

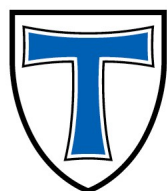
The moat regime in QCD

QuantFunc 2024

Shi Yin

Justus Liebig University Giessen

JUSTUS-LIEBIG-



UNIVERSITÄT
GIESSEN

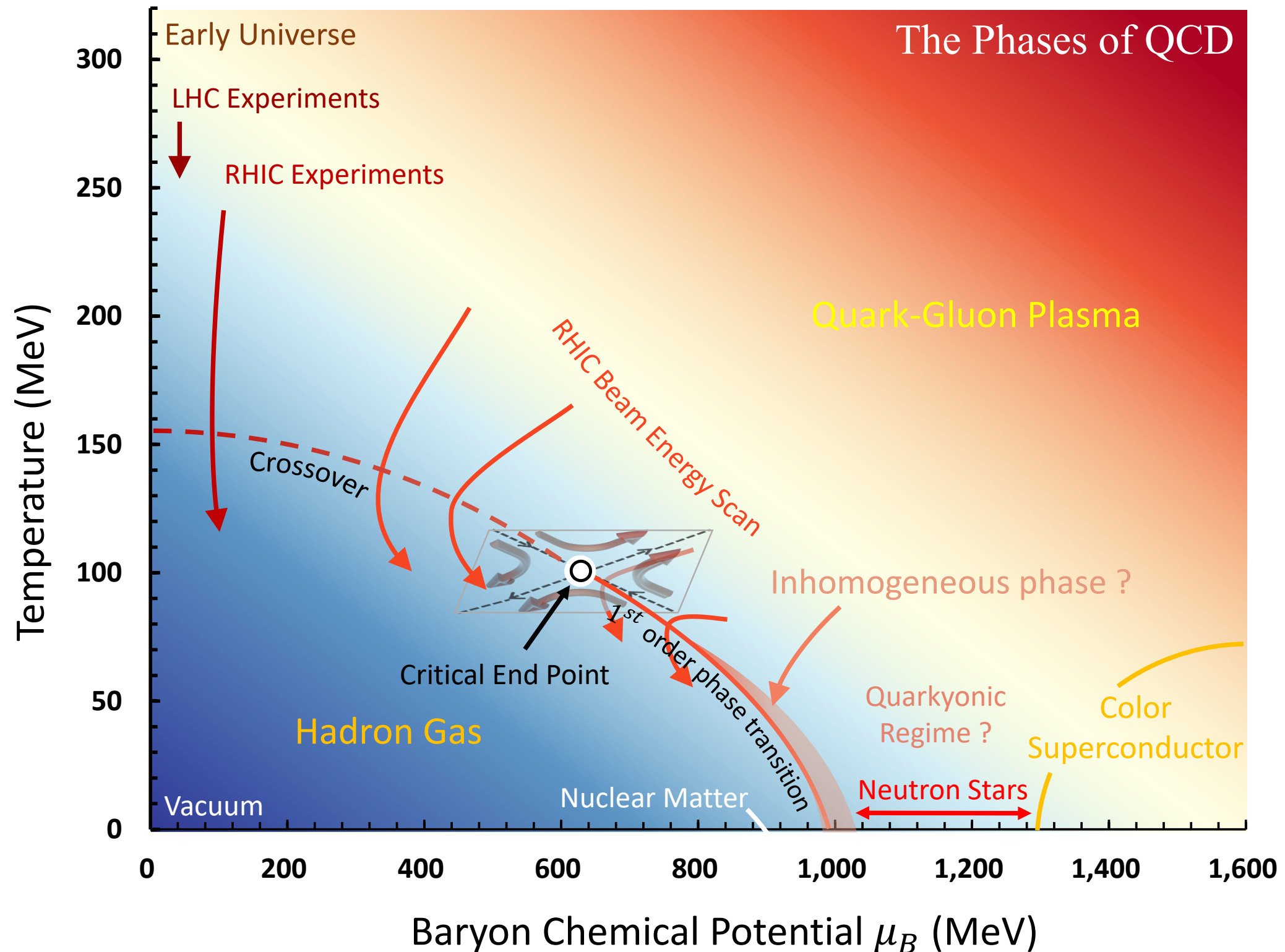


Alexander von
HUMBOLDT
STIFTUNG

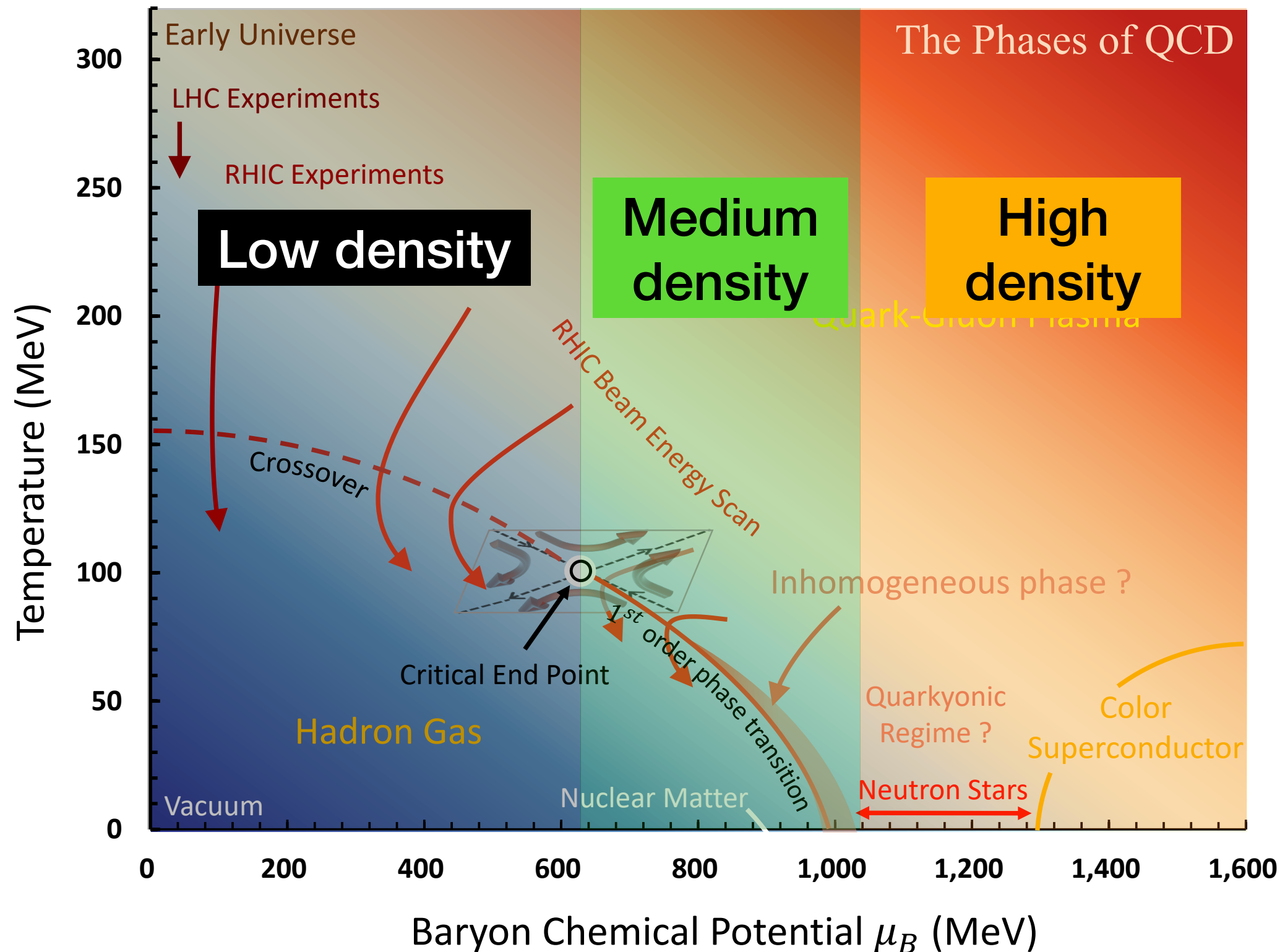
Outline

- QCD phase structure
- Moat regime in QCD
- Summary and outlook

QCD phase structure



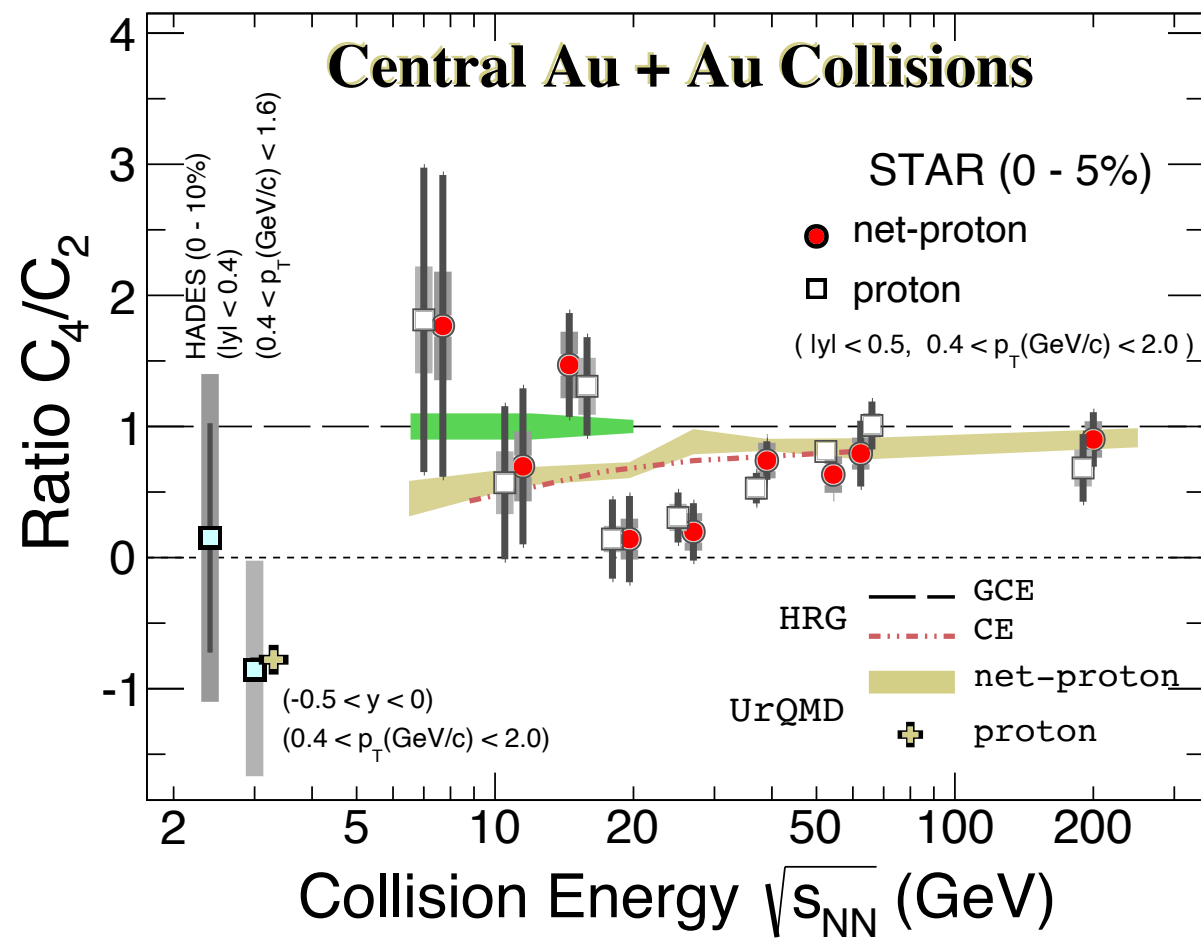
QCD phase structure



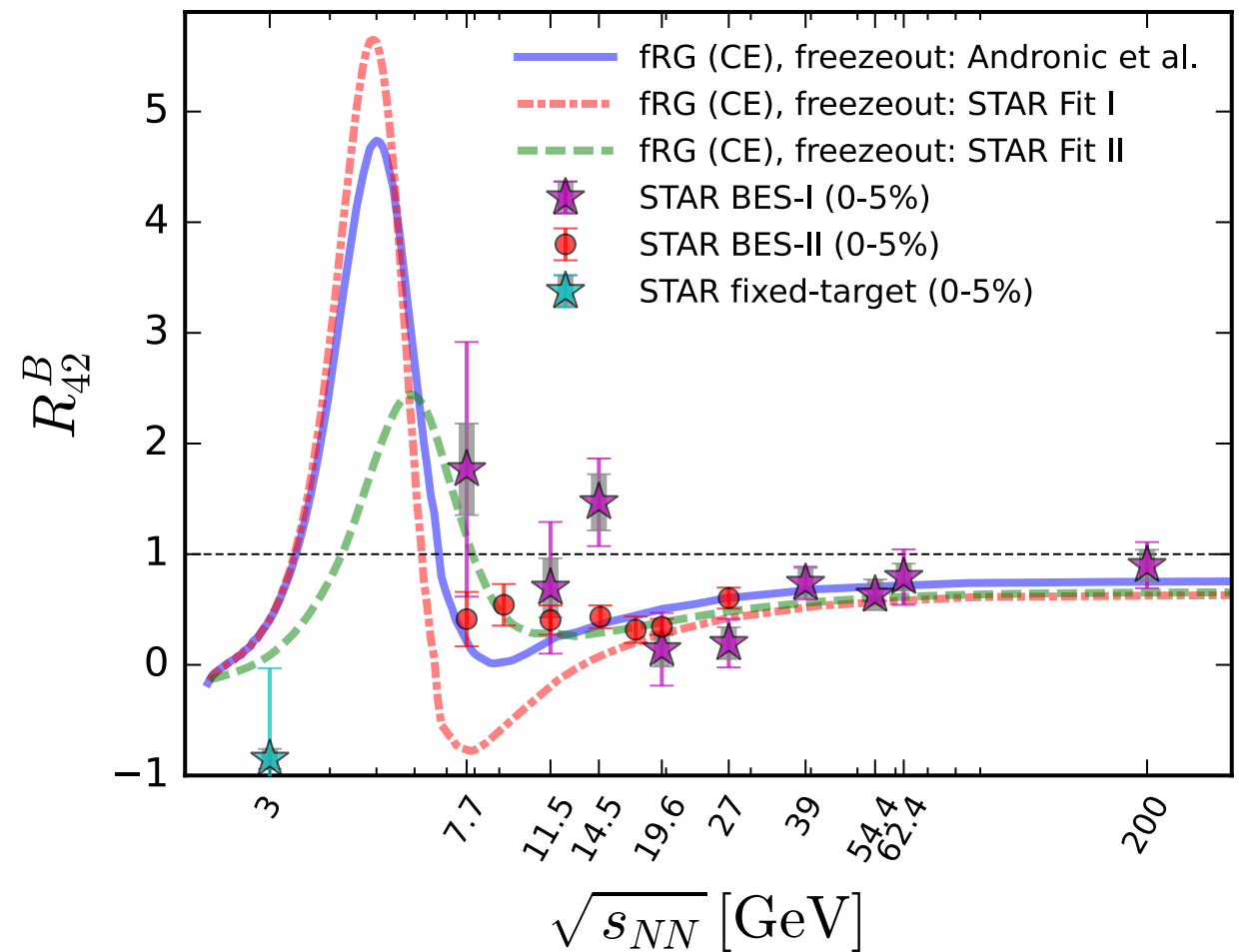
Looking for CEP

From low density to medium density

Proton number fluctuations

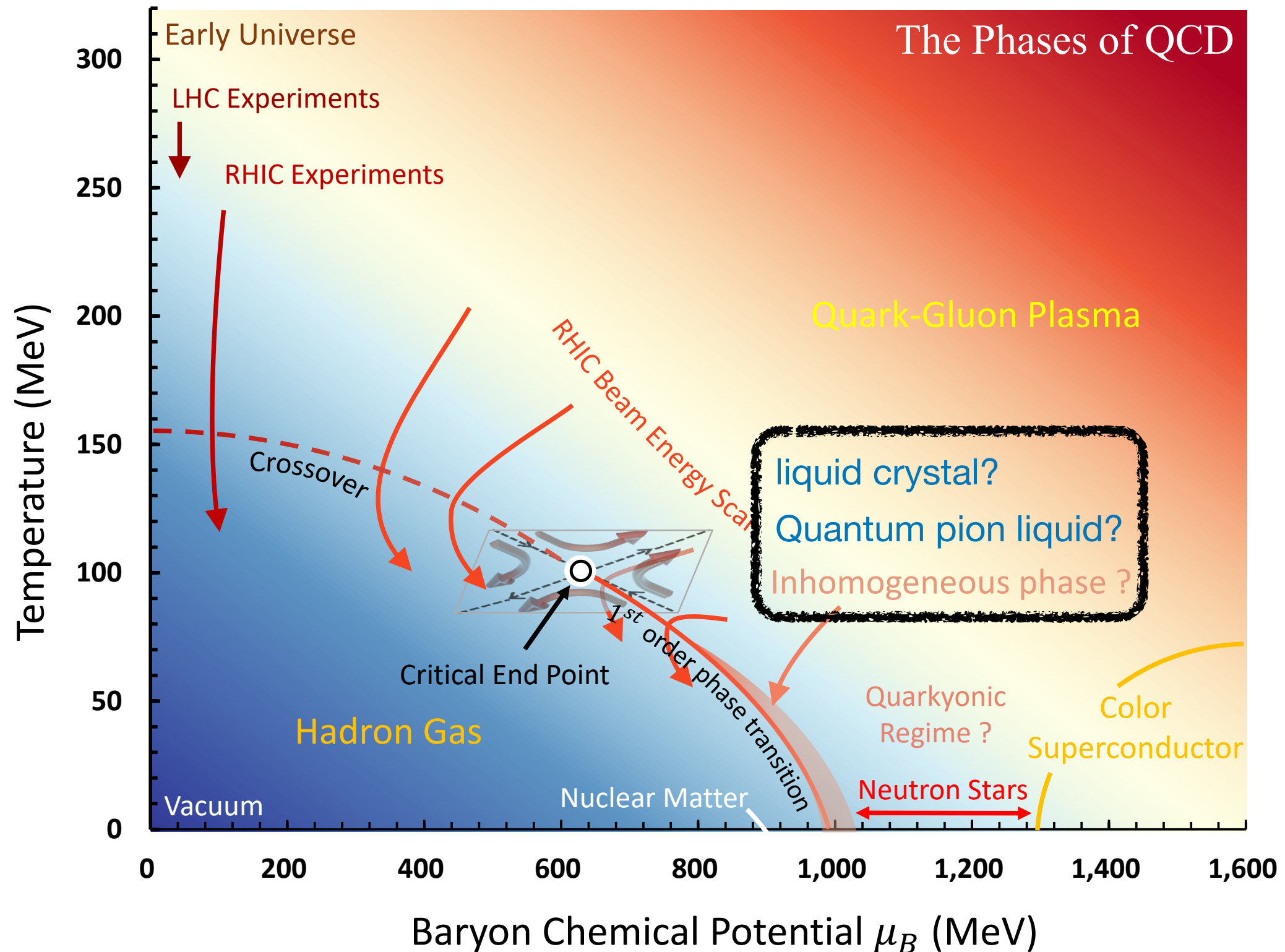


M. S. Abdallah et. al., *Phys.Rev.Lett.* 128 (2022) 20, 202303

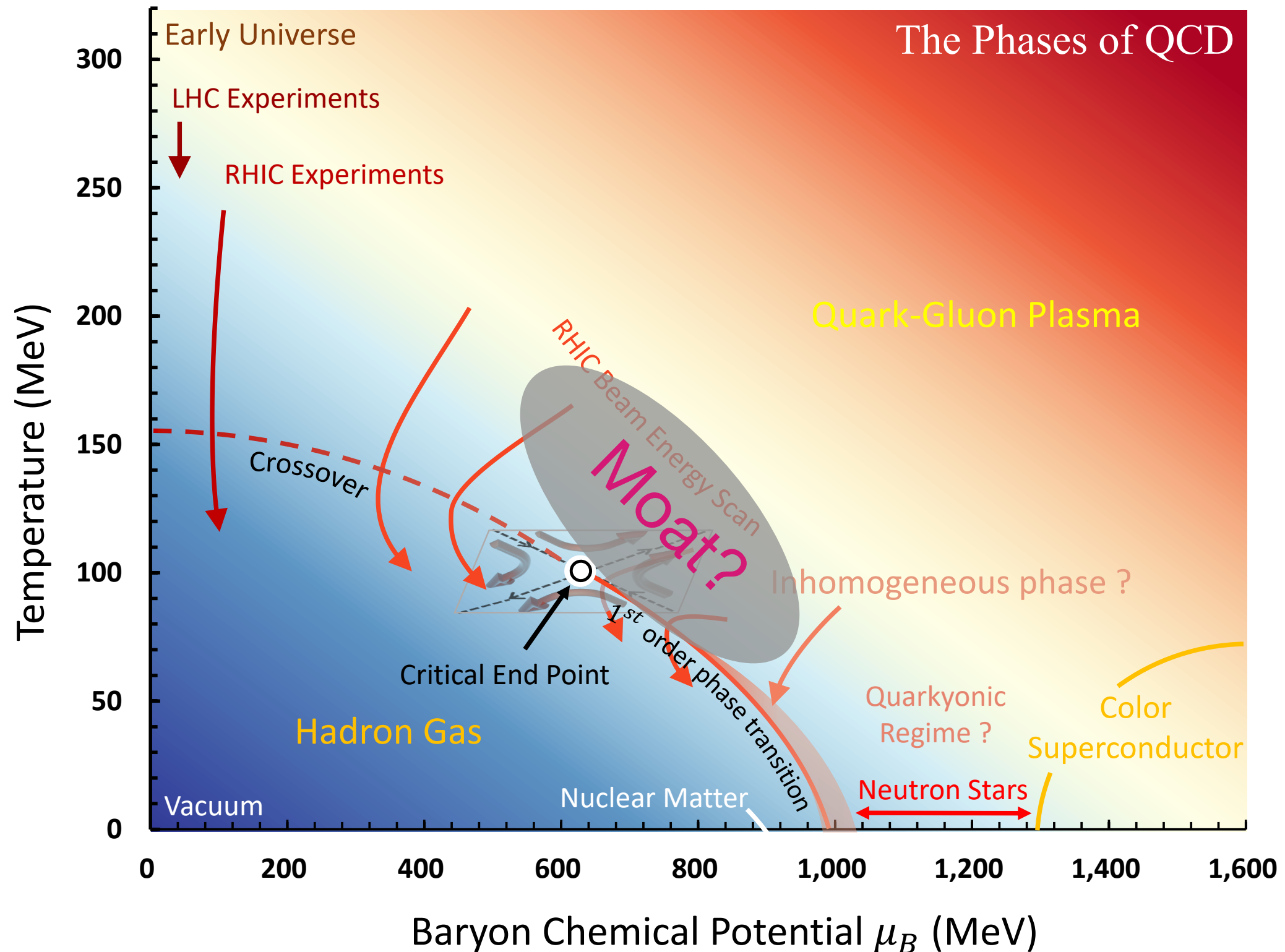


W.-j. Fu, XF Luo, J. M. Pawlowski, F. Rennecke and SY, arXiv:2308.15508

QCD phase structure



Moat regime



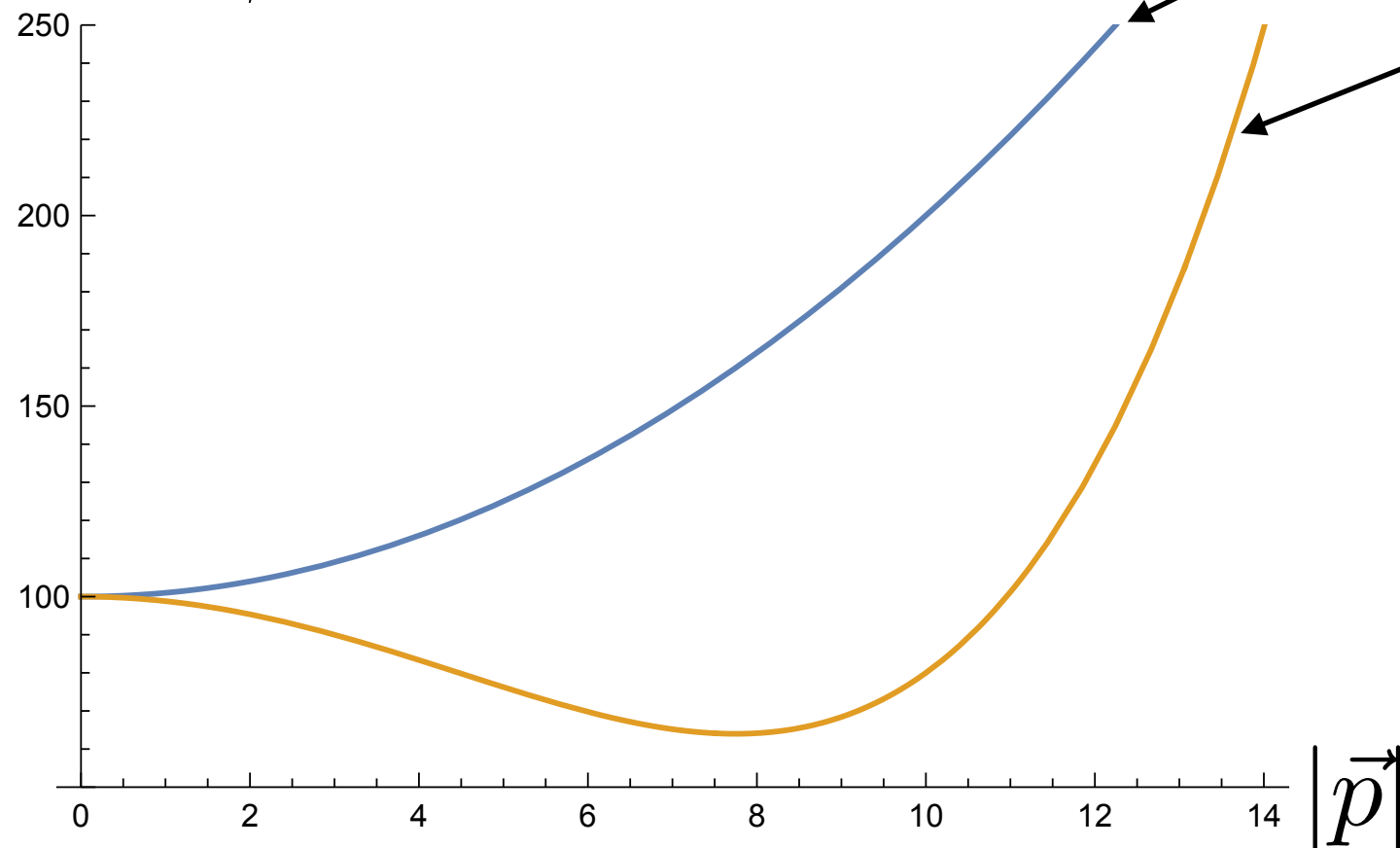
Moat behavior

Dispersion relation

$$E_\phi(\vec{p}^2, m_\phi^2) = \sqrt{\vec{p}^2 + m_\phi^2 + \Pi_{1\text{-loop}}(\vec{p}^2, m_\phi^2)}$$
