Four-quark scatterings and quasi-PDA for pion from fRG

Wei-jie Fu

Dalian University of Technology

QuantFunc2024, Valencia, Sep. 2-6, 2024

Based on:
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, '*Four-quark scatterings in QCD I*', SciPost Phys. 14 (2023) 069, arXiv:2209.13120;
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, '*Four-quark scatterings in QCD II*', arXiv:2401.07638;
WF, Chuang Huang, Jan M. Pawlowski, Yang-yang Tan, Li-jun Zhou, '*Four-quark scatterings in QCD III*', in preparation;
Lei Chang, WF, Chuang Huang, Jan M. Pawlowski, Dao-yu Zhang, '*Quasi-parton distributions for pion from fRG*', in preparation.

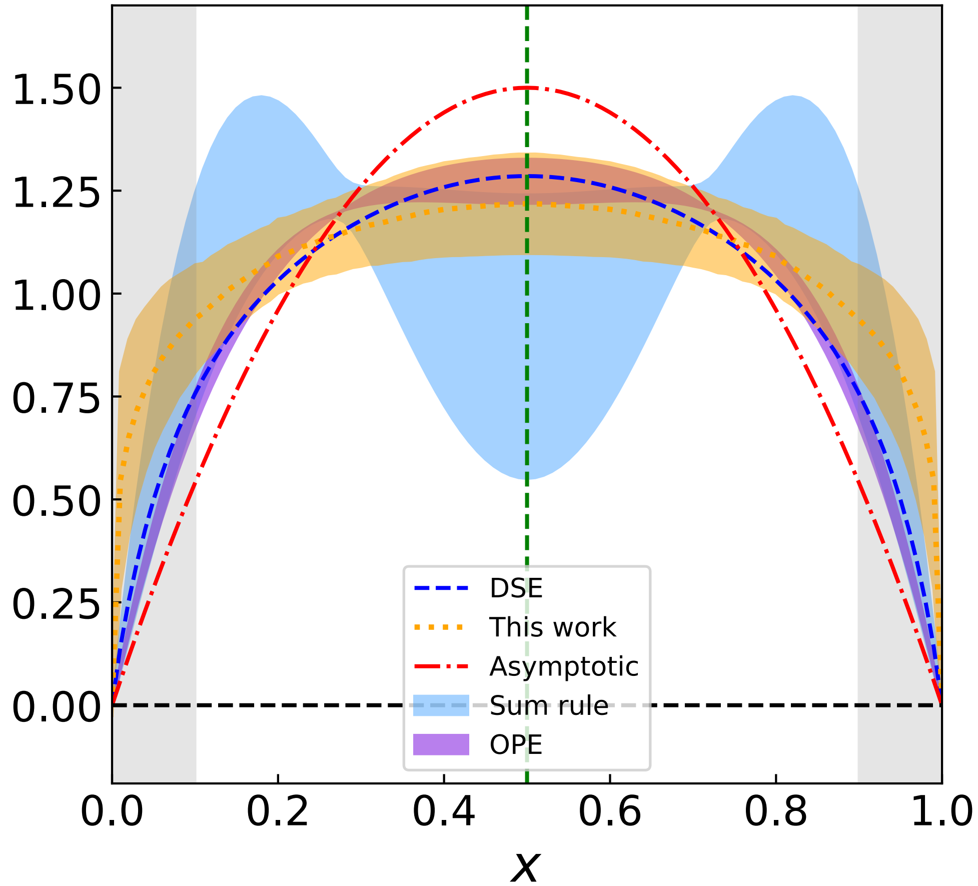
fQCD collaboration:
Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

Basic questions in nuclear physics

Mass generation

Image from BNL website

Distribution amplitudes for pion



Lattice: J. Hua *et al.* (LPC), *PRL* 129 (2022) 132001;
DSE: C. Roberts *et al.*, *PPNP* 120 (2021) 103883;
Sum rules: P. Ball *et al.*, *JHEP* 08 (2007) 090;
OPE: G. Bali *et al.* (RQCD), *JHEP* 08 (2019) 065; 11 (2020) 37.

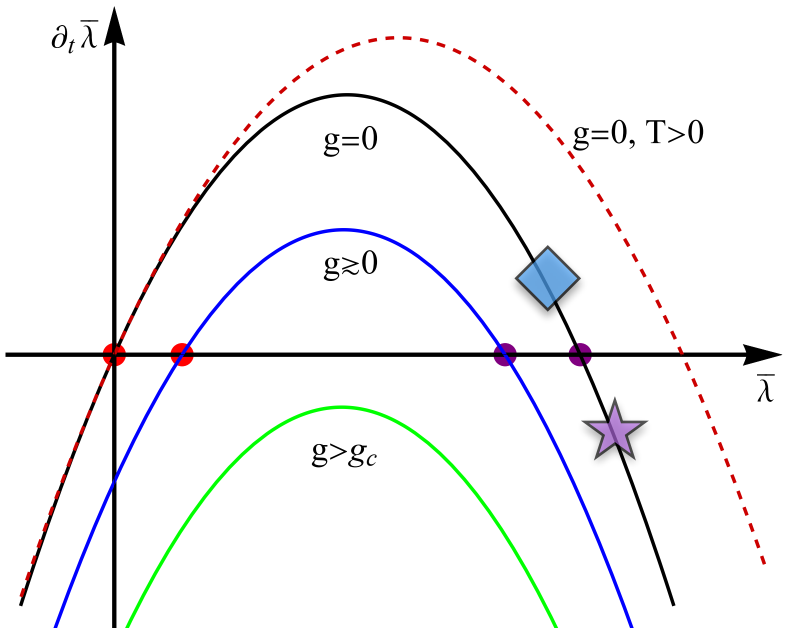
- How can we understand mass generation and hadron structure from first-principles QCD?

Outline

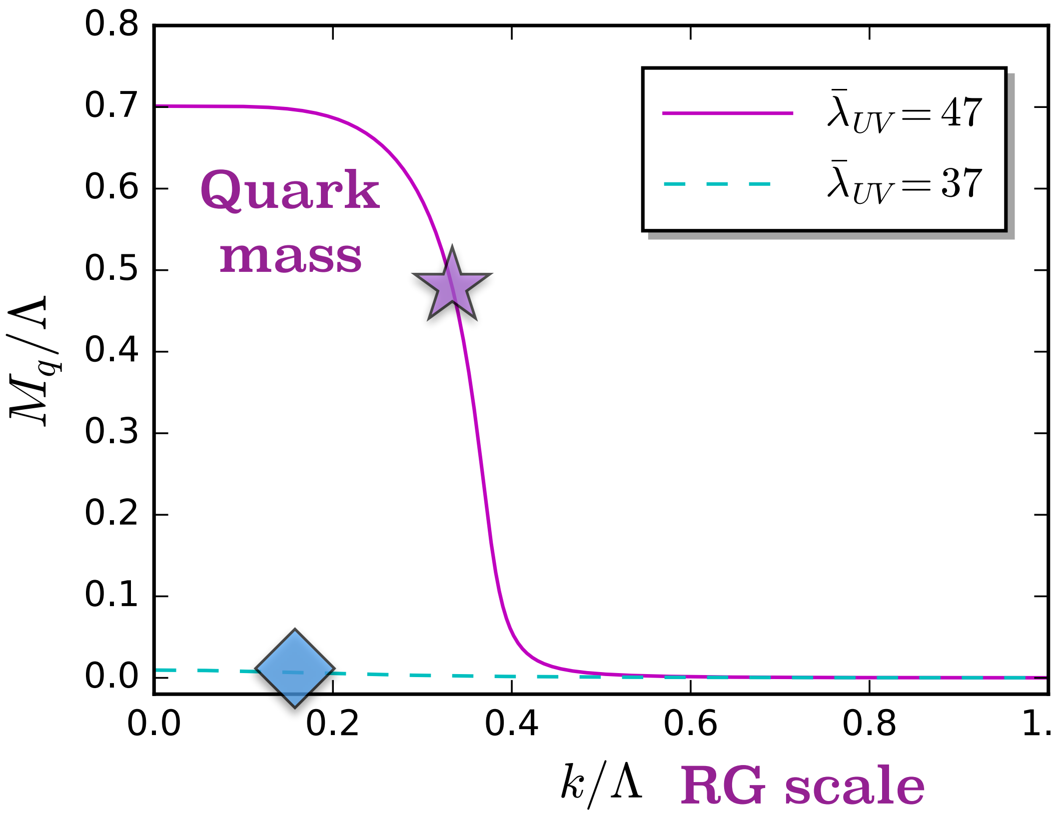
* Introduction
* Four-quark scatterings in LEFT and QCD
* Pion quasi-parton distribution amplitudes
* Summary and outlook

Chiral symmetry breaking in RG

- β **function of 4-quark coupling:**

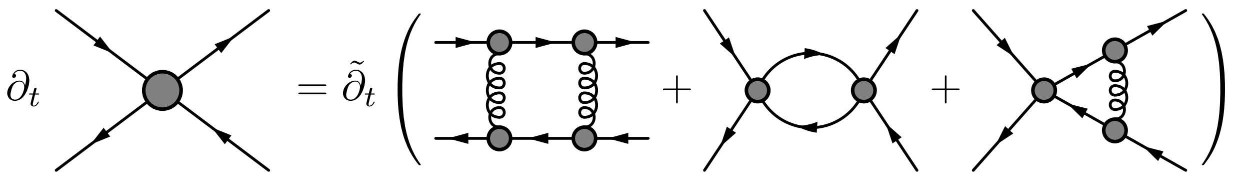


Braun, Gies, *JHEP* 06 (2006) 024.



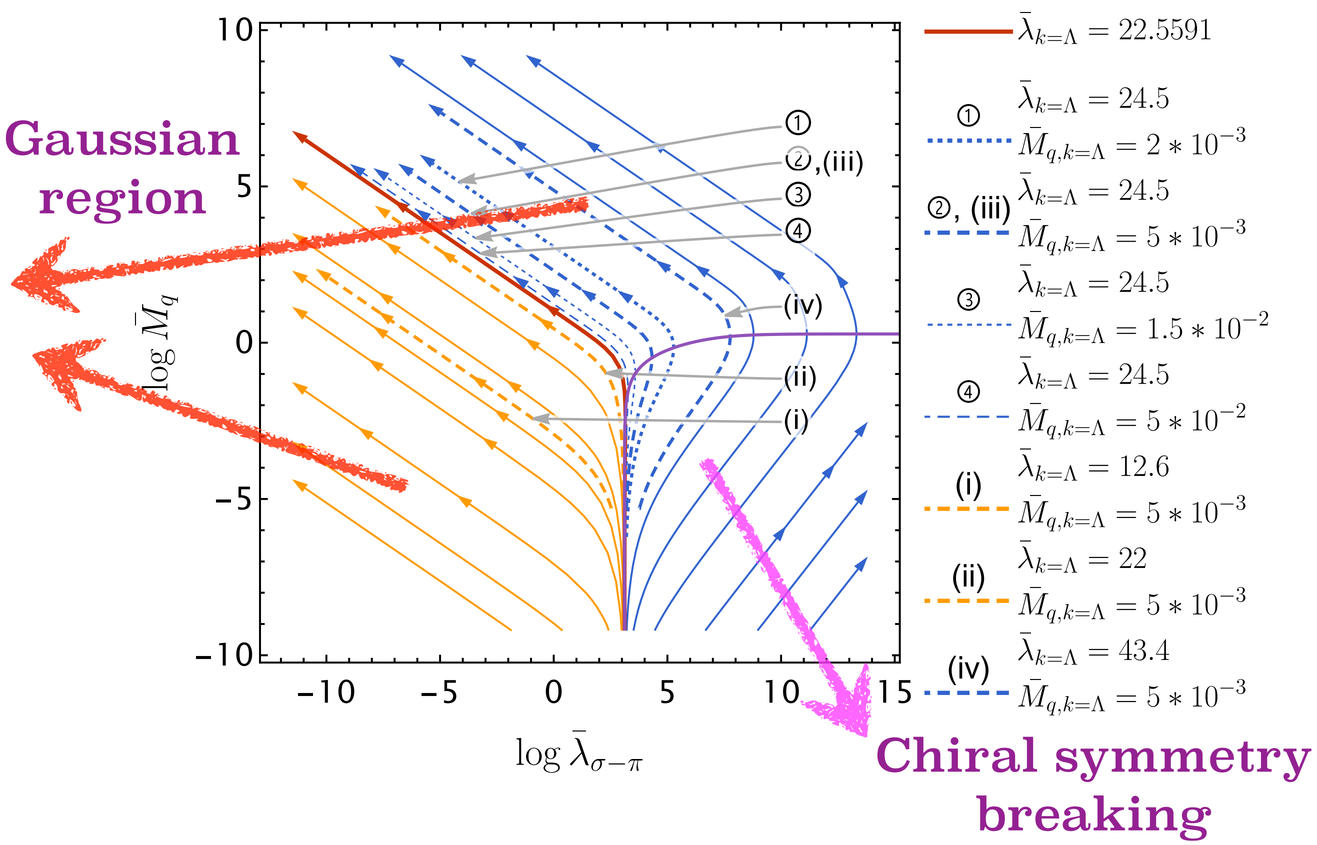
WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

$$\partial_t \bar{\lambda} = (d-2)\bar{\lambda} - a\bar{\lambda}^2 - b\bar{\lambda}g^2 - cg^4 \,,$$

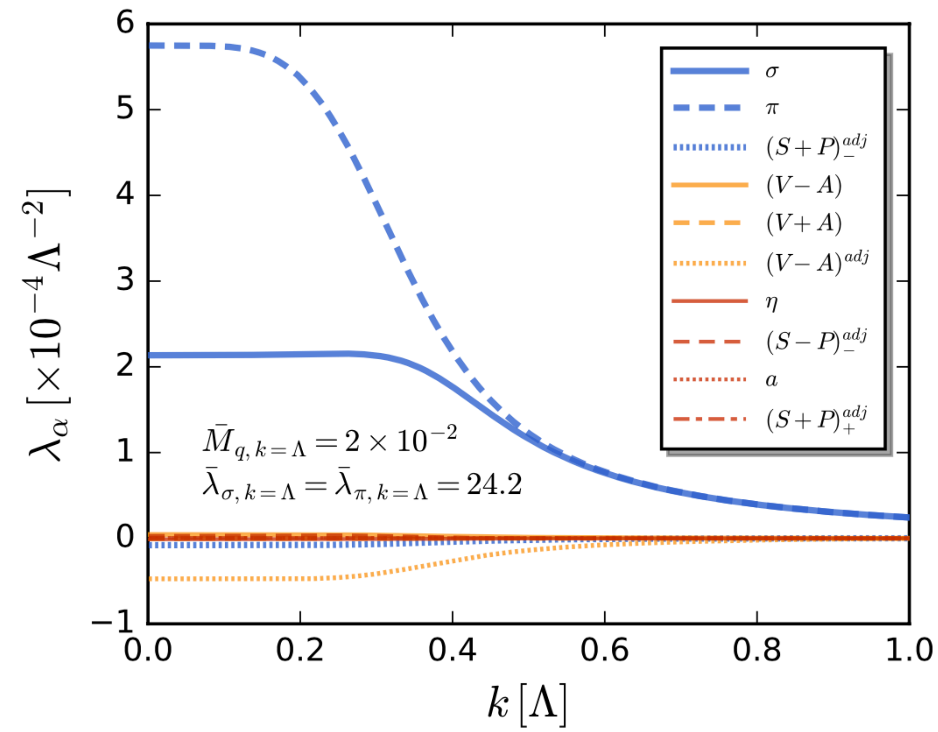


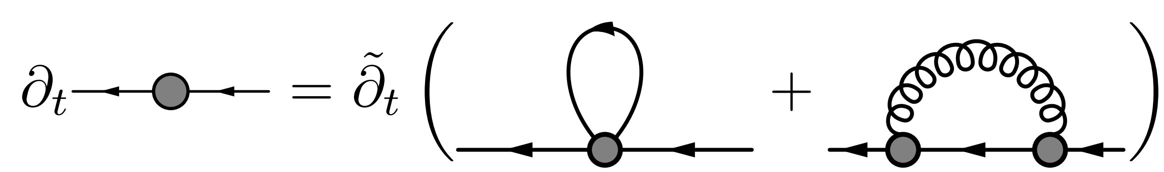
Mass generation in RG

- flow in the plane of the mass and coupling:

