UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

Towards the phase diagram of QCD and its critical endpoint

Based on
[arXiv:2408.08413]

Franz R. Sattler
ITP Heidelberg

In Collaboration with
Friederike Ihssen, Jan M. Pawlowski, Nicolas Wink

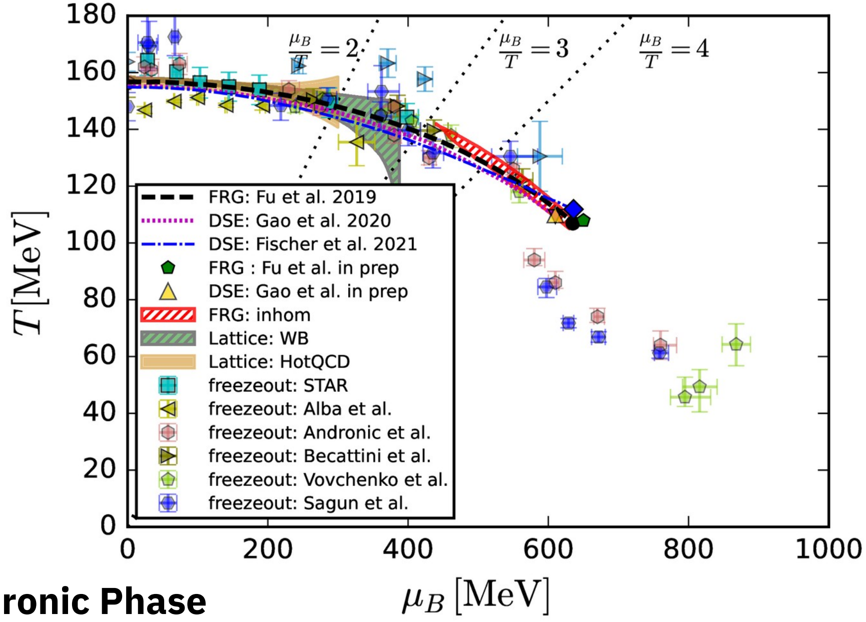
QuantFunc2024,

Valencia

September 05, 2024

The QCD phase diagram

Quark-Gluon Plasma



Hadronic Phase

Fu, Pawlowski, Rennecke [Phys. Rev. D 101 (2020), 054032]
Gao, Pawlowski [Phys.Lett.B820(2021) 136584]
Gunkel, Fischer [Phys.Rev.D 104 (2021) 5, 054022]
Bellwied et al. (WB) [Phys.Lett.B 751 (2015) 559-564]
Bazavov et al. (HotQCD) [Phys.Lett.B 795 (2019) 15-21]

Phase diagram shows
Chiral symmetry breaking
i.e. condensation of $\langle \bar{q} q \rangle$

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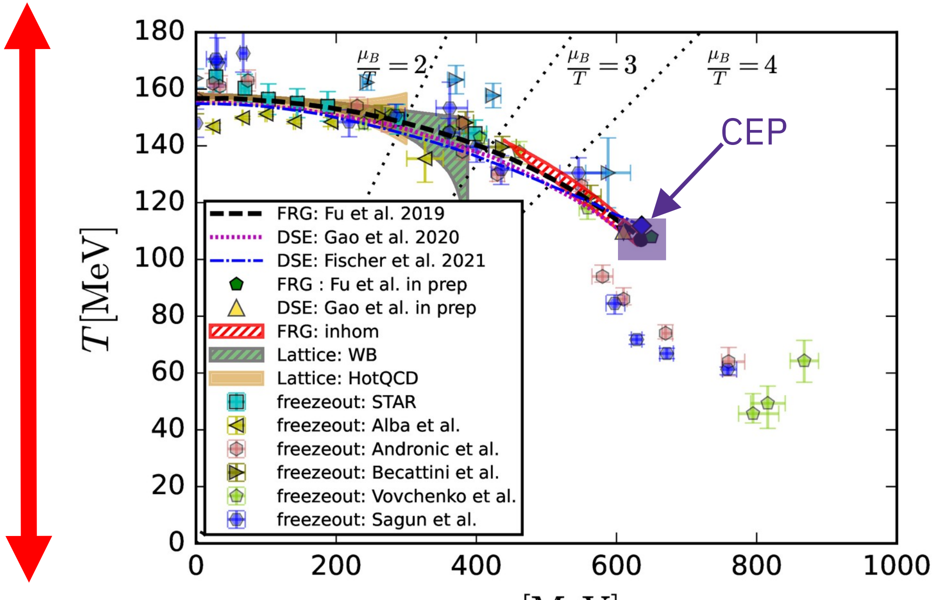

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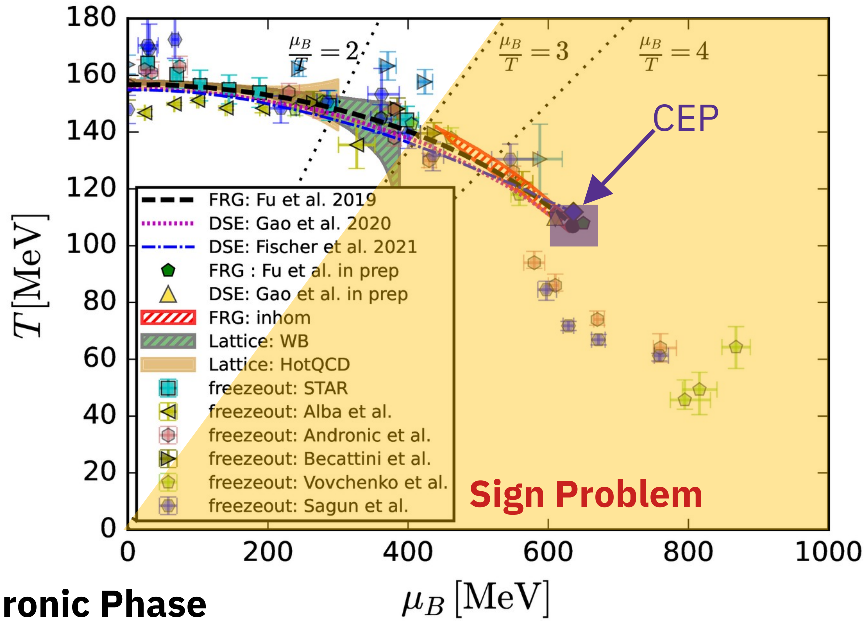