$$= \sqrt{Z_\phi(\vec{p}^2) \vec{p}^2 + m_\phi^2}$$



$$E_\phi(\vec{p}^2, m_\phi^2)$$



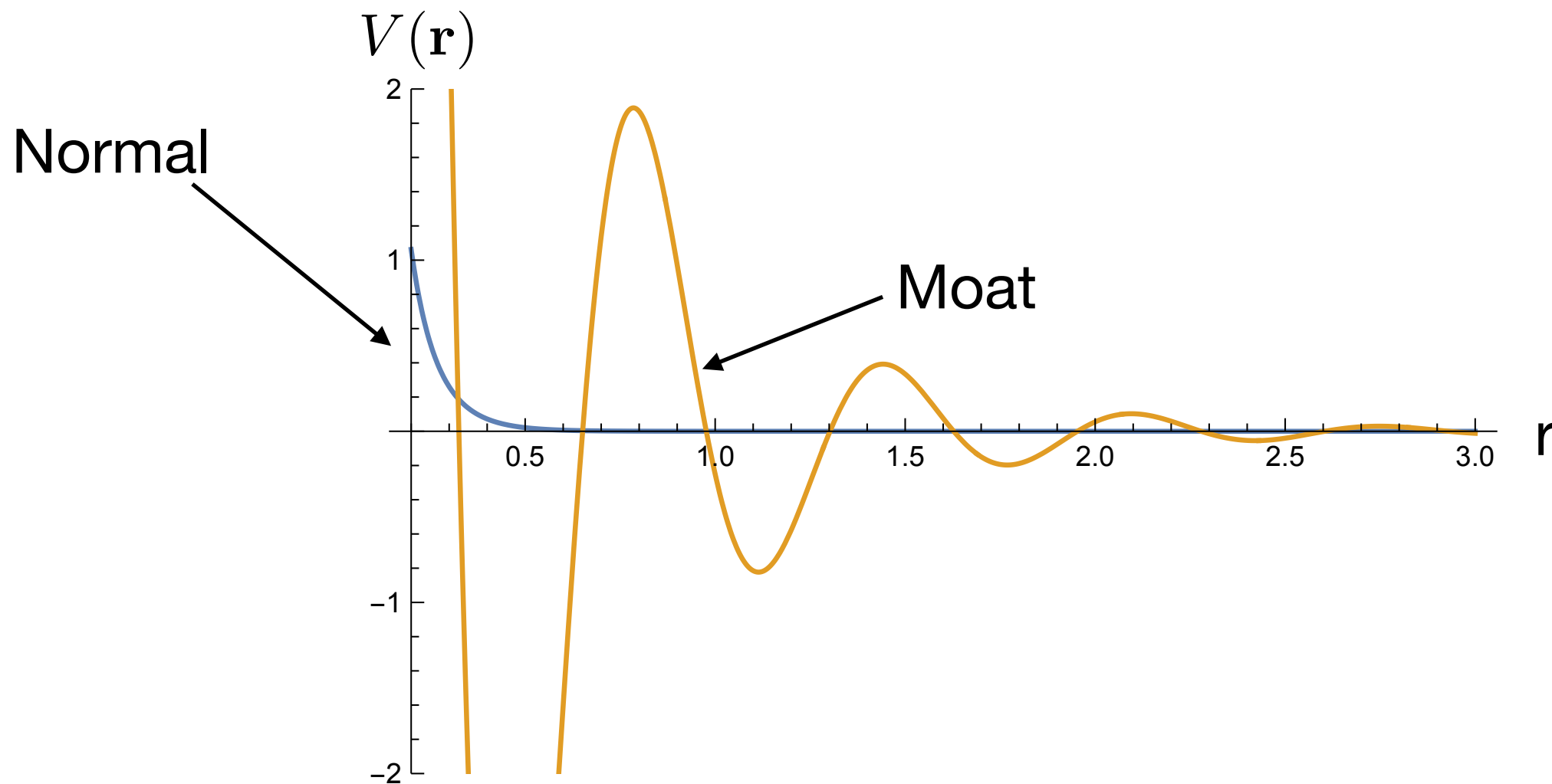
Finite temperature
and density !!!

Friedel Oscillations

Analogy to condensed matter:

$$V(\mathbf{r}) = Q_1 Q_2 \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{r}} \frac{1}{\mathbf{k}^2 + m^2 + \Pi(0, \mathbf{k})}$$

J. Kapusta and T. Toimela, *Phys.Rev.D* 37 (1988) 3731



Fabian Rennecke, SY, in preparation

Observables in moat

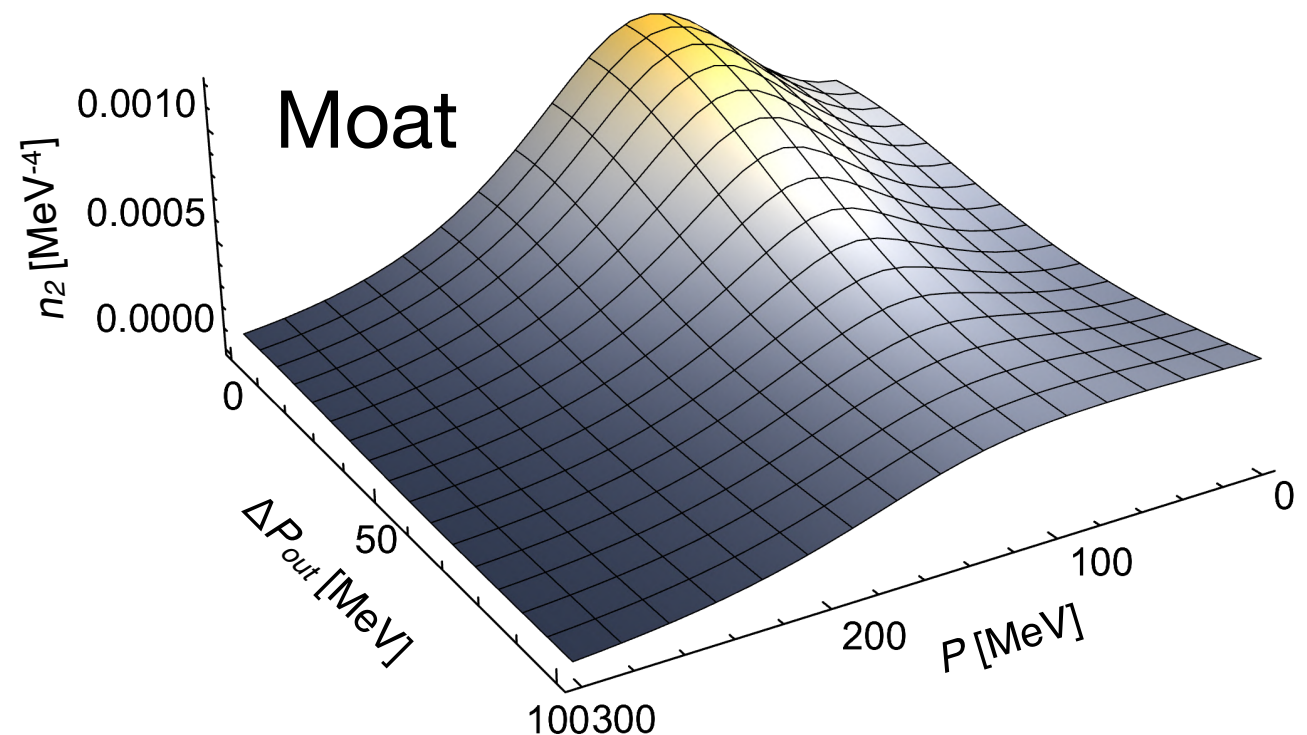
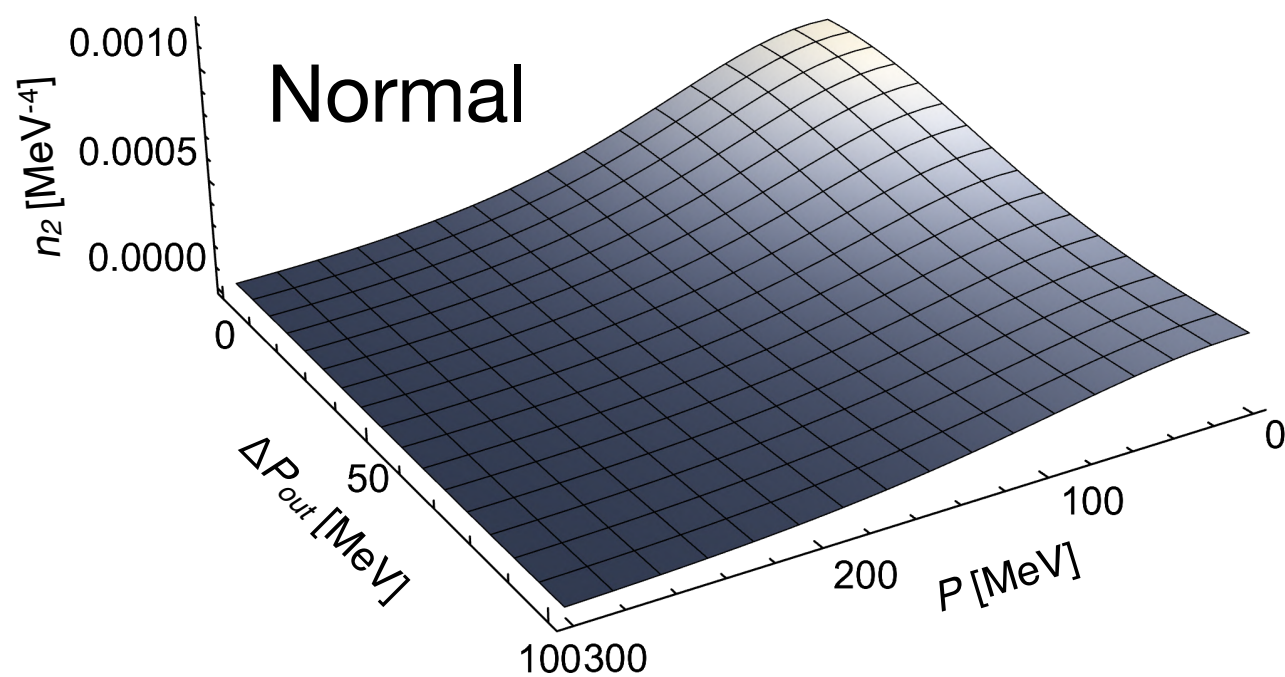
First look on what
might be happening
in the moat:

R. D. Pisarski, F. Rennecke, *PRL* 127 (2021), 152302

R. D. Pisarski, F. Rennecke, D. H. Rischke, *PRD* 107 (2023) 11, 116011

$$C(\mathbf{P}, \Delta\mathbf{P}) = \int_X e^{-i\Delta\mathbf{P} \cdot \mathbf{X}} f(X, \mathbf{P}) \rho(X, \mathbf{P})$$

Spectral function at
finite temperature
and density !!!



