n=1,2} \left(t_{1,\mathbf{e}_n} a_2^\dagger \sigma_{1,\mathbf{e}_n}^z a_1 + \text{H.c.} \right) + \sum_{n=1,2} h \sigma_{1,\mathbf{e}_n}^x,$$

$$e^{i\frac{e}{\hbar} \int_{\mathcal{C}} d\mathbf{x} \cdot \mathbf{A}_{\text{eff}}}$$

Dynamical Aharonov-Bohm effect

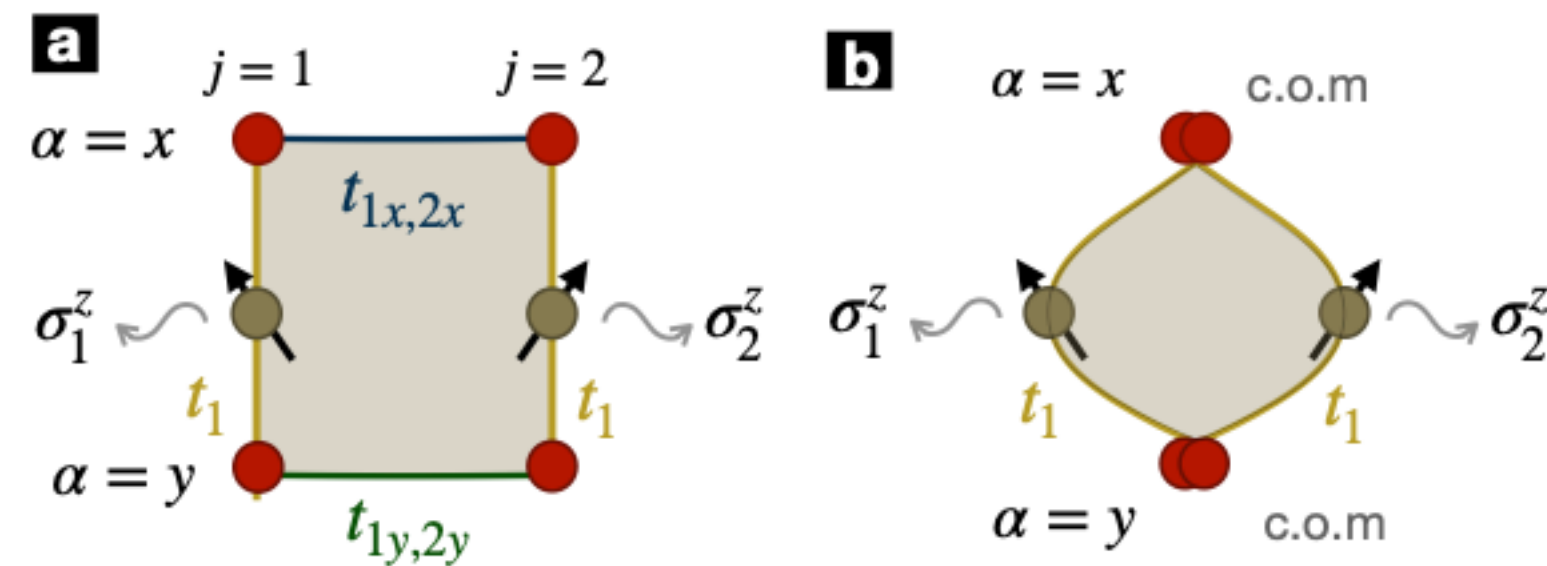
ex S. Saner, O. Bazavan *et al.*, [arXiv:24xx.SOON](#)



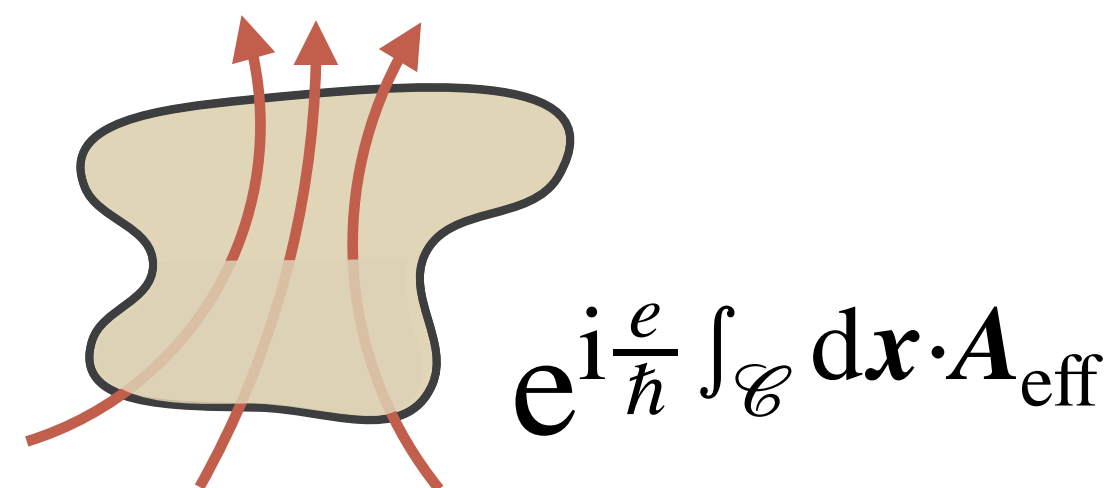
$\mathbb{Z}_2$ LGT in a chain and a chain of loops

More ions and normal modes

Gauge theory on a loop

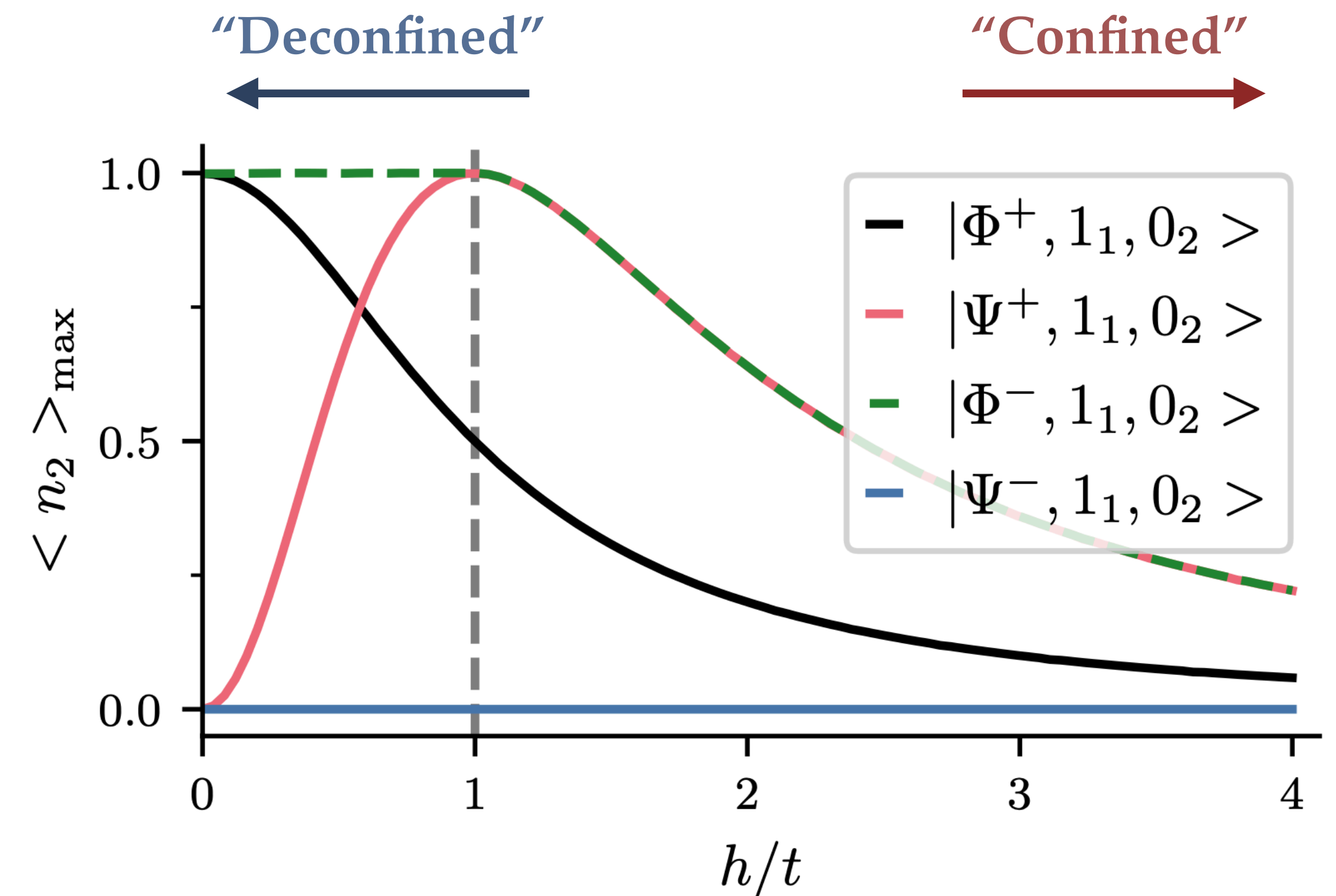


$$H_{\text{eff}} = \sum_{n=1,2} \left(t_{1,e_n} a_2^\dagger \sigma_{1,e_n}^z a_1 + \text{H.c.} \right) + \sum_{n=1,2} h \sigma_{1,e_n}^x,$$



Dynamical Aharonov-Bohm effect

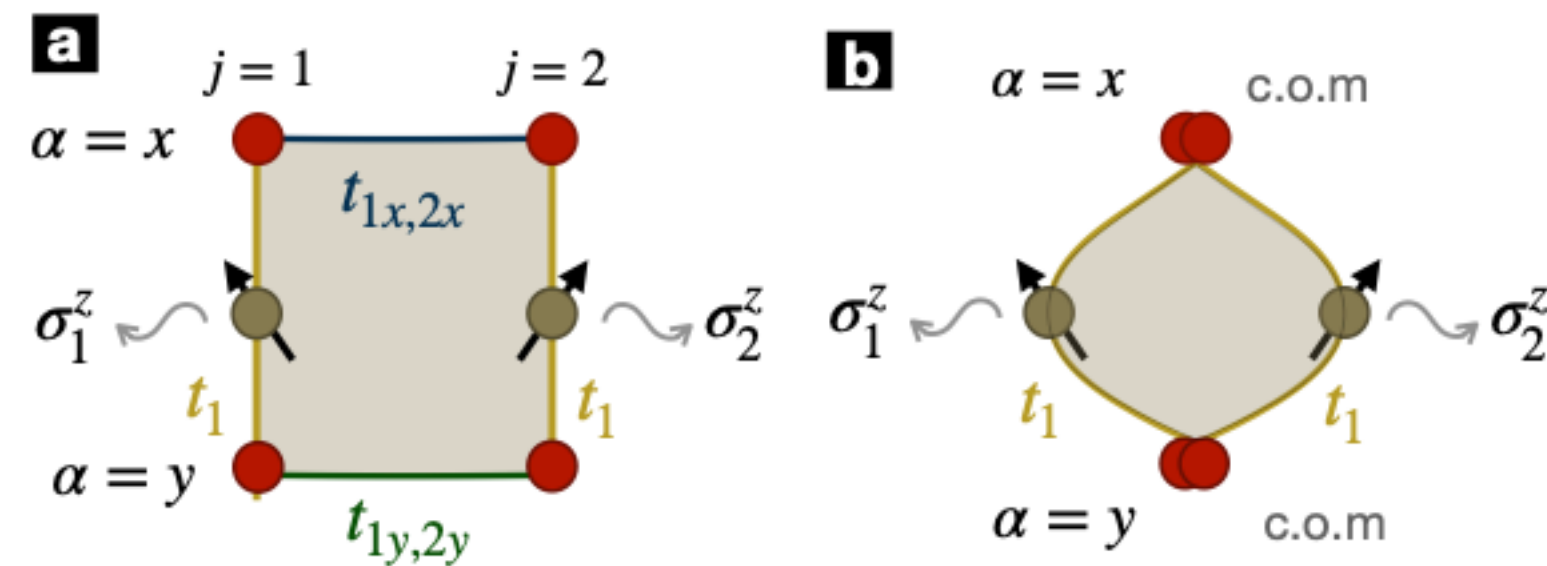
E-field competes with the AB interference in the B-field basis



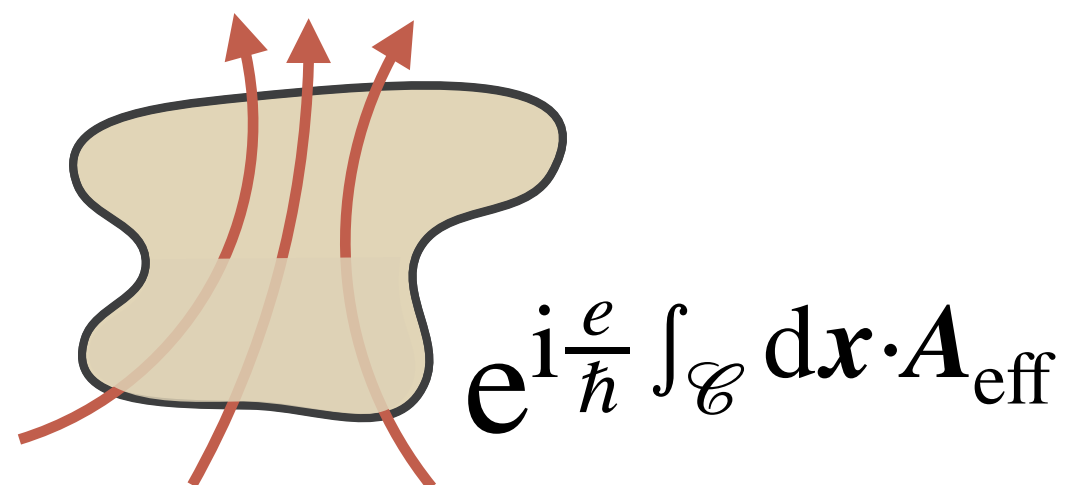
$\mathbb{Z}_2$ LGT in a chain and a chain of loops