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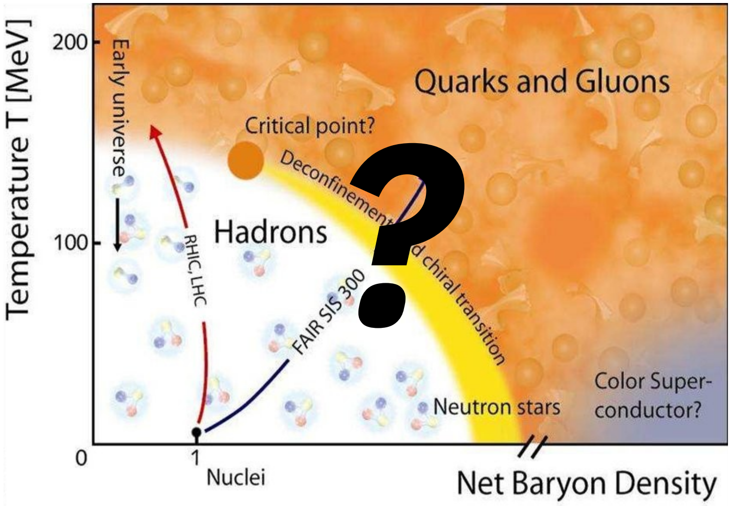
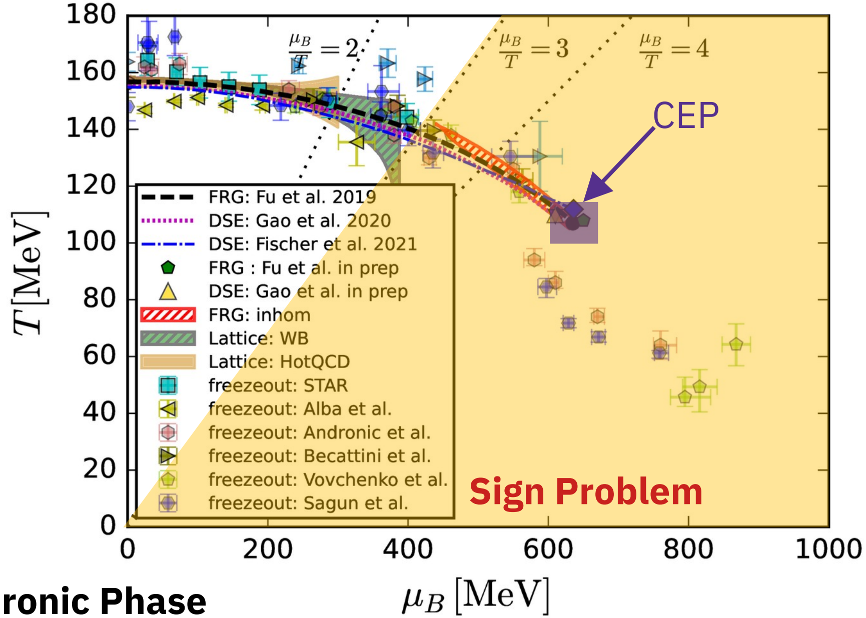
Phase Diagram for intermediate μ not know



Sketch by GSI Darmstadt

The QCD phase diagram

Quark-Gluon Plasma



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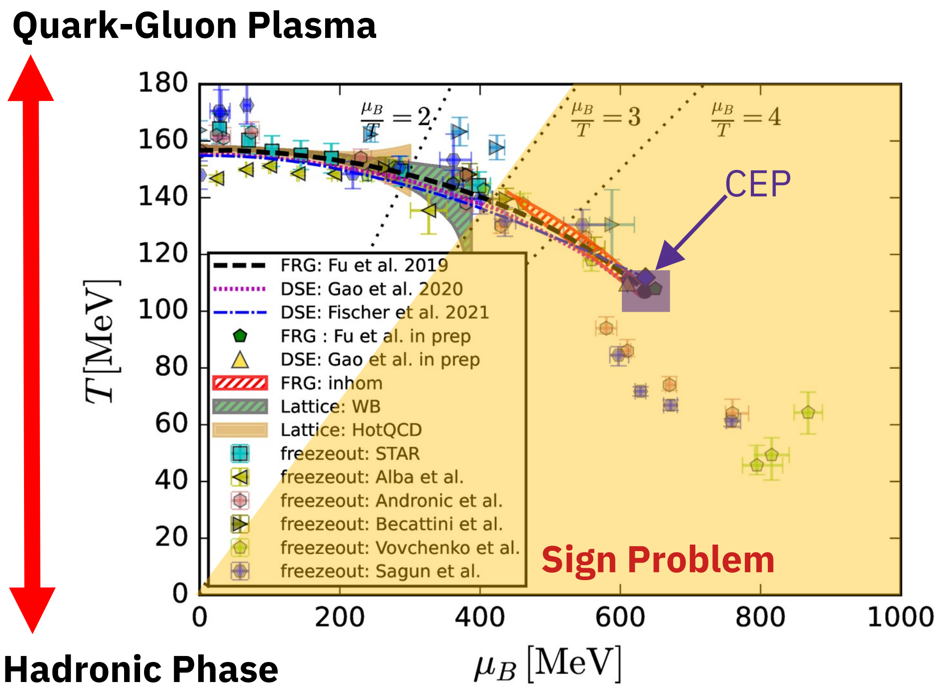

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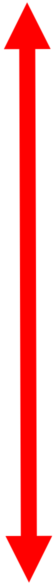
Direct access to phase structure using the