Outline

- QCD phase structure
- **Moat regime in QCD**
- Summary and outlook

QCD within functional renormalization group

Flow equations of QCD

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \left(\text{orange loop} - \text{dotted loop} - \text{solid loop} + \frac{1}{2} \text{blue loop} \right)$$

$$\partial_t \text{quark self-energy} = \tilde{\partial}_t \left(\frac{1}{2} \text{orange loop} + \frac{1}{2} \text{orange loop} - \text{dotted loop} - \text{solid loop} \right)$$

$$\partial_t \text{gluon self-energy} = \tilde{\partial}_t \left(\text{orange loop} + \text{blue loop} \right)$$

$$\partial_t \text{ghost self-energy} = \tilde{\partial}_t \left(-\text{solid loop} + \frac{1}{2} \text{blue loop} + \frac{1}{2} \text{blue loop} \right)$$

$$\partial_t \text{quark-gluon vertex} = \tilde{\partial}_t \left(-\text{triangle} - \text{triangle} + \text{triangle} + \frac{1}{2} \text{triangle} \right)$$

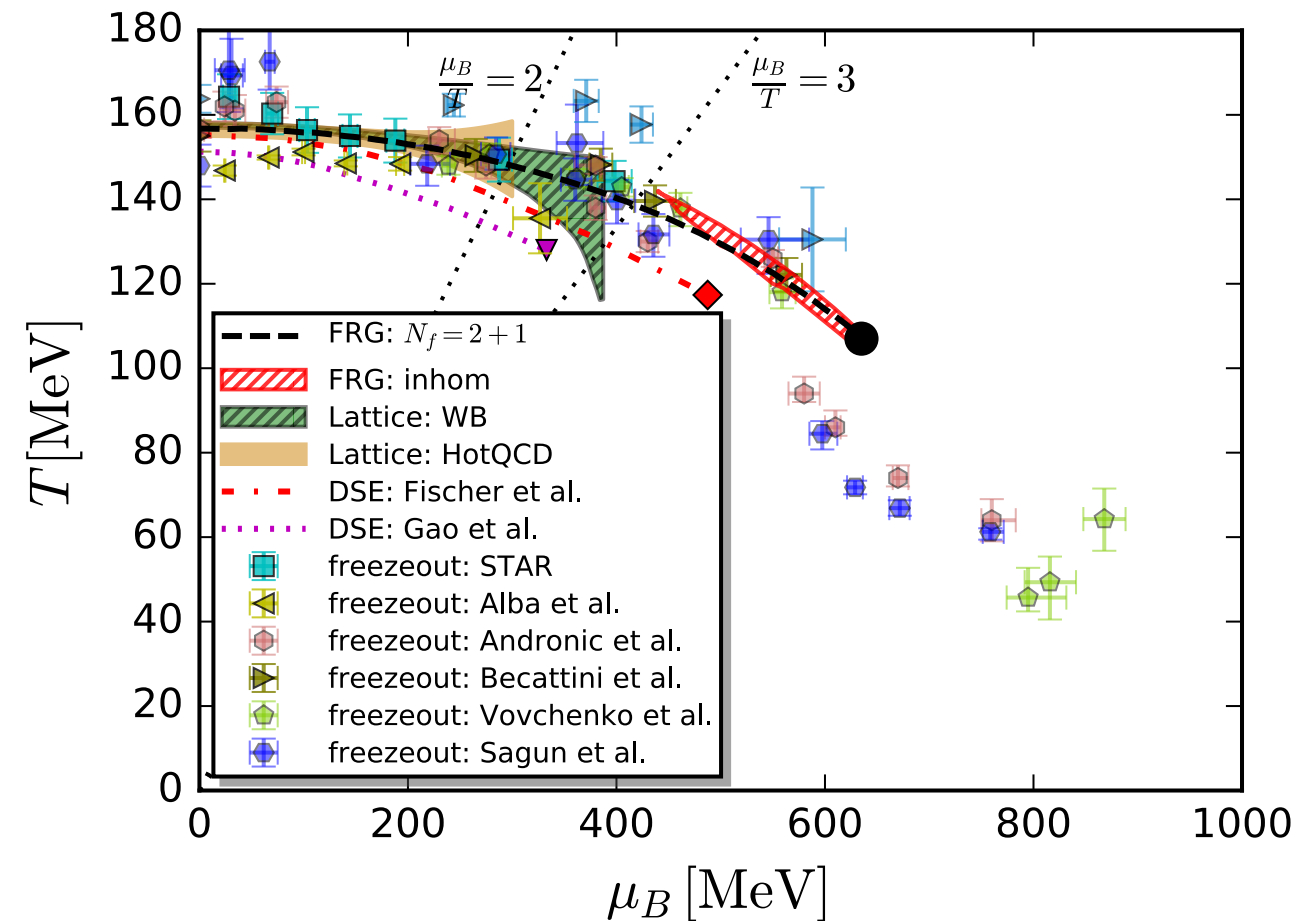
$$\partial_t \text{gluon-gluon vertex} = \tilde{\partial}_t \left(\text{triangle} + \text{triangle} + \text{triangle} \right)$$

$$\partial_t \text{ghost-gluon vertex} = \tilde{\partial}_t \left(\text{triangle} + \text{triangle} + \text{triangle} \right)$$

$$\partial_t \text{quark-quark vertex} = \tilde{\partial}_t \left(\text{triangle} + \text{triangle} + \text{triangle} \right)$$

$$\partial_t \text{gluon-quark-gluon vertex} = \tilde{\partial}_t \left(\text{triangle} + \text{triangle} + \text{triangle} \right)$$

$$\partial_t \text{gluon-gluon-gluon vertex} = \tilde{\partial}_t \left(\text{triangle} + \text{triangle} + \text{triangle} \right)$$



Critical endpoint

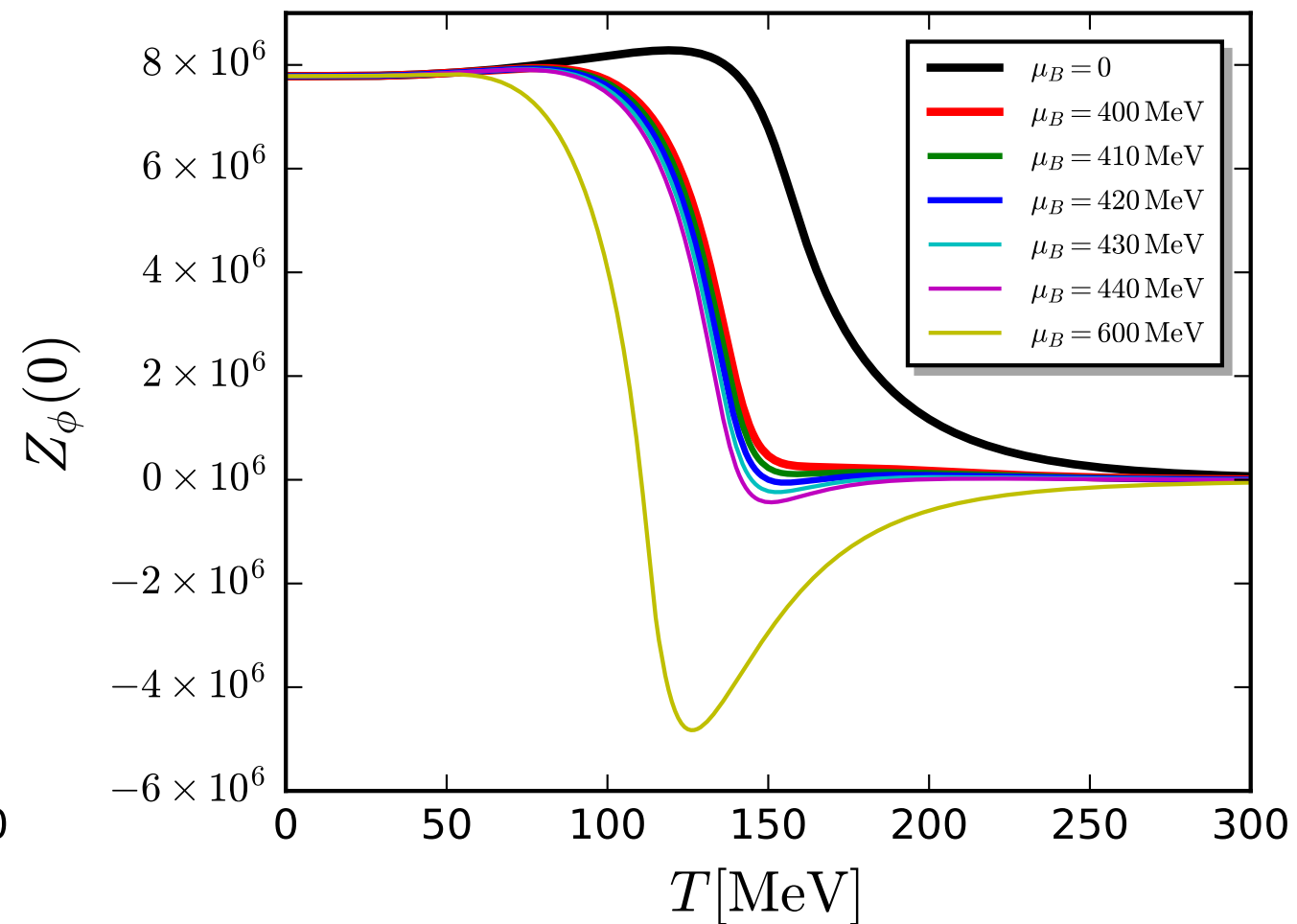
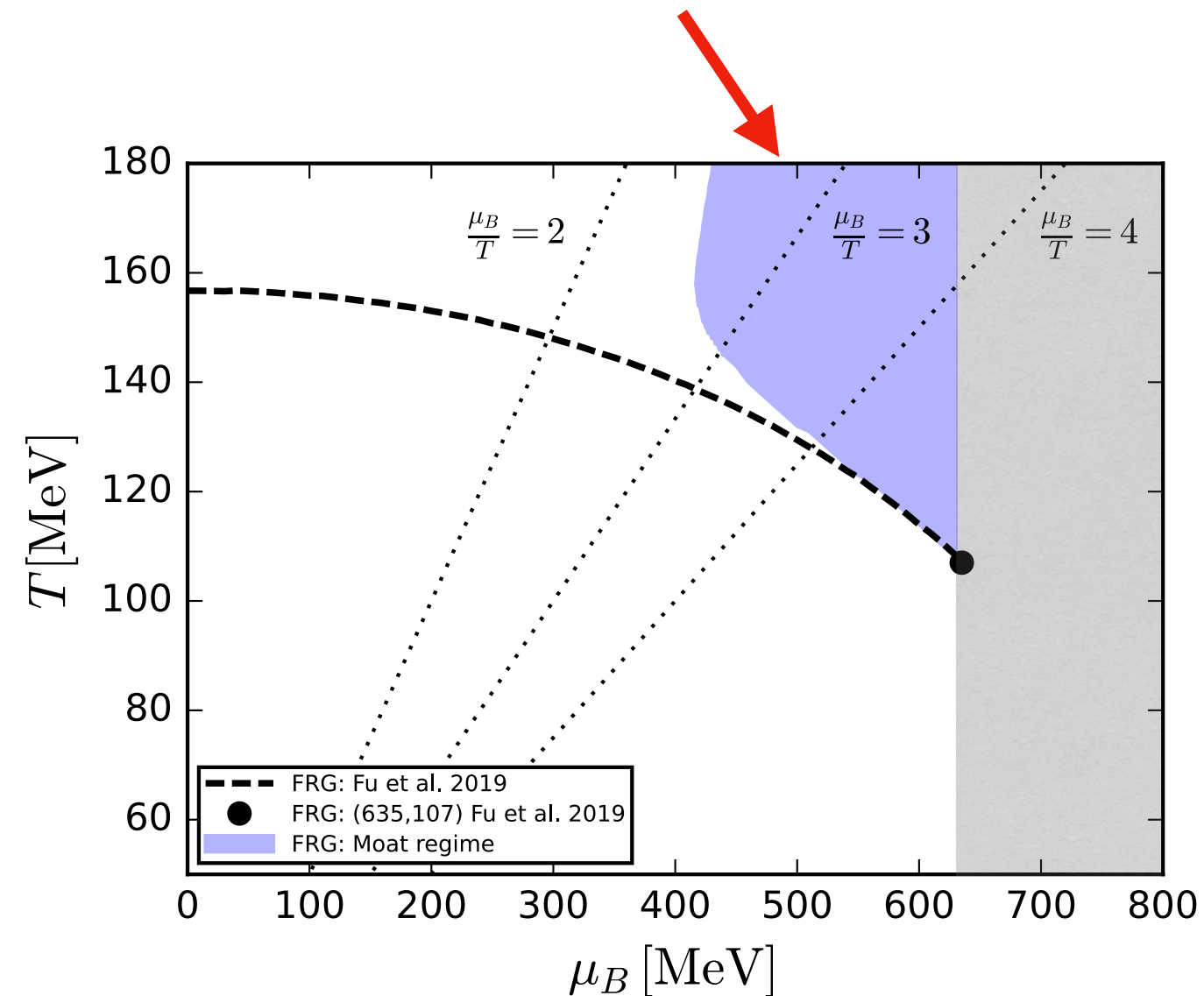
$$(T_{CEP}, \mu_{B_{CEP}}) = (107, 635) \text{ MeV}$$

Quantitative improvement on the way !!!