More ions and normal modes

Gauge theory on a loop

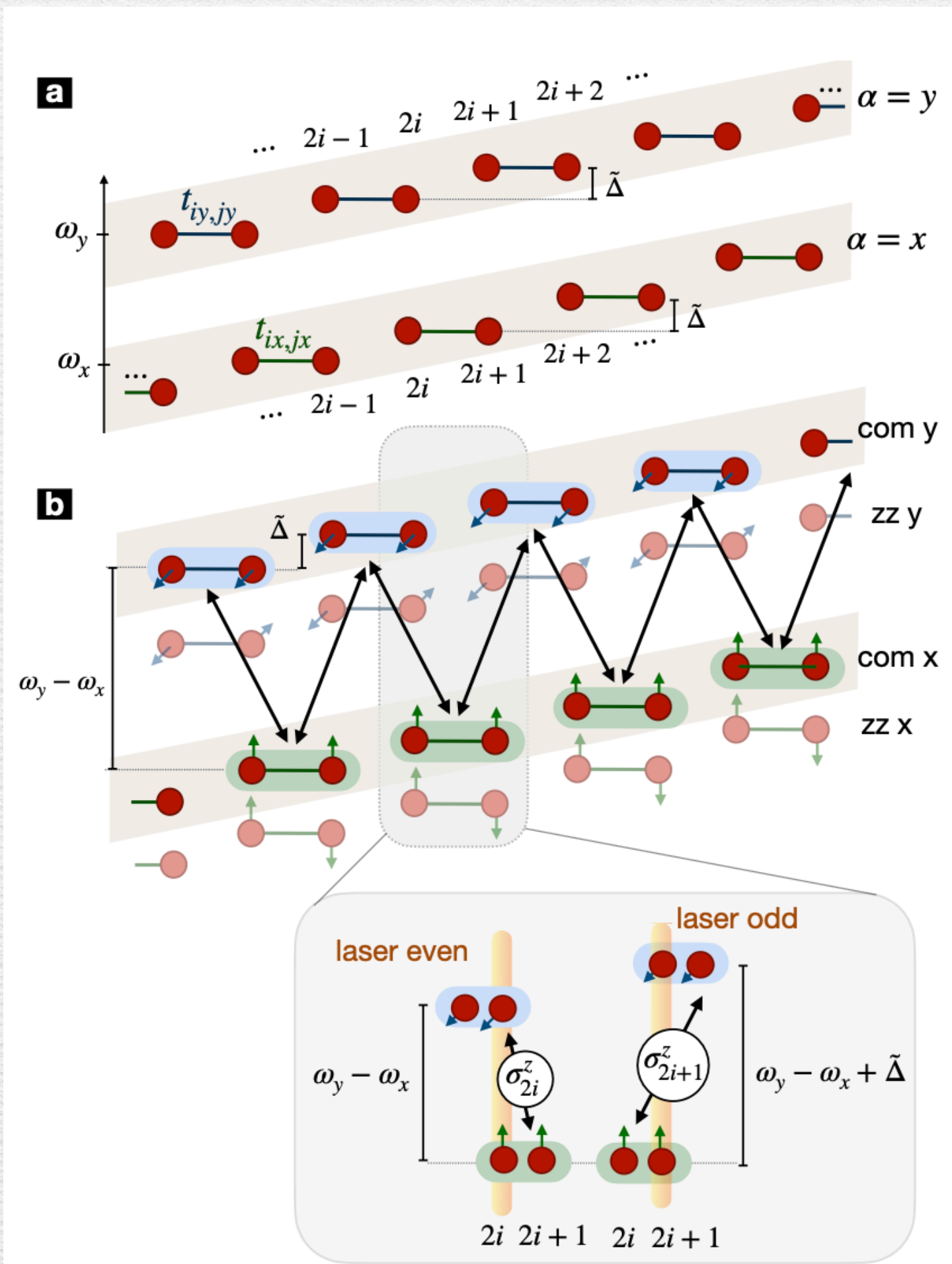


$$H_{\text{eff}} = \sum_{n=1,2} \left(t_{1,e_n} a_2^\dagger \sigma_{1,e_n}^z a_1 + \text{H.c.} \right) + \sum_{n=1,2} h \sigma_{1,e_n}^x,$$

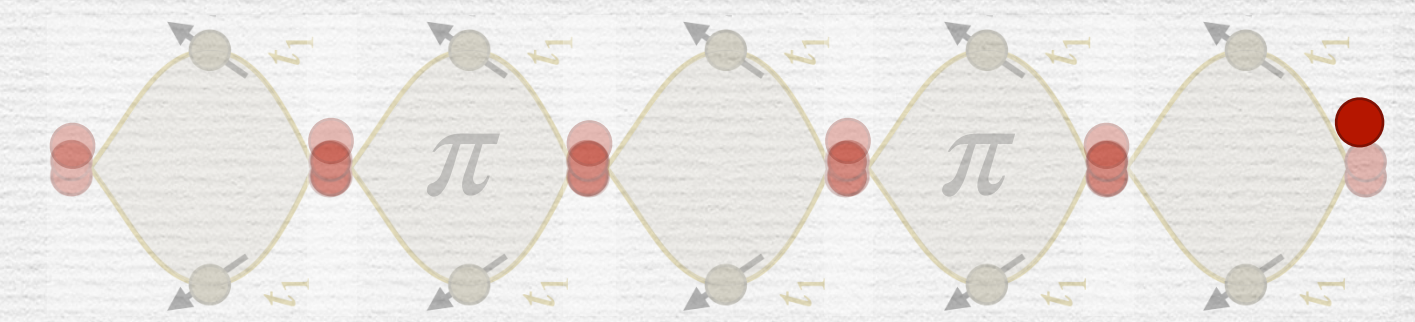


→ “dimensional reduction” →

Gauge theory on a chain

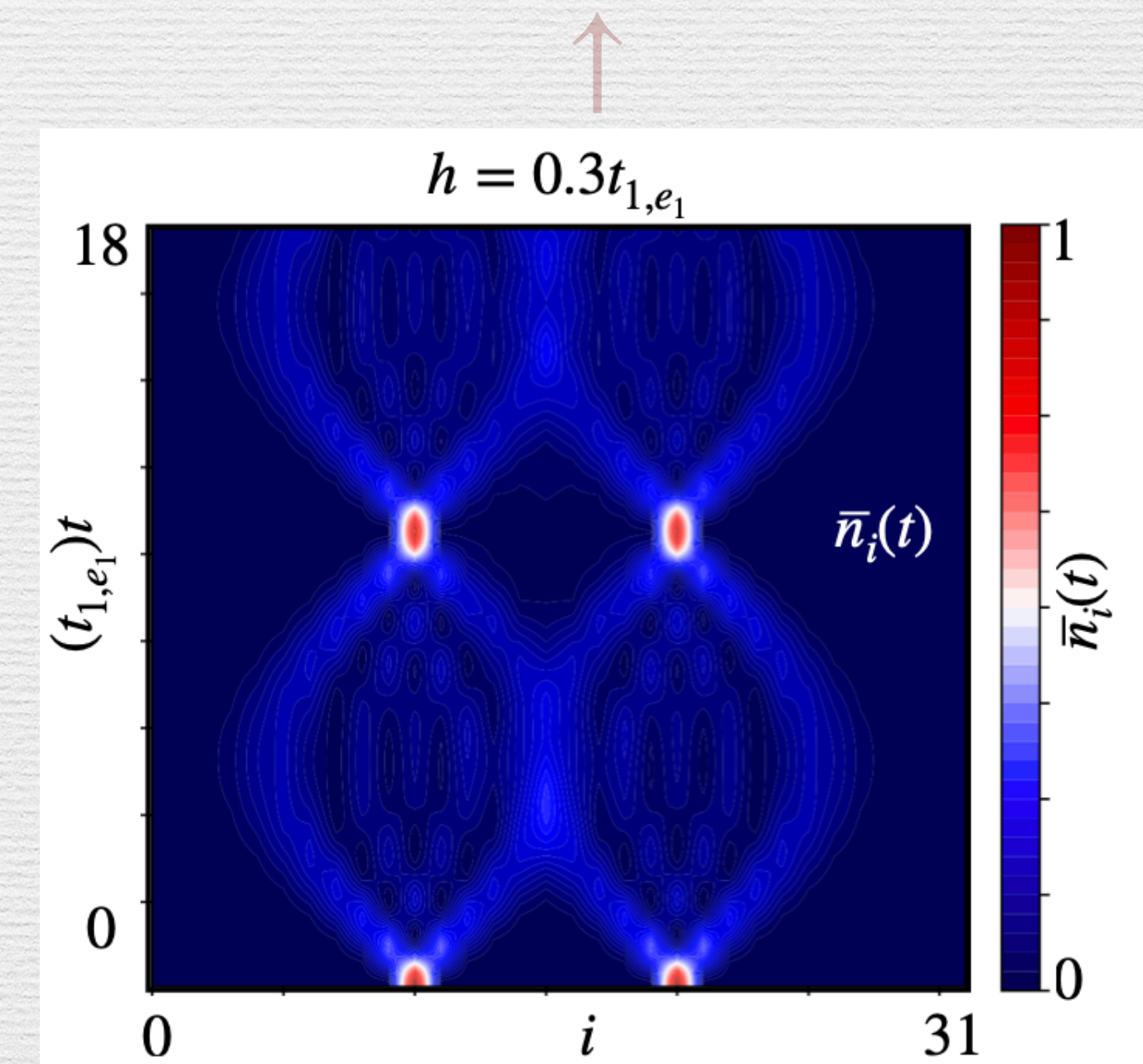


Gauge theory on a chain loops



“AB interf. & deconfined mesons”

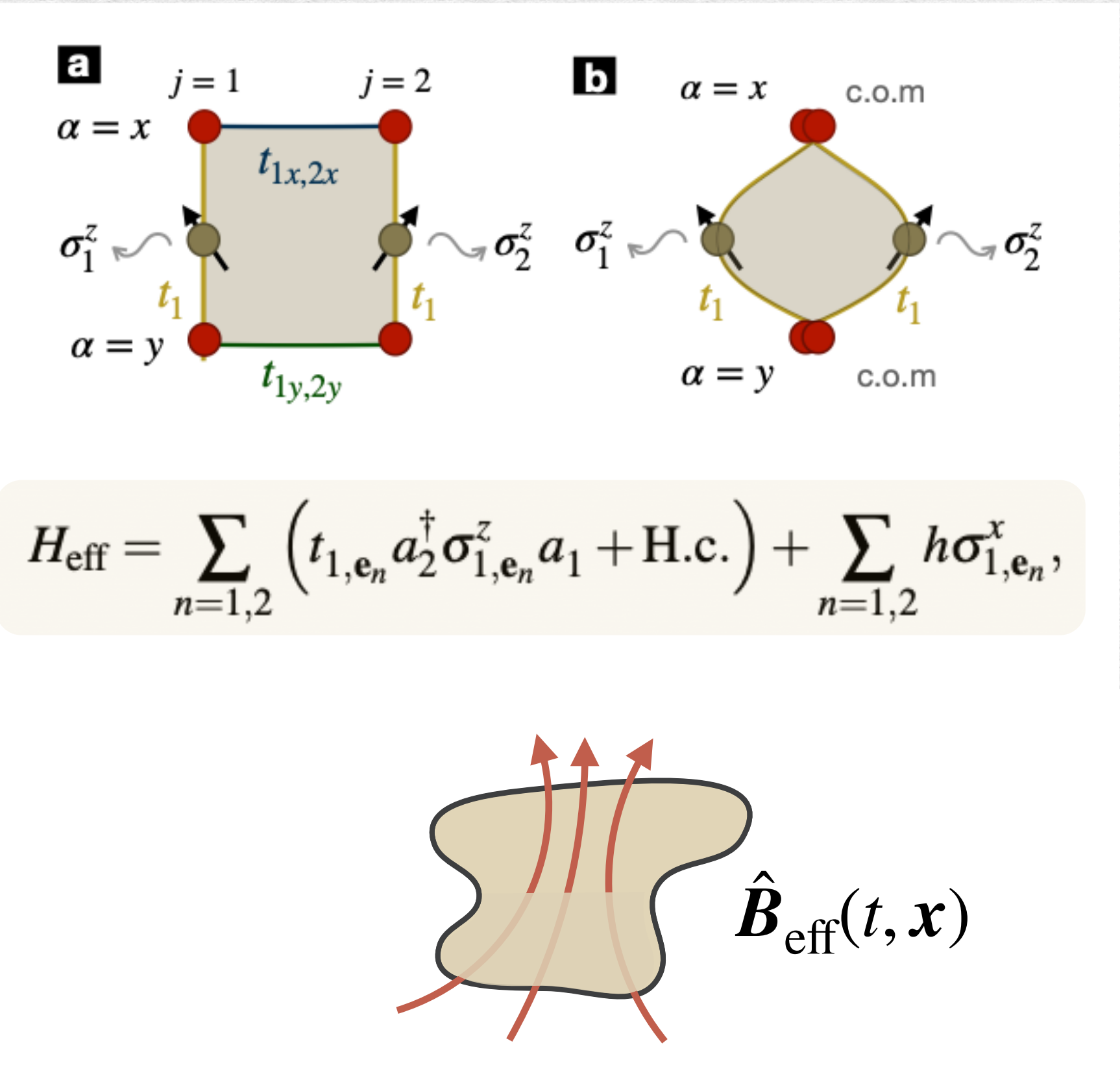
E. Domanti, et al, [arXiv:24xx.SOON](#)