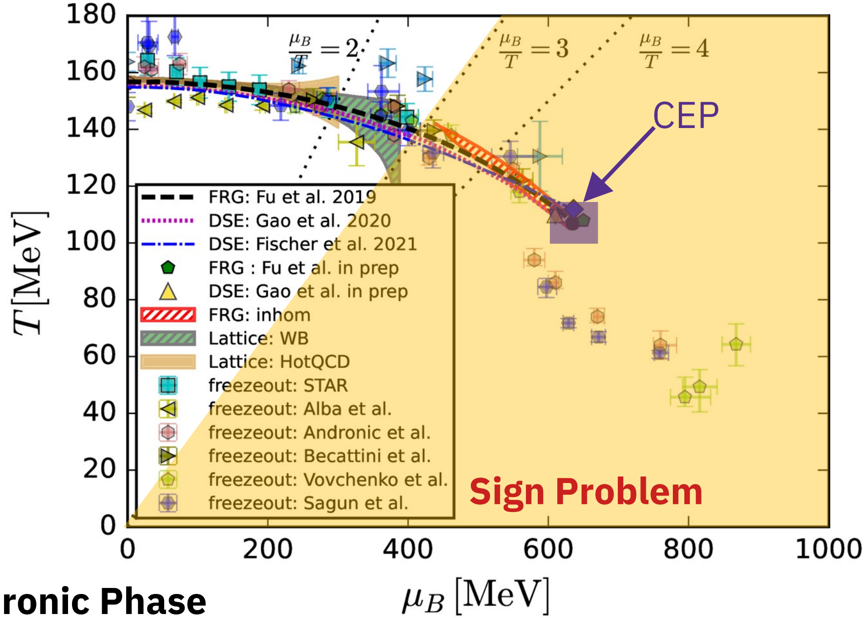
functional Renormalization Group

Diagrammatic, non-perturbative method with direct access to finite μ.

The QCD phase diagram

Quark-Gluon Plasma





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Diagrammatic, non-perturbative method with direct access to finite μ.

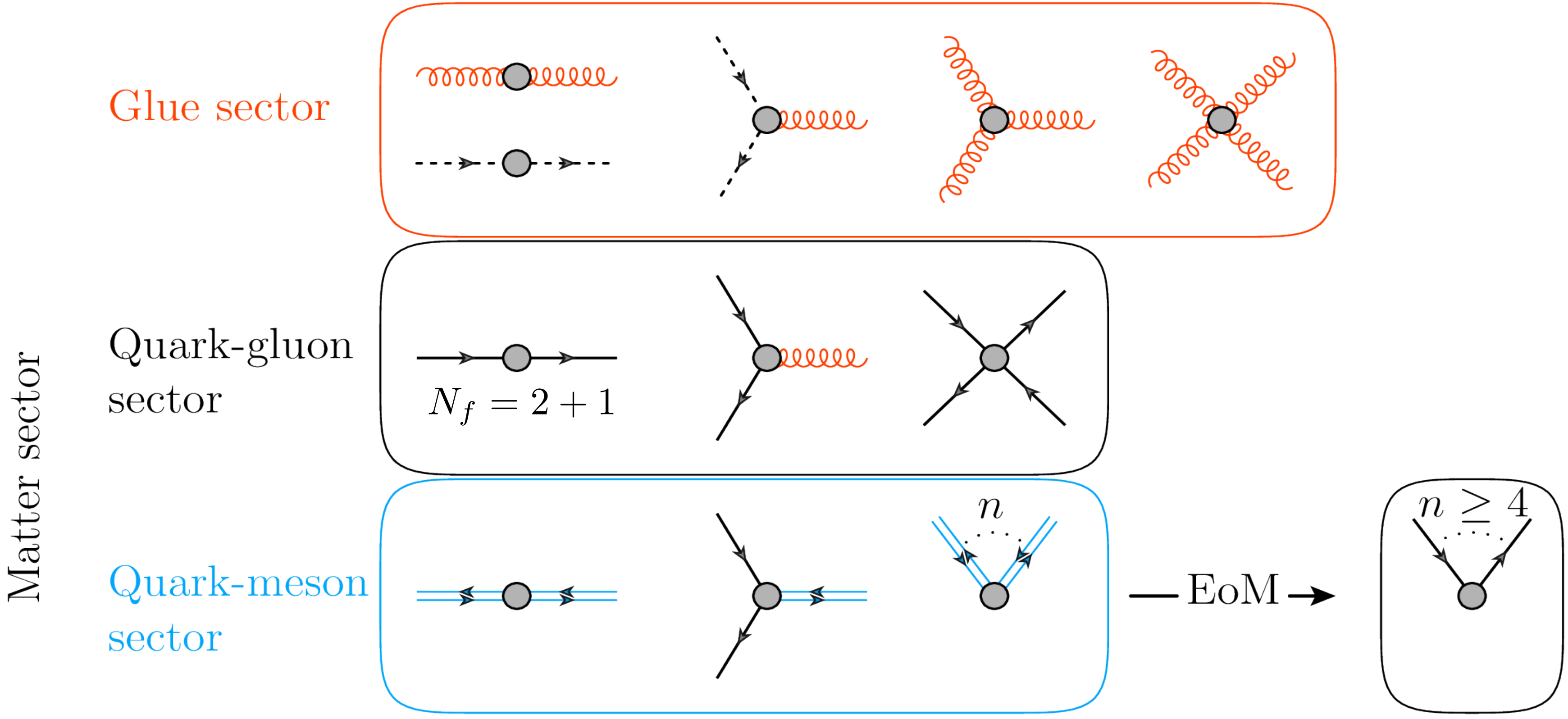
Here, first step:

- Setup
- Systematics

Vacuum

Ihssen, Pawlowski, Sattler, Wink [arXiv:2408.08413]