Moat regime in QCD

Negative meson wave function renormalisation

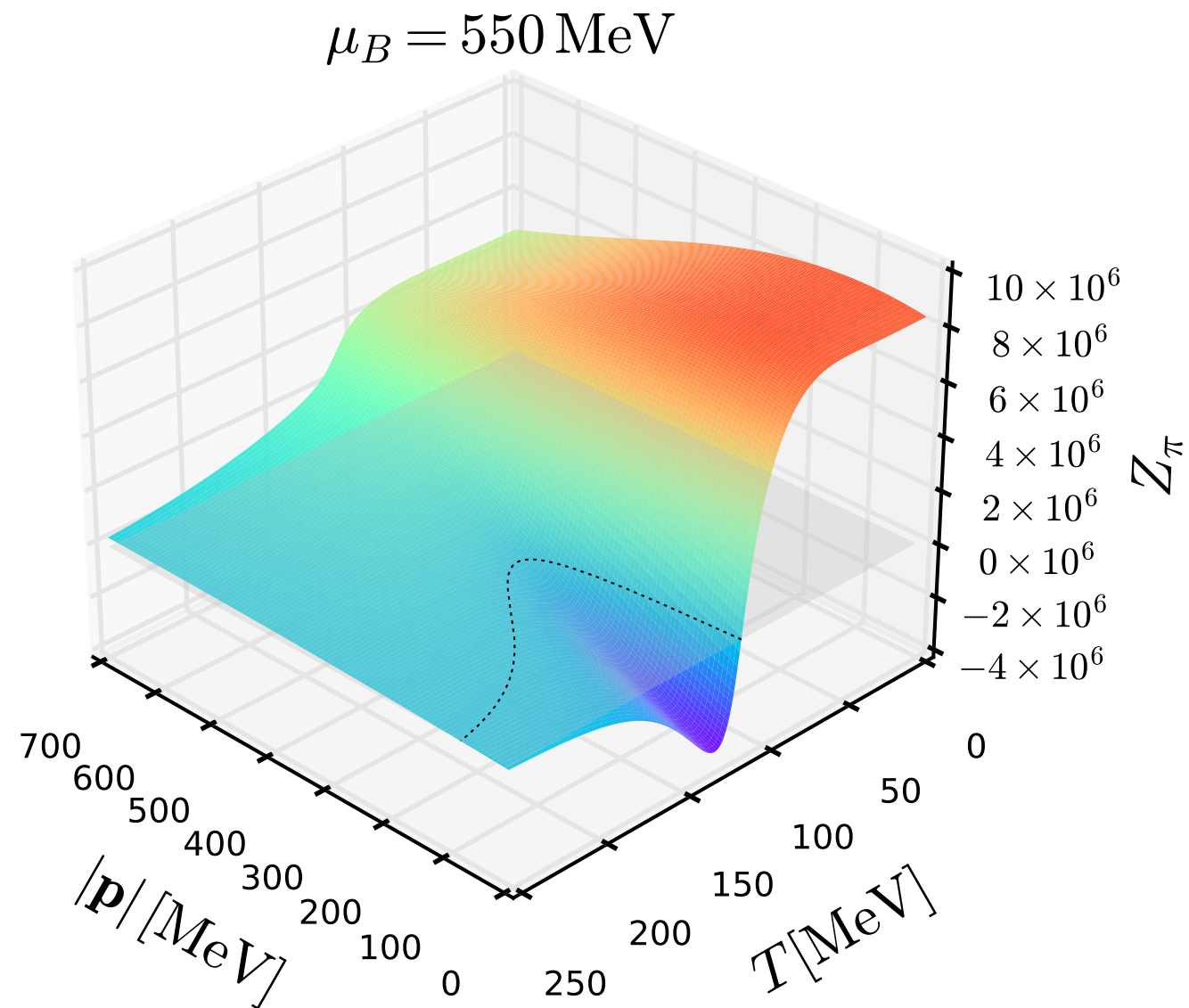
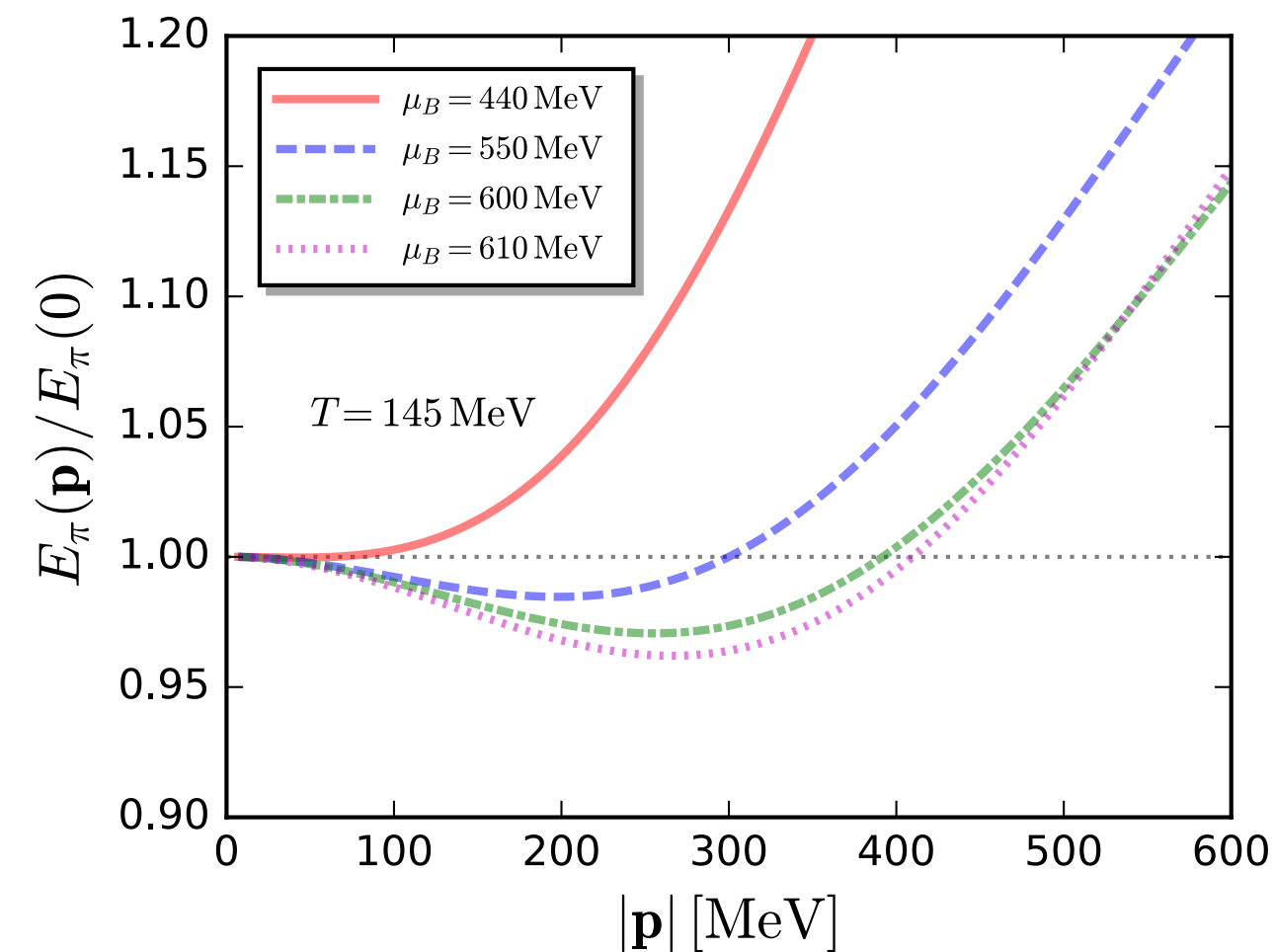
“Moat”



Moat regime in QCD

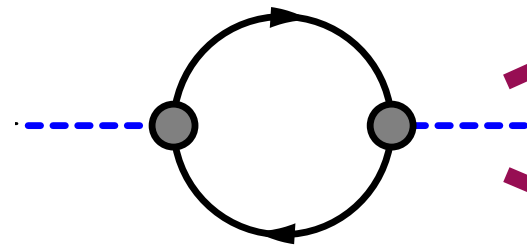
Dispersion relation

$$E_\pi(\vec{p}) = \sqrt{Z_\pi^\perp(\vec{p}) \vec{p}^2 + m_\pi^2}$$



Generation of moat behavior

Quark loop

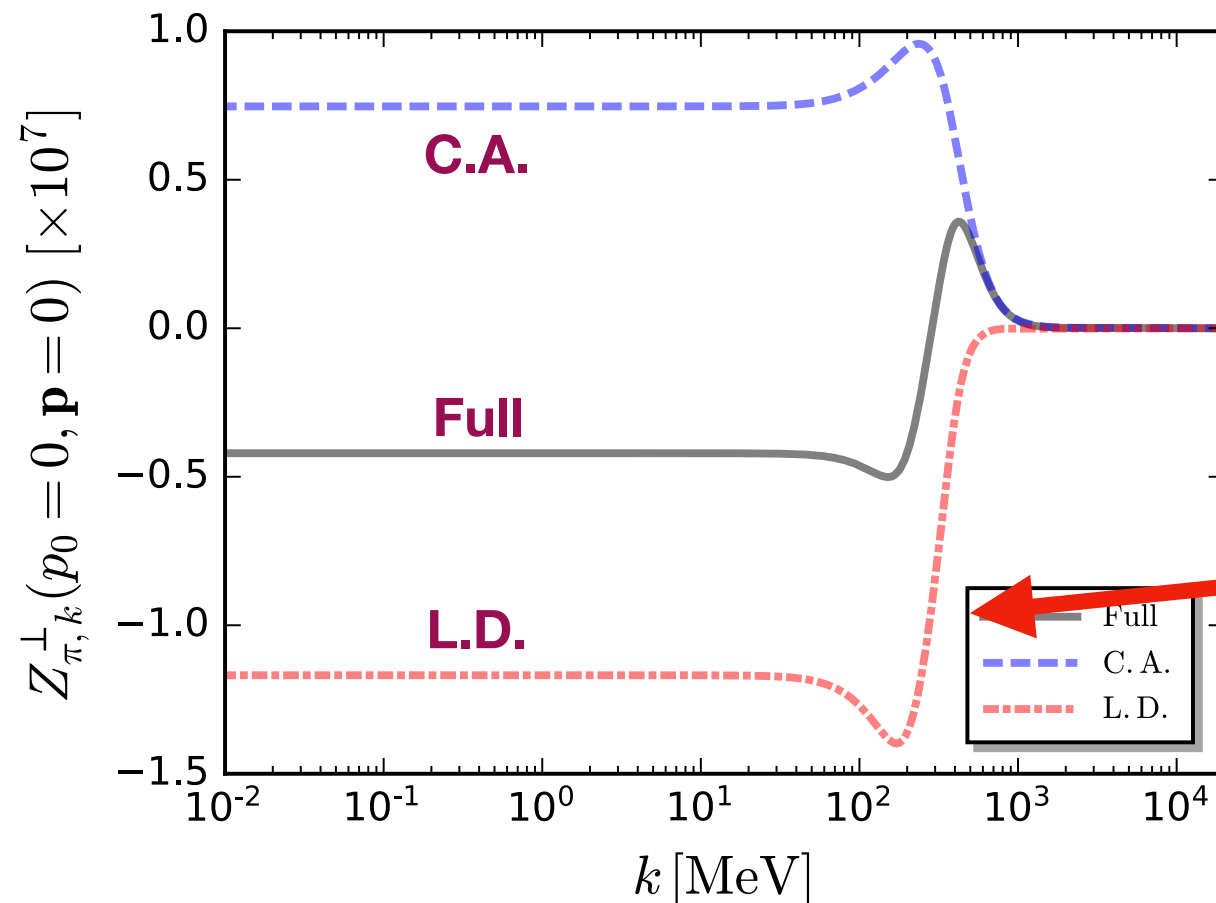


particle creation and annihilation (C.A.)

$$\mathcal{F}\mathcal{F}_{(1,1)}^{\text{CA}}(p) = \frac{k^3}{4E(|\vec{q}|)E(|\vec{q}-\vec{p}|)} \left\{ \frac{1}{ip_0 - E(|\vec{q}|) - E(|\vec{q}-\vec{p}|)} \left[-1 + n_F(E(|\vec{q}|); T, \mu) + n_F(E(|\vec{q}-\vec{p}|); T, -\mu) \right] \right. \\ \left. + \frac{1}{ip_0 + E(|\vec{q}|) + E(|\vec{q}-\vec{p}|)} \left[1 - n_F(E(|\vec{q}|); T, \mu) - n_F(E(|\vec{q}-\vec{p}|); T, -\mu) \right] \right\}.$$

$$\mathcal{F}\mathcal{F}_{(1,1)}^{\text{LD}}(p) = \frac{k^3}{4E(|\vec{q}|)E(|\vec{q}-\vec{p}|)} \left\{ \frac{1}{ip_0 - E(|\vec{q}|) + E(|\vec{q}-\vec{p}|)} \left[-n_F(E(|\vec{q}|); T, -\mu) + n_F(E(|\vec{q}-\vec{p}|); T, -\mu) \right] \right. \\ \left. + \frac{1}{ip_0 + E(|\vec{q}|) - E(|\vec{q}-\vec{p}|)} \left[n_F(E(|\vec{q}|); T, \mu) - n_F(E(|\vec{q}-\vec{p}|); T, \mu) \right] \right\}.$$

Landau damping (L.D.)



The negative meson wave function renormalisation is caused by **Landau damping**

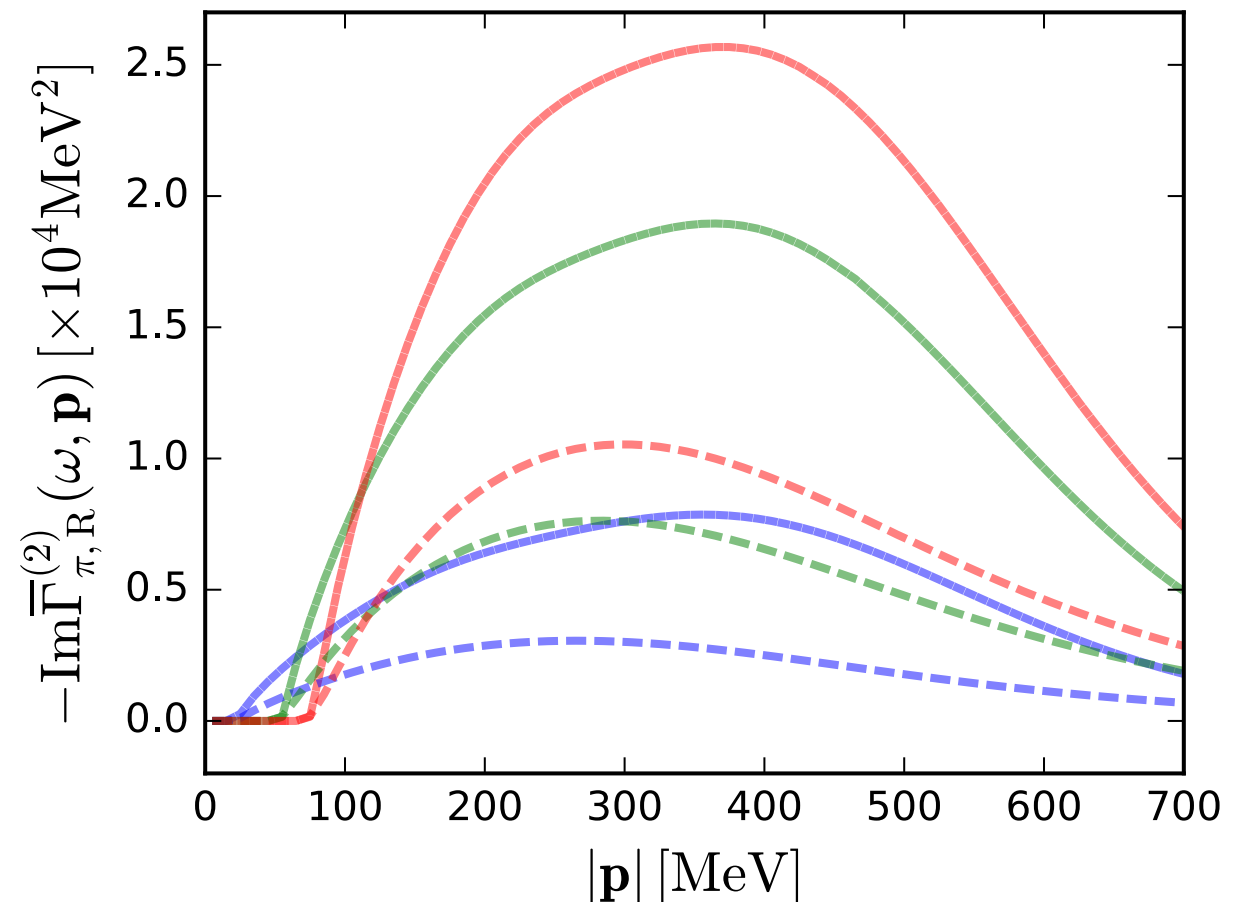
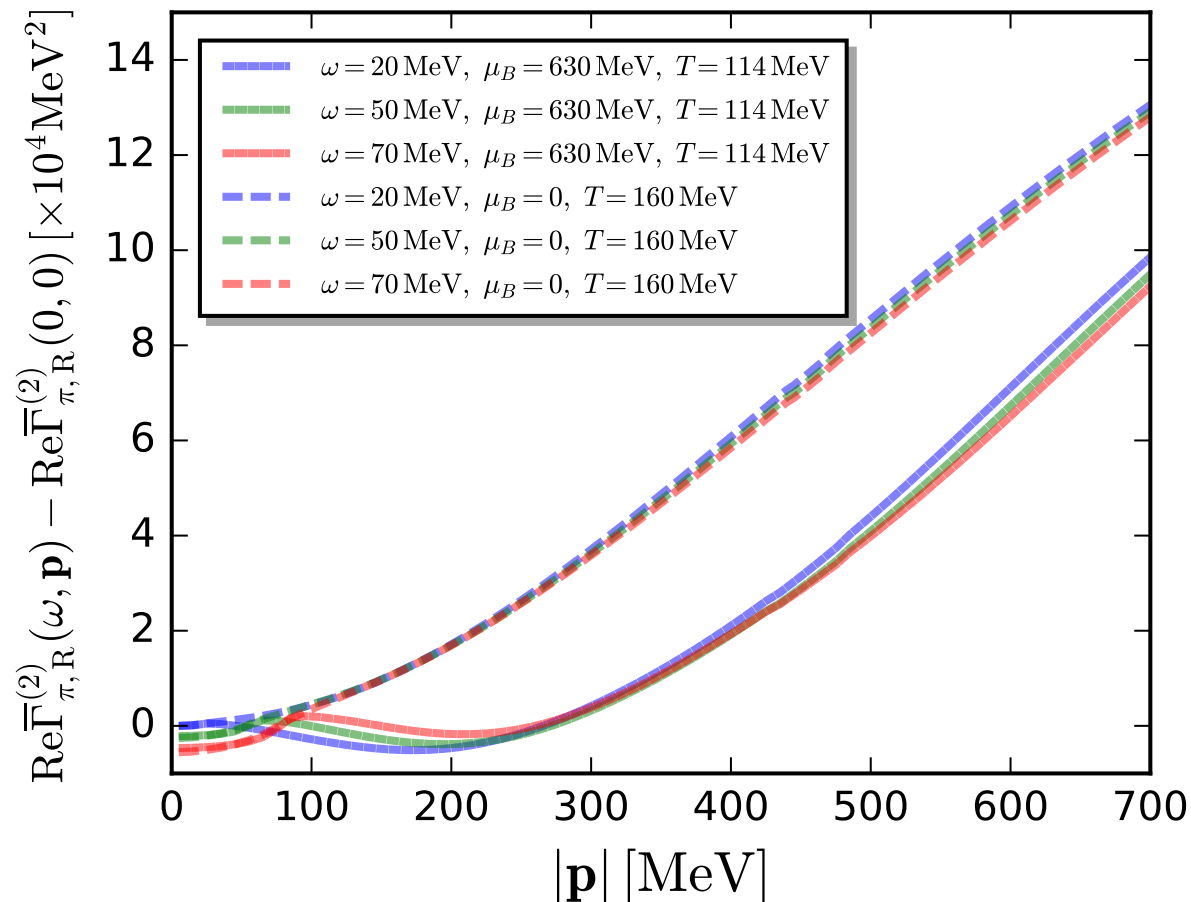
Pion spectral function

Analytical continuation $p_0 \rightarrow -i(\omega + i\epsilon)$ $\Gamma_{\phi\phi,R}^{(2)}(\omega, \vec{p}) = \lim_{\epsilon \rightarrow 0^+} \Gamma_{\phi\phi}^{(2)}(-i(\omega + i\epsilon), \vec{p})$

$$\text{Re}\Gamma_{\pi}^{(2)}(\omega, \mathbf{p}) = \text{Re}(-Z_{\pi}^{\parallel}(\omega, \mathbf{p})\omega^2 + Z_{\pi}^{\perp}(\omega, \mathbf{p})\mathbf{p}^2) + m_{\pi}^2$$

$$\text{Im}\Gamma_{\pi}^{(2)}(\omega, \mathbf{p}) = \text{Im}(-Z_{\pi}^{\parallel}(\omega, \mathbf{p})\omega^2 + Z_{\pi}^{\perp}(\omega, \mathbf{p})\mathbf{p}^2)$$