$\mathbb{Z}_2$ LGT in a chain and a loop chain

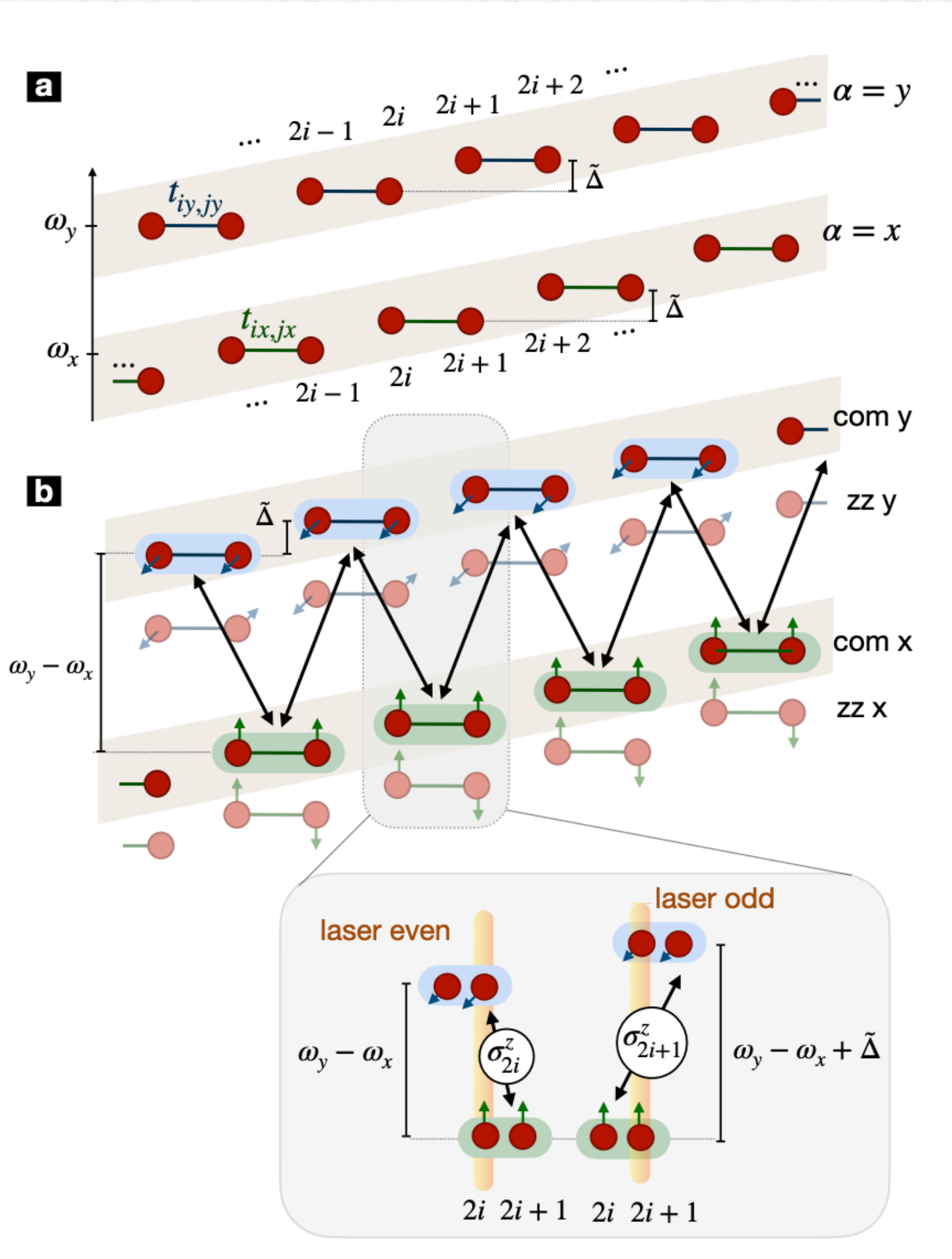
More ions and normal modes

Gauge theory on a loop

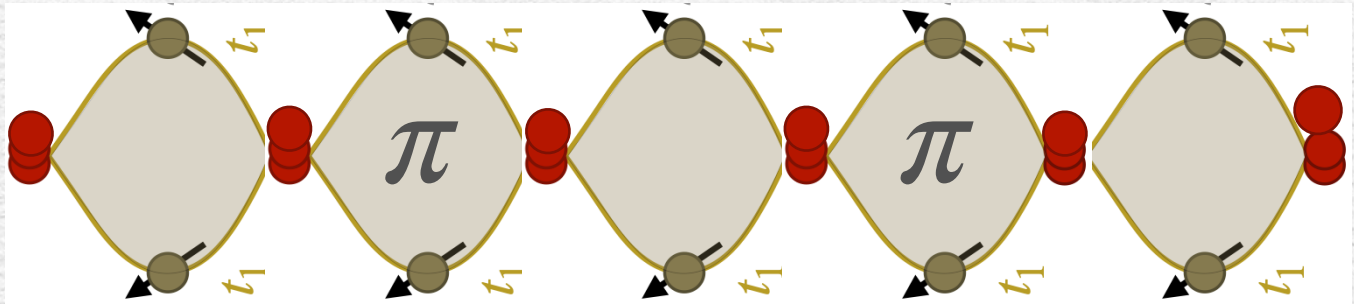


↪ “dimensional reduction” ↪

Gauge theory on a chain

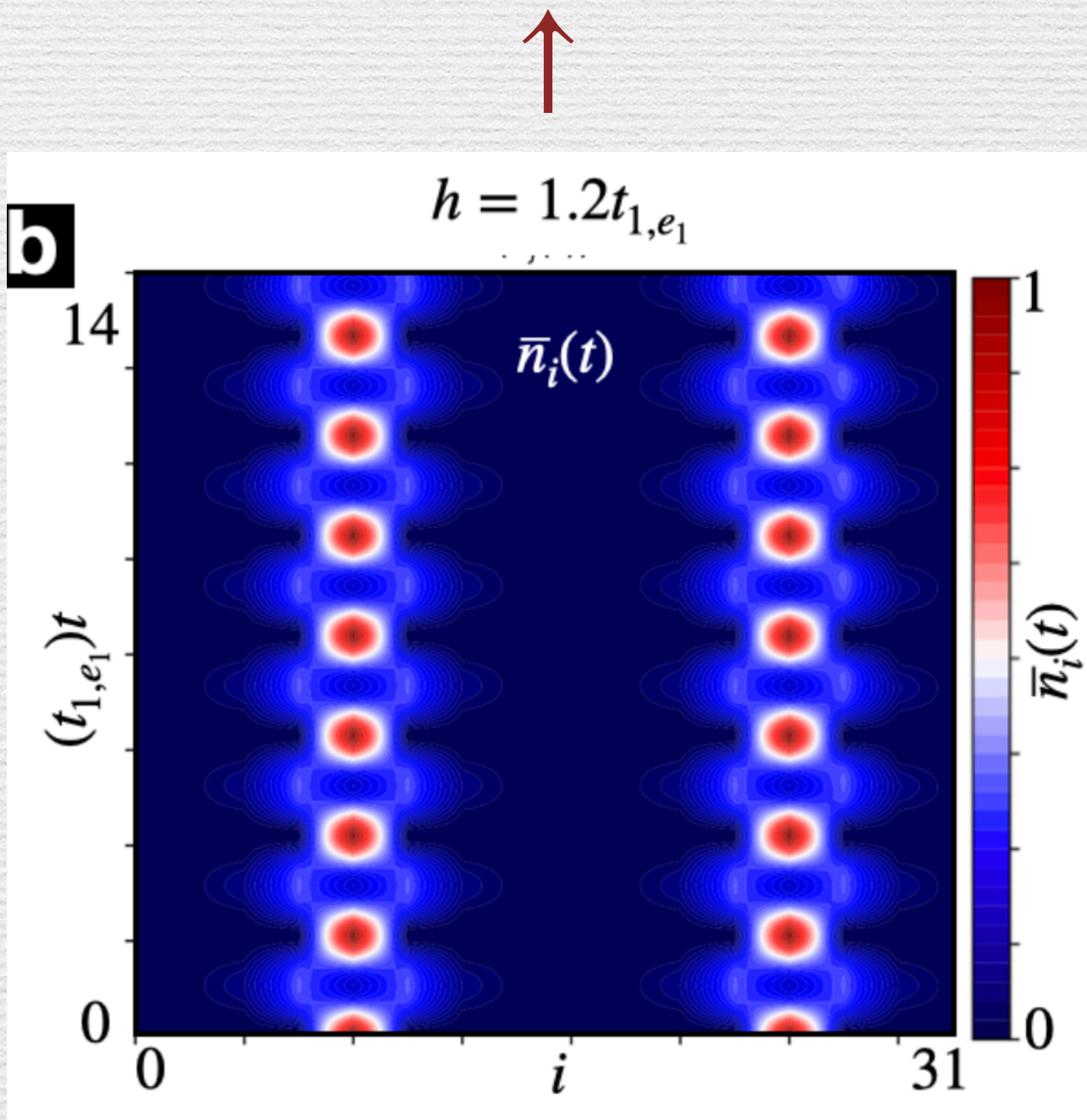


Gauge theory on a chain loops



AB interf. & deconfined “mesons”

E. Domanti, et al, [arXiv:24xx.SOON](#)



$\mathbb{Z}_2$ LGT in a chain and a loop chain

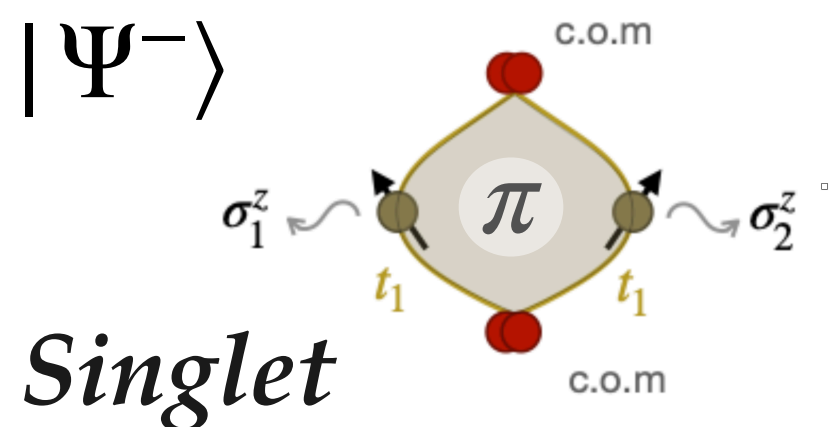
We can now study the **competition** of **electric** & **magnetic** terms in the presence of **AB interference**

$$H_{\text{loop}\mathbb{Z}_2} = t \sum_i \sum_{n=1,2} (a_i^\dagger Z_{i,n} a_{i+1} + \text{H.c.}) + h \sum_i \sum_{n=1,2} X_{i,n} + J \sum_i Z_{i,1} Z_{i,2}$$

$$S_i = \frac{1}{2} (\sigma_{i,1} + \sigma_{i,2})$$

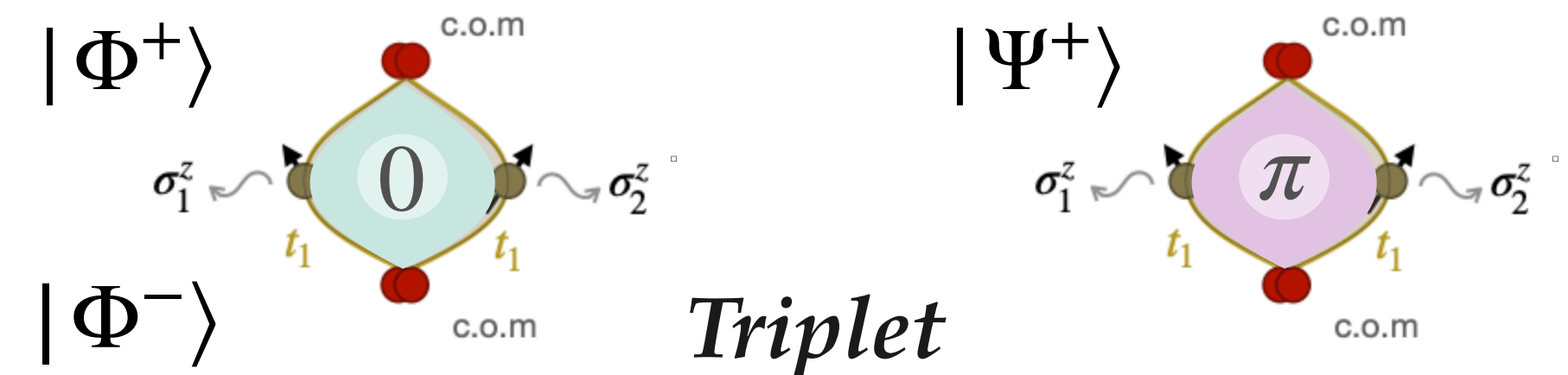
Effective $S=1$ gauge theory (not q -link model)

