Making Big Lattices Bigger: Bloch's Theorem and The Lattice Gluon Propagator (Part II)

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IFSC – University of São Paulo

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Special emphasis is given to the rôle played by boundary conditions.

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- Here, $h(\vec{x})$ has periodicity N and Θ_μ are commuting matrices, i.e. they can be written as $\Theta_\mu = \sum_{a=1}^{N_c - 1} \lambda_C^a \, \theta_\mu^a$, where the λ_C^a matrices are Cartan generators

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From Tereza's Talk (II)

- The usual minimizing functional

$$\mathcal{E}_U[g] = \frac{\Re\ \mathrm{Tr}}{N_c\, d\, m^d\, V} \sum_{\mu=1}^{d} \sum_{\vec{z}\in\Lambda_z} \left[1\!\!1 - U_\mu(g;\vec{z}) \right]$$

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which is independent of $\vec{y}$ and we can write

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- The numerical minimization can now be carried out on the original lattice Λ_x

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- Is this telling us something about the relevant configurations for the QCD vacuum?

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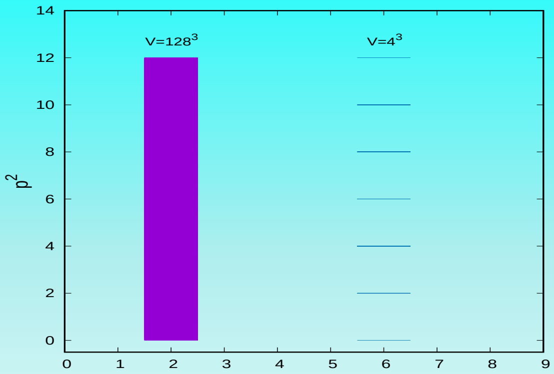
  $\Longrightarrow$ several definitions of minima
- Can we relate Gribov copies in a large lattice volume $V = (mN)^d$ with those obtained using Bloch waves in a volume $V = N^d \times m^d$?

Gluon Propagator "Spectrum" (I)



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For $V = 128^3$ there are ~ 45000 different momenta (with degeneracy)

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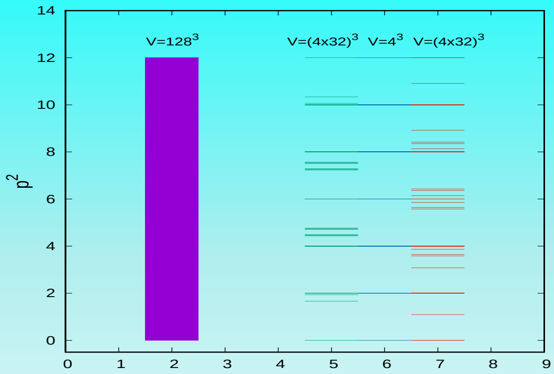

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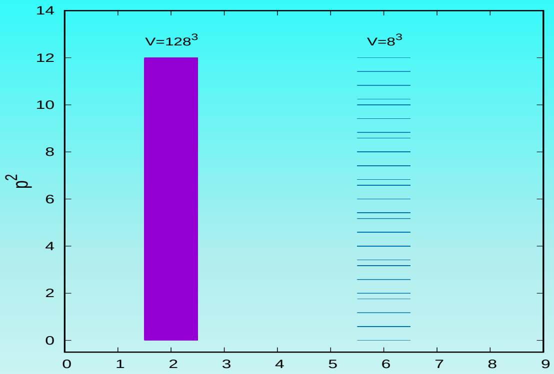

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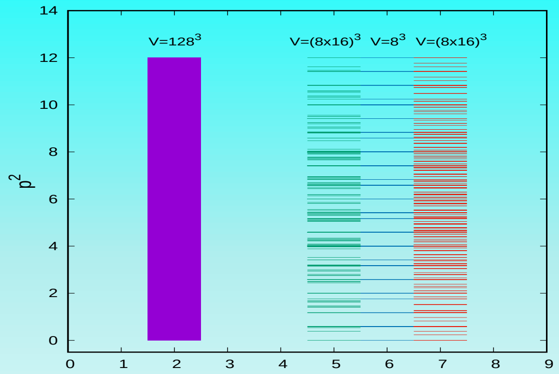

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The Math of Bloch Waves (I)

In the $SU(2)$ case we can write the Θ_μ matrices as

$$\Theta_\mu = \theta_\mu \, v^\dagger \sigma_3 \, v \ ,$$

where $v \in SU(2)$, $\theta_\mu \in \Re$ and σ_3 is the third Pauli matrix. Then, they have eigenvectors

$$w_1 = v^\dagger \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad w_2 = v^\dagger \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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In like manner, in the $SU(3)$ case, which has rank two, we can write

$$\Theta_\mu \;=\; v^\dagger \; (\, \theta_{\mu,3} \, \lambda_3 \,+\, \theta_{\mu,8} \, \lambda_8 \,) \; v \;,$$

with real parameters $\theta_{\mu,3}$ and $\theta_{\mu,8}$, $v \in SU(3)$, and where λ_3, λ_8 are the two diagonal Gell-Mann matrices