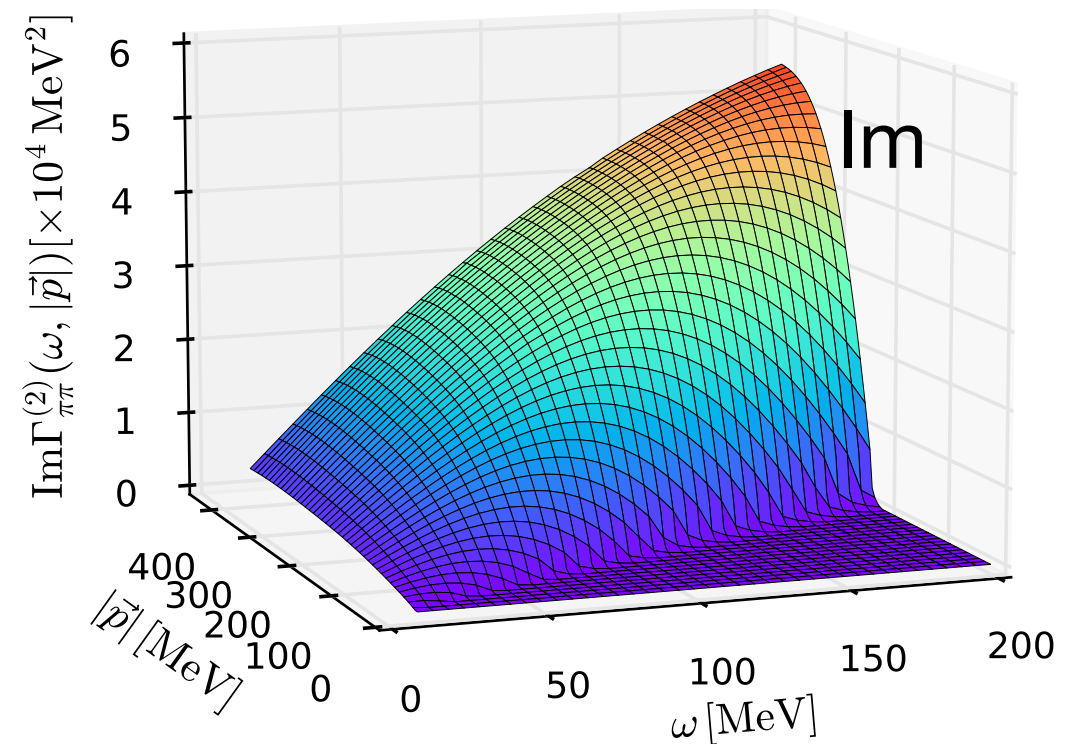
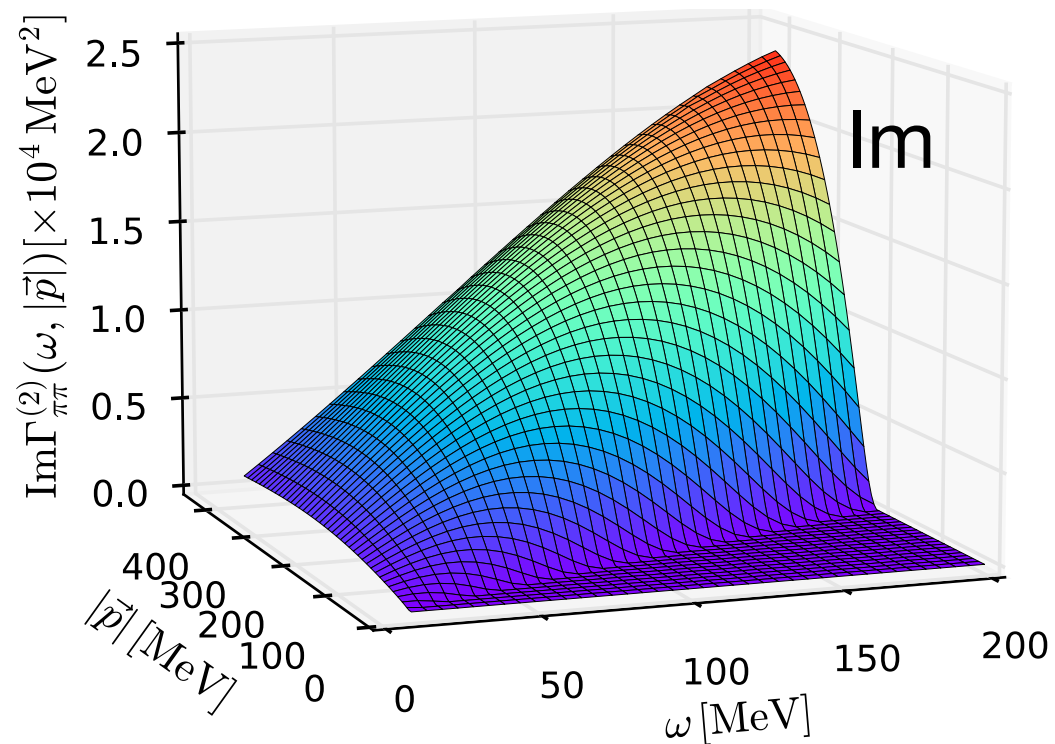
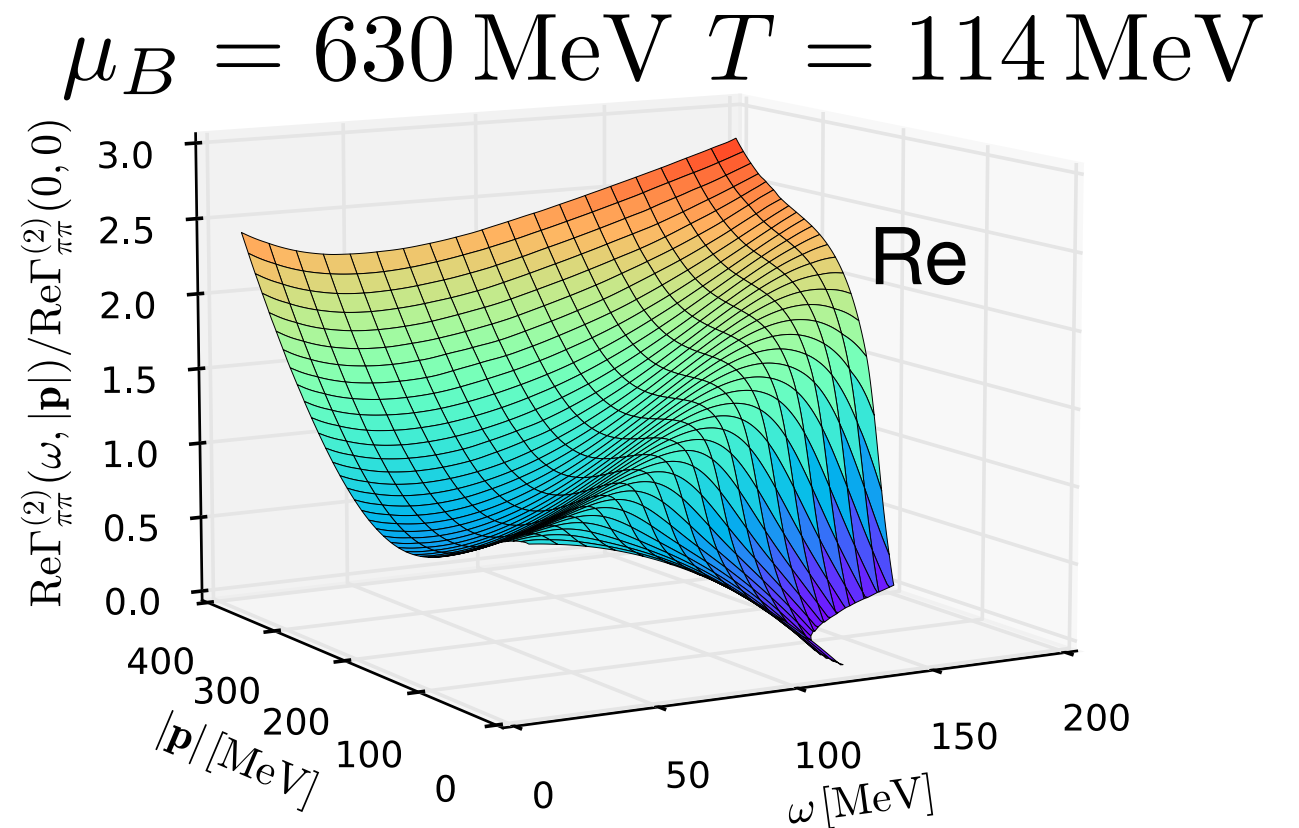
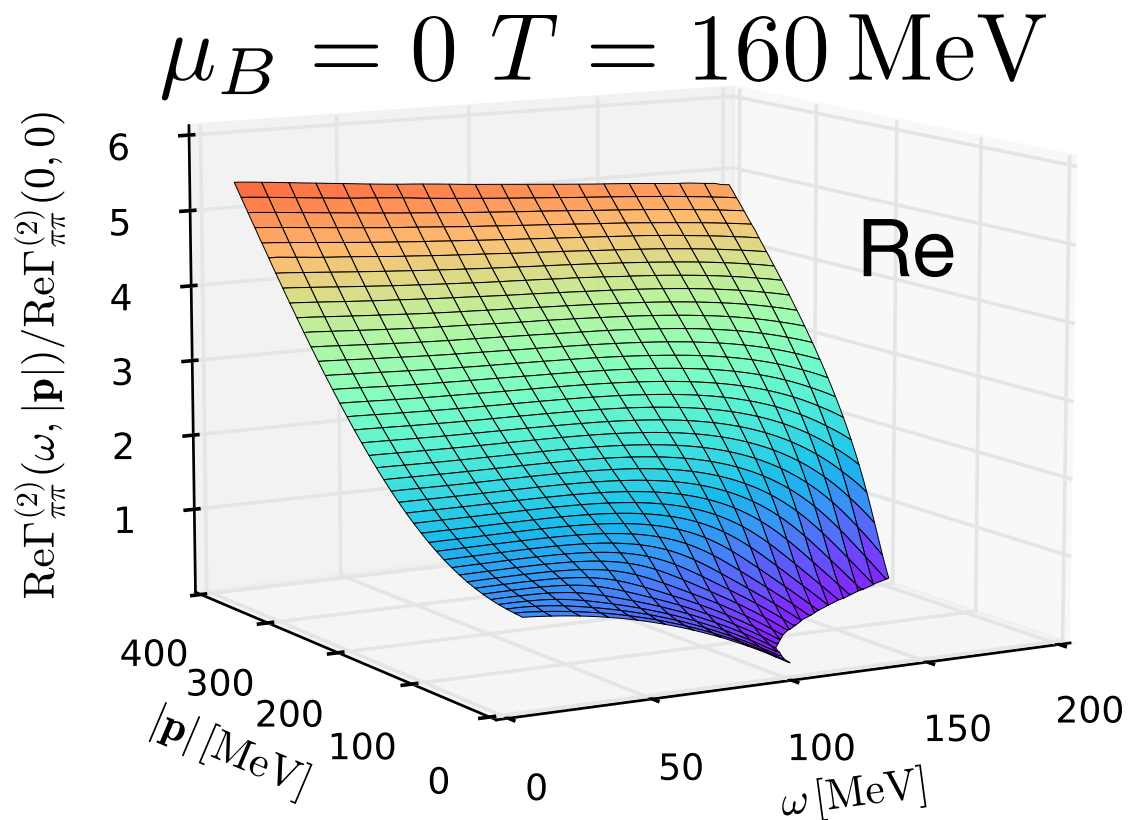
$$\rho_{\phi}(\omega, \mathbf{p}) = -\frac{1}{\pi} \frac{\text{Im}\Gamma_{\phi\phi,R}^{(2)}(\omega, \mathbf{p})}{(\text{Re}\Gamma_{\phi\phi,R}^{(2)}(\omega, \mathbf{p}))^2 + (\text{Im}\Gamma_{\phi\phi,R}^{(2)}(\omega, \mathbf{p}))^2}$$



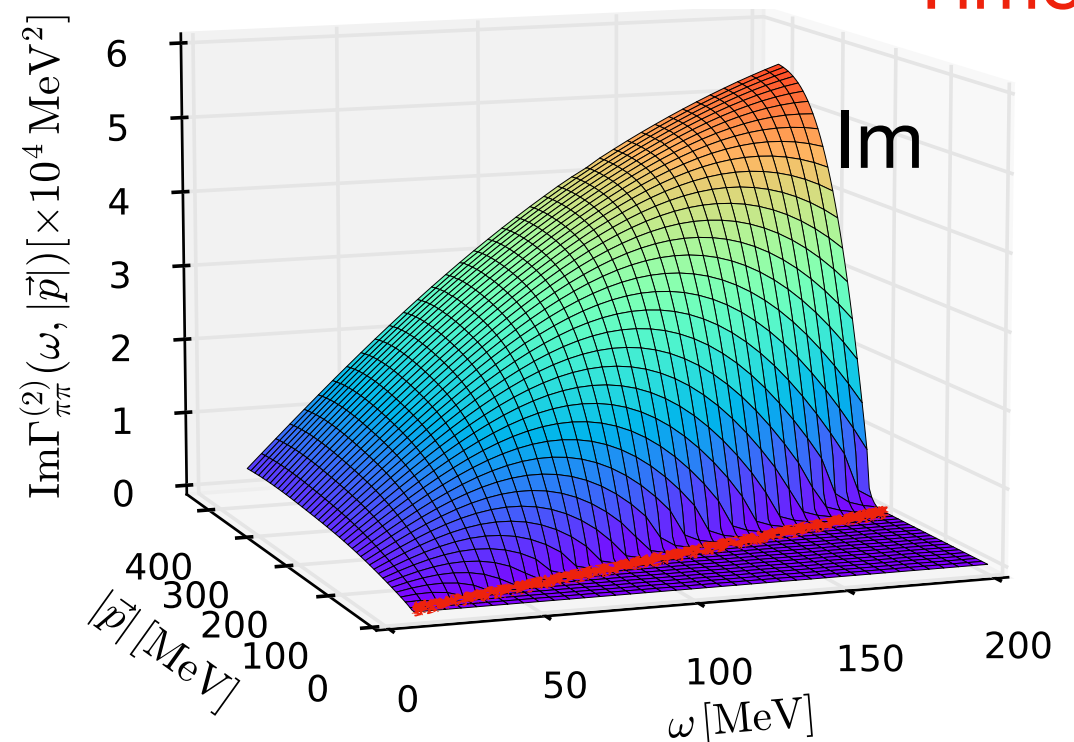
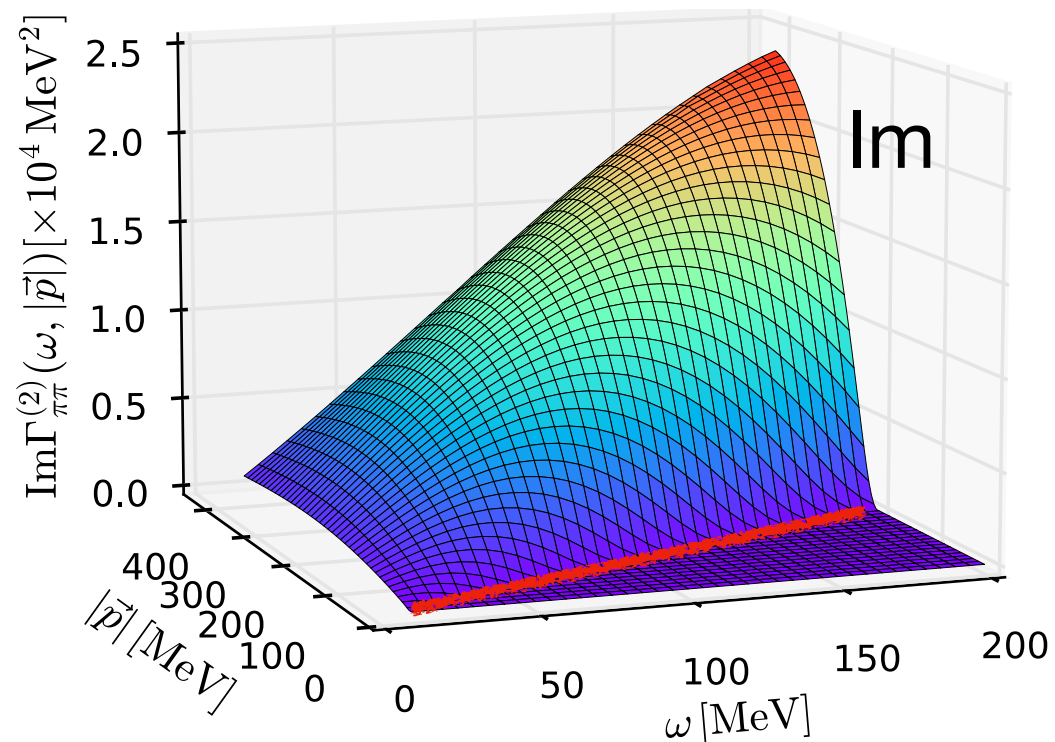
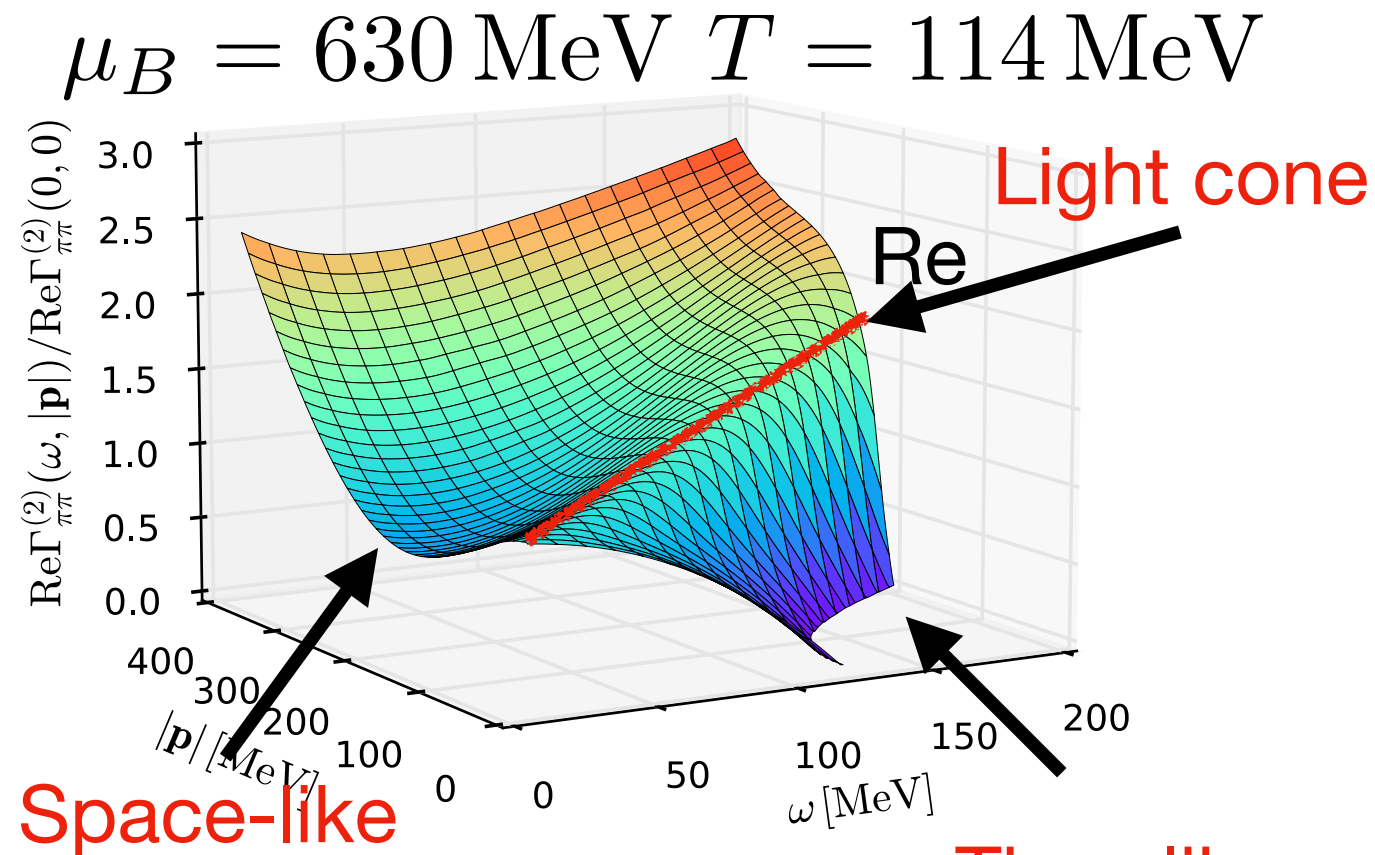
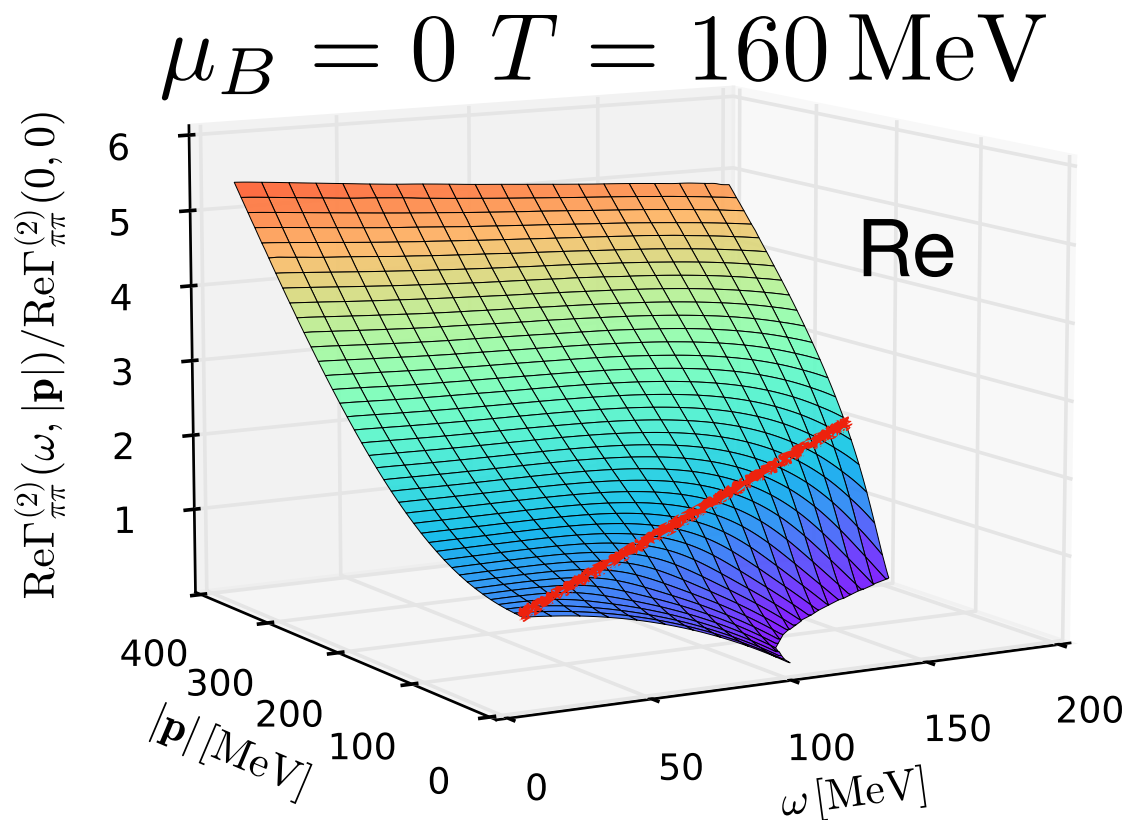
Moat effect is in the space-like regime