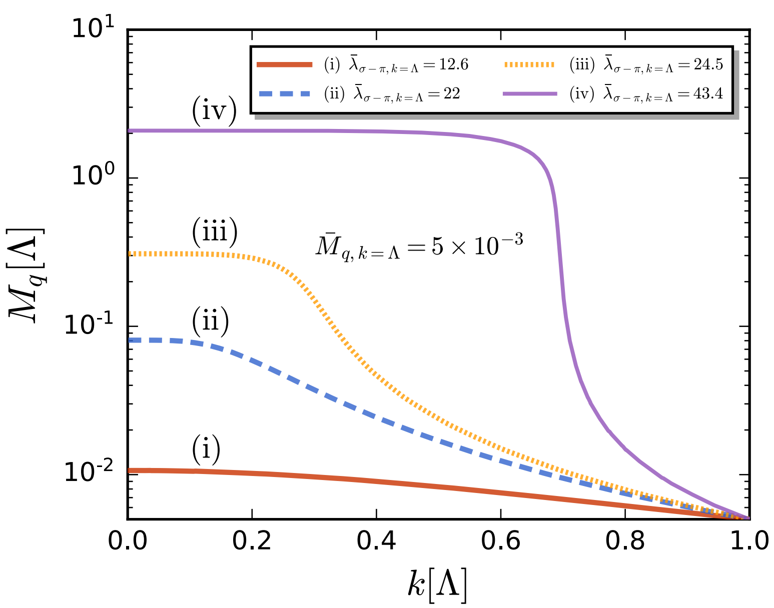


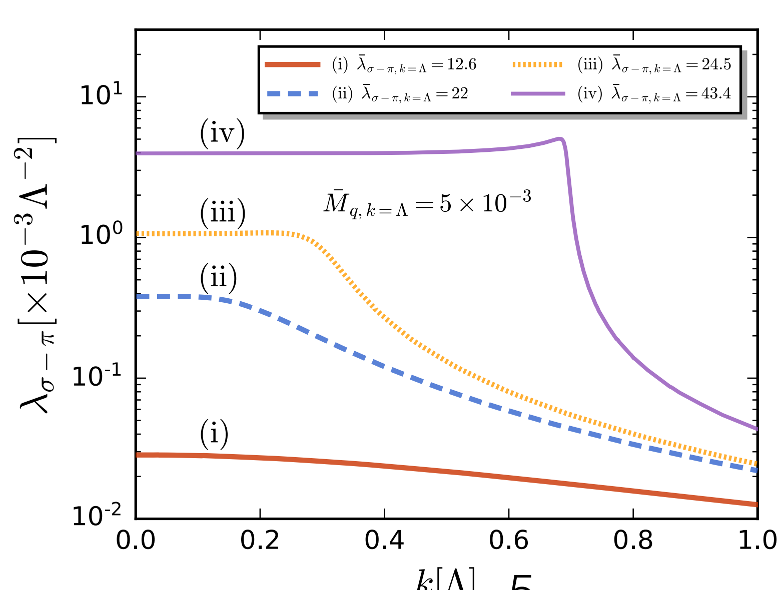
- couplings of different channels vs RG scale:





- quark mass and couplings vs RG scale:



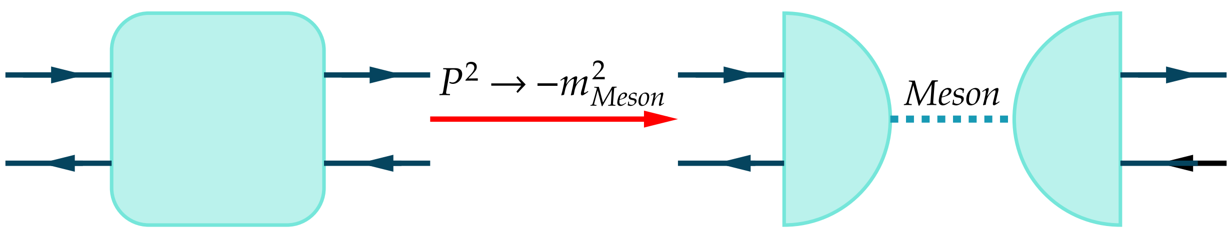


- Understanding quark mass production from the viewpoint of phase transition.
- Analogue of gap equation in terms of RG flow.

WF, Huang, Pawlowski, Tan, *SciPost Phys.* 14 (2023) 069, arXiv:2209.13120

Bound states in RG

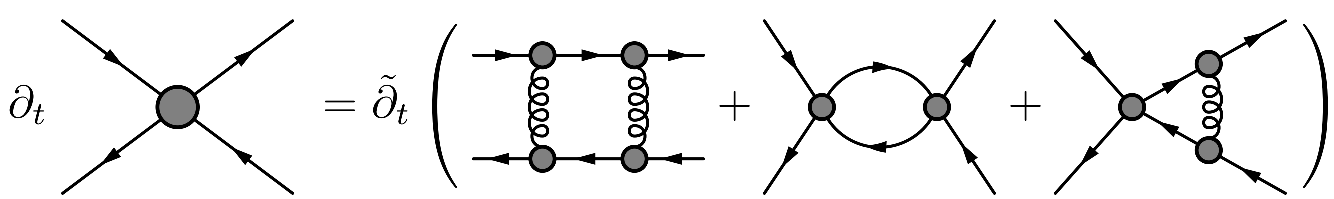
- **Bound states encoded in n-point correlation functions:**



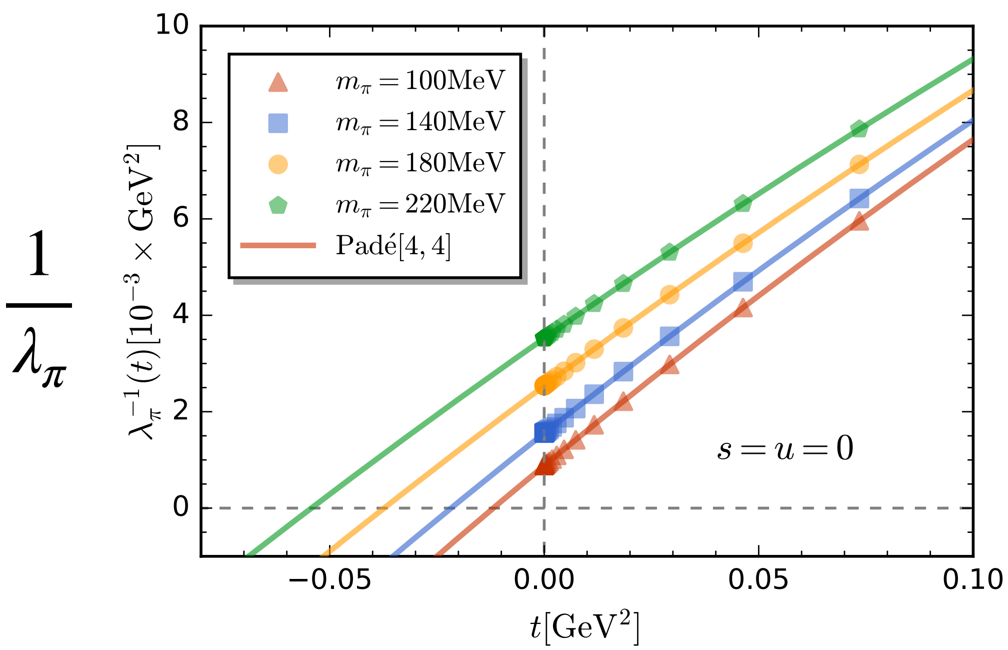
$$\partial_t \lambda_{\pi,k}(P^2) = C_k(P^2)\lambda^2_{\pi,k}(P^2) + A_k(P^2),$$

$$\lambda_{\pi,k=0}(P^2) = \frac{\lambda_{\pi,k=\Lambda}}{1 - \lambda_{\pi,k=\Lambda}\int_\Lambda^0 C_k(P^2)\frac{dk}{k}},$$

- **Flow equation of 4-quark interaction:**

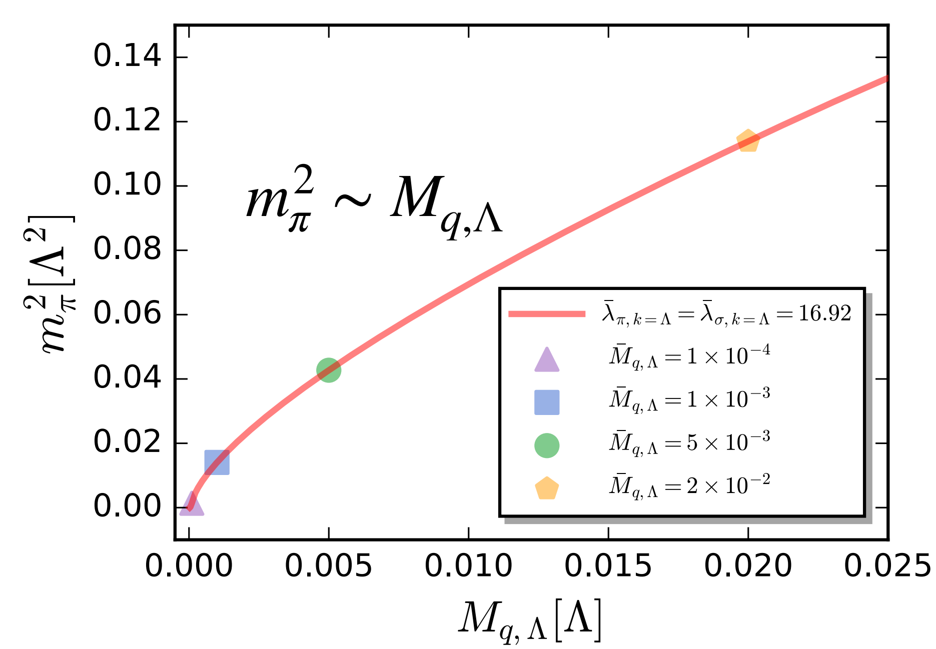


Note: playing the same role as the Bethe-Salpeter equation.



WF, Huang, Pawlowski, Tan, arXiv:2401.07638

Gell-Mann--Oakes--Renner relation



Four-quark vertices

- 4-quark effective action:

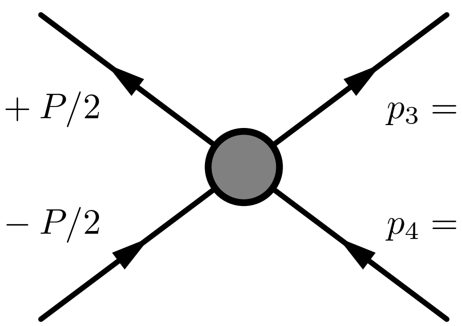
$$\Gamma_{4q,k} = -\int \frac{d^4p_1}{(2\pi)^4}\cdots\frac{d^4p_4}{(2\pi)^4}(2\pi)^4\delta(p_1+p_2+p_3+p_4)$$
$$\times \sum_\alpha \lambda_\alpha(\boldsymbol{p})\,\mathcal{T}^{(\alpha)}_{ijlm}(\boldsymbol{p})\,\bar{q}_i(p_1)q_j(p_2)\bar{q}_l(p_3)q_m(p_4)\,,$$

With $\boldsymbol{p}=(p_1,p_2,p_3,p_4)$, $\mathcal{T}^{(\alpha)}(\boldsymbol{p})$ is comprised of 512 tensors. Eichmann, *PRD* 84 (2011) 014014
A basis of the lowest momentum-independent order includes ten elements

$$\alpha \in \Big\{\sigma\,,\pi\,,a\,,\eta\,,(V\pm A)\,,(V-A)^{\text{adj}}\,,$$
$$(S\pm P)^{\text{adj}}_-\,,(S+P)^{\text{adj}}_+\Big\}\,,$$

- 4-quark vertex:

$$\Gamma^{(4)}_{\bar{q}_iq_j\bar{q}_lq_m}(\boldsymbol{p}) = \frac{\delta^4\Gamma_k[q,\bar{q}]}{\delta\bar{q}_i(p_1)\delta q_j(p_2)\delta\bar{q}_l(p_3)\delta q_m(p_4)}$$
$$= -4\,(2\pi)^4\delta(p_1+p_2+p_3+p_4)$$
$$\times\sum_\alpha\Big[\lambda^+_\alpha(\boldsymbol{p})\mathcal{T}^{(\alpha^-)}_{ijlm}+\lambda^-_\alpha(\boldsymbol{p})\mathcal{T}^{(\alpha^+)}_{ijlm}\Big]$$



where we have used 4-quark dressings and tensor structures with definite symmetries, viz.,

$$\lambda^\pm_\alpha(\boldsymbol{p}) \equiv \frac{1}{2}\Big[\lambda_\alpha(p_1,p_2,p_3,p_4)\pm\lambda_\alpha(p_3,p_2,p_1,p_4)\Big]\,,$$

and

$$\mathcal{T}^{(\alpha^\pm)}_{ijlm}\equiv\frac{1}{2}\big(\mathcal{T}^{(\alpha)}_{ijlm}\pm\mathcal{T}^{(\alpha)}_{ljim}\big)$$

with the symmetry relations

$$\lambda^+_\alpha(p_1,p_2,p_3,p_4) = \lambda^+_\alpha(p_3,p_2,p_1,p_4)$$
$$= \lambda^+_\alpha(p_1,p_4,p_3,p_2) = \lambda^+_\alpha(p_3,p_4,p_1,p_2)\,,$$
$$\lambda^-_\alpha(p_1,p_2,p_3,p_4) = -\,\lambda^-_\alpha(p_3,p_2,p_1,p_4)$$
$$= -\lambda^-_\alpha(p_1,p_4,p_3,p_2) = \lambda^-_\alpha(p_3,p_4,p_1,p_2)$$

and similar relations for the tensors.