Current vertex expansion



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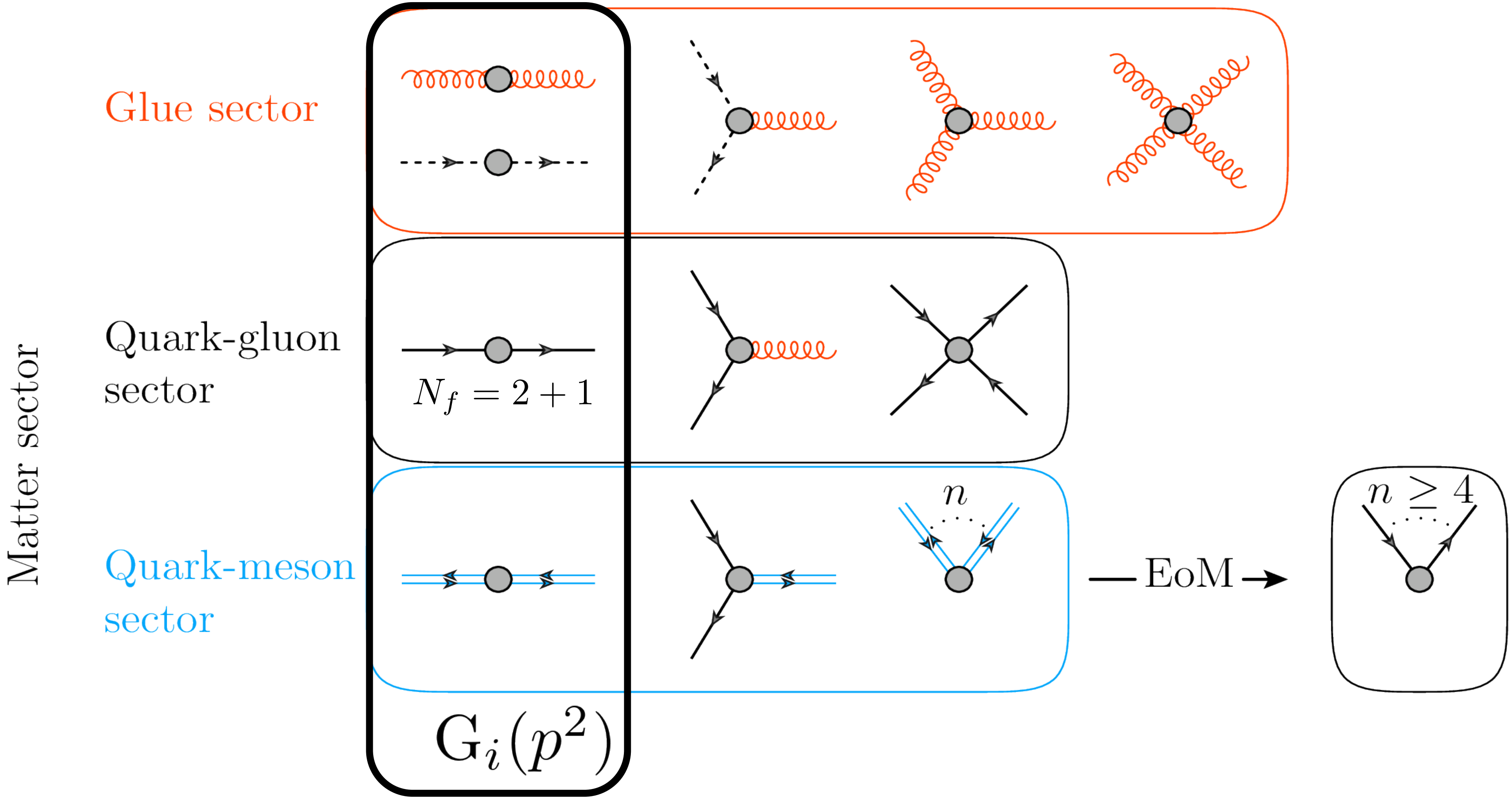

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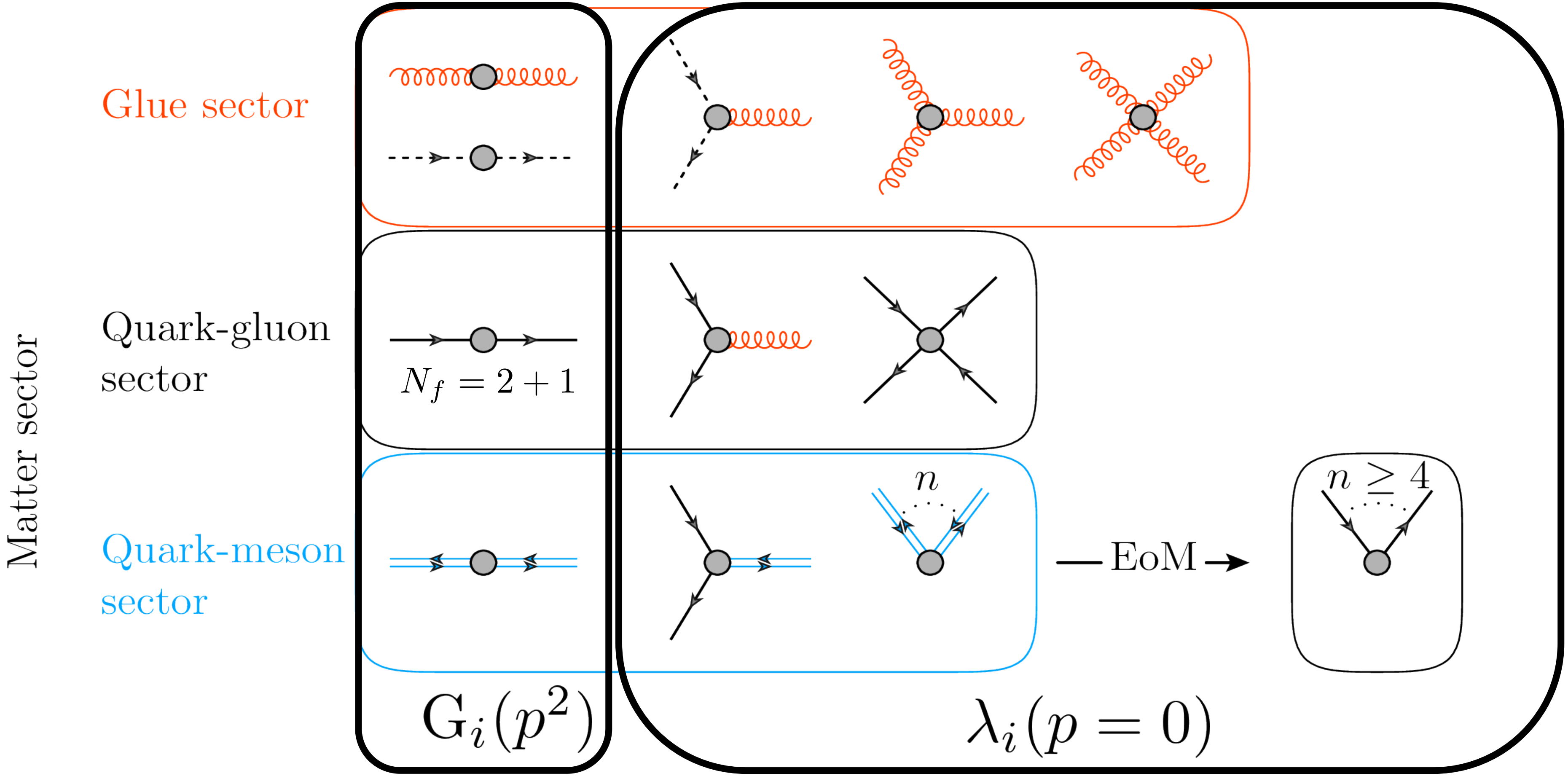