Wei-jie Fu, Jan M. Pawłowski, Robert D. Pisarski, Fabian Rennecke, Rui Wen, SY. In preparation.

Real part and Imaginary part

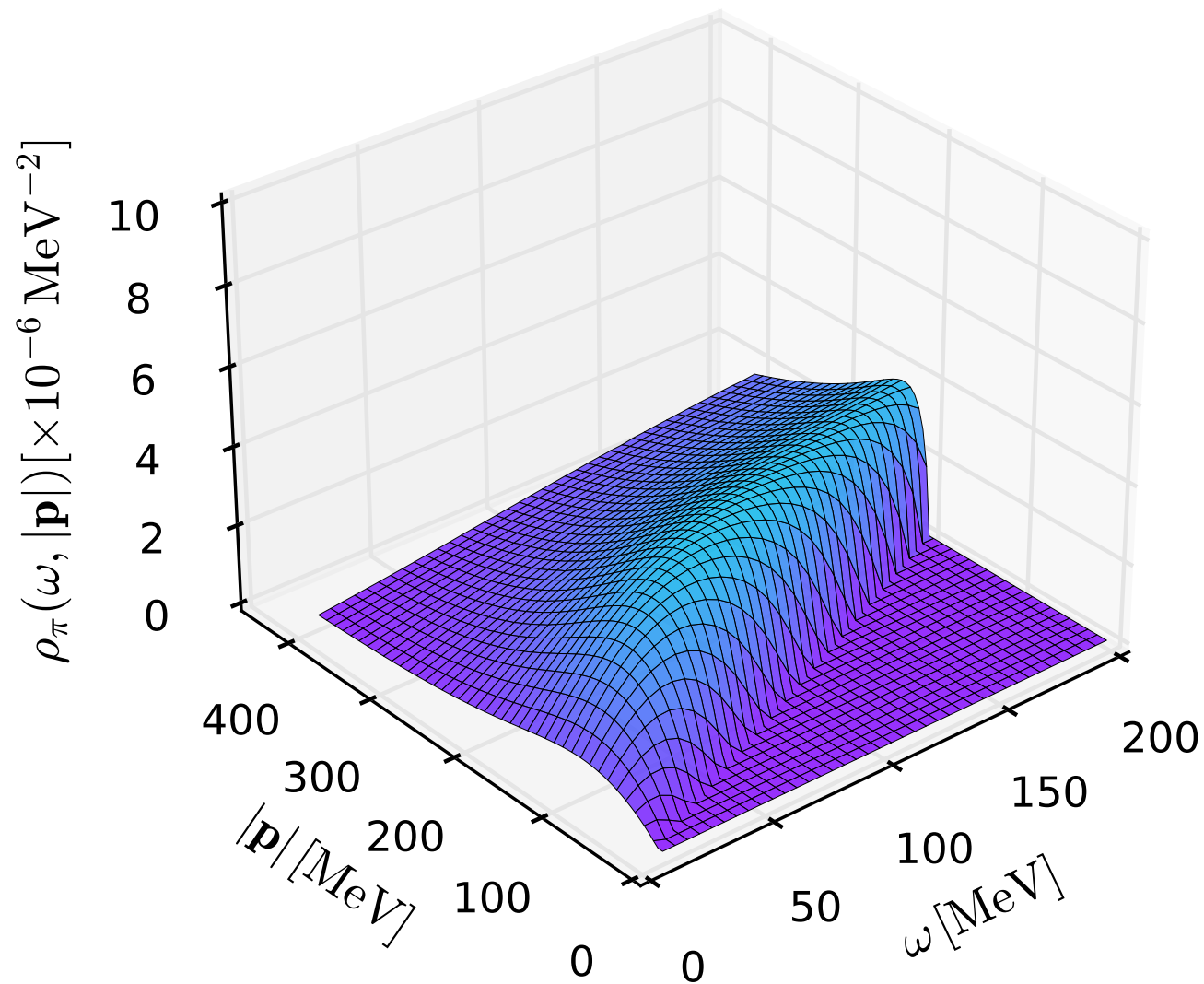


Real part and Imaginary part

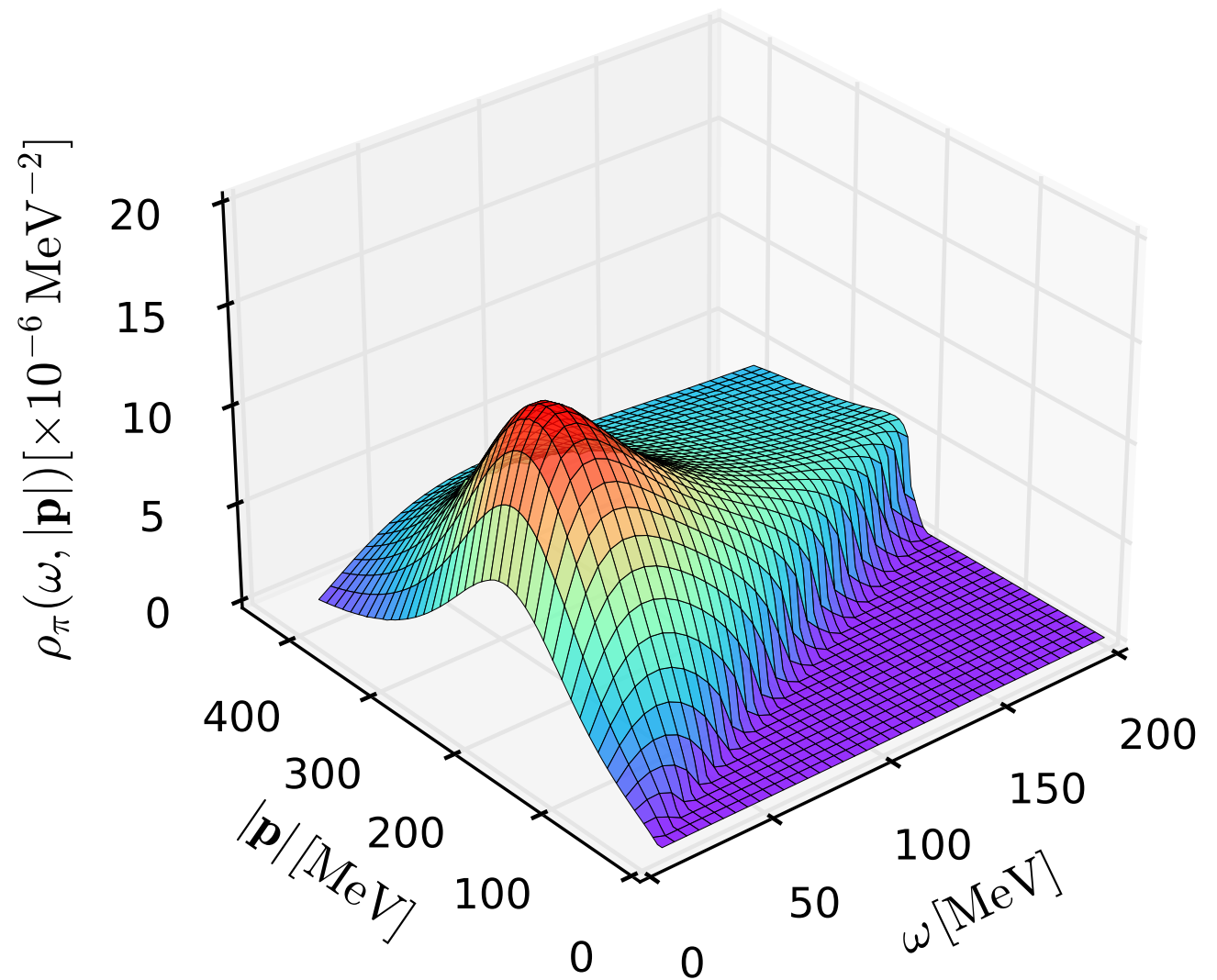


Pion spectral function

$$\mu_B = 0 \quad T = 160 \text{ MeV}$$



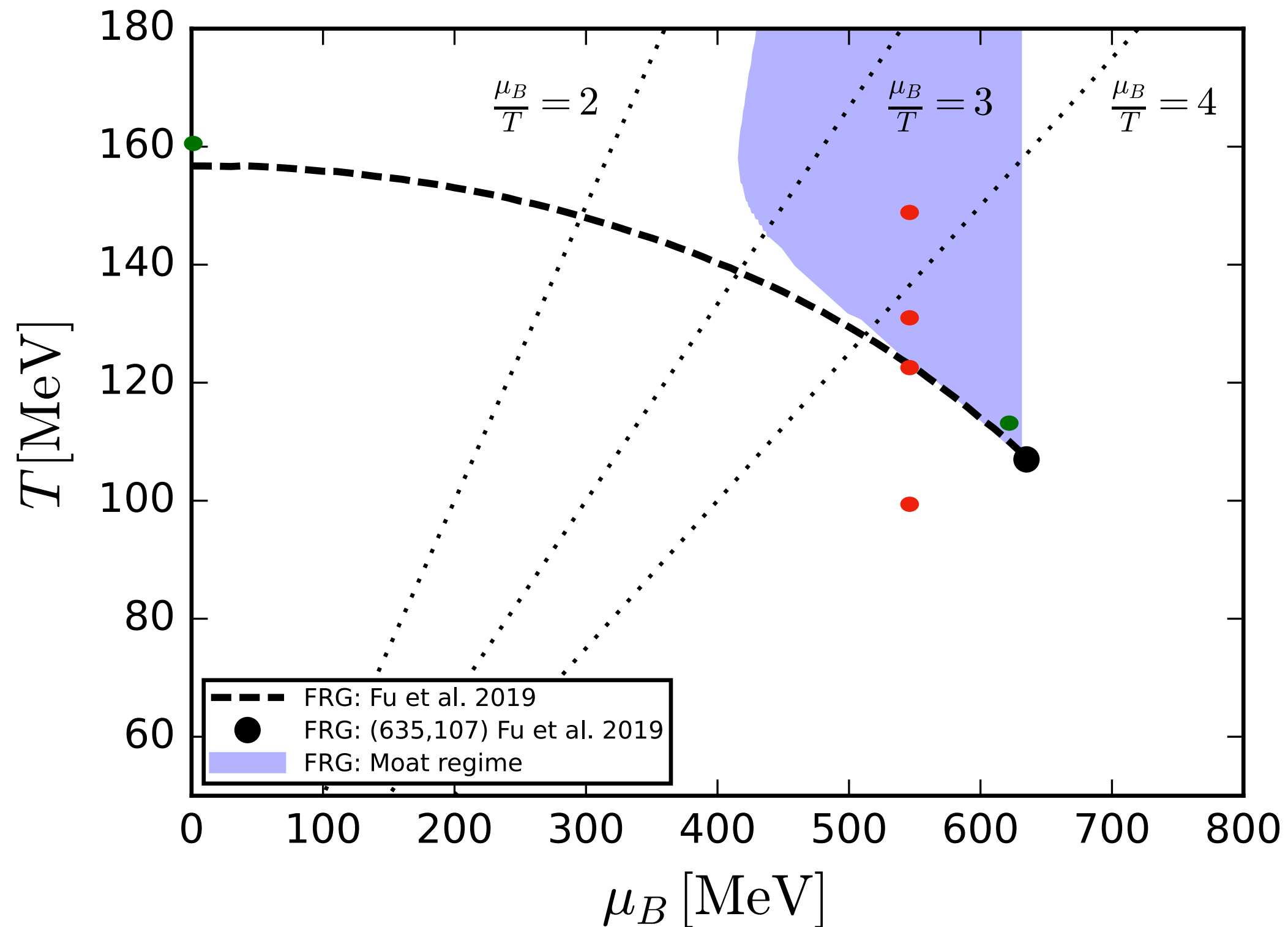
$$\mu_B = 630 \text{ MeV} \quad T = 114 \text{ MeV}$$