s, t, u-channel truncation

- s, t, u-channel approximation for 4-quark vertices:

$$\lambda_\alpha^\pm(p_1,p_2,p_3,p_4) = \lambda_\alpha^\pm(s,t,u) + \Delta\lambda_\alpha^\pm(p_1,p_2,p_3,p_4) \approx \lambda_\alpha^\pm(s,t,u)$$

with

$$t = (p_1 - p_2)^2 = P^2\,,$$
$$u = (p_1 - p_4)^2 = (\bar{p} - \bar{p}')^2\,,$$
$$s = (p_1 + p_3)^2 = (\bar{p} + \bar{p}')^2$$

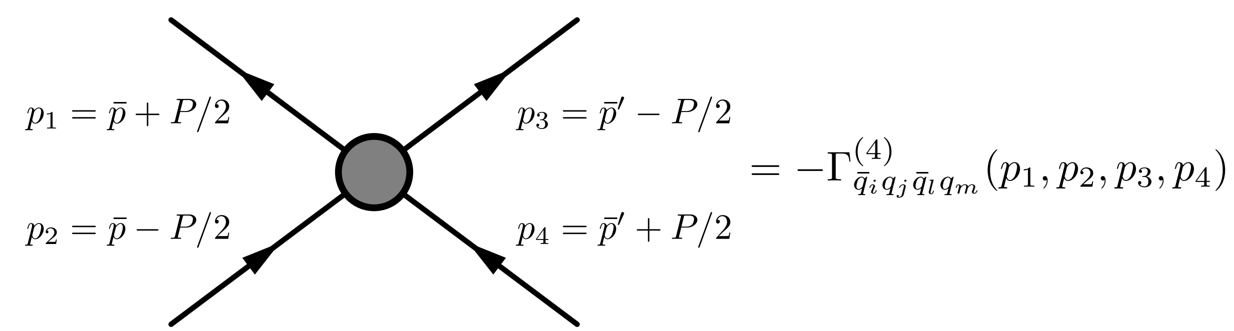
- We choose a subspace of the full momentum of 4-quark vertices as follows

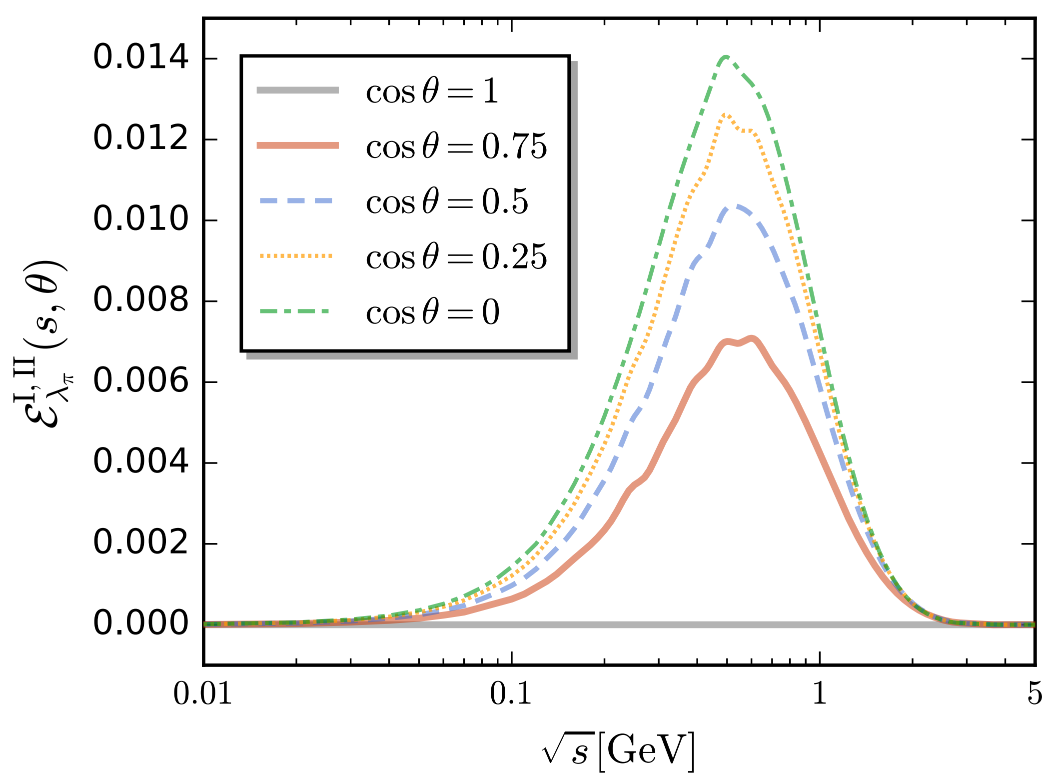
$$P_\mu = \sqrt{P^2}\,\Big(1,0,0,0\Big)\,,$$
$$\bar{p}_\mu = \sqrt{p^2}\,\Big(1,0,0,0\Big)\,,$$
$$\bar{p}'_\mu = \sqrt{p^2}\,\Big(\cos\theta,\ \sin\theta,0,0\Big)$$

one is led to

$$t = P^2\,, \quad u = 2p^2(1-\cos\theta)\,, \quad s = 2p^2(1+\cos\theta)$$

Here, $\{\sqrt{P^2}, \sqrt{p^2}, \cos\theta\}$ is in one-by-one correspondence with respect to $\{t, u, s\}$



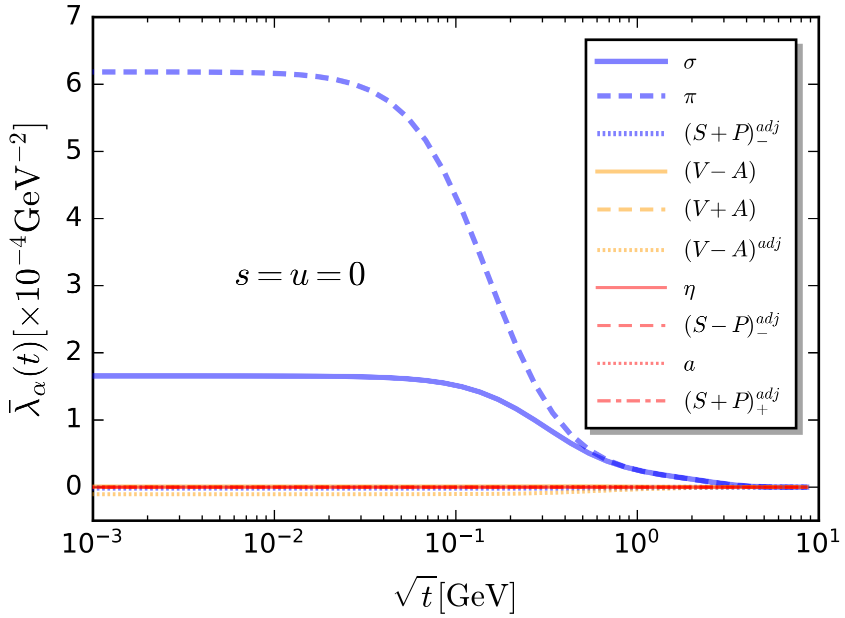


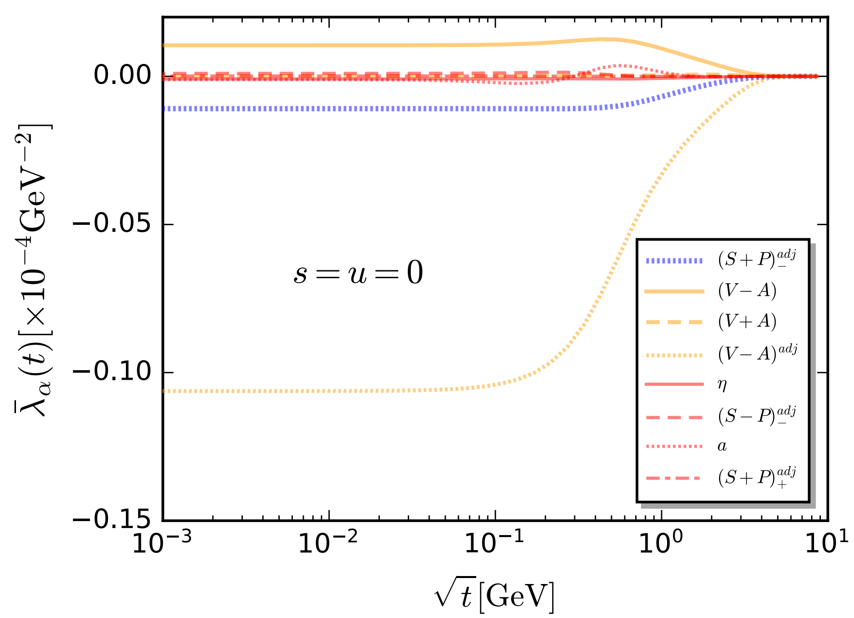
WF, Huang, Pawlowski, Tan, arXiv:2401.07638

The error for the truncation is smaller than 1.5%

Four-quark dressings

Dressings of different tensors:





Symmetry relations

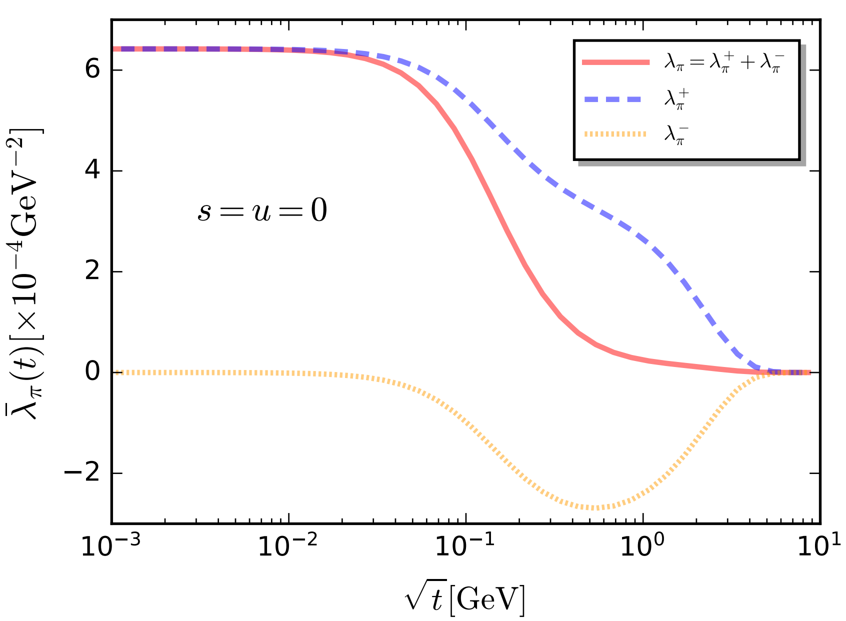
$$\lambda^+_\alpha(s,t,u) = \lambda^+_\alpha(s,u,t)\,,$$
$$\lambda^-_\alpha(s,t,u) = -\lambda^-_\alpha(s,u,t)$$

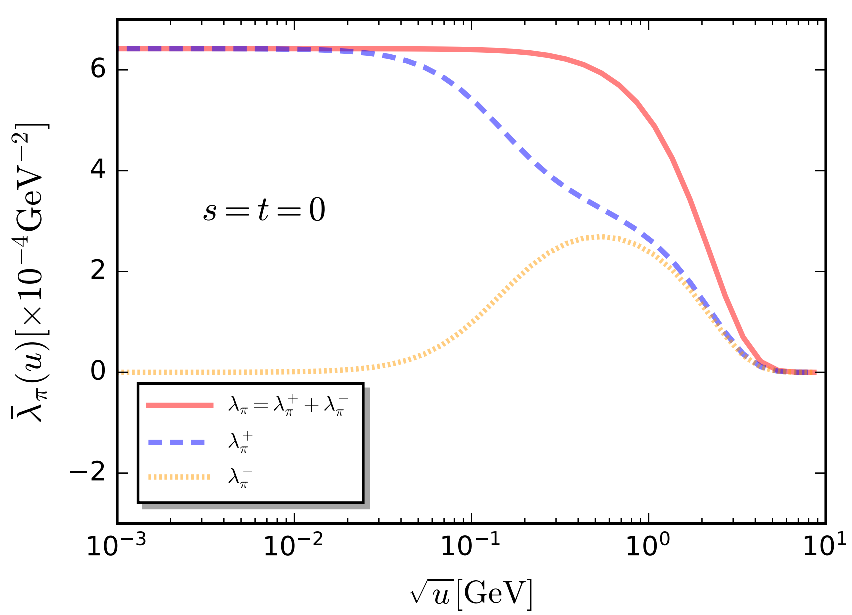
then

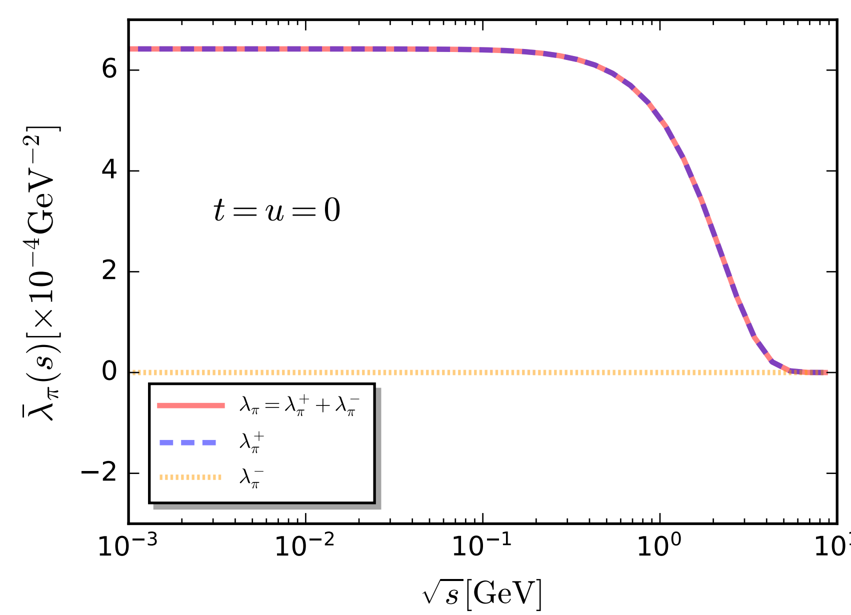
$$\lambda^+_\alpha(0,p^2,0) = \lambda^+_\alpha(0,0,p^2)\,,$$
$$\lambda^-_\alpha(0,p^2,0) = -\lambda^-_\alpha(0,0,p^2)$$

Pion channel:

WF, Huang, Pawlowski, Tan, arXiv:2401.07638

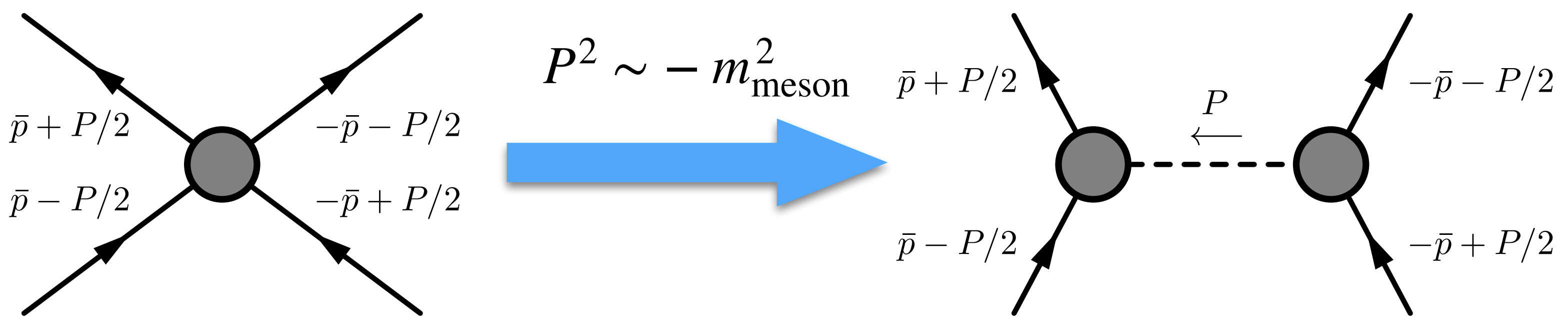


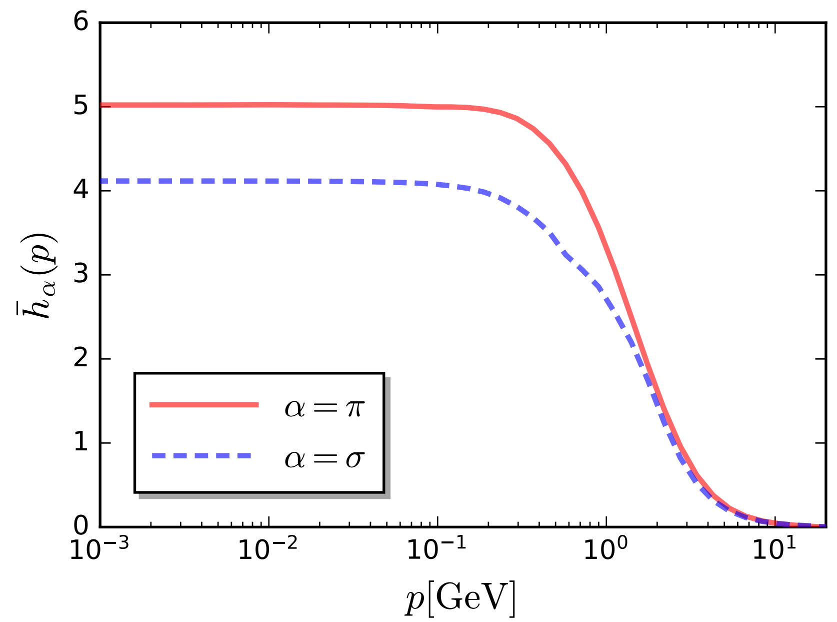




Bethe-Salpeter amplitude

- **Bethe-Salpeter amplitude can be extracted from the 4-quark vertex in the proximity of on-shell momentum of bound states:**





The 4-quark vertex near the on-shell momentum

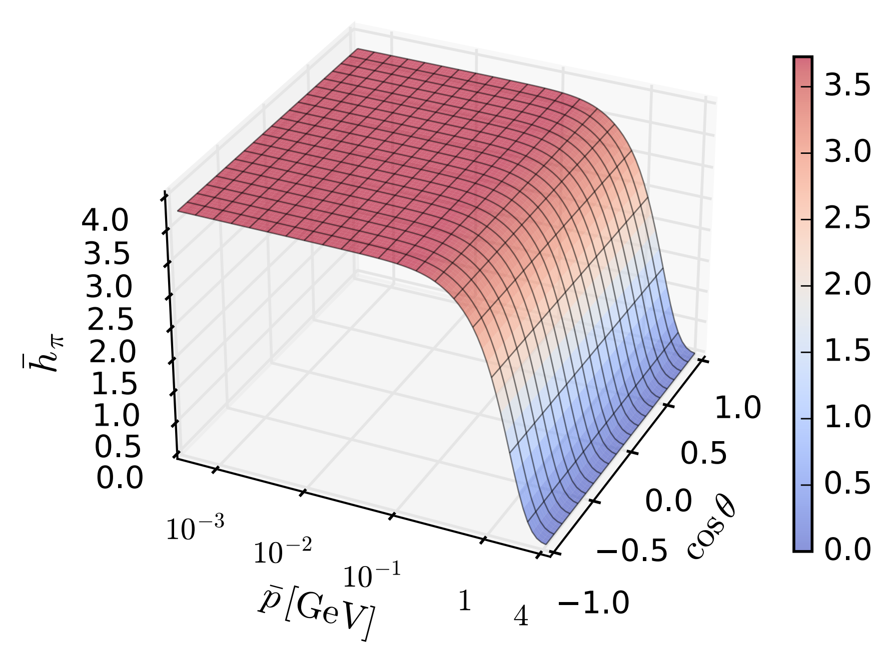
$$\lambda_\pi(P^2, p, \cos\theta) \sim \frac{h_\pi^2(p,\cos\theta)}{P^2+m_\pi^2}$$

with the BS amplitude

$$h_\pi(p,\cos\theta) = \lim_{P^2\to -m_\pi^2}\left[\lambda_\pi(P^2,p,\cos\theta)\,(P^2+m_\pi^2)\right]^{1/2}$$

and

$$P_\mu = \sqrt{P^2}\,\Big(1,0,0,0\Big)$$
$$\bar{p}_\mu = -\bar{p}'_\mu = \sqrt{p^2}\,\Big(\cos\theta,\ \sin\theta,0,0\Big)$$



WF, Huang, Pawlowski, Tan, arXiv:2401.07638

Pion decay constant

The pion weak decay constant is defined as

$$\langle 0 | J^a_{5\mu}(x) | \pi^b \rangle = i P_\mu f_\pi \delta^{ab}$$

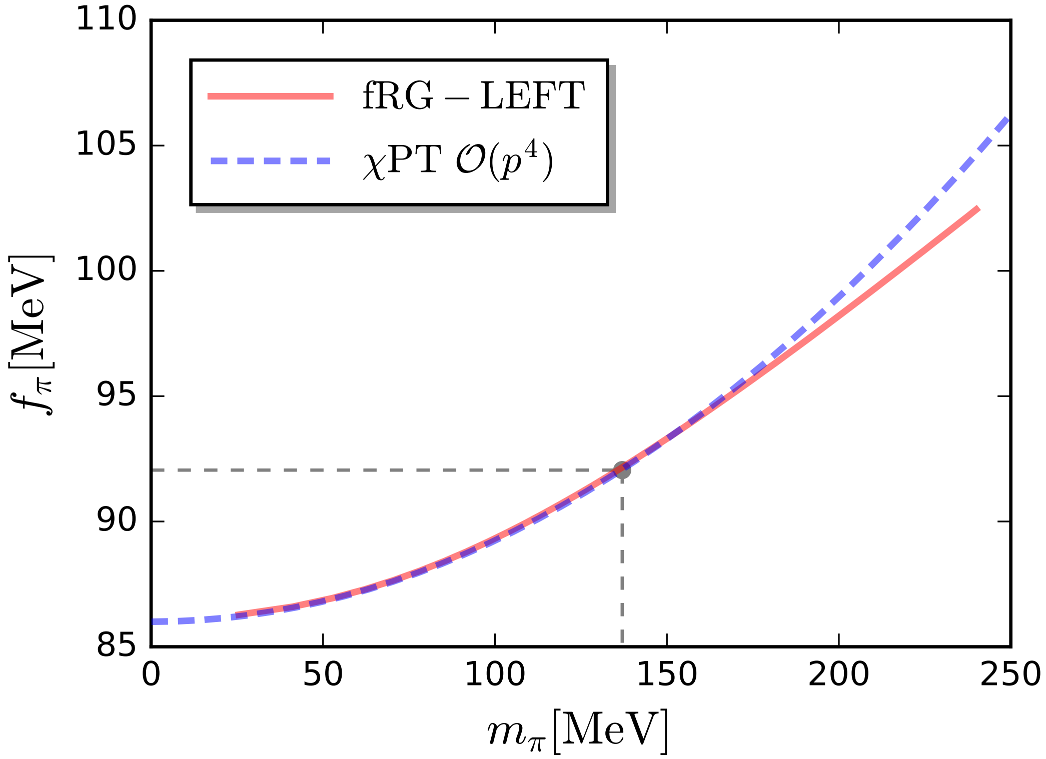
where the left hand side reads

$$\langle 0 | J^a_{5\mu}(x) | \pi^b \rangle$$
$$= \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr}\left[\gamma_\mu \gamma_5 T^a \, \bar{G}_q(q+P) \, \bar{h}_\pi(q) \, \gamma_5 T^b \, \bar{G}_q(q) \right],$$

then

$$f_\pi = 2N_c \int \frac{d^4 q}{(2\pi)^4} \frac{\bar{h}_\pi(q) M_q(q)}{\left[q^2 + M_q^2(q)\right]^2}$$

up to leading order in powers of $P^2 = -m_\pi^2$

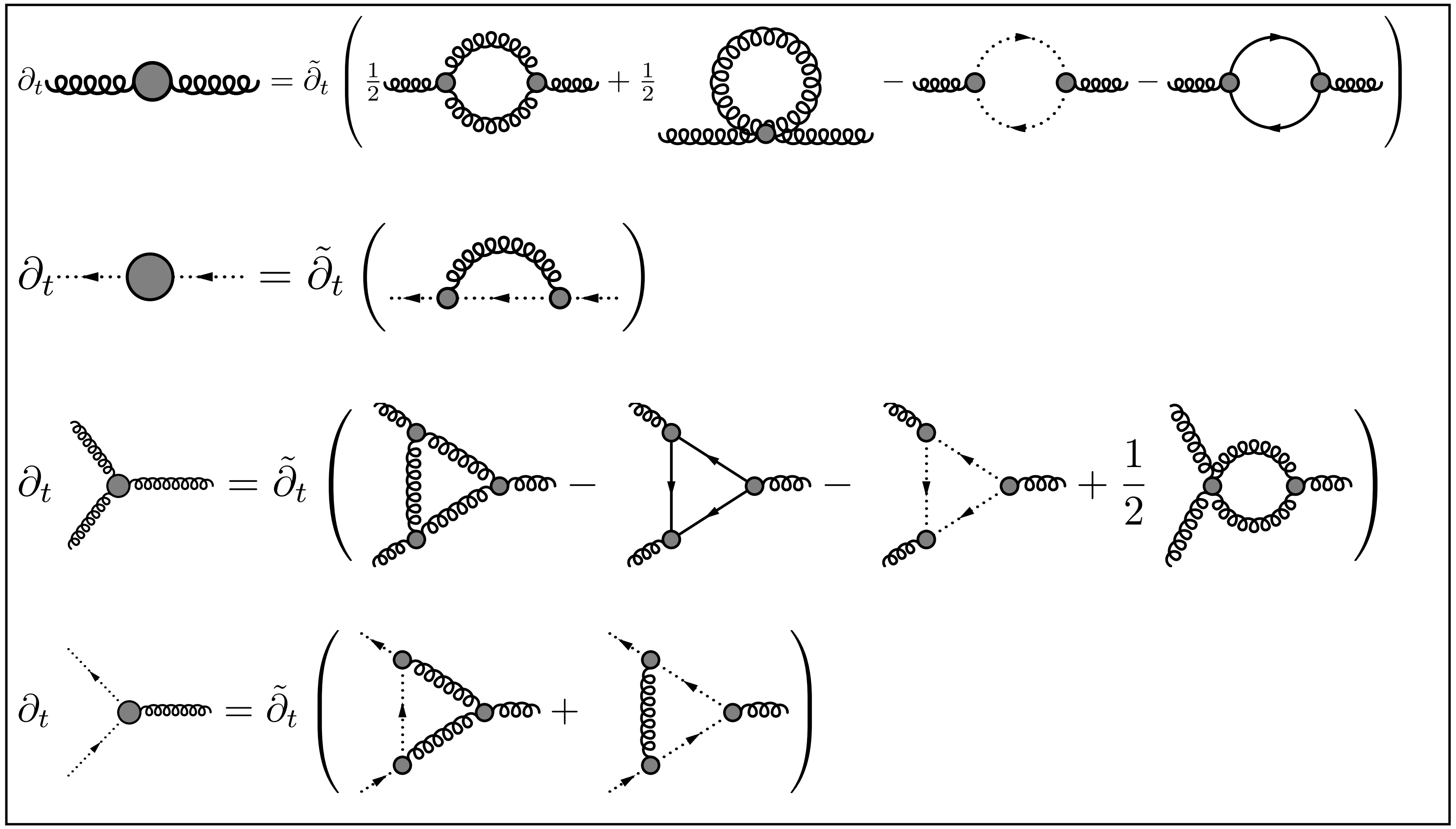


fRG: WF, Huang, Pawlowski, Tan, arXiv:2401.07638

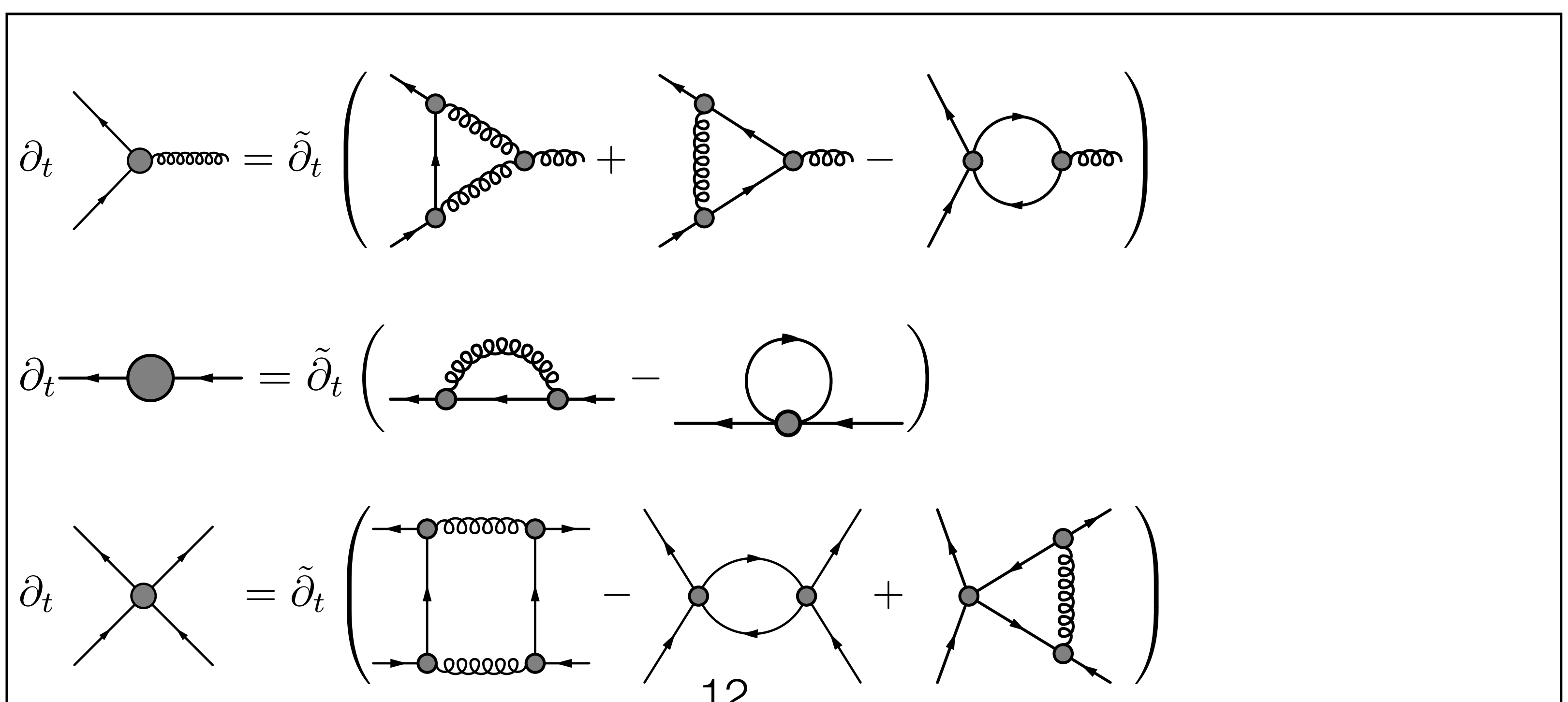
chiPT: Gasser and Leutwyler, *Annals Phys*. 158 (1984) 142