TensorBases Mathematica package

With J. Braun,
J. Pawlowski, A. Geißel,
N. Wink

- Automatically derived projectors
- Library of tensor bases, extendable by everyone

```
Needs["TensorBases`"]

In[2]:= TBGetProjector["transAqbq", 1, {p1, mu, a}, {p2, d2, A2, F2}, {p3, d3, A3, F3}]
        TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 1] // FormTrace // Simplify
        TBGetInnerProduct["transAqbq"][TBGetProjector, 1, TBGetBasisElement, 8] // FormTrace // Simplify
```

Out[2]= $\frac{1}{6\,Nf - 6\,Nc^2\,Nf}\, i\, \text{deltaFundFlav}[F2, F3] \times \text{gamma}[nu\$20834, d2, d3] \times \text{TCol}[a, A2, A3] \times \text{transProj}[p1, mu, nu\$20834]$

Out[3]= 1

Out[4]= 0

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DiFfRG framework

- Automatic derivation and code generation for large fRG systems
- Hydrodynamic methods for full field dependeces
- GPU accelerated

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Open Source available October/November

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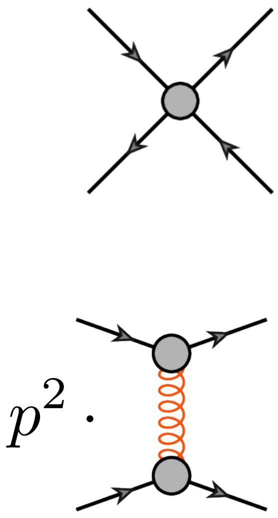
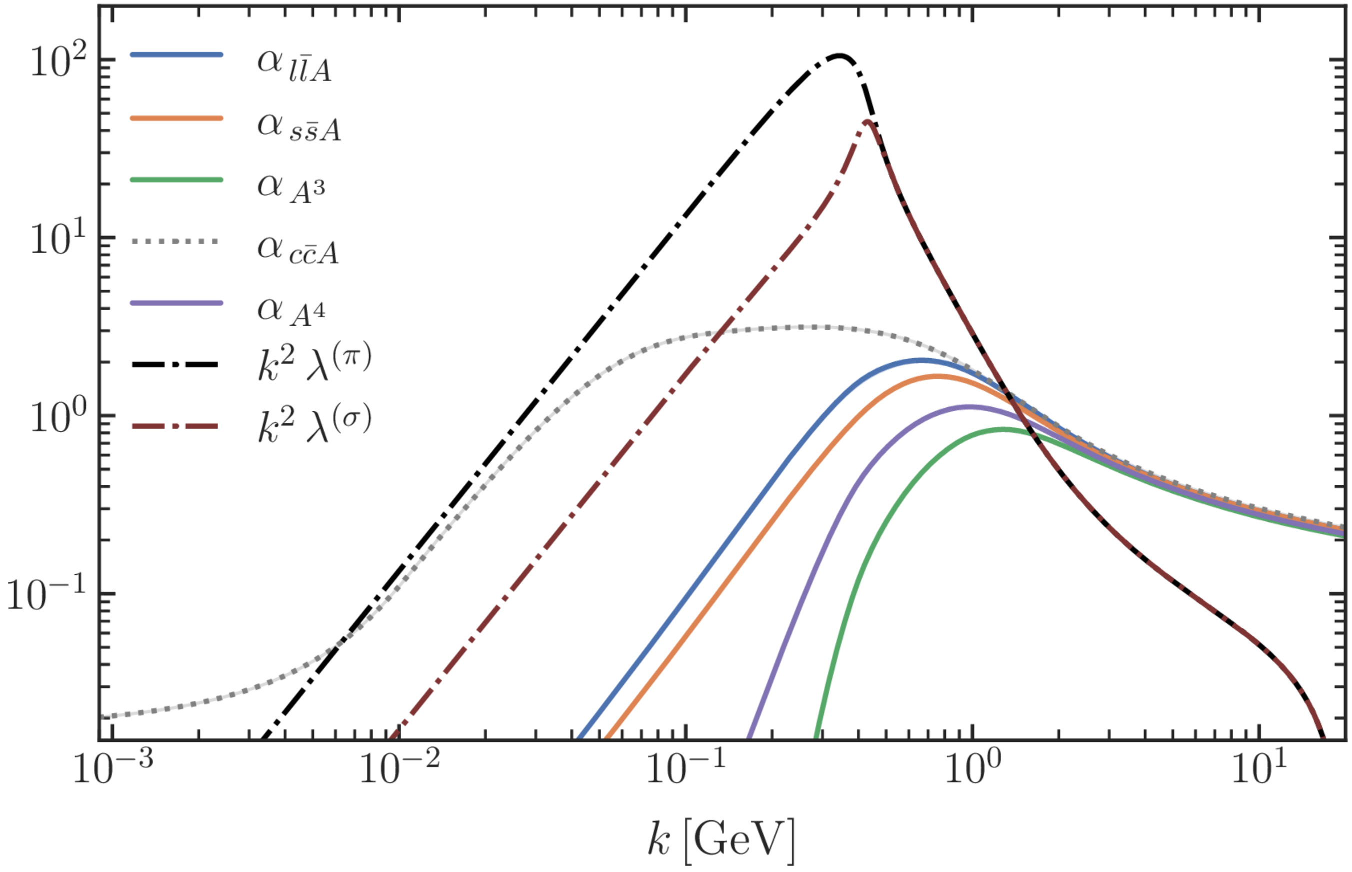
Dynamical Hadronisation in fRG