Competition between real-part
and Imaginary part

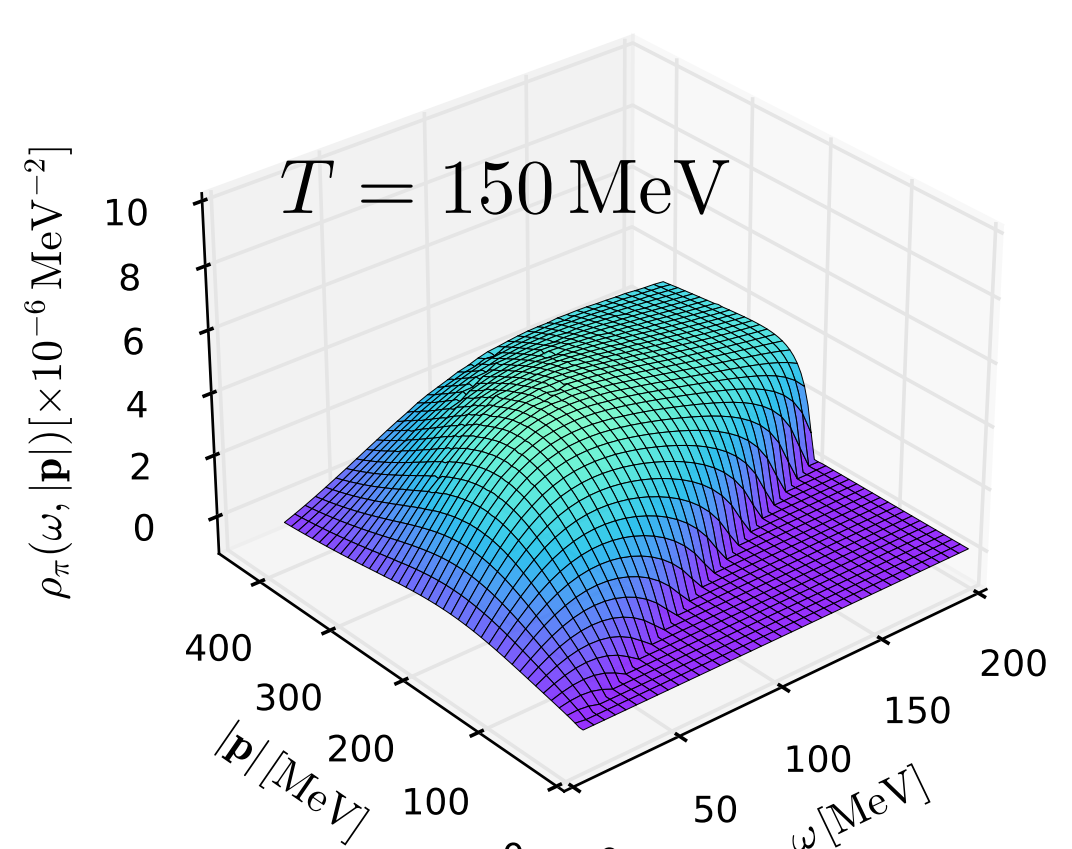
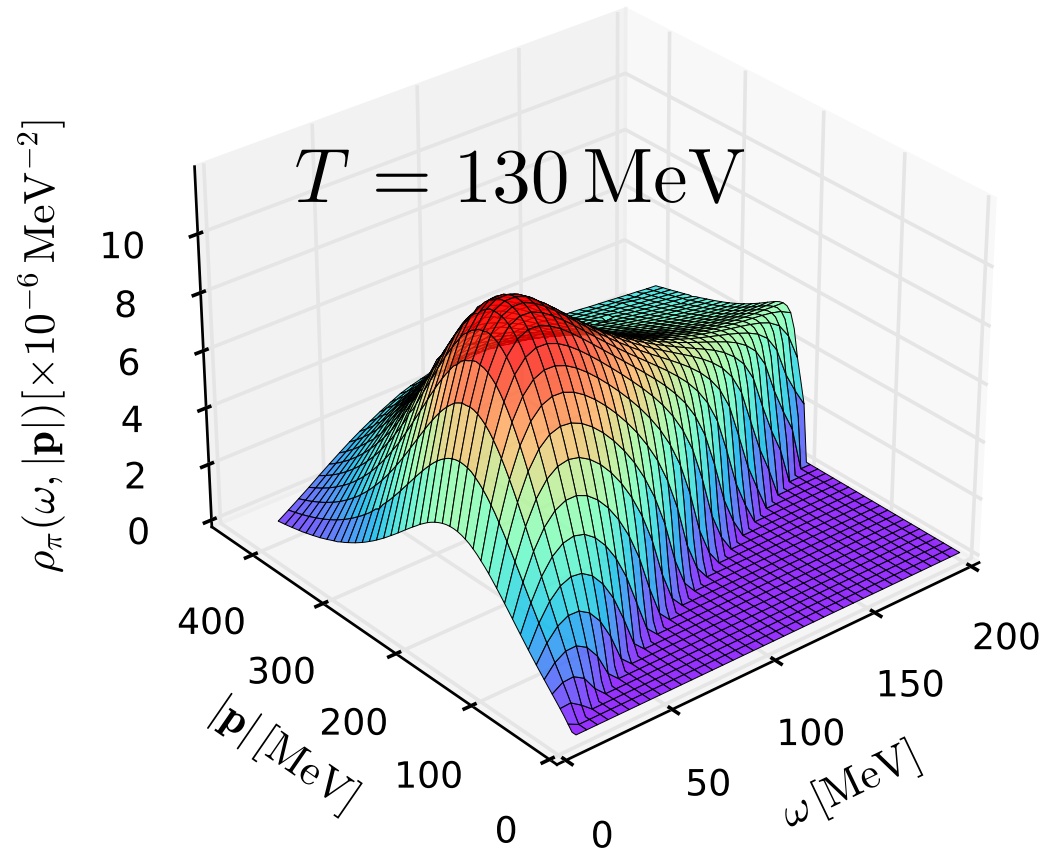
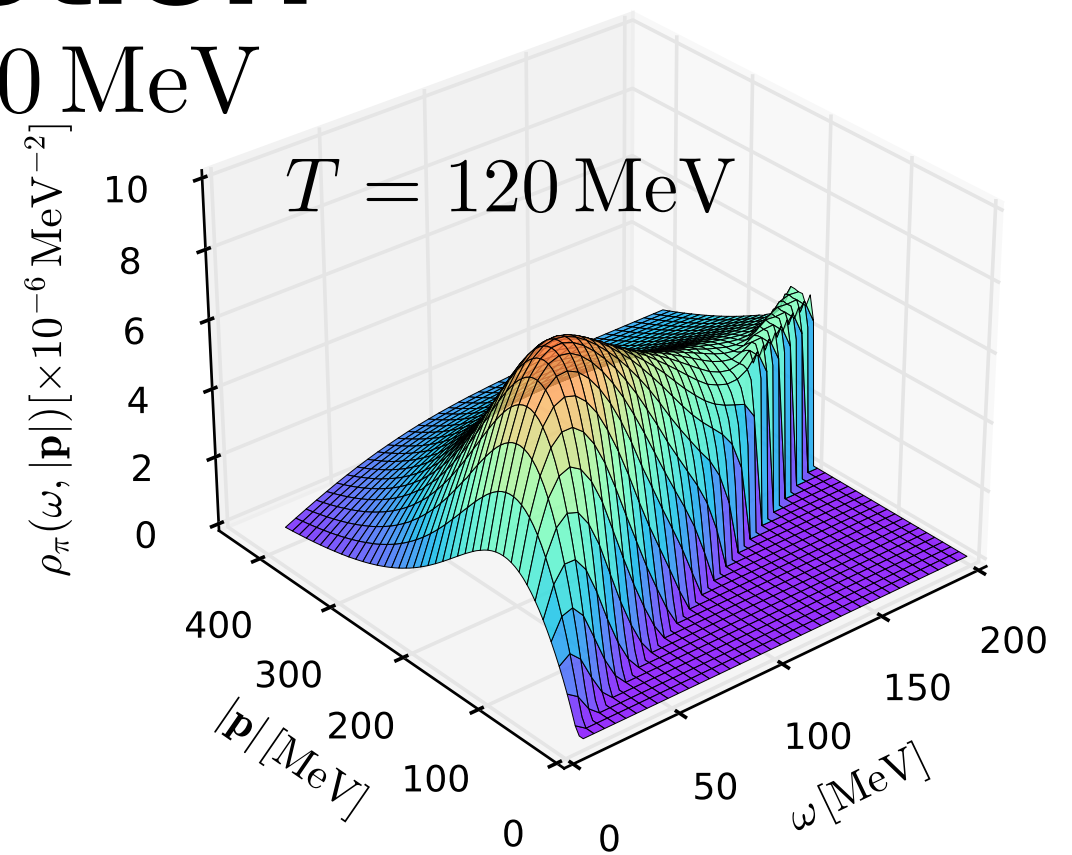
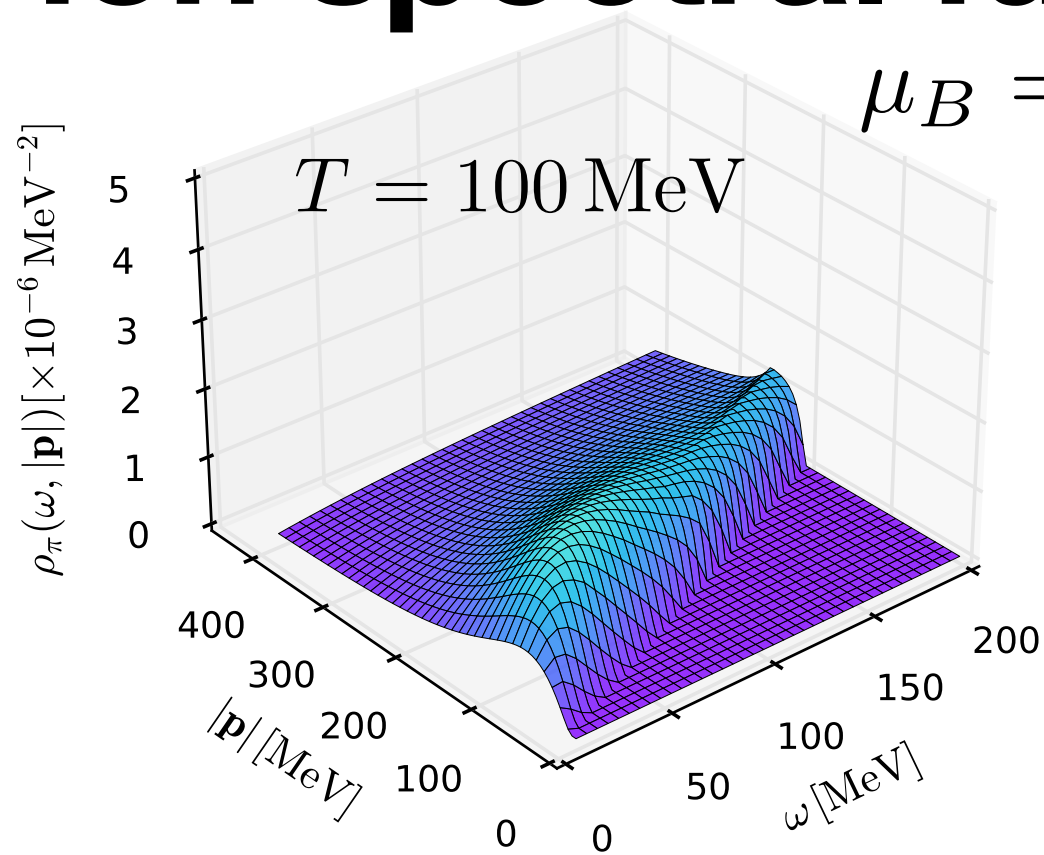
Moat peak

Moat regime



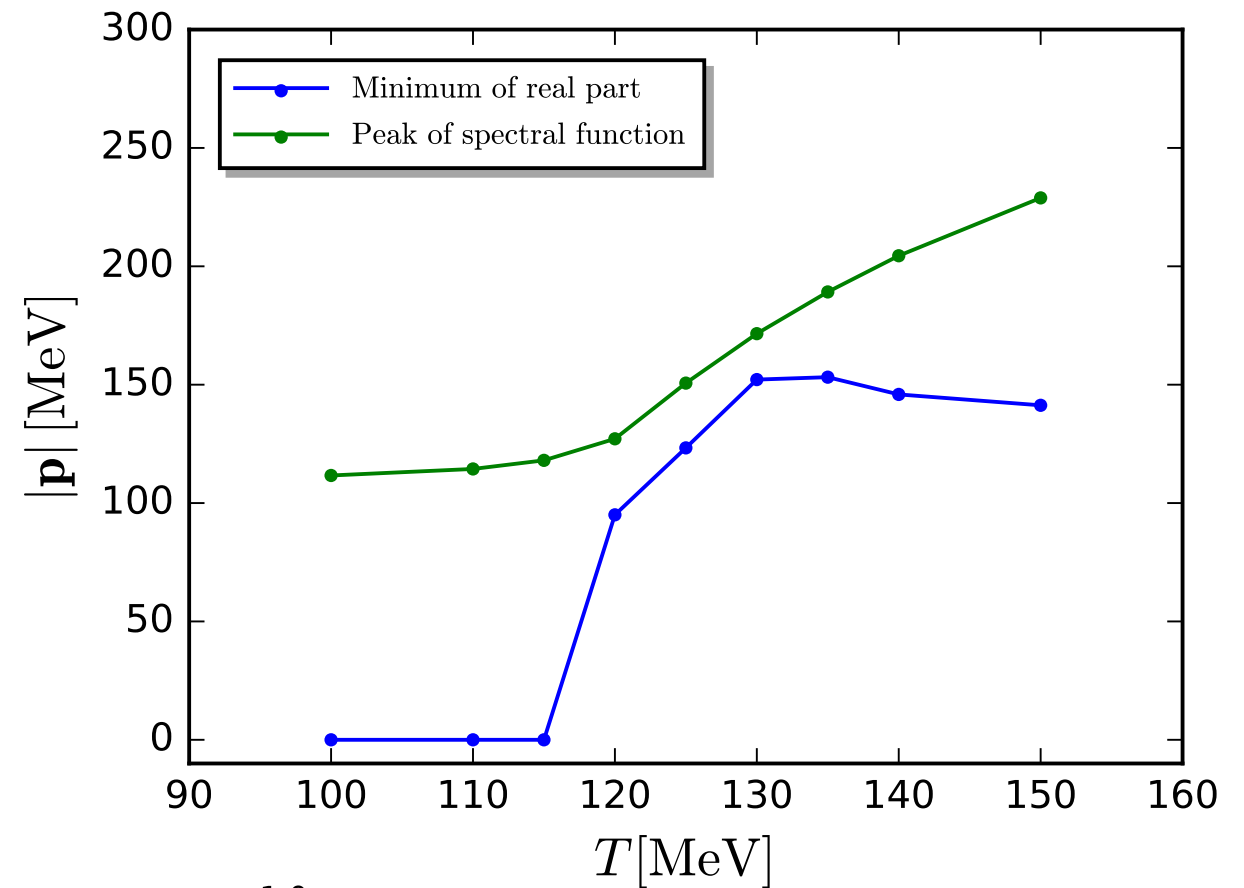
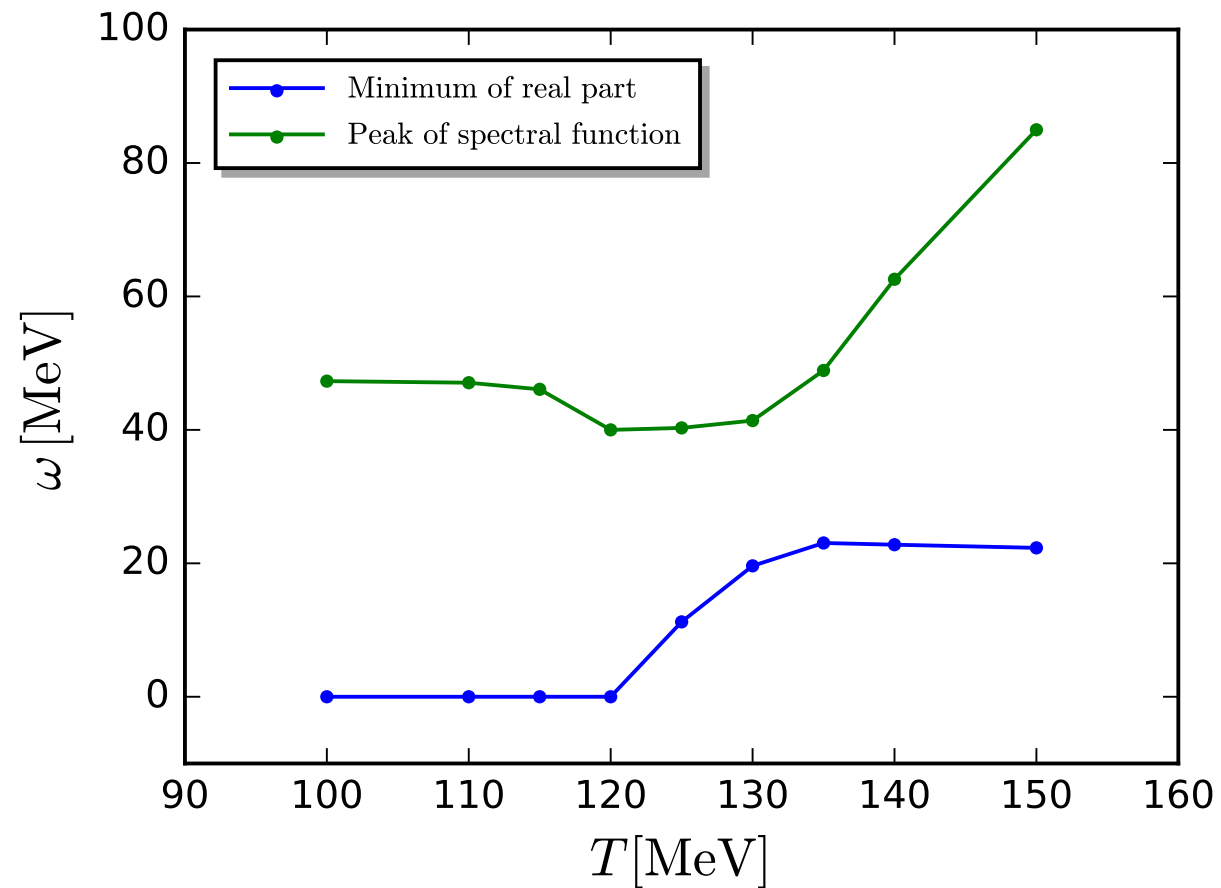
Pion spectral function