QCD within fRG

Glue sector:



Matter sector:



QCD with dynamical hadronization

Introducing a RG scale dependent composite field:

$$\hat{\phi}_k(\hat{\varphi}),\ \textbf{with}\ \hat{\varphi} = (\hat{A}, \hat{c}, \hat{\bar{c}}, \hat{q}, \hat{\bar{q}}),$$

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k\, \bar{q}\tau q + \dot{B}_k\, \phi + \dot{C}_k\, \hat{e}_\sigma\,,$$

Gies, Wetterich , *PRD* 65 (2002) 065001; 69 (2004) 025001; Pawlowski, *AP* 322 (2007) 2831; Flörchinger, Wetterich, *PLB* 680 (2009) 371

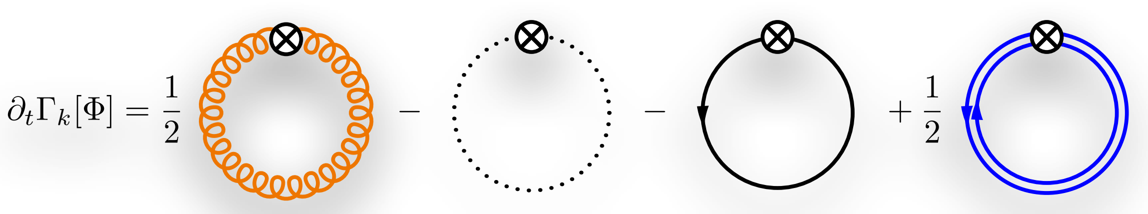
Wetterich equation is modified as

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\big(G_k[\Phi]\,\partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_a}[\Phi]\frac{\delta\langle\partial_t\hat{\phi}_k\rangle}{\delta\Phi_a}\,R_\phi\right)$$

$$-\int \langle \partial_t \hat{\phi}_{k,i}\rangle \left(\frac{\delta\Gamma_k[\Phi]}{\delta\phi_i} + c_\sigma \delta_{i\,\sigma}\right),$$

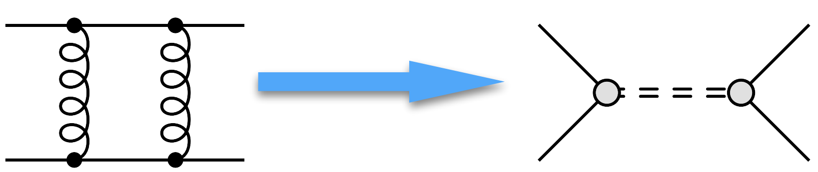
Mitter, Pawlowski, Strodthoff, *PRD* 91 (2015) 054035, arXiv:1411.7978; Braun, Fister, Pawlowski, Rennecke, *PRD* 94 (2016) 034016, arXiv:1412.1045; Cyrol, Mitter, Pawlowski, Strodthoff, *PRD* 97 (2018) 054006, arXiv:1706.06326; WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Flow equation:



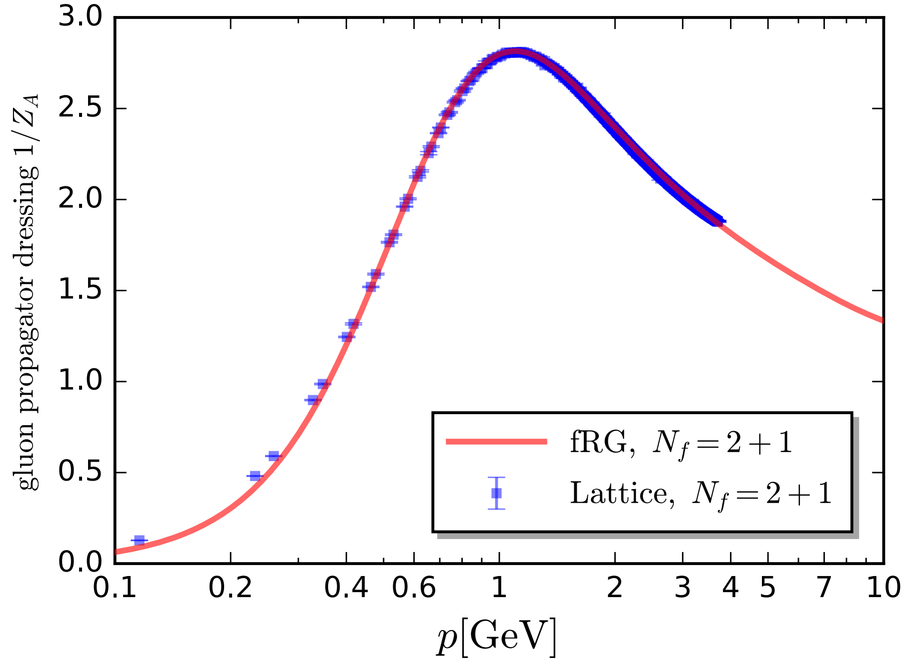
See also recent work:
Ihssen, Pawlowski, Sattler, Wink, arXiv:2408.08413

four-quark interaction encoded in Yukawa coupling:



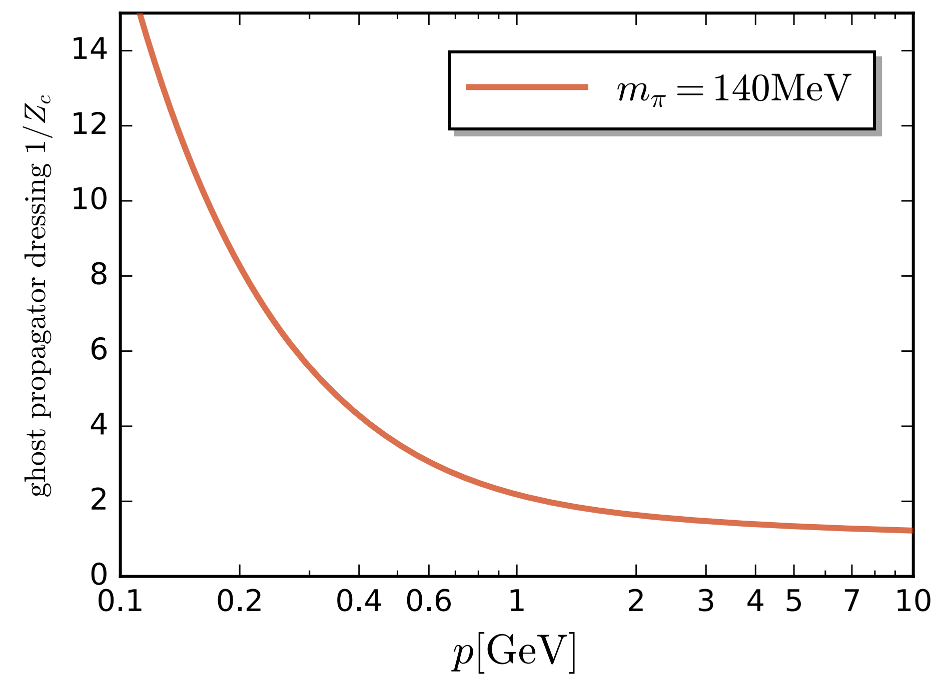
QCD within fRG in vacuum

Gluon dressing:



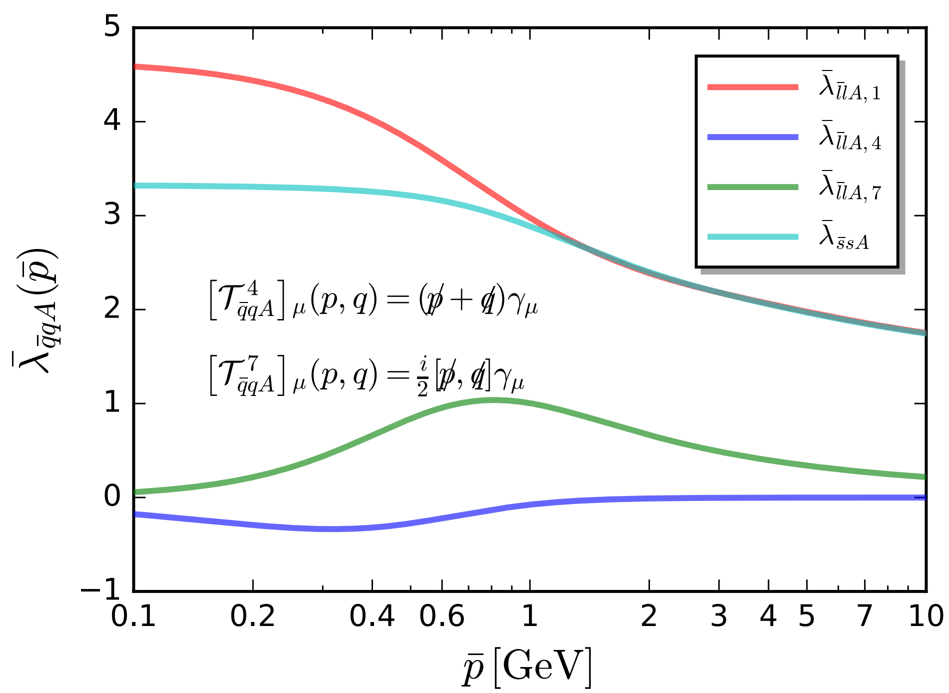
Lattice: Boucaud *et al.*, *PRD* 98 (2018) 114515

Ghost dressing:

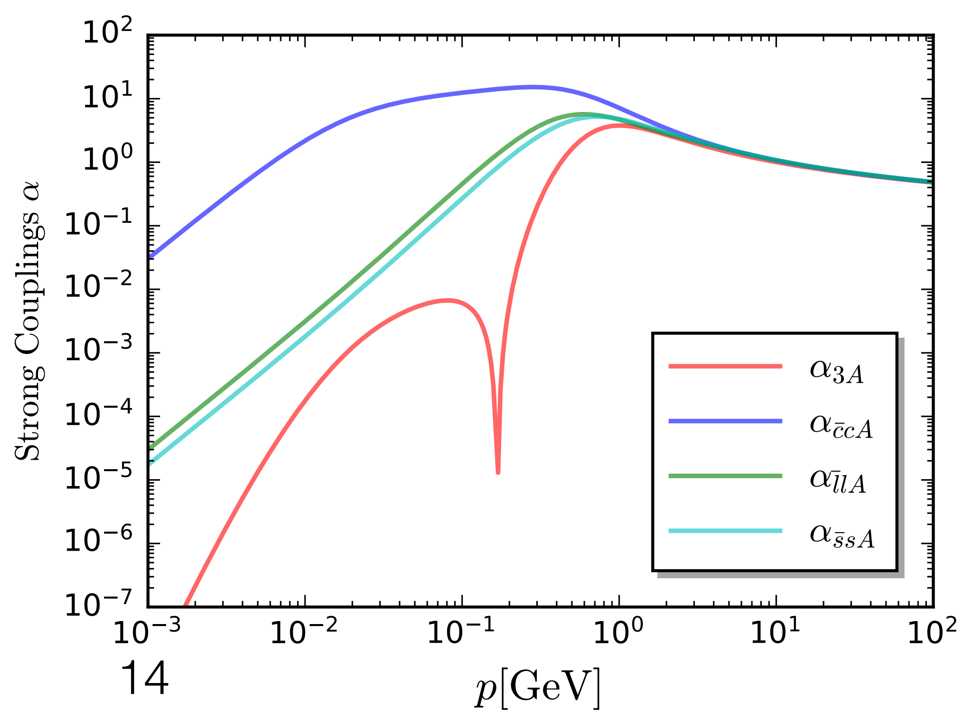


fRG: WF, Huang, Pawlowski, Tan, Zhou, in preparation

Quark-gluon vertex:

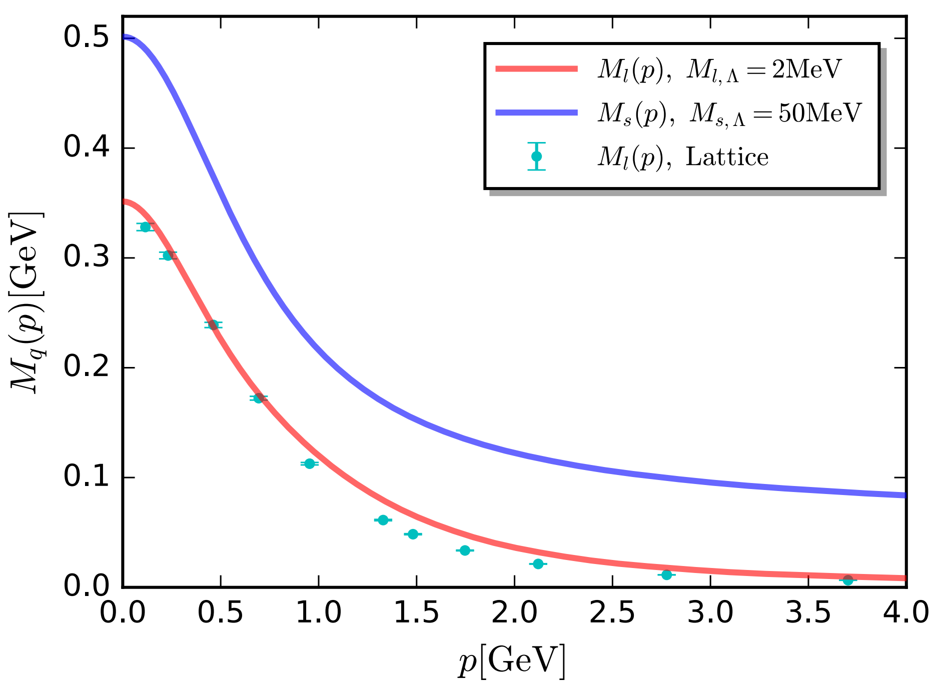


Strong couplings:



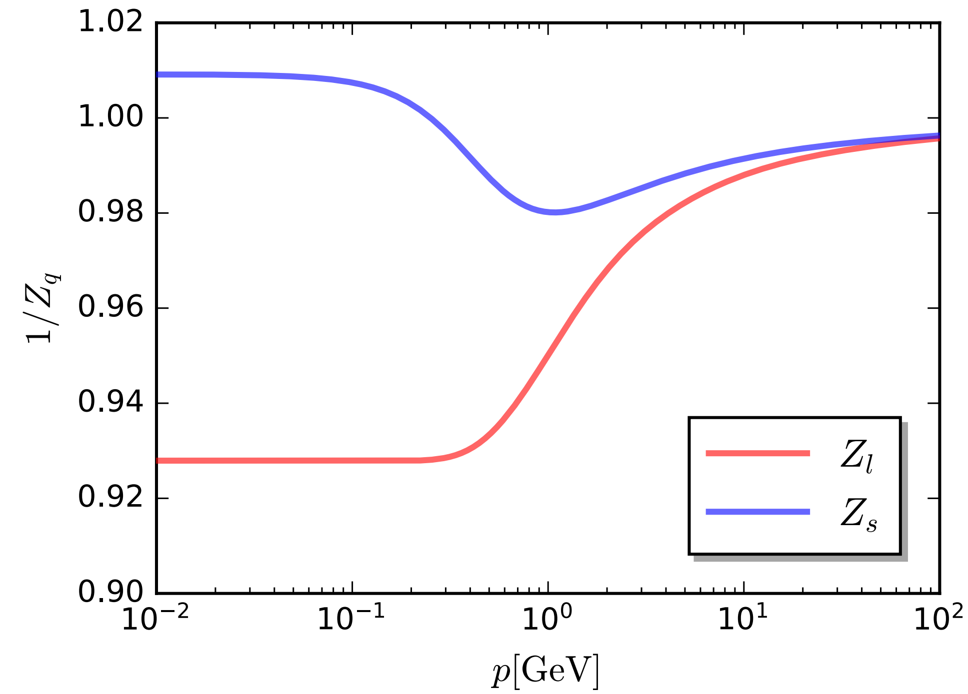
QCD within fRG in vacuum

Quark mass:

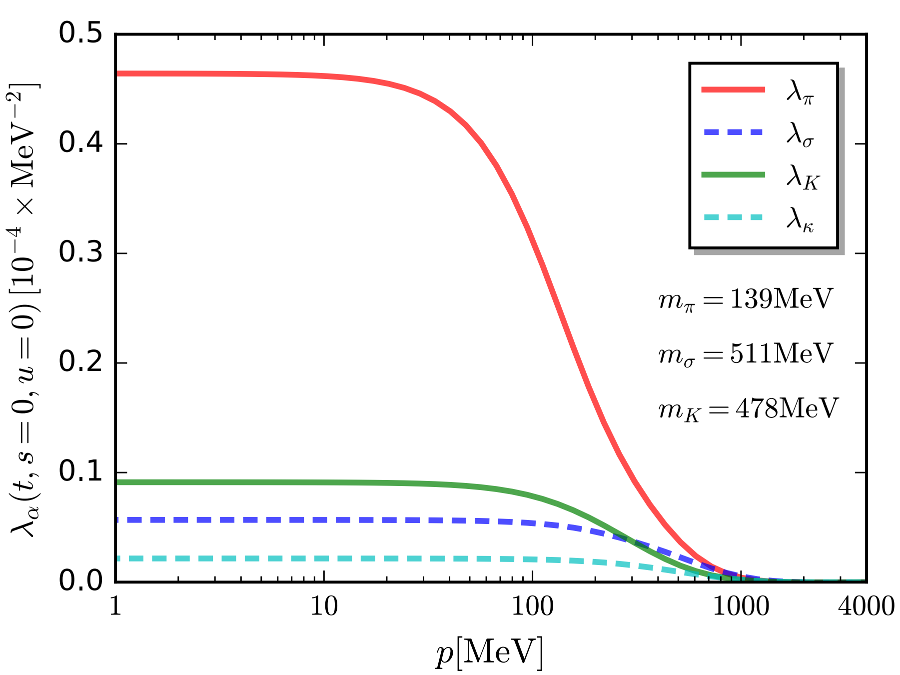


Lattice: Chang *et al.*, *PRD* 104 (2021) 094509

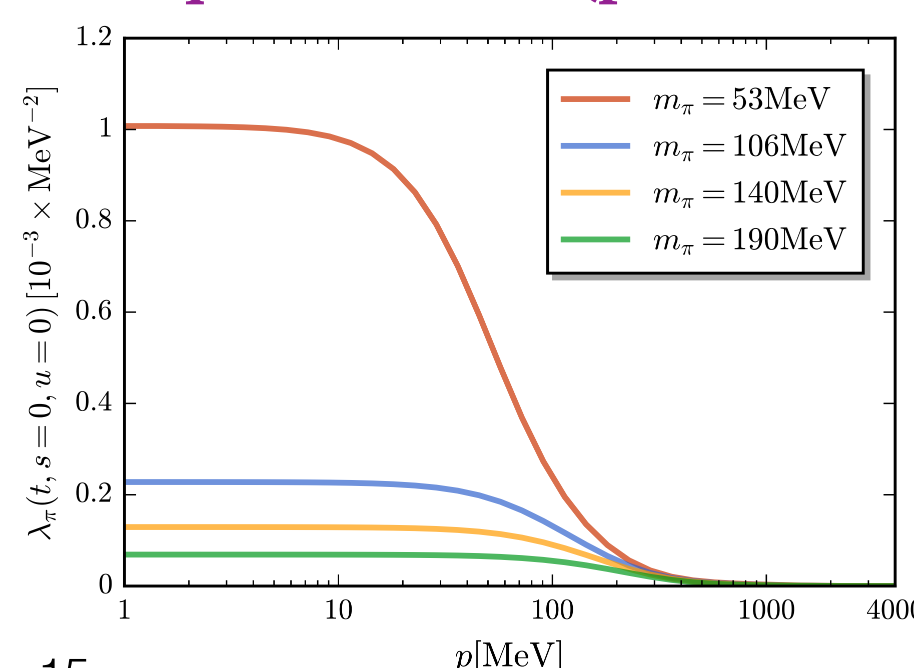
Quark wave function:



Four-quark vertex:



Four-quark vertex (pion channel):



fRG: WF, Huang, Pawlowski, Tan, Zhou, in preparation

Quasi-PDA of pion

- **Bethe-Salpeter amplitude** (unamputated):

$$\chi_\pi(k;P) = G_q(k_+)\Gamma(k;P)G_q(k_-)$$

with

$$\Gamma(k;P) = i\gamma_5 h_\pi(k;P)$$

and $P = (iE_\pi, P_z, 0, 0)$, $E_\pi = \sqrt{P_z^2 + m_\pi^2}$ and $k_\pm = k \pm P/2$

- **Quasi parton distribution amplitude** (qPDA) reads

$$\phi_\pi(x, P_z) = \frac{1}{f_\pi}\mathrm{Tr}_{\mathrm{CD}}\left[\int \frac{d^4k}{(2\pi)^4}\delta(\tilde{n}\cdot k_+ - x\tilde{n}\cdot P)\gamma_5\gamma\cdot\tilde{n}\chi_\pi(k;P)\right]$$

with $\tilde{n} = (0,1,0,0)$. Integrating k_3 firstly by using the delta function, one is led to

$$\phi_\pi(x, P_z) = \frac{1}{f_\pi}\frac{4N_c}{(2\pi)^4}\int d^2k_\perp dk_0\, h_\pi(k;P)P_z\left[xM_q(k_-^2) + (1-x)M_q(k_+^2)\right]$$
$$\times \frac{1}{Z_q(k_+^2)Z_q(k_-^2)}\frac{1}{k_+^2 + M_q^2(k_+^2)}\frac{1}{k_-^2 + M_q^2(k_-^2)}$$

$$k_\mu = \left(k_0, (x - 1/2)P_z, k_\perp\right),$$
$$k_{+\mu} = \left(k_0 + iE_\pi/2, xP_z, k_\perp\right),$$
$$k_{-\mu} = \left(k_0 - iE_\pi/2, (x-1)P_z, k_\perp\right)$$

Contour of k_0 integral

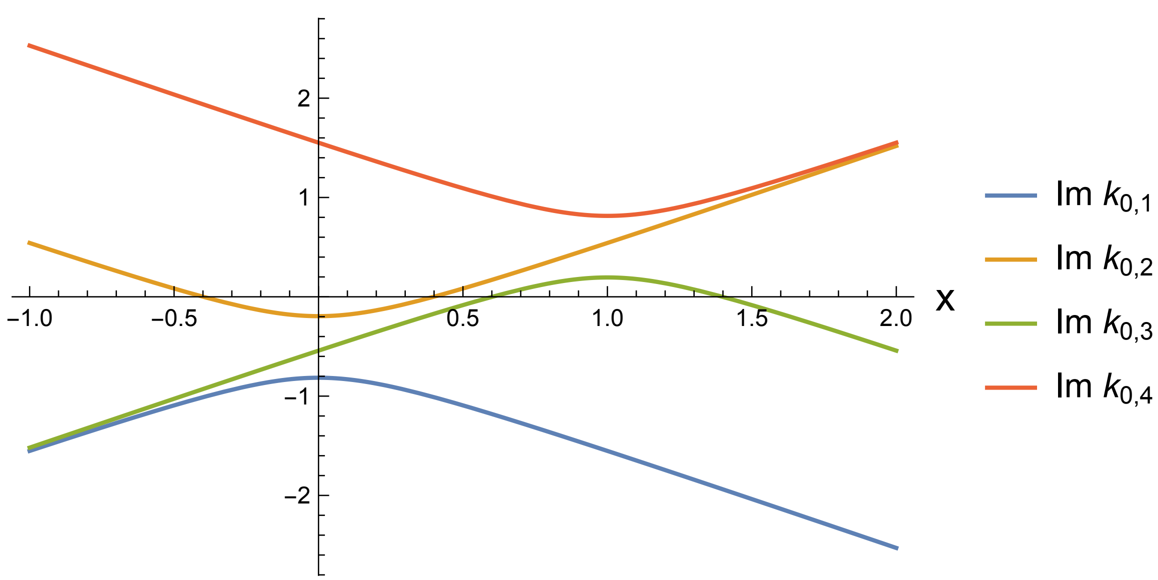
- **Poles of two quark propagators**:

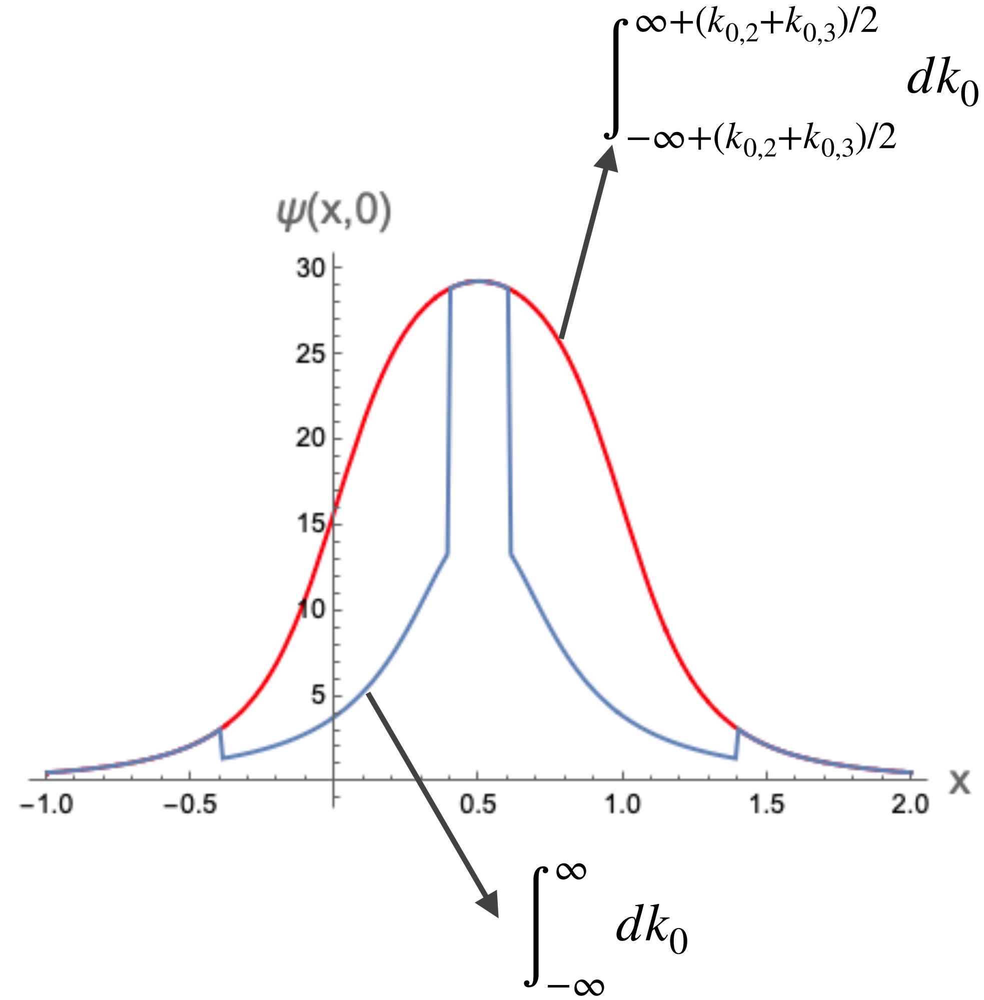
$$k_{0,1} = i\left[-\sqrt{k_\perp^2 + (xP_z)^2 + M_q^2(k_+^2)} - \sqrt{P_z^2 + m_\pi^2}/2\right],$$

$$k_{0,2} = i\left[\sqrt{k_\perp^2 + (xP_z)^2 + M_q^2(k_+^2)} - \sqrt{P_z^2 + m_\pi^2}/2\right],$$

$$k_{0,3} = i\left[-\sqrt{k_\perp^2 + (x-1)^2 P_z^2 + M_q^2(k_-^2)} + \sqrt{P_z^2 + m_\pi^2}/2\right],$$

$$k_{0,4} = i\left[\sqrt{k_\perp^2 + (x-1)^2 P_z^2 + M_q^2(k_-^2)} + \sqrt{P_z^2 + m_\pi^2}/2\right]$$