trivial $S=0$



Singlet

Fully disconnected sites

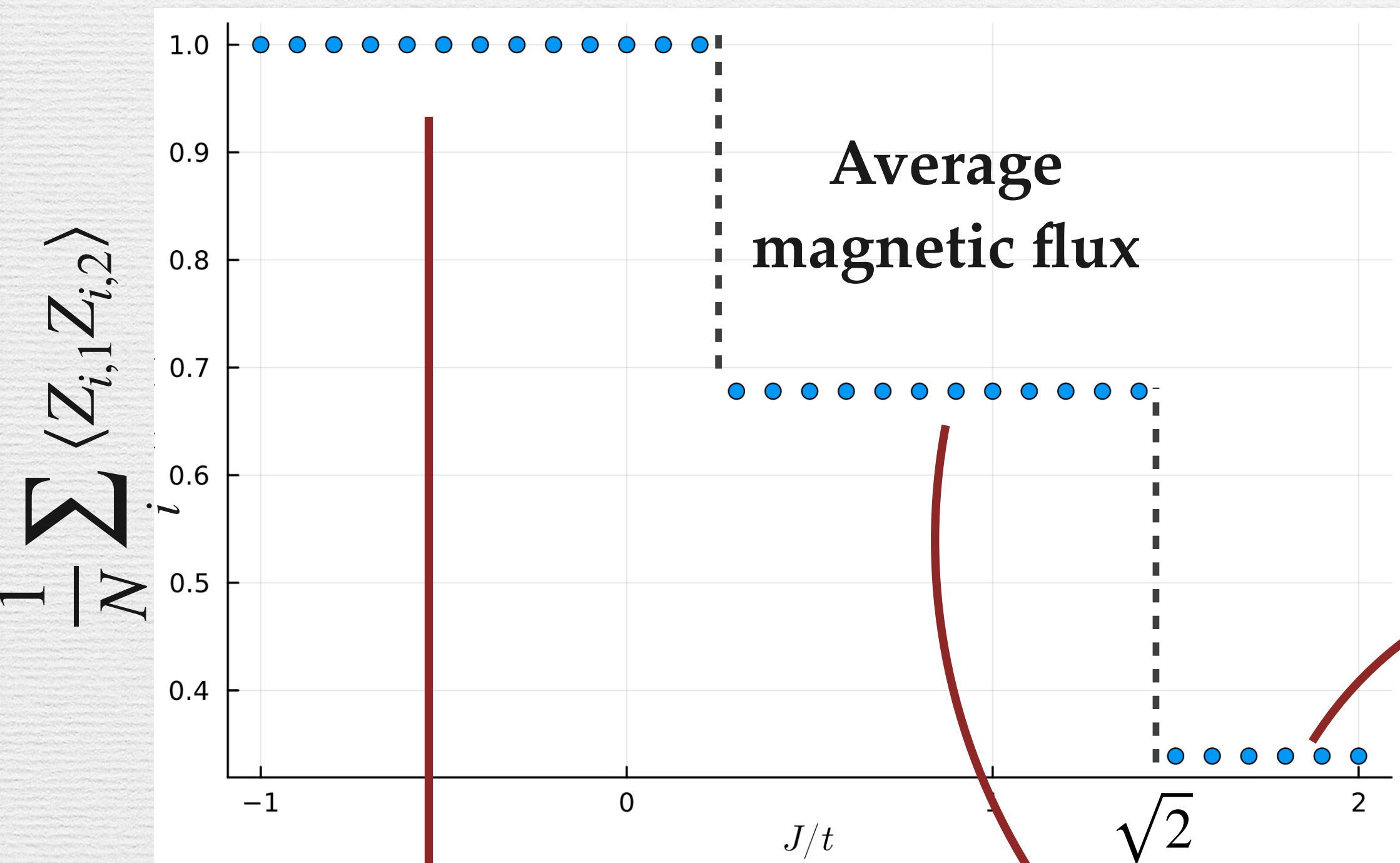


Triplet

Connected / disconnected sites depending J/t , J/h

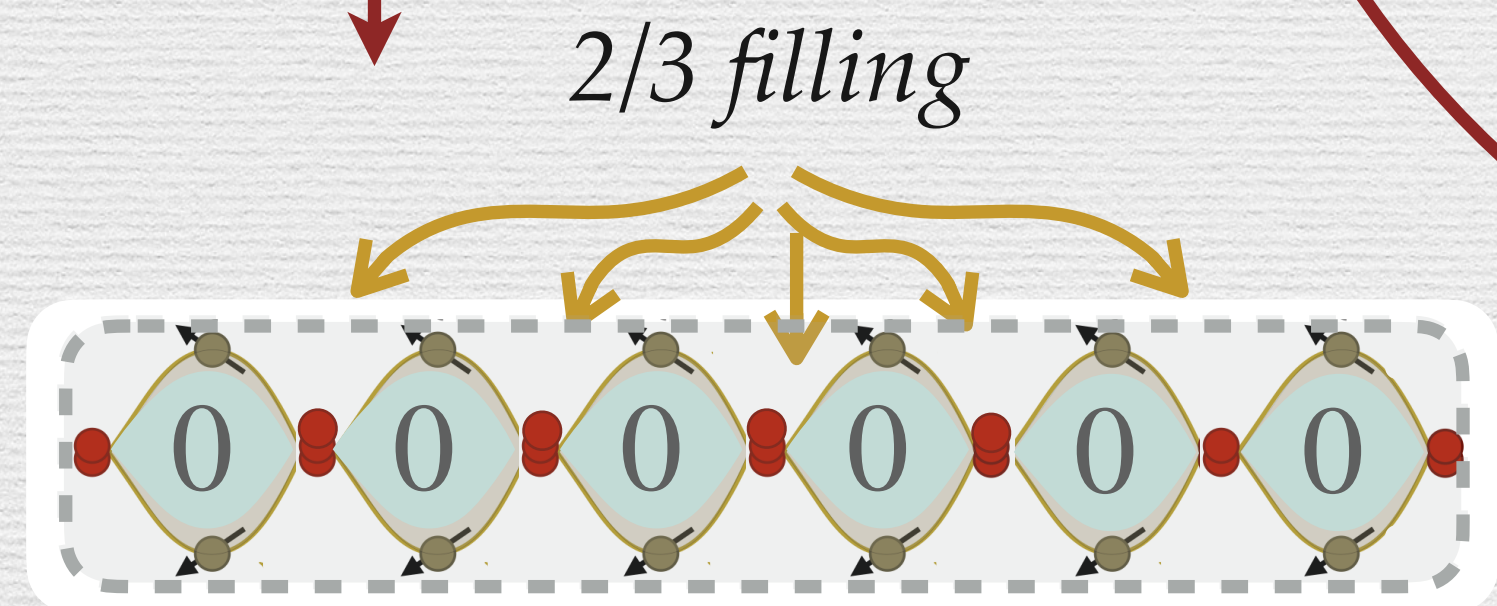
$$(2(S_{i-1}^x)^2 - 1)(-1)^{a_i^\dagger a_i} (2(S_i^x)^2 - 1) |\psi\rangle = |\psi\rangle$$

$\mathbb{Z}_2$ LGT in a chain and a loop chain

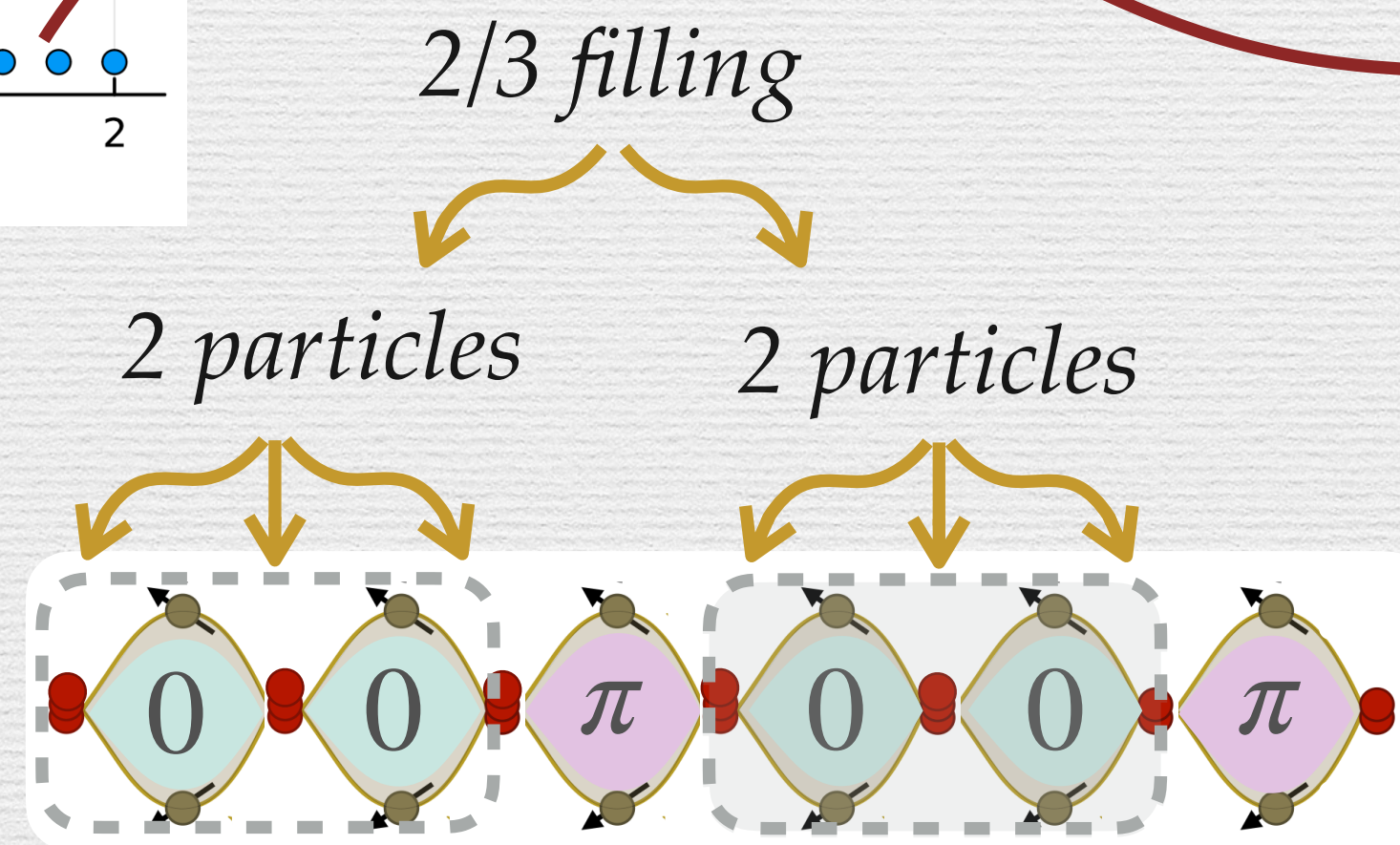


Let's focus first **classical competition** of **magnetic** terms in the presence of **AB interference** $h = 0$

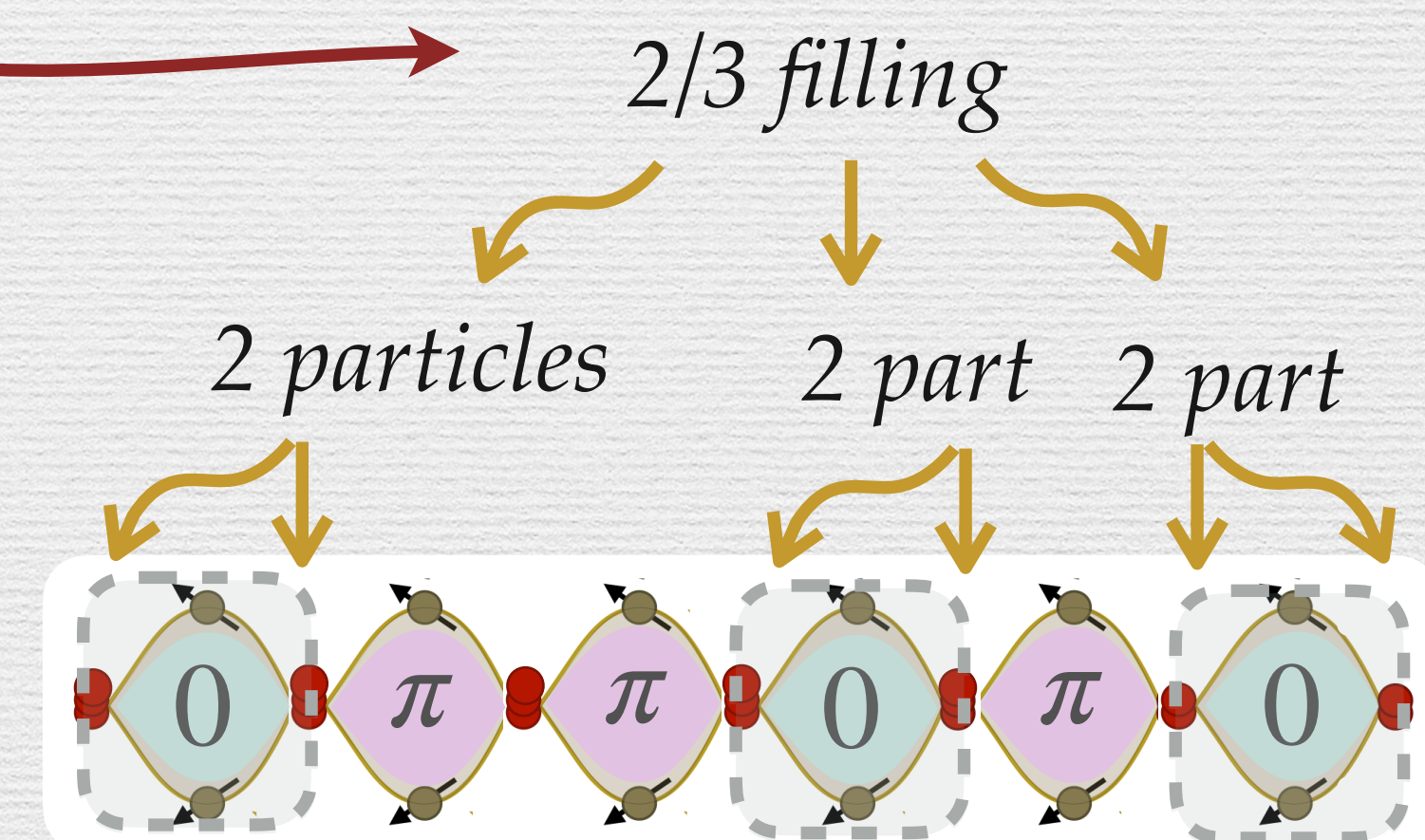
Break down of **translational invariance**
(recalls Peierls' instability)



Fully connected



Packed 3-site trimers



Diluted 2-site dimers

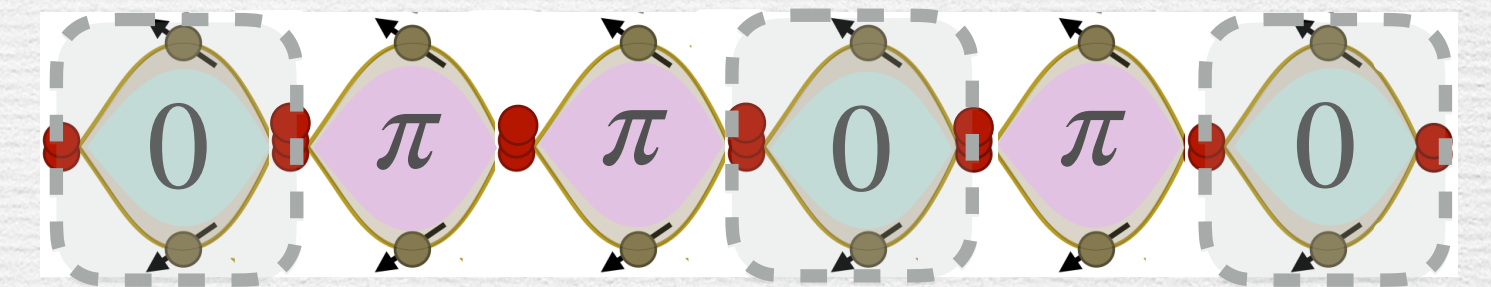
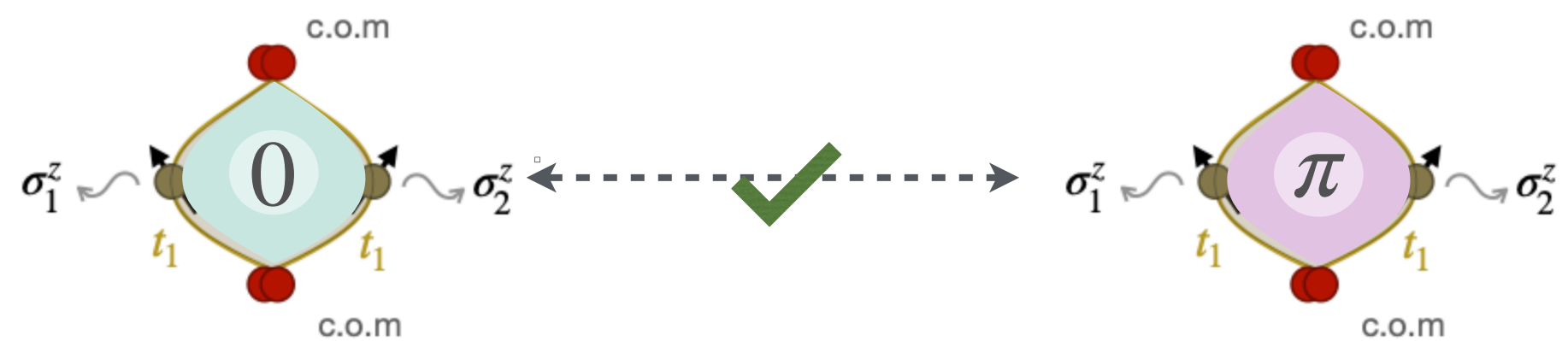
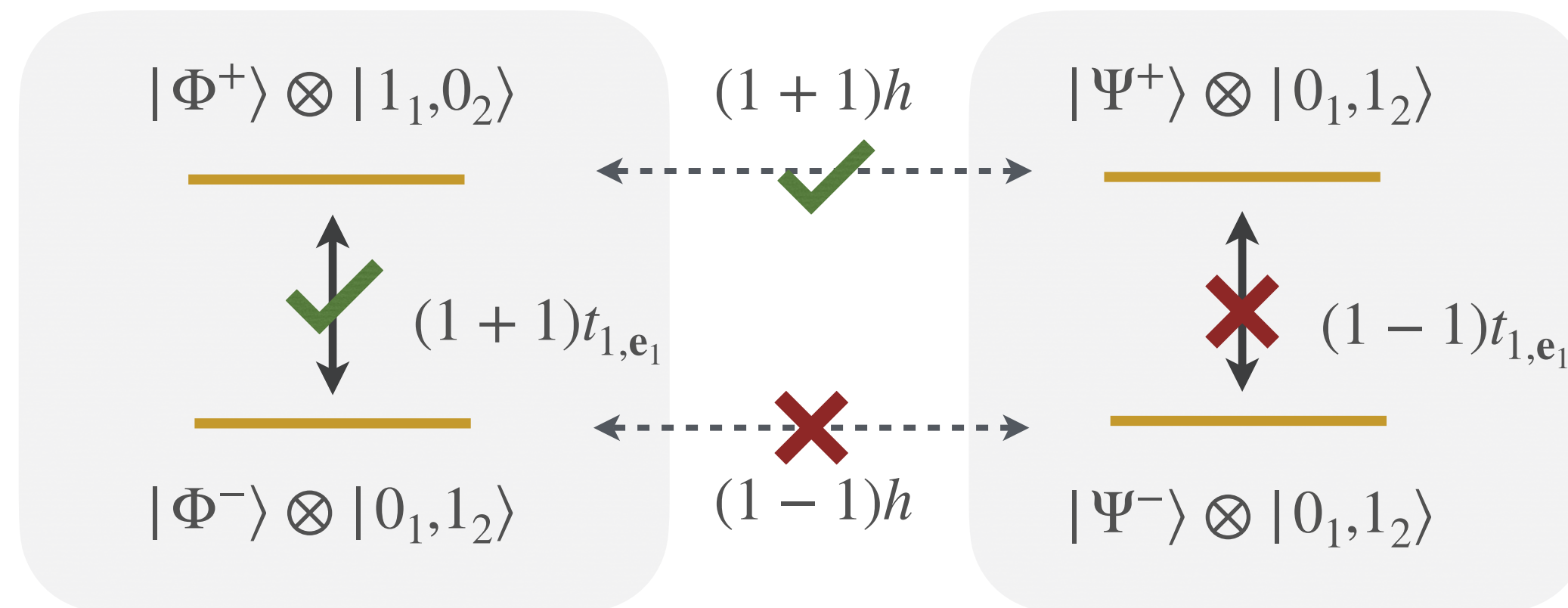
$\mathbb{Z}_2$ LGT in a chain and a loop chain