$$\mu_B = 550 \text{ MeV}$$

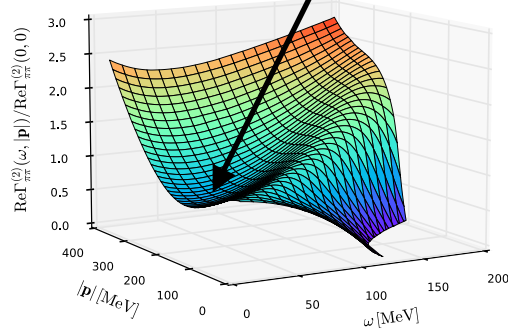


Minimum of Real-part and peak of spectral function

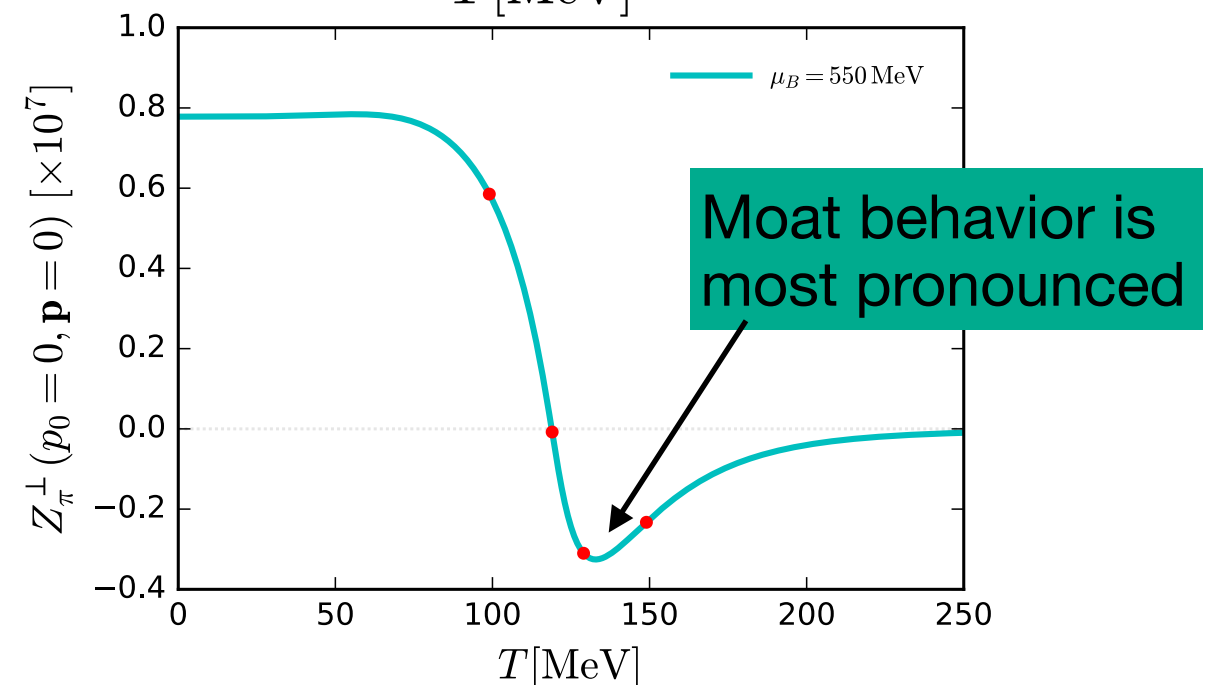
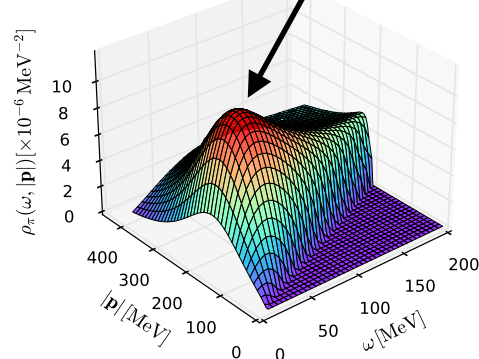
$$\mu_B = 550 \text{ MeV}$$



Minimum point

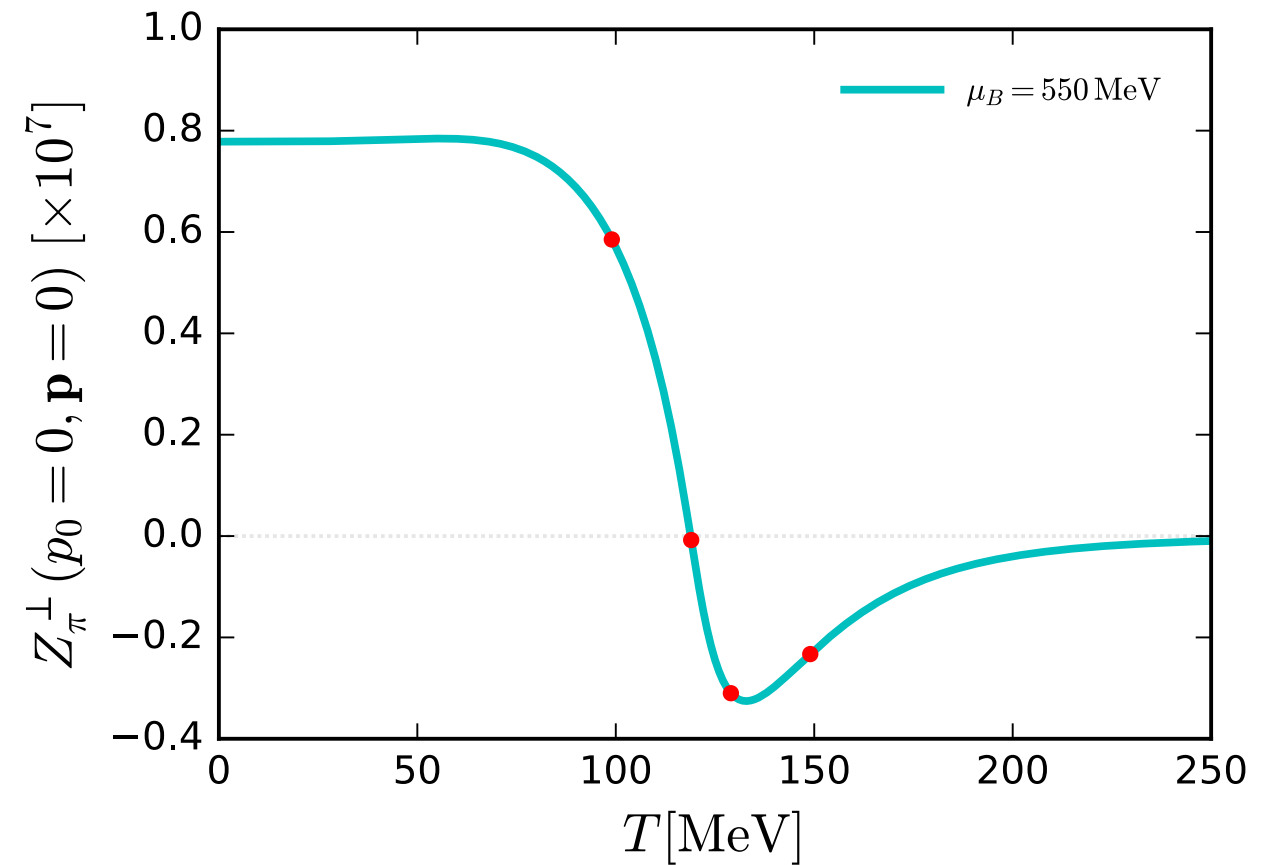
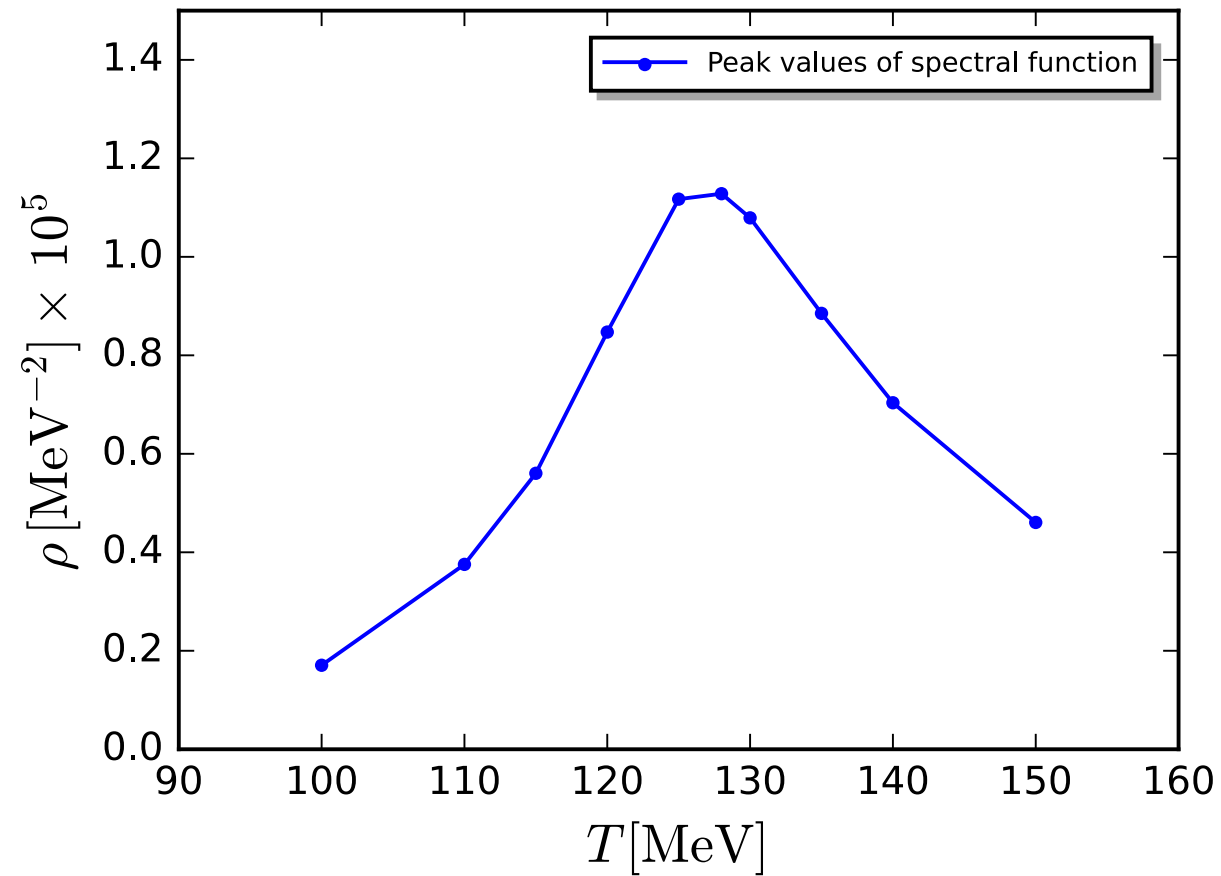


Peak



Peak height of spectral function

$$\mu_B = 550 \text{ MeV}$$



Outline

- QCD phase structure
- Moat regime in QCD
- **Summary and outlook**

Summary and outlook

- * Moat behavior is found in medium density area.
- * Moat is generated by Landau damping in quark loop.
- * Moat exists in the space-like region of the spectral function.
- * Moat will enhance the peak of the spectral function in the space-like region.
- * Study the influence of moat on experimental observables.

Summary and outlook

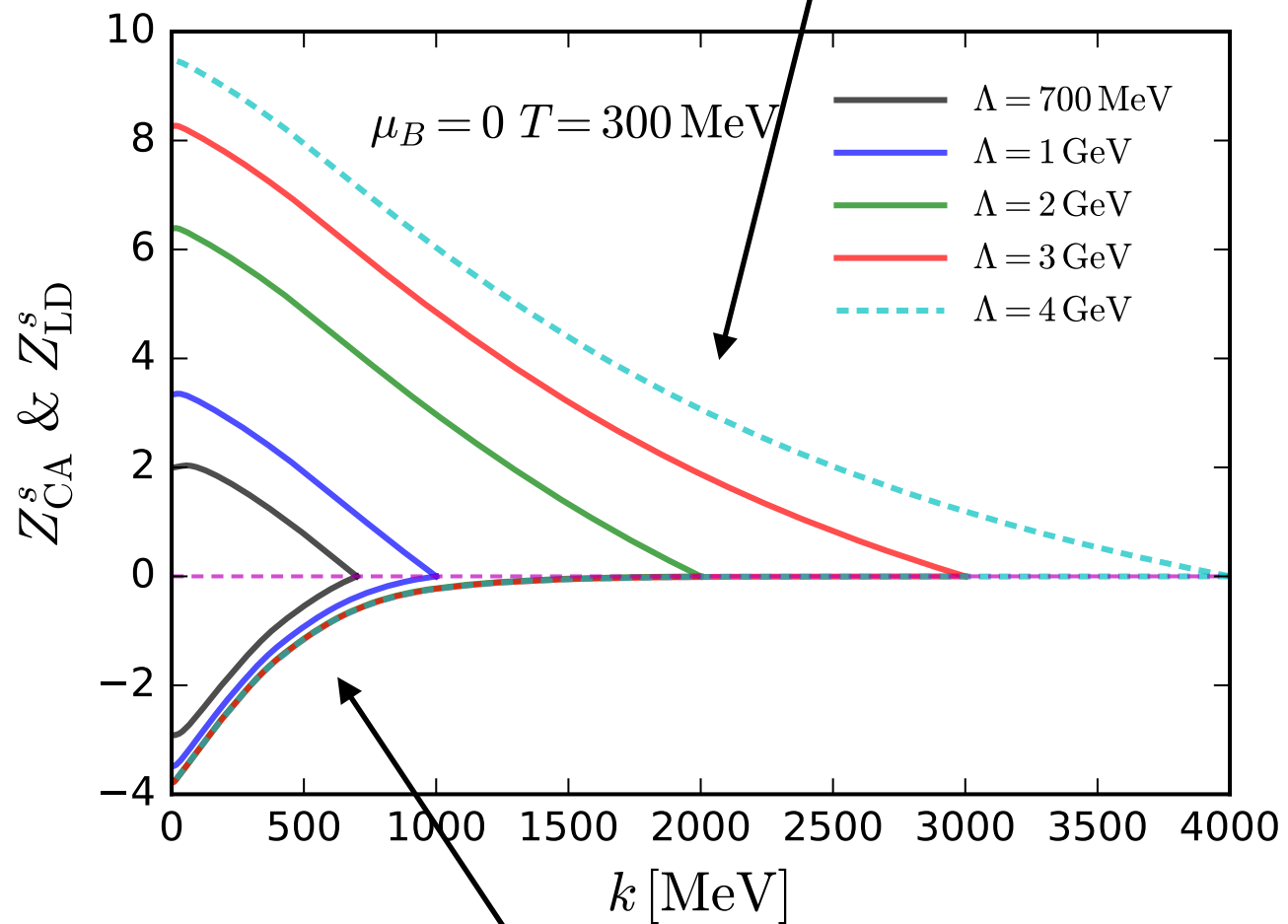
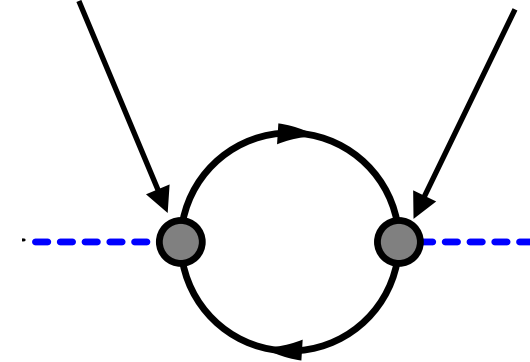
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Thank you !!!

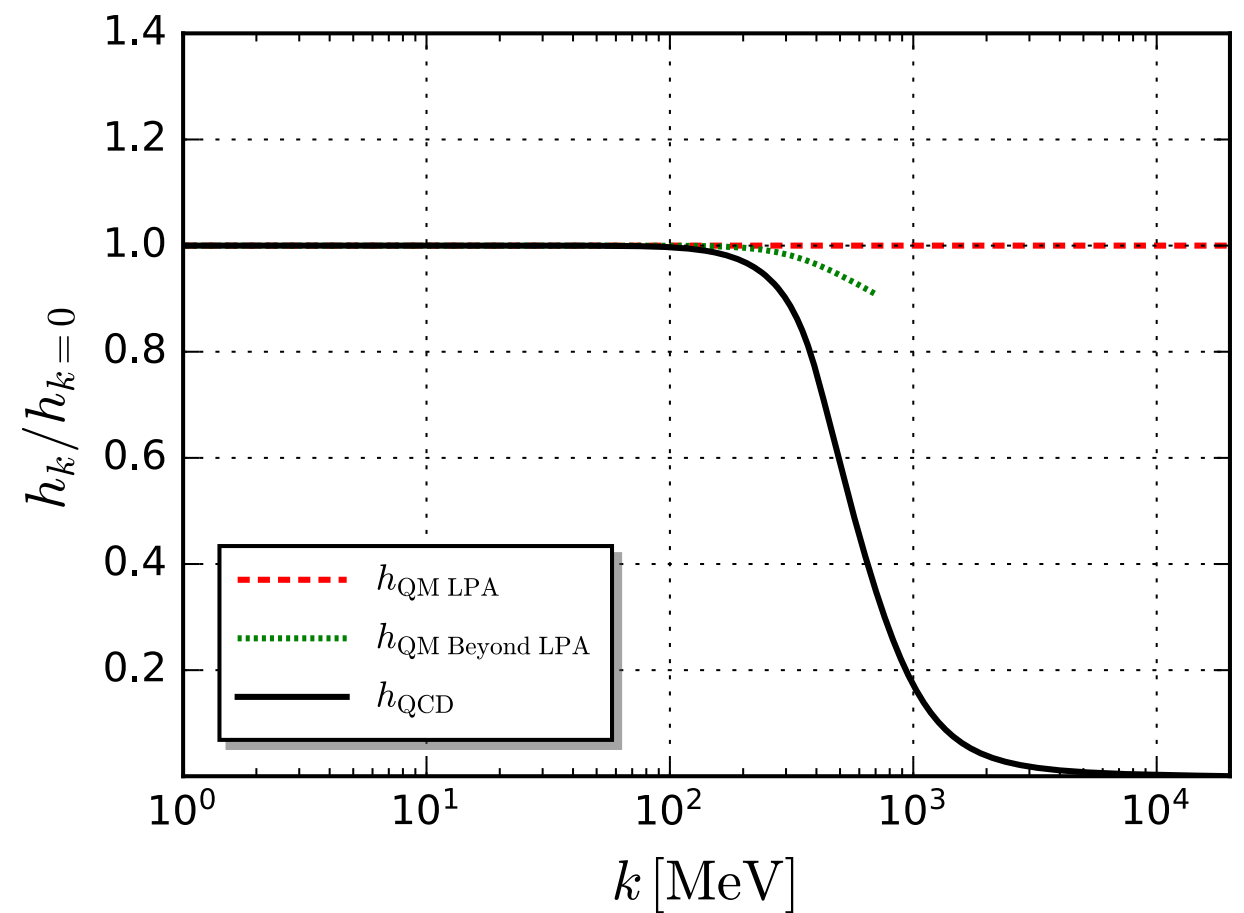
Back up

particle creation and annihilation (C.A.)

Yukawa coupling



Landau damping (L.D.)



Back up

PQM model