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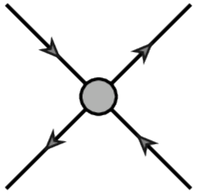
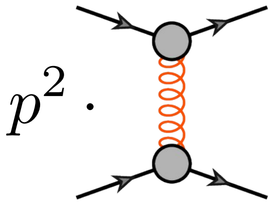
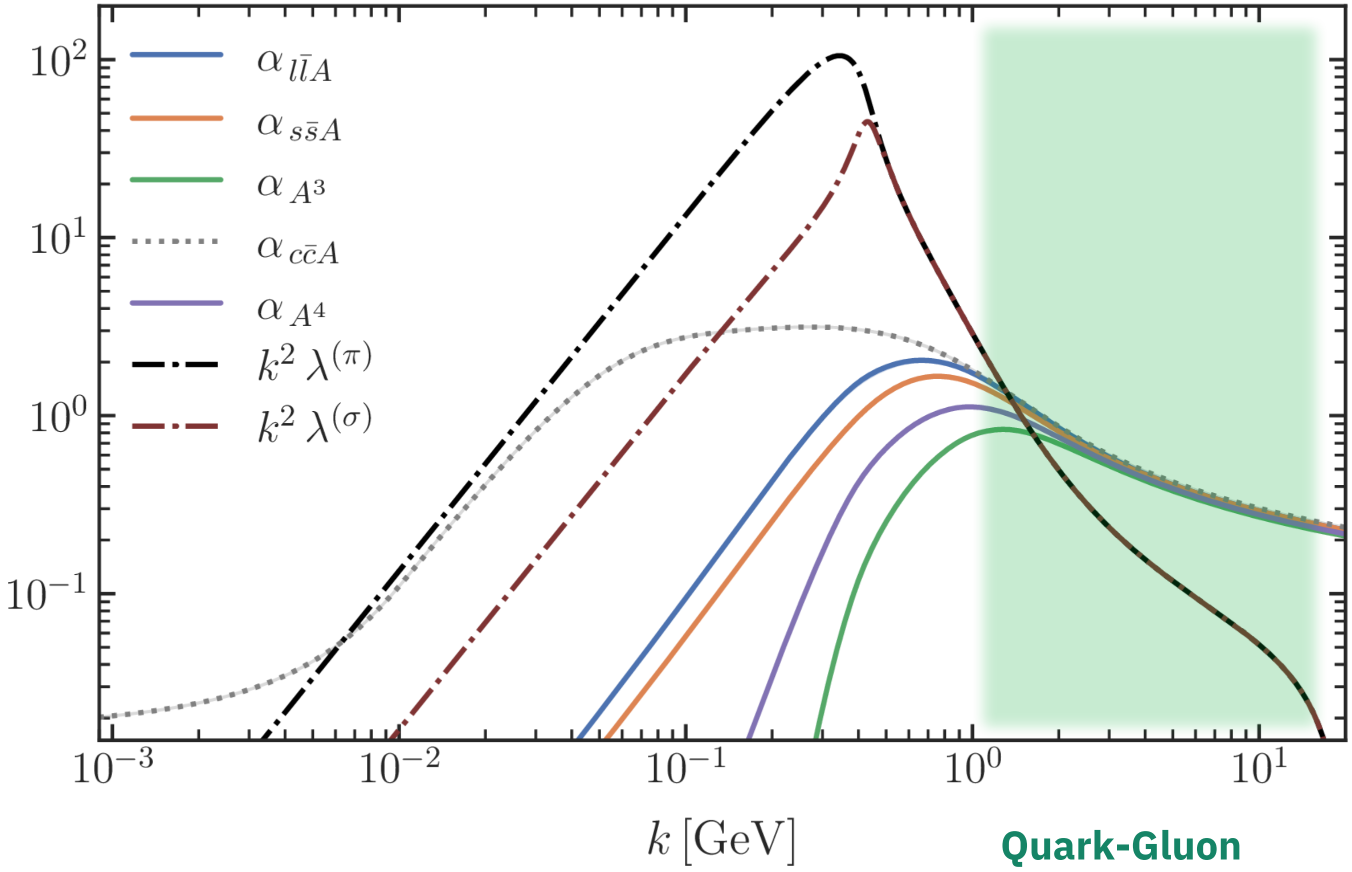
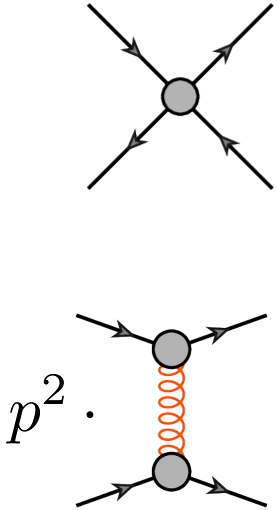