With the increase of P_z, $k_{0,2}$ or $k_{0,3}$ cross the x axis, one has to shift the integral of k_0 towards finite imaginary part, such that one can pick up the desired pair of poles, e.g., $k_{0,1}$ and $k_{0,3}$ or $k_{0,2}$ and $k_{0,4}$

Analytic continuation

- We use Taylor expansion to continue h_π, M_q, Z_q in the complex plane of k_0:

$$h_\pi(k^2, P^2, \cos\theta) = h_\pi(\bar{k}^2, P^2, \cos\theta) + \frac{\partial}{\partial k^2} h_\pi \bigg|_{k^2=\bar{k}^2} k_0^2 + \cdots$$

and

$$M_q(k_+^2) = M_q(\bar{k}_+^2) + \frac{\partial}{\partial k_+^2} M_q \bigg|_{k_+^2=\bar{k}_+^2} (k_0 + iE_\pi/2)^2 + \cdots$$

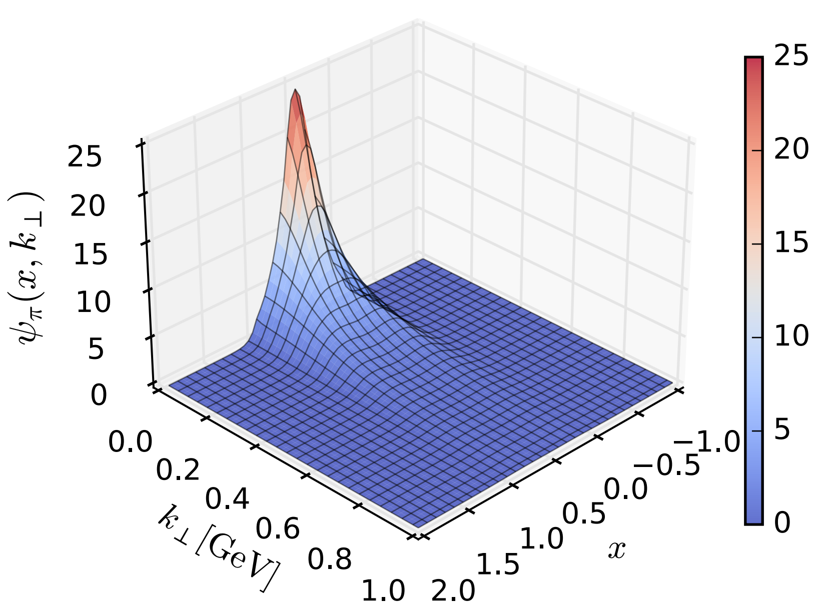
$$M_q(k_-^2) = M_q(\bar{k}_-^2) + \frac{\partial}{\partial k_-^2} M_q \bigg|_{k_-^2=\bar{k}_-^2} (k_0 - iE_\pi/2)^2 + \cdots$$

$$k^2 = \bar{k}^2 + k_0^2$$
$$\bar{k}^2 = k_\perp^2 + (x - 1/2)^2 P_z^2$$

$$k_+^2 = \bar{k}_+^2 + (k_0 + iE_\pi/2)^2$$
$$\bar{k}_+^2 = k_\perp^2 + x^2 P_z^2$$

$$k_-^2 = \bar{k}_-^2 + (k_0 - iE_\pi/2)^2$$
$$\bar{k}_-^2 = k_\perp^2 + (x - 1)^2 P_z^2$$

Pion wave function amplitudes:



Chang, WF, Huang, Pawlowski, Zhang, in preparation

Quasi-PDA and PDA

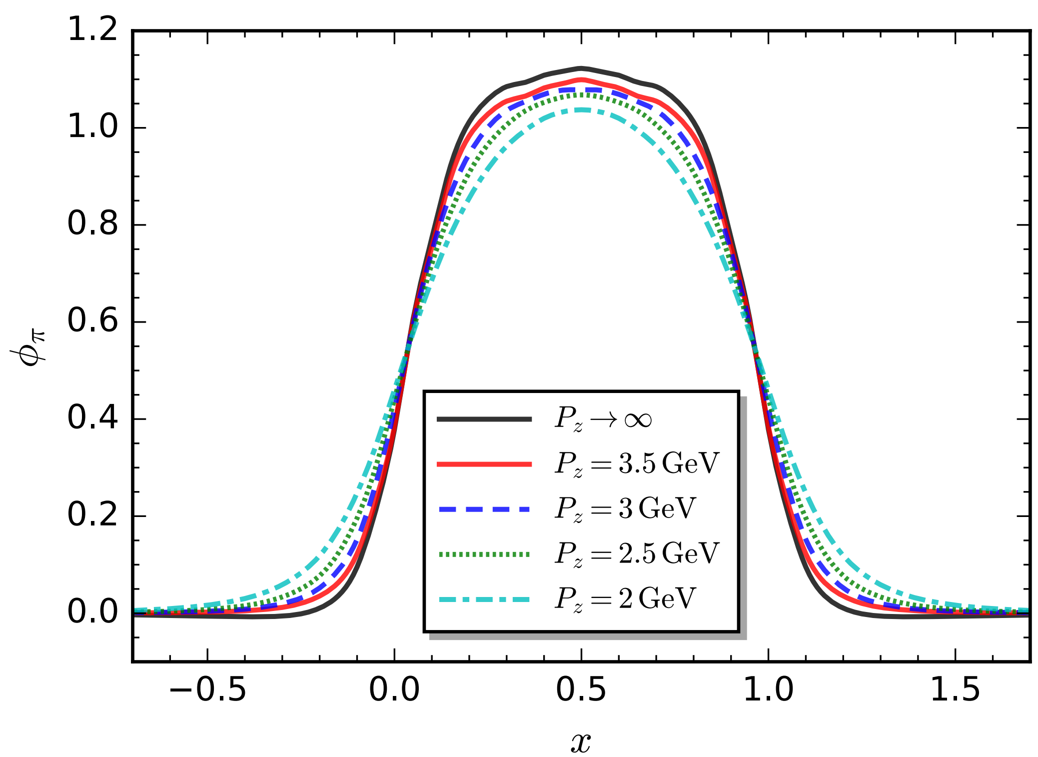
- Quasi-PDA at finite P_z can be used to extrapolate the PDA with $P_z \to \infty$ based on LaMET Ji, *PRL* 110 (2013) 262002

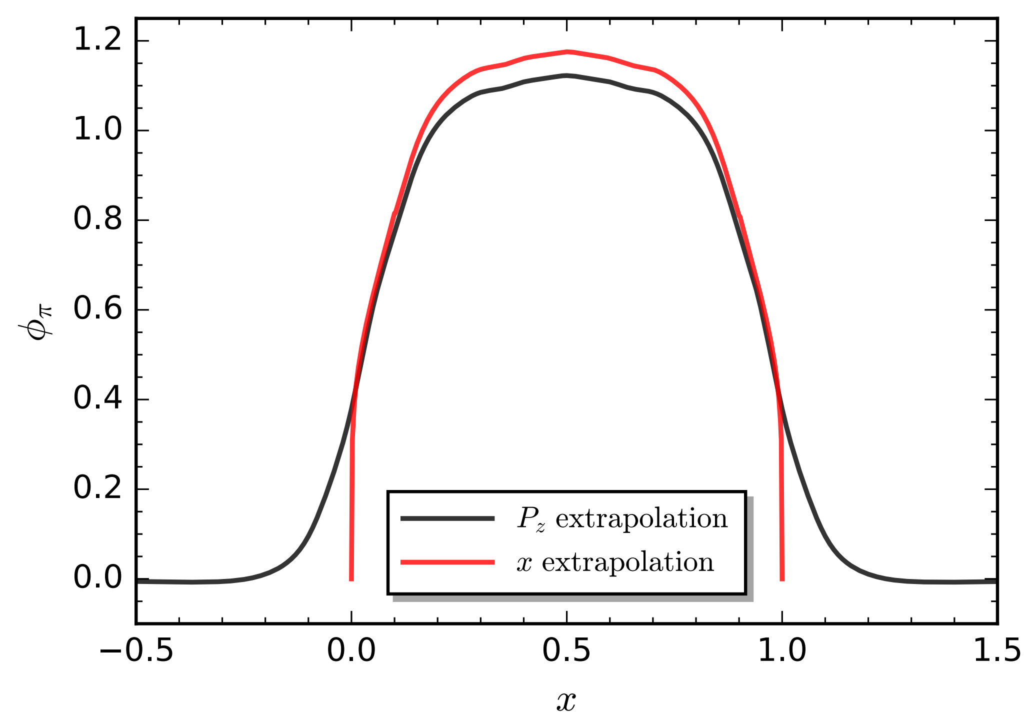
$$\phi_\pi(x, P_z) = \phi_\pi(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2} + \mathcal{O}\left(\frac{1}{P_z^4}\right)$$

- But, in the endpoint region, say $0 < x < 0.1$ and $0.9 < x < 1$, LaMET cannot be reliably used, we adopt a phenomenological extrapolation $cx^a(1-x)^a$ J. Hua *et al.* (LPC), *PRL* 129 (2022) 132001

Quasi-PDA:

fRG: Chang, WF, Huang, Pawlowski, Zhang, in preparation



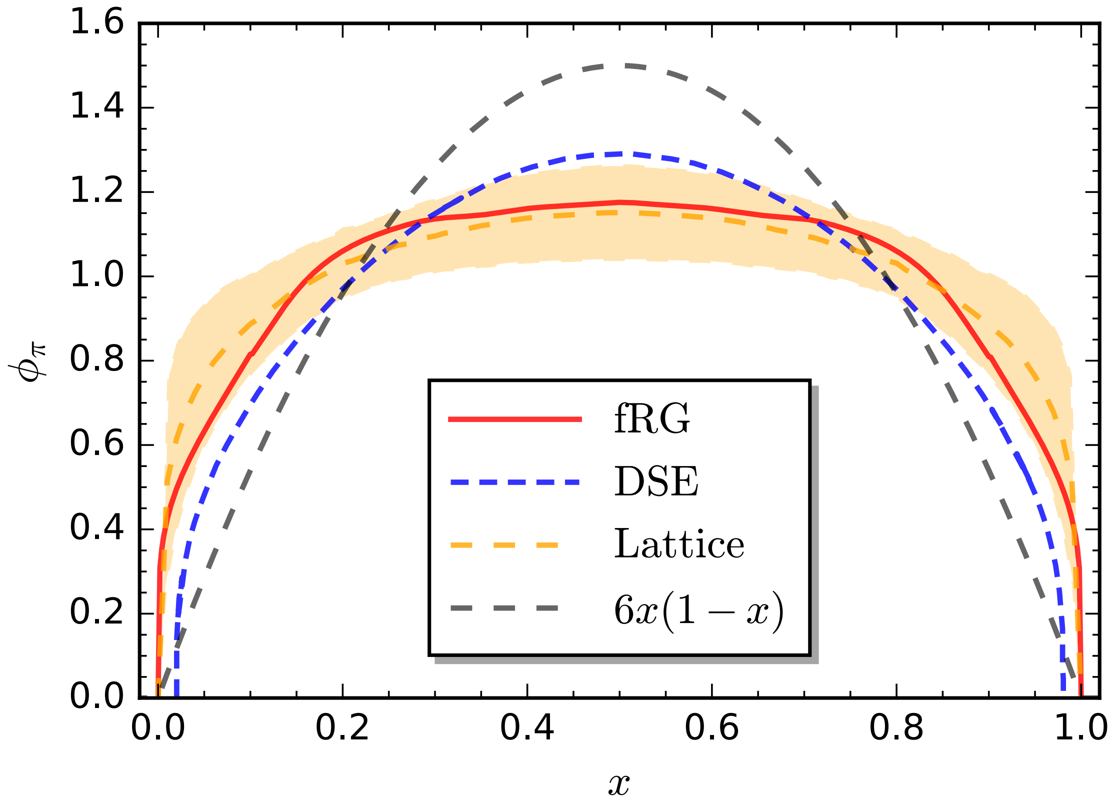


PDA and its moments

- Moments of pion PDA

$$\langle \xi^n \rangle \equiv \int_0^1 dx(2x-1)^n \phi_\pi(x)$$

Pion PDA:



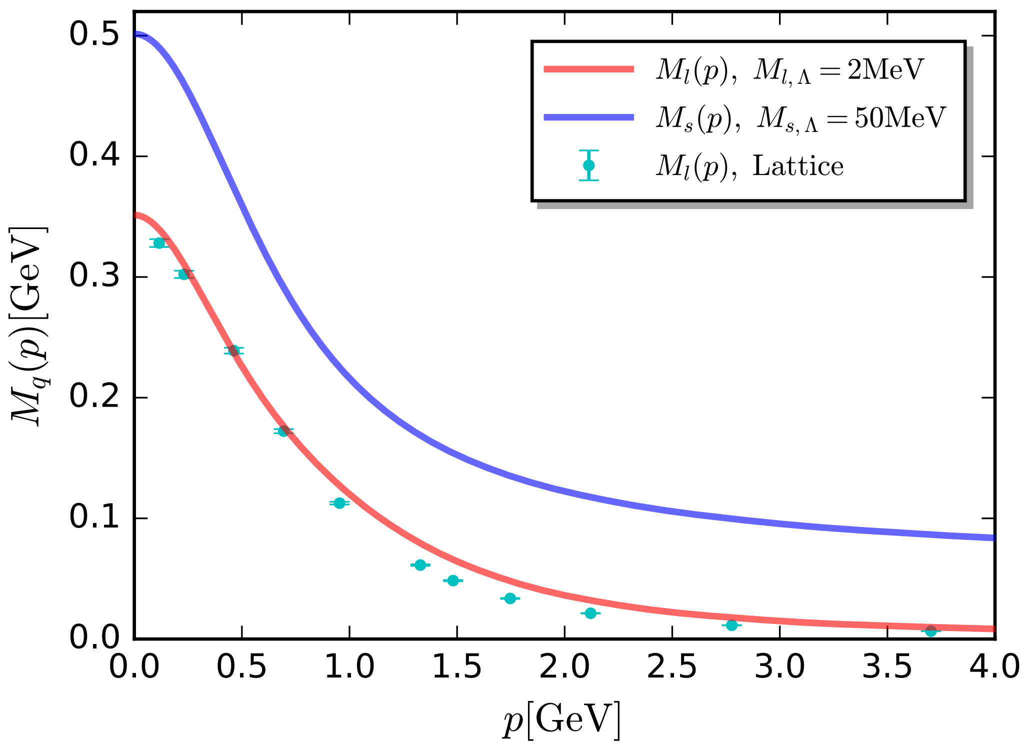
fRG: Chang, WF, Huang, Pawlowski, Zhang, in preparation