Let's now switch on **quantum fluctuations** (MPS simulations & effective mappings) $h \neq 0$

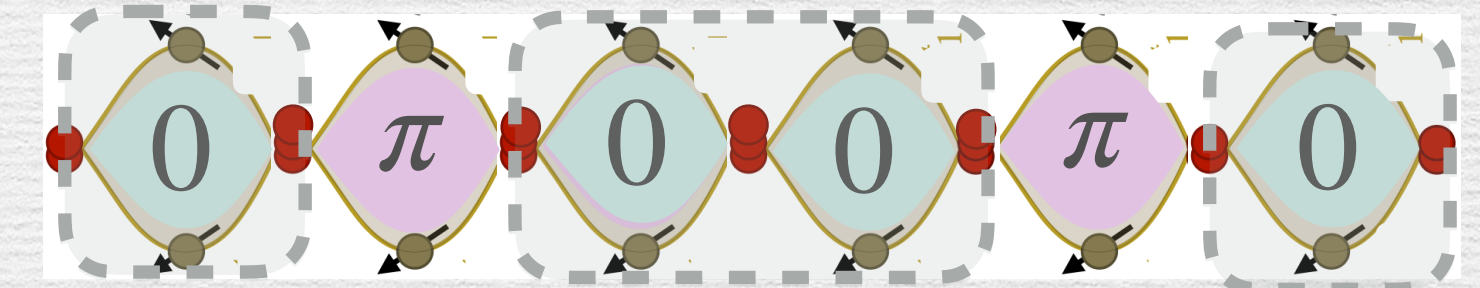
$$|\Phi^\pm\rangle = (|\uparrow, \uparrow\rangle \pm |\downarrow, \downarrow\rangle) / \sqrt{2} \quad |\Psi^\pm\rangle = (|\uparrow, \downarrow\rangle \pm |\downarrow, \uparrow\rangle) / \sqrt{2}$$

Const. AB interference

Dest. AB interference

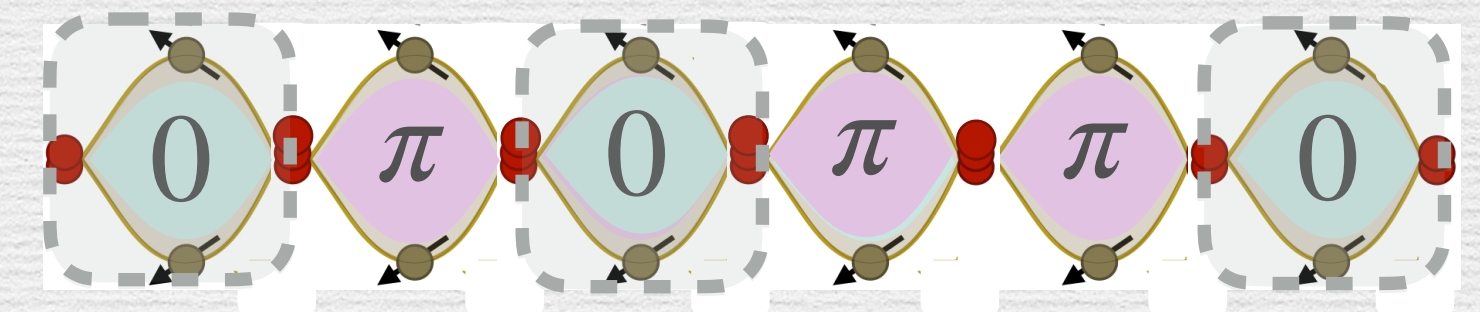


$h \ll J, t$



$h \ll J, t$

Dimers start moving



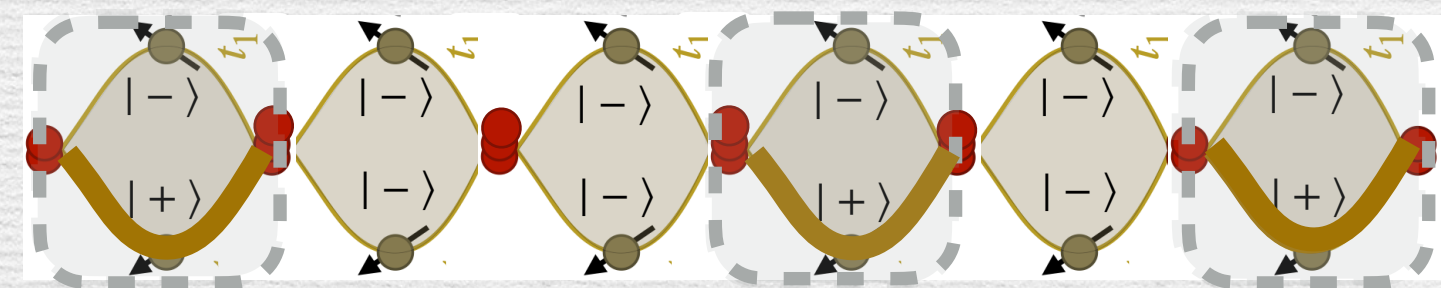
$$H_{\text{eff}} = t_{\text{eff}} \sum_i (c_i^\dagger c_{i+1} + \text{H.c.}) + V_{\text{eff}} \sum_i n_i n_{i+2}$$

$$c_i^\dagger = a_i^\dagger S_i^z a_{i+1}^\dagger \quad \text{'mesons' move with them}$$

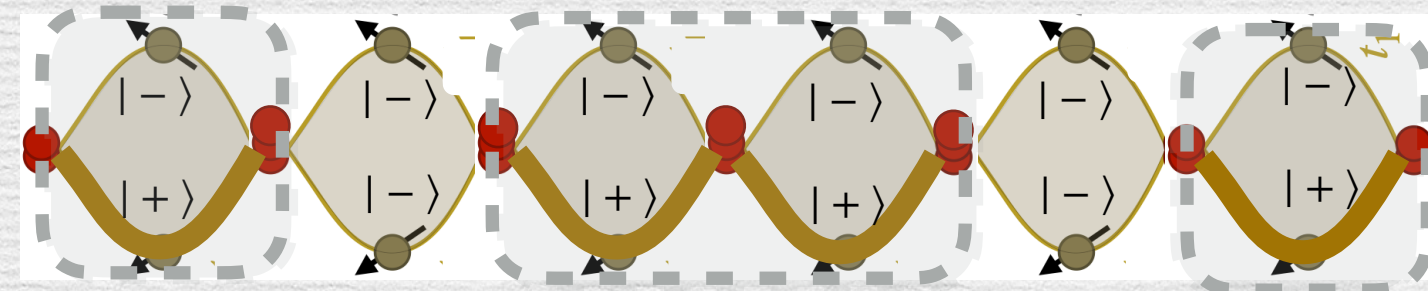
$\mathbb{Z}_2$ LGT in a chain and a loop chain

This **connects adiabatically** with the standard confined mesons in the limit $h \gg J, t$

Instead of moving by flipping fluxes (dimer-trimer), mesons move by stretching & compressing **electric field lines**

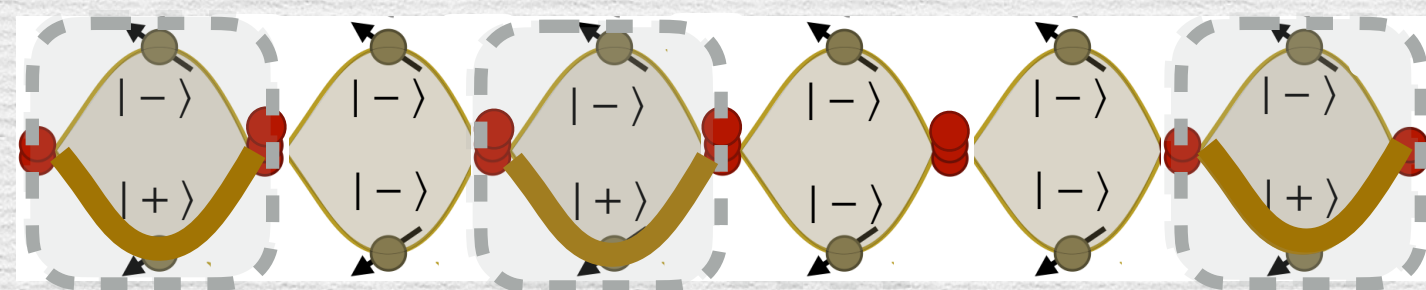


$t, J \ll h$



$t, J \ll h$

'Mesons' start moving



Similar to the standard $\mathbb{Z}_2$ chain

*U. Borla, et al, **PRL 124, 120503 (2020)***

$$c_i^\dagger = a_i^\dagger S_i^z a_{i+1}^\dagger$$

$$H_{\text{eff}} = t_{\text{eff}} \sum_i (c_i^\dagger c_{i+1} + \text{H.c.}) + V_{\text{eff}} \sum_i n_i n_{i+2}$$

In both cases

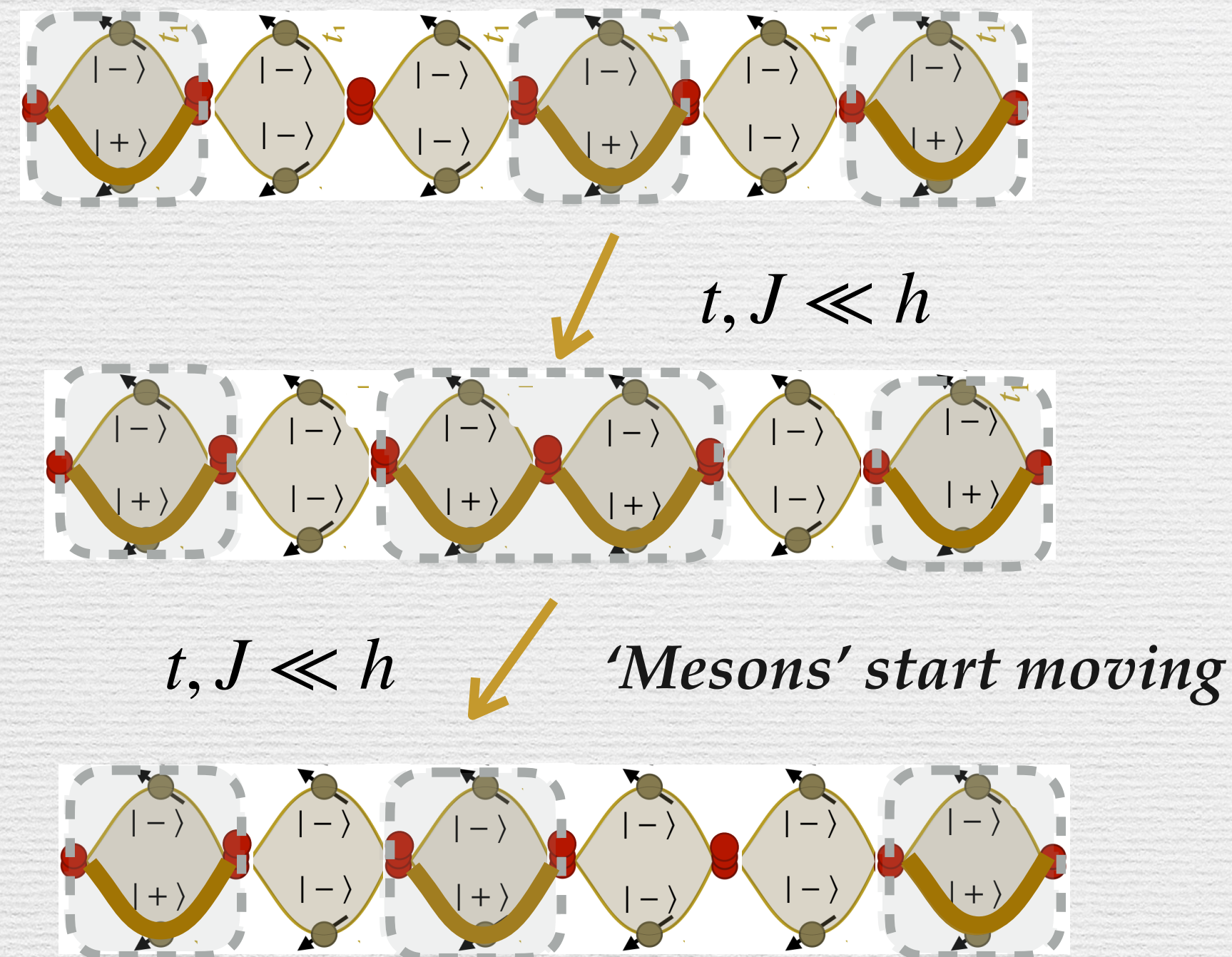
$$V_{\text{eff}} = 2t_{\text{eff}}$$

Luttinger liquid phase for the repulsive Bose-Hubbard model

$\left\{ \begin{array}{l} \mathbb{Z}_2 \text{ charges confined, (exp decay Green's function)} \\ \text{mesons show a power-law decay (free-ish quasiparticles)} \end{array} \right.$