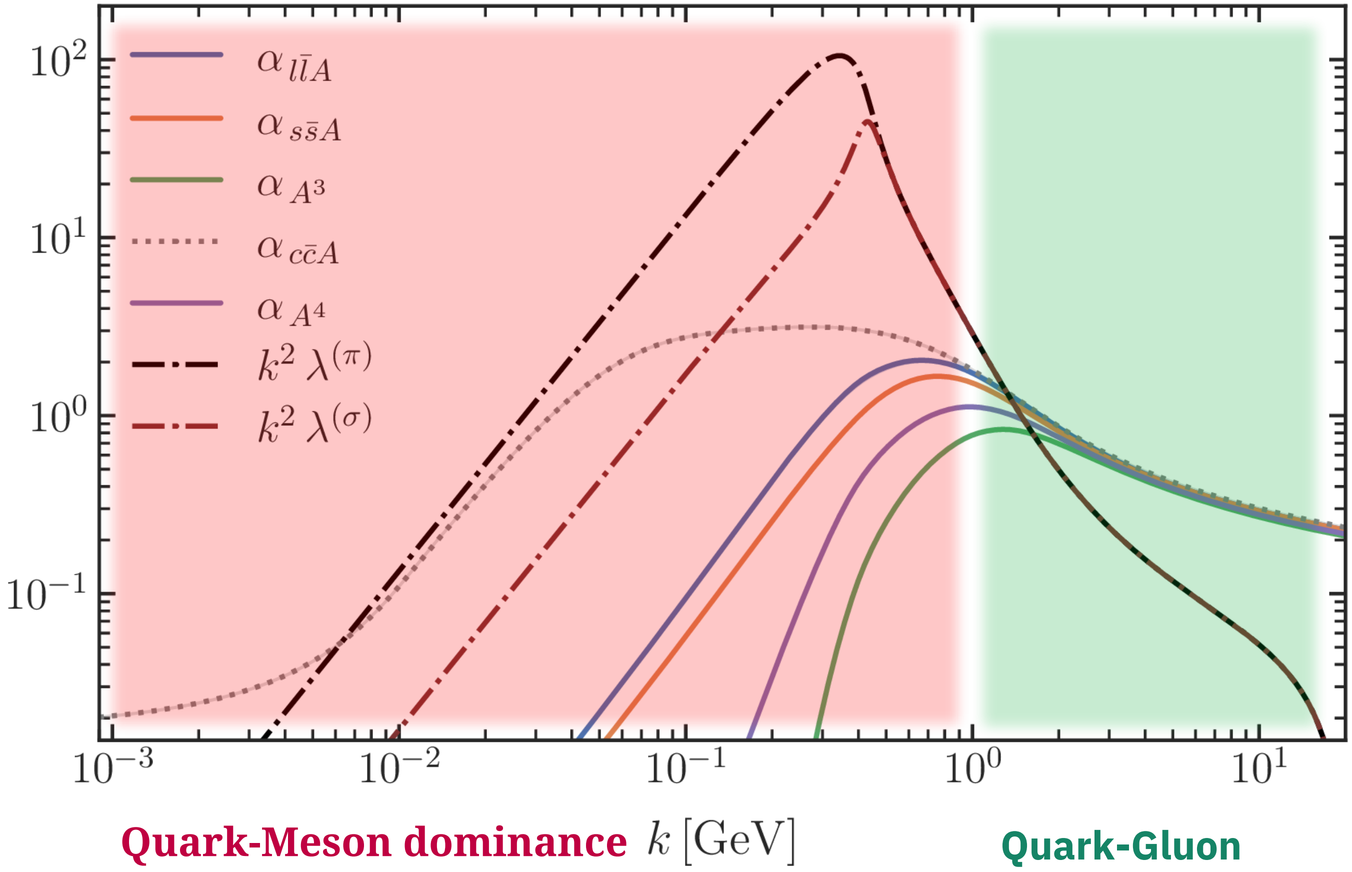




Quark-Gluon dominance

Dynamical Hadronisation in fRG

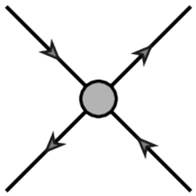




Quark-Meson dominance

Quark-Gluon dominance

$\sigma - \pi -$ four-quark flow



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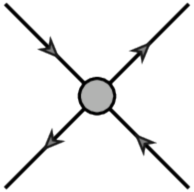

(Resonant)

Fu, Huang, Pawlowski, Tan
[SciPost Phys. 14 (2023) 4, 069]
[arxiv:2401.07638 (2024)]

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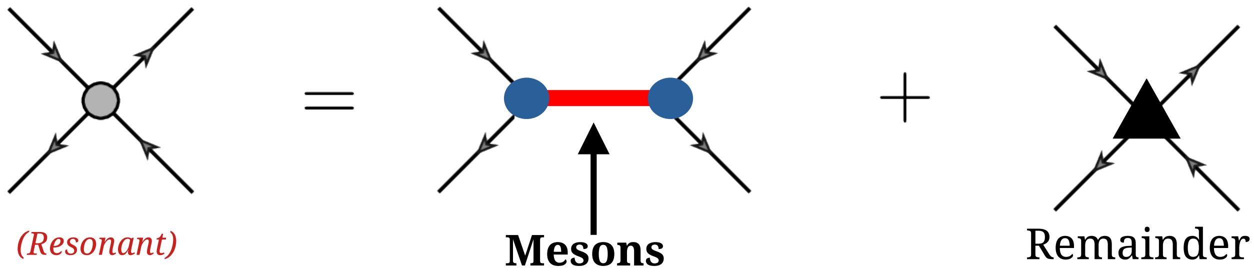