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With the above setup, we also have to impose the constraint

$$\Theta_\mu \, w_j \; = \; \alpha_\mu^{(j)} \, w_j \; = \; \frac{2\pi \, n_\mu^{(j)}}{m} \, w_j$$

so that

$$\exp\left(-i \sum_{\nu=1}^{d} \Theta_\nu \, y_\nu \right) w_j \; = \; \exp\left(-i \sum_{\nu=1}^{d} \frac{2\pi \, n_\nu^{(j)}}{m} \, y_\nu \right) w_j$$

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Then, it is natural to consider the basis $\lambda_{jk} \equiv w_j \, w_k^\dagger = v^\dagger \, M_{jk} \, v$, where the $N_c \times N_c$ matrices M_{jk} have elements $(M_{jk})_{gh} = \delta_{jg}\delta_{kh}$, and write

$$U_\mu(g;\vec{z}) \;=\; v^\dagger \left\{ \sum_{h,j=1}^{N_c} \left[U_\mu(g;\vec{z})\right]_{hj} \, M_{hj} \right\} v$$

The Fourier Transform (I)

We can now evaluate the Fourier transform

$$\widetilde{U}_\mu(g;\vec{k}) = \sum_{\vec{z}\in\Lambda_z} U_\mu(g;\vec{z}) \exp\left[-\frac{2\pi i}{m N}\left(\vec{k}\cdot\vec{z}\right)\right]$$

of the gauge-fixed link variables $U_\mu(g;\vec{z})$ and find

$$\left[\widetilde{U}_\mu(g;\vec{k})\right]_{hj} \equiv w_h^\dagger \, \widetilde{U}_\mu(g;\vec{k}) \, w_j \propto \sum_{\vec{y}\in\Lambda_y} \exp\left[-\frac{2\pi i}{m}\sum_{\nu=1}^{d}\left(k_\nu + n_\nu^{(j)} - n_\nu^{(h)}\right) y_\nu\right]$$

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NOTE: different matrix elements require different conditions!

The Fourier Transform (II)

The same result applies to the Fourier transform of the gauge-fixed gluon field

$$A_\mu(g;\vec{z}) \equiv \frac{1}{2\,i}\left[\,U_\mu(g;\vec{z}) - U_\mu^\dagger(g;\vec{z})\,\right]_{\text{traceless}}$$

and to the gluon propagator

$$D(\vec{k}) = \frac{\mathrm{Tr}}{2\,(d-1)\,(N_c^2-1)\,m^d\,V}\sum_{\mu=1}^{d}\langle\,\widetilde{A}_\mu(g;\vec{k})\,\widetilde{A}_\mu(g;-\vec{k})\,\rangle$$

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For the zero momentum we need $n_\nu^{(j)} - n_\nu^{(h)} \propto m$, which usually implies $h=j$

Results

- We verified that the only non-zero values of the gluon propagator $D(p^2)$, evaluated using Bloch waves, satisfy the condition $k_\nu + n_\nu^{(j)} - n_\nu^{(h)} \propto m$ for any direction ν

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- This explains the global factor m^d obtained in numerical simulations
- The zero-momentum gluon propagator becomes

$$D(\vec{0}) \approx \frac{m^d}{2\,d\,(N_c^2-1)\,V} \sum_{\mu=1}^{d} \sum_{j=1}^{N_c} \langle \left[\sum_{\vec{x}\in\Lambda_x} A_\mu(h; \Theta_\mu; \vec{x}) \right]_{jj}^2 \rangle$$

  and only the diagonal components of the zero modes usually gives a contribution

Our Old Explanation is Confirmed!

Question: why is $D(0)$ so suppressed?

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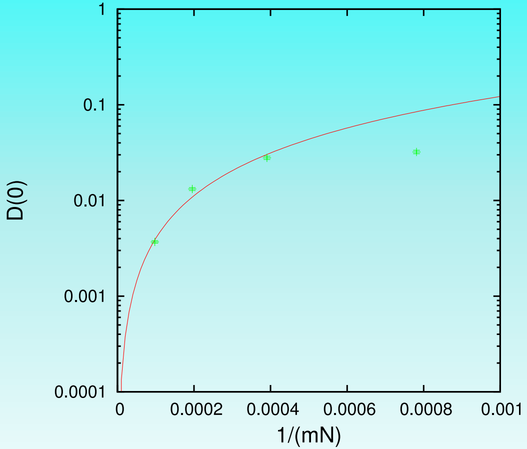
Then we can remove the Abelian zero modes!

The $m \to \infty$ Limit



$SU(2)$ Gluon propagator at zero momentum $D(0)$, in the two-dimensional case, as a function of the inverse lattice side $1/(mN)$ with $N = 320$ and $m = 2, 4, 8$ and 16 at $\beta = 10.0$. The fit is $\sim 1/(mN)^{1.5}$.

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It seems very difficult to relate Gribov copies in the "unit cell" with those obtained by gauge fixing a configuration that is directly thermalized on the extended lattice Λ_z

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- We probably need to move part of the simulation to GPUs
- We also plan to extend this analysis to the ghost propagator

THANKS!


Cantabria
Pais Vasco
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Galicia
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Cataluna
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Aragon
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Castile
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