$\mathbb{Z}_2$ LGT in a chain and a loop chain

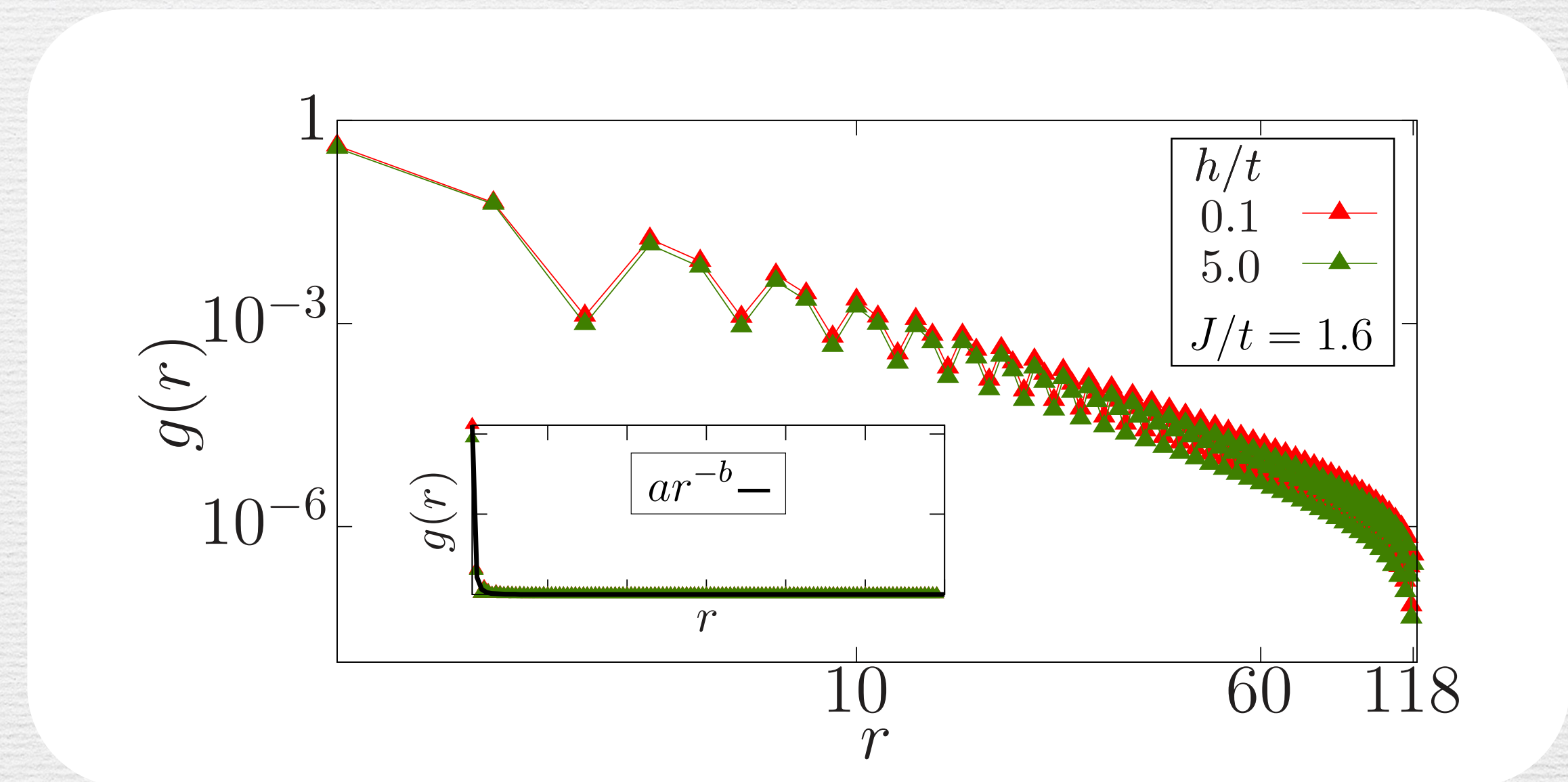
This **connects adiabatically** with the standard confined mesons in the limit $h \gg J, t$



Similar to the standard $\mathbb{Z}_2$ chain

U. Borla, et al, PRL 124, 120503 (2020)

$$c_i^\dagger = a_i^\dagger S_i^z a_{i+1}^\dagger \rightarrow g(r) = \langle c_i^\dagger c_{i+r} \rangle$$

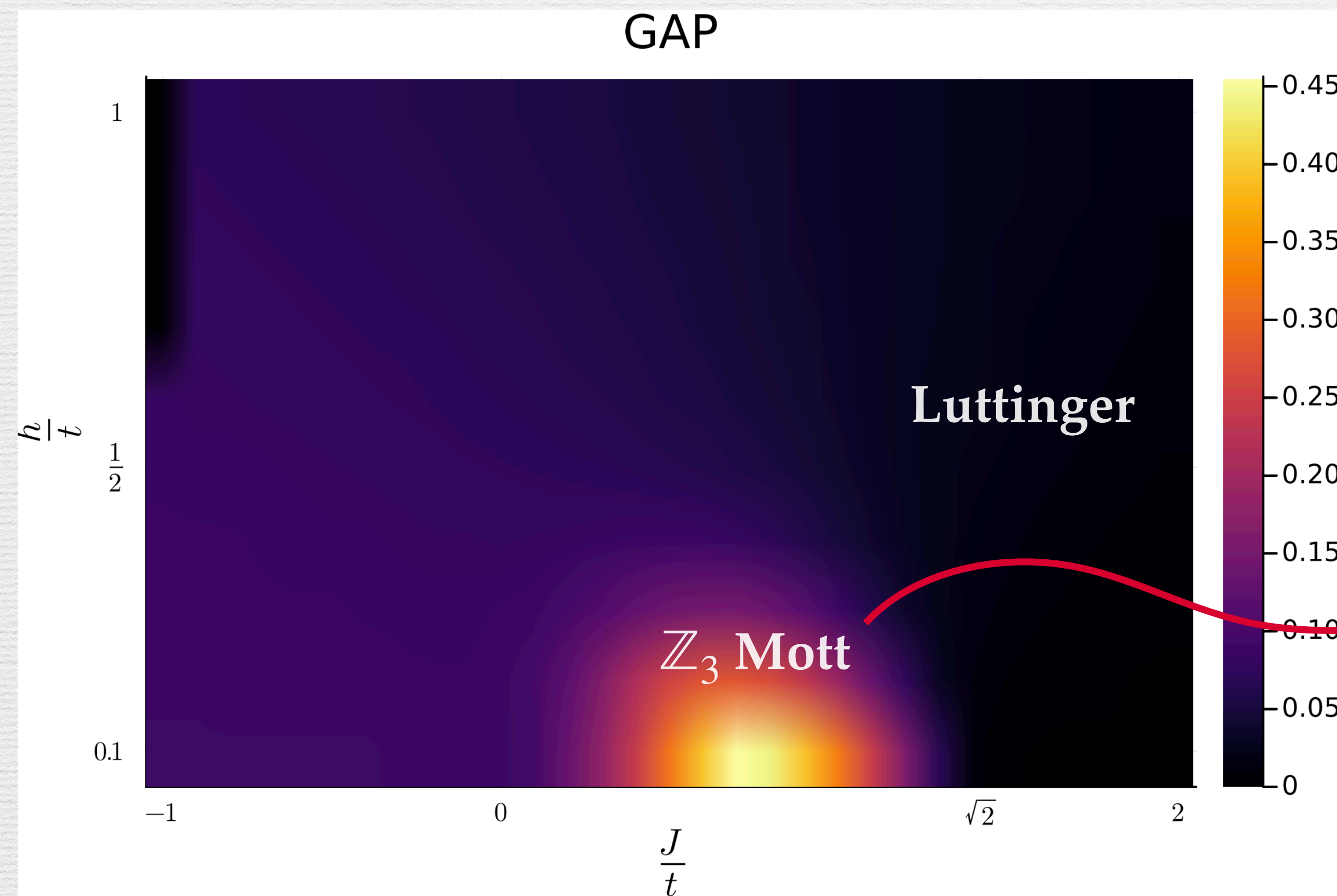


Luttinger liquid phase for the repulsive Bose-Hubbard model

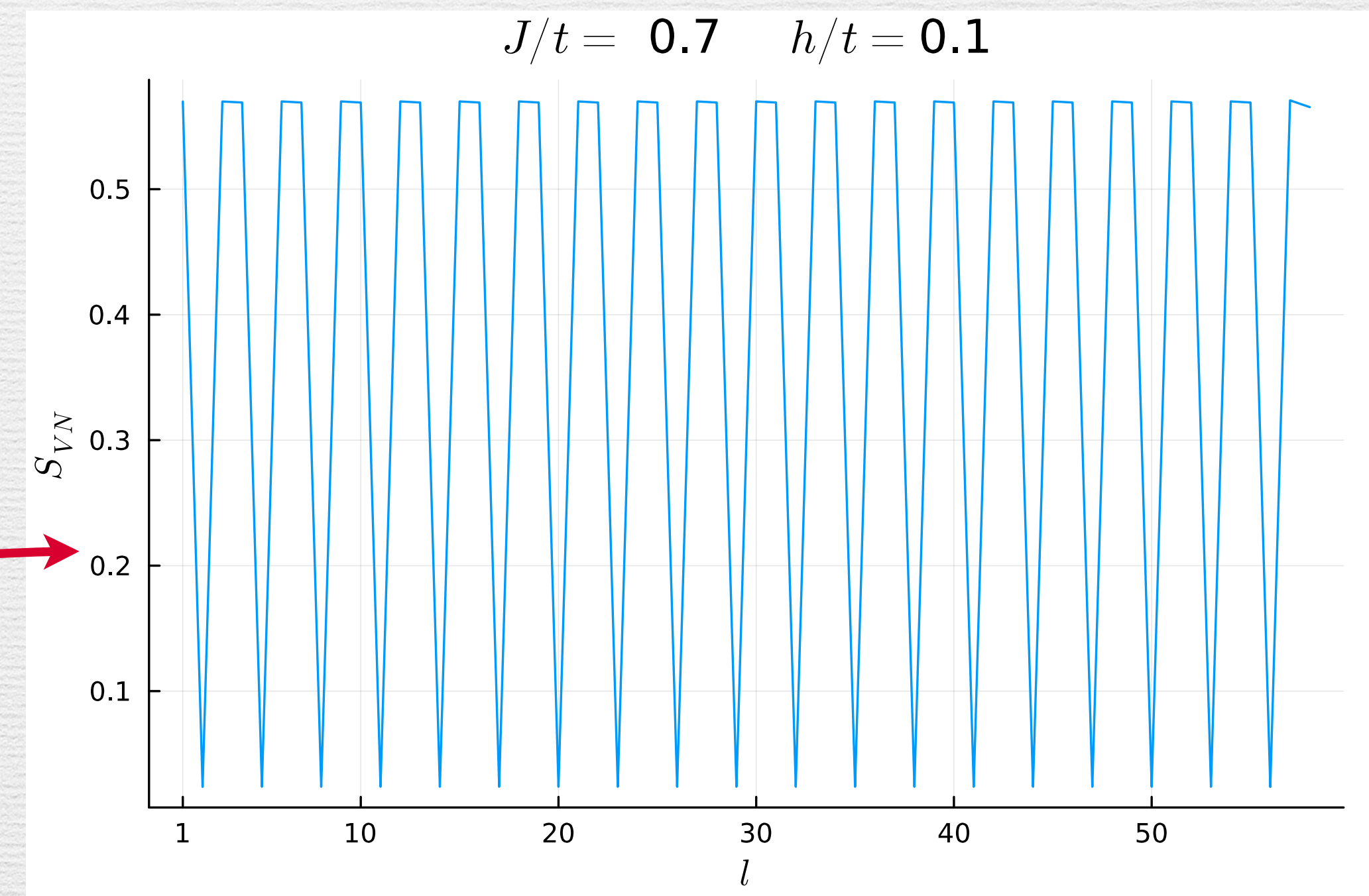
- $\mathbb{Z}_2$ charges confined, (exp decay Green's function)
- mesons show a **power-law decay** (free-ish quasiparticles)

$\mathbb{Z}_2$ LGT in a chain and a loop chain

A novelty is that lowering the magnetic plaquette term J , we find a $\mathbb{Z}_3$ Mott insulator