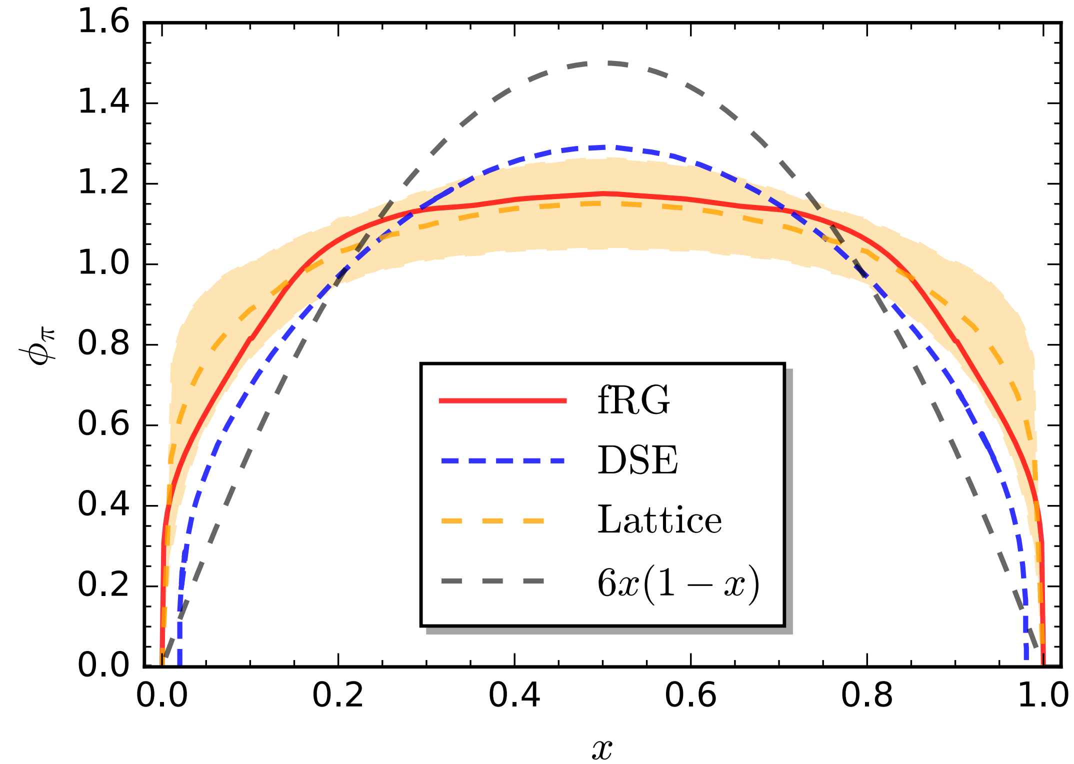
Moments:

Method	ξ_π^2	ξ_π^4	ξ_π^6
fRG (This Work)	0.271	0.142	0.092
Lattice LaMET (LPC)	0.300(41)	-	-
DSE	0.251	0.128	-
Lattice OPE (RQCD)	$0.234^{+6}_{-6}(4)(4)(2)$	-	-
Lattice OPE (RBC and UKQCD)	0.28(1)(2)	-	-
Sum Rule	0.271(13)	0.138(10)	0.087(6)

Lattice LaMET: J. Hua *et al.* (LPC), *PRL* 129 (2022) 132001.
DSE: C. Roberts *et al.*, *PPNP* 120 (2021) 103883; Chang *et al.*, *PRL* 110 (2013) 132001.
Lattice OPE: G. Bali *et al.* (RQCD), *JHEP* 08 (2019) 065; 11 (2020) 37.
Lattice OPE: R. Arthur *et al.* (RBC and UKQCD), *PRD* 83 (2011) 074505.
Sum rules: P. Ball *et al.*, *JHEP* 08 (2007) 090; T. Zhong *et al.*, *PRD* 104 (2021) 016021
Asymptotic: $6x(1-x)$

Summary

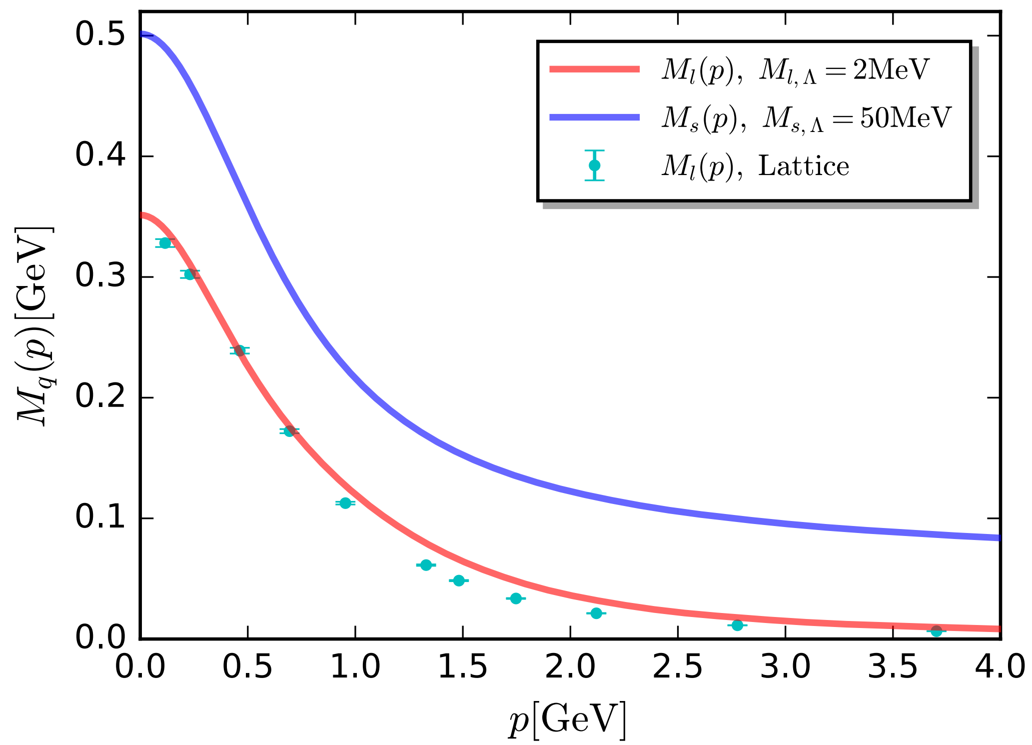


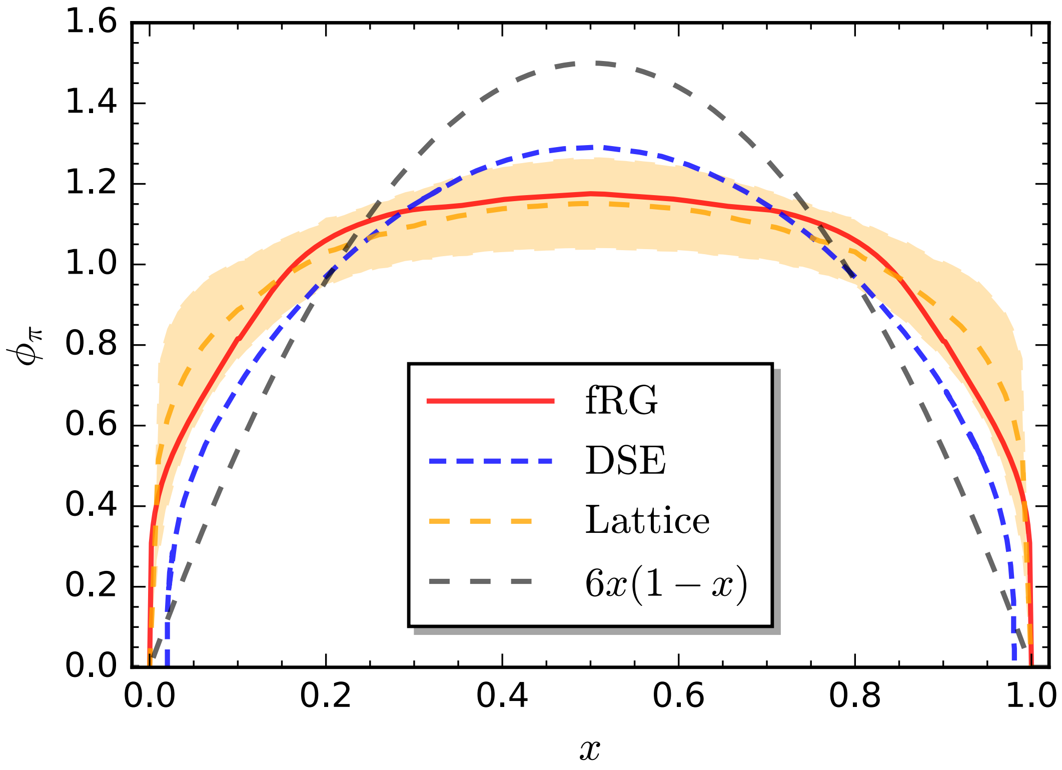


★ Functional renormalization group provides us with a powerful approach to study basic problems in physics, e.g., mass generation and hadron structure, from first-principles QCD.

★ There are also things to be done, better analytic continuations, larger Pz, etc.

Summary





★ Functional renormalization group provides us with a powerful approach to study basic problems in physics, e.g., mass generation and hadron structure, from first-principles QCD.

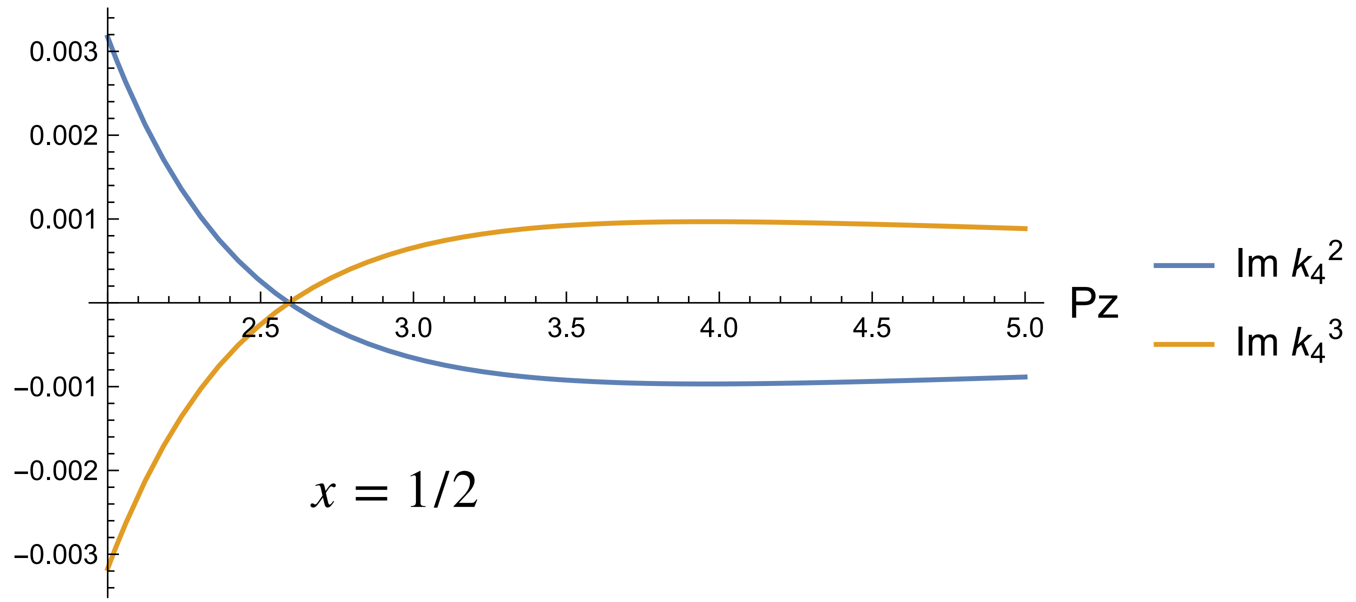
★ There are also things to be done, better analytic continuations, larger Pz, etc.

Thank you very much for your attentions!

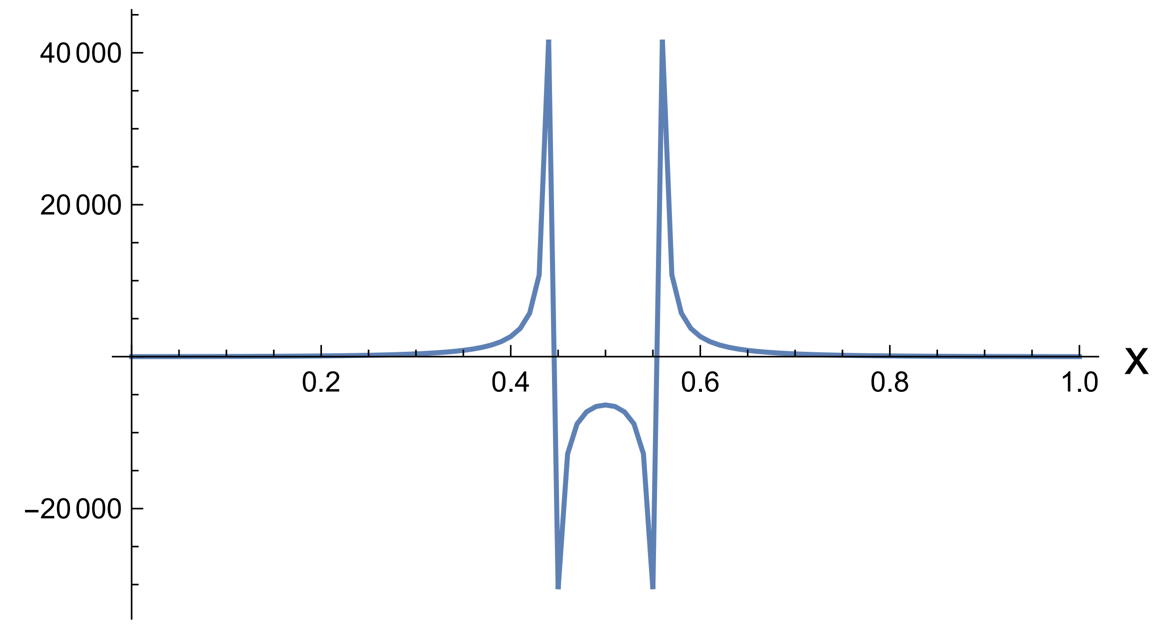
Backup

Larger P_z?

Pole crossing:



Wave function:



Poles of $k_{0,2}$ and $k_{0,3}$ interchange their potions, when $P_z \gtrsim 3.5$ GeV

fRG: Chang, WF, Huang, Pawlowski, Zhang, in preparation