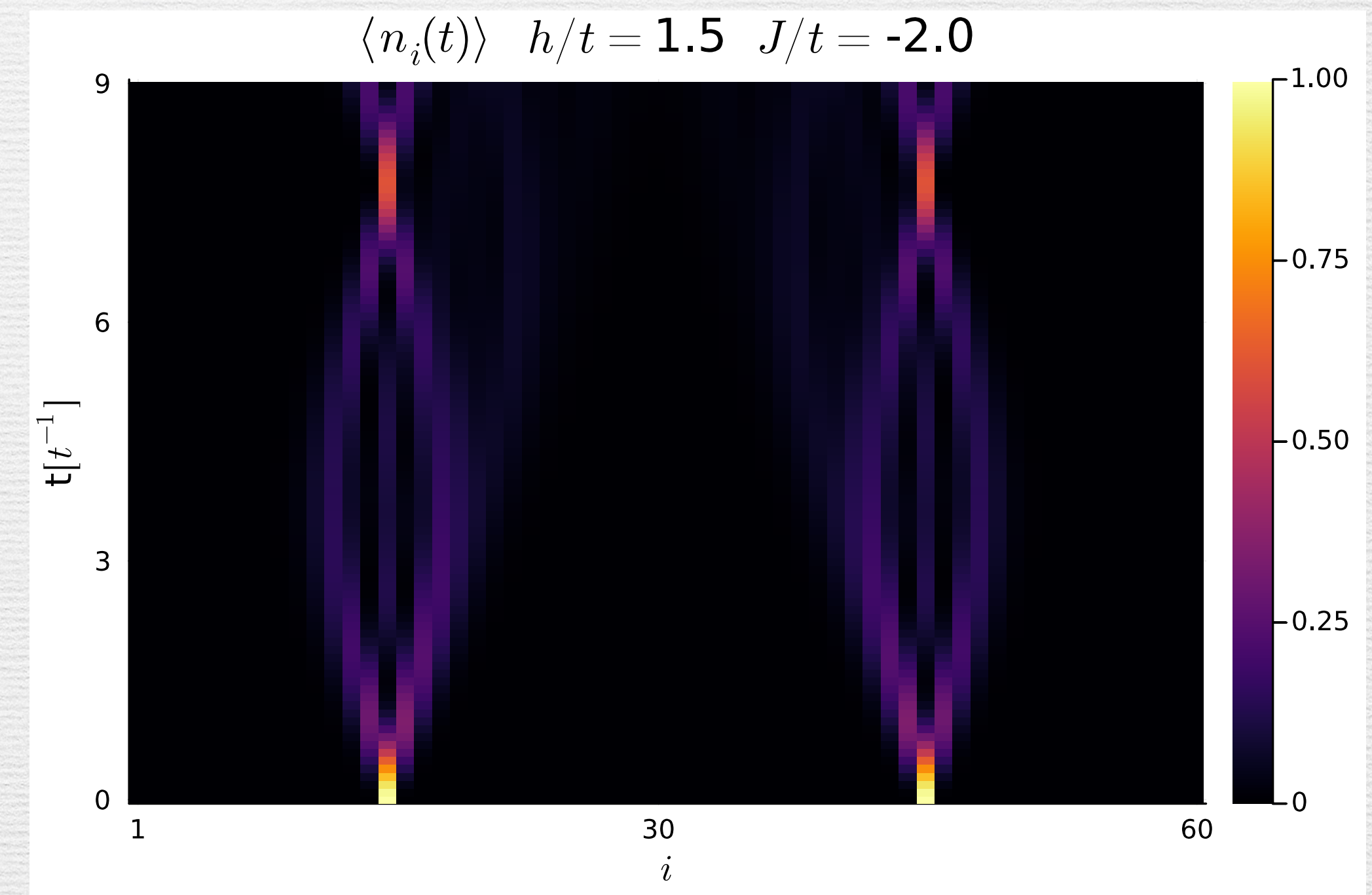
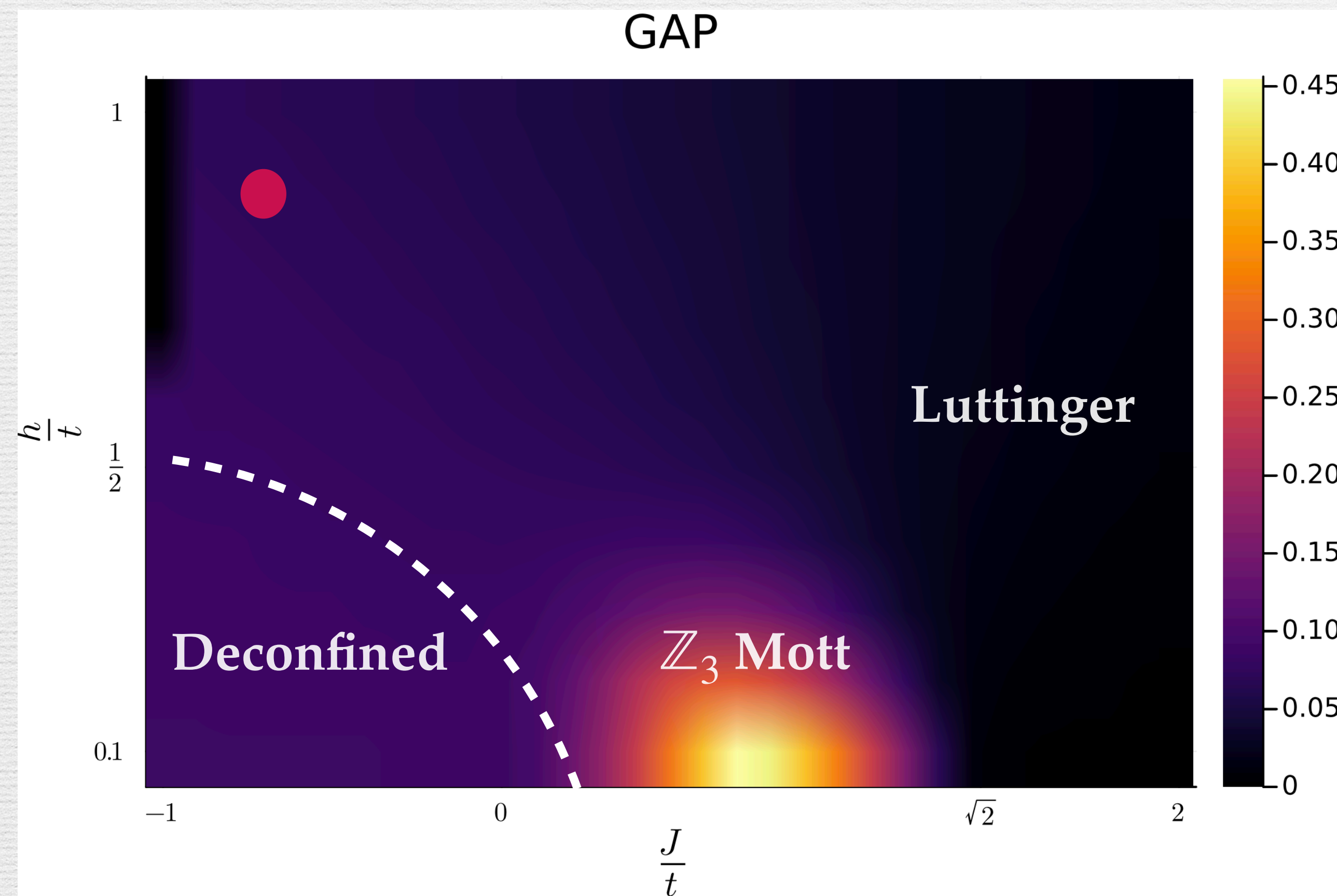


Trimerization of entanglement entropy



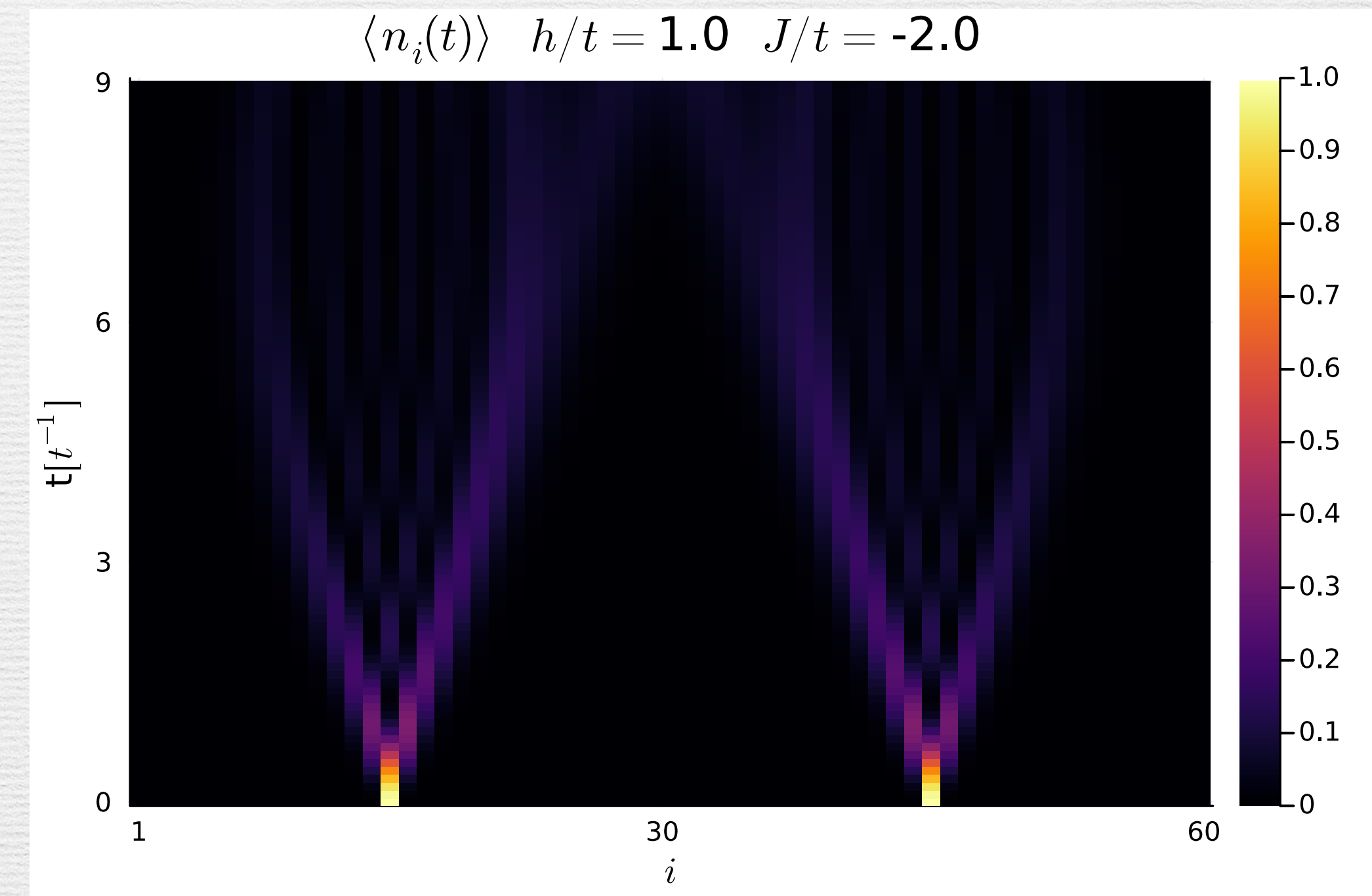
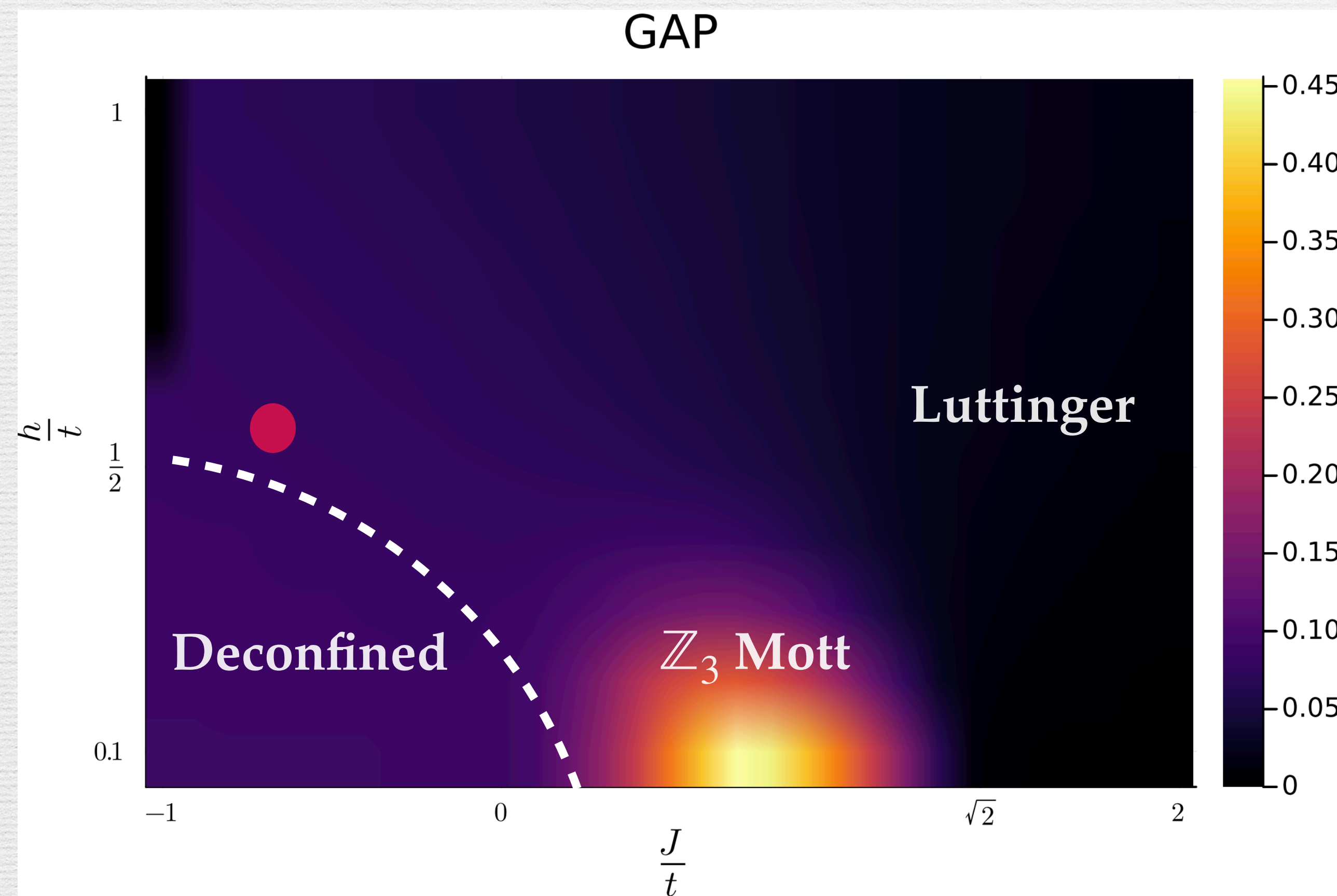
$\mathbb{Z}_2$ LGT in a chain and a loop chain

A novelty is that lowering the magnetic plaquette term J , we find a **deconfined phase**



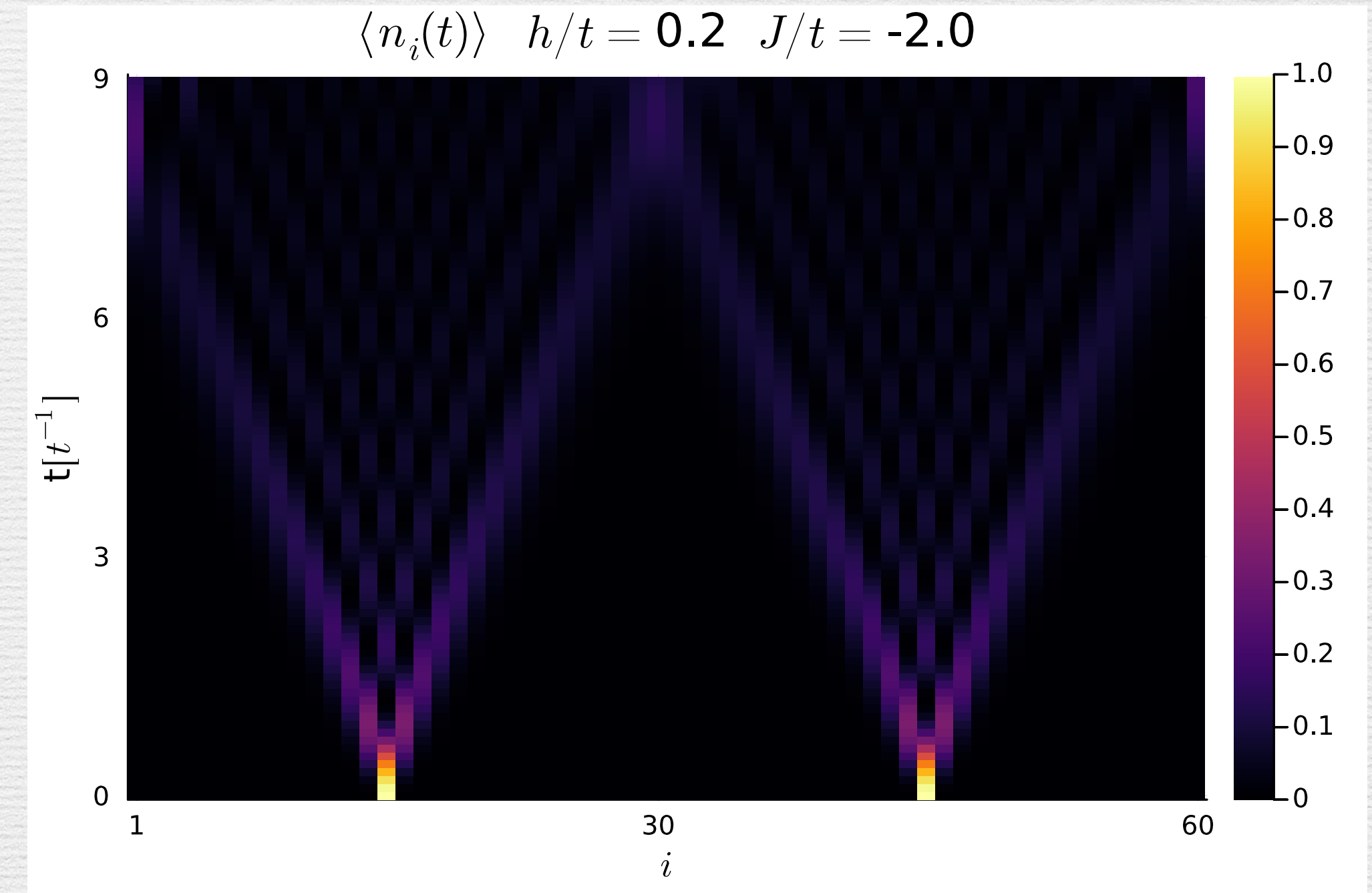
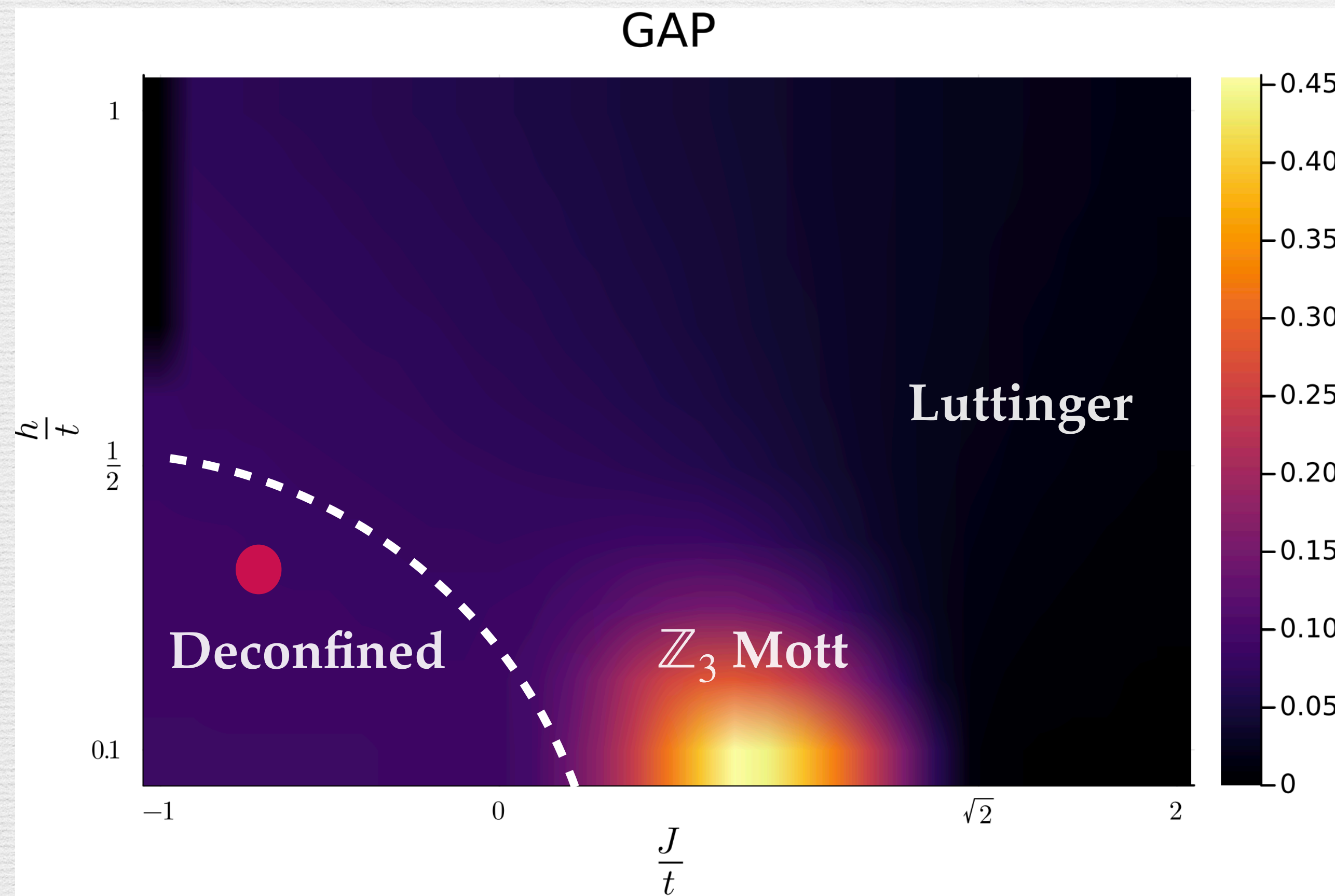
$\mathbb{Z}_2$ LGT in a chain and a loop chain

A novelty is that lowering the magnetic plaquette term J , we find a **deconfined phase**



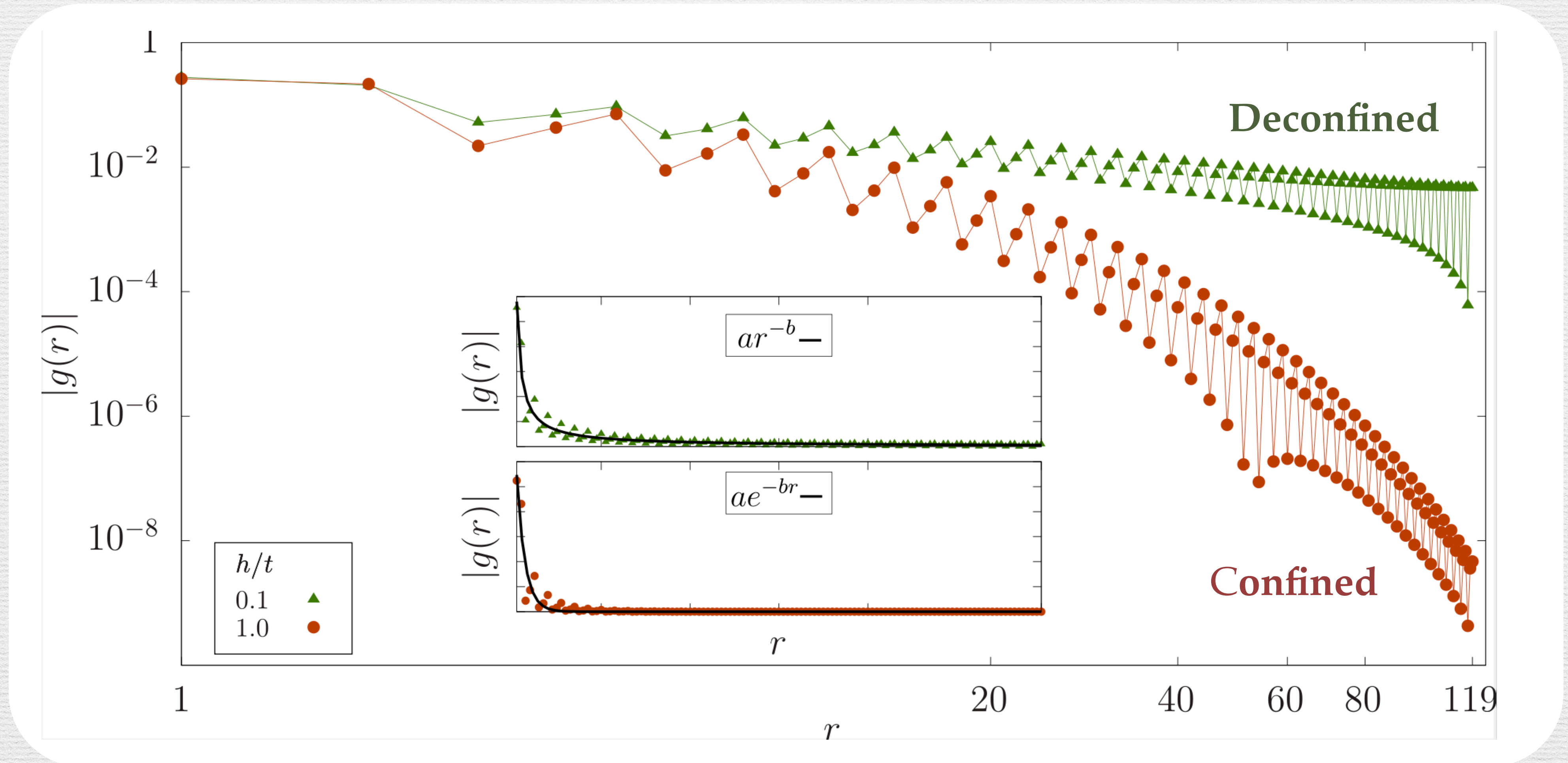
$\mathbb{Z}_2$ LGT in a chain and a loop chain

A novelty is that lowering the magnetic plaquette term J , we find a **deconfined phase**



$\mathbb{Z}_2$ LGT in a chain and a loop chain

A novelty is that **lowering the plaquette term J**, we find a **deconfined phase** $g(r) = \langle a_i^\dagger \prod_{\ell} S_{i+\ell}^z a_{i+r} \rangle$

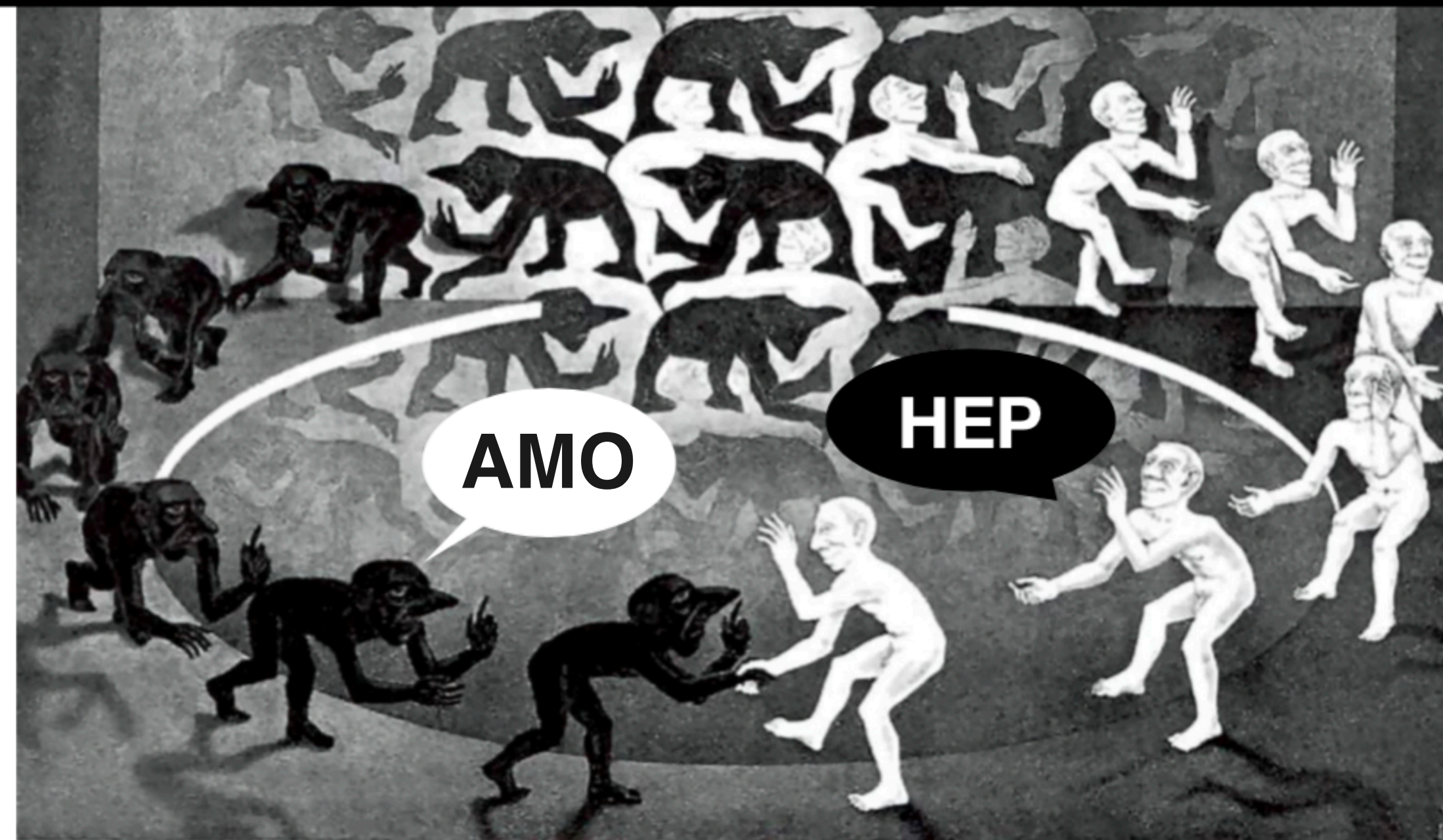


Conclusion and outlook

Can we exploit trapped ions for lattice gauge theories?

My take on

*at the very least, we'll provide an **interdisciplinary** look at the problem
and **enjoy the ride***




The ride