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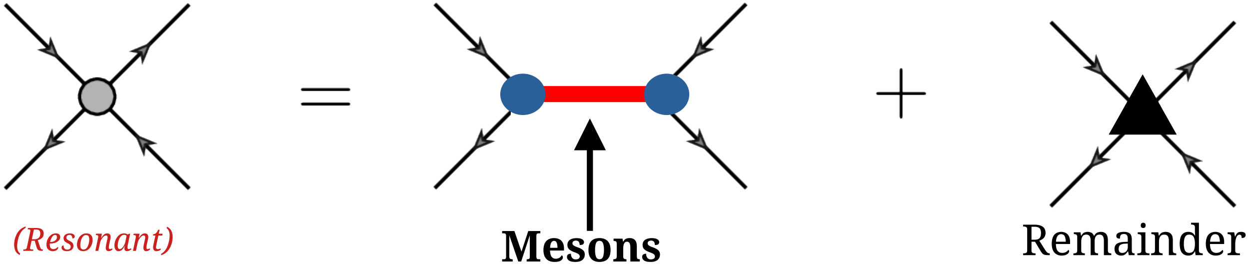

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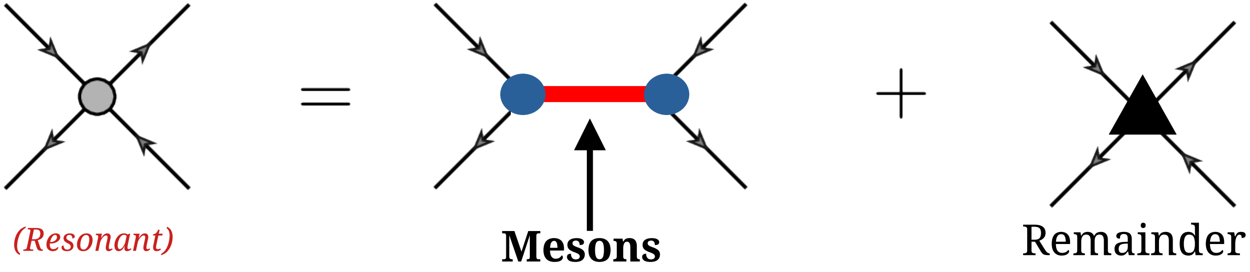
BS WF

$$h_\phi(p,q) \cdot \frac{1}{Z_\phi(t^2)(t^2+m_\phi^2)} \cdot h_\phi(p,q)$$

Meson propagator

$\sigma - \pi -$ four-quark flow



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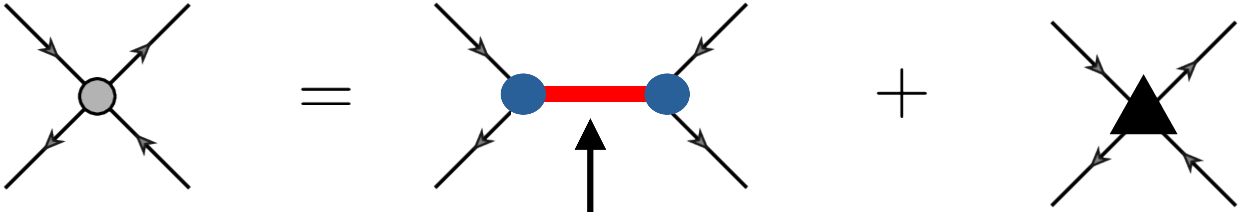
Meson propagator

Full scattering potential

$$V\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{2}\right)$$

All orders of n-meson scatterings

$\sigma - \pi -$ four-quark flow



(Resonant) **Mesons** Remainder

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Physical DoFs emergent from full QCD

Dynamical hadronization / Emergent Composites

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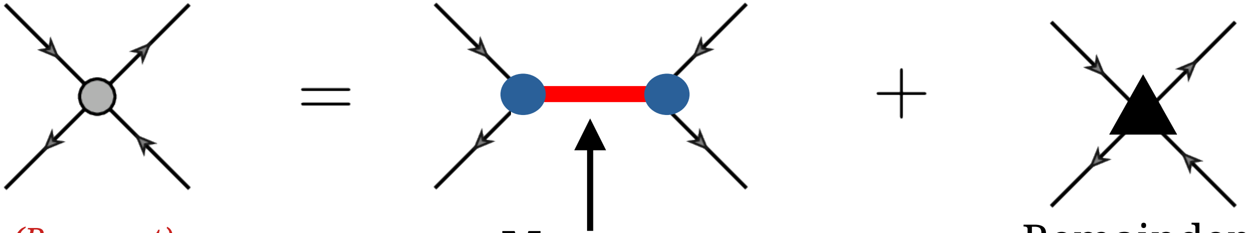

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Physical DoFs emergent from full QCD

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Full mesonic potential of QCD

Field space:

Finite element method

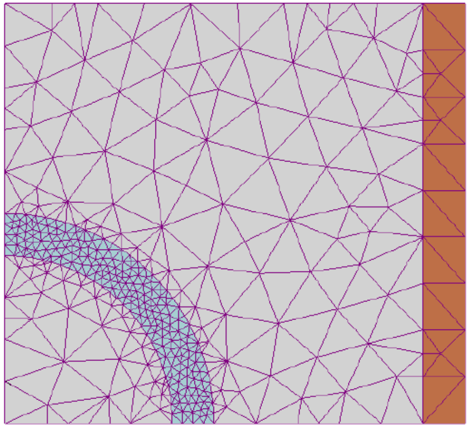

Image source: wikipedia.org

Grossi, Wink [SciPost Phys.Core 6 (2023) 071]
Grossi, Ihssen, Pawlowski, Wink [Phys.Rev.D 104 (2021) 1, 016028]
Ihssen, Pawlowski, Sattler, Wink [arXiv:2309.07335], [Comput.Phys.Commun. 300 (2024) 109182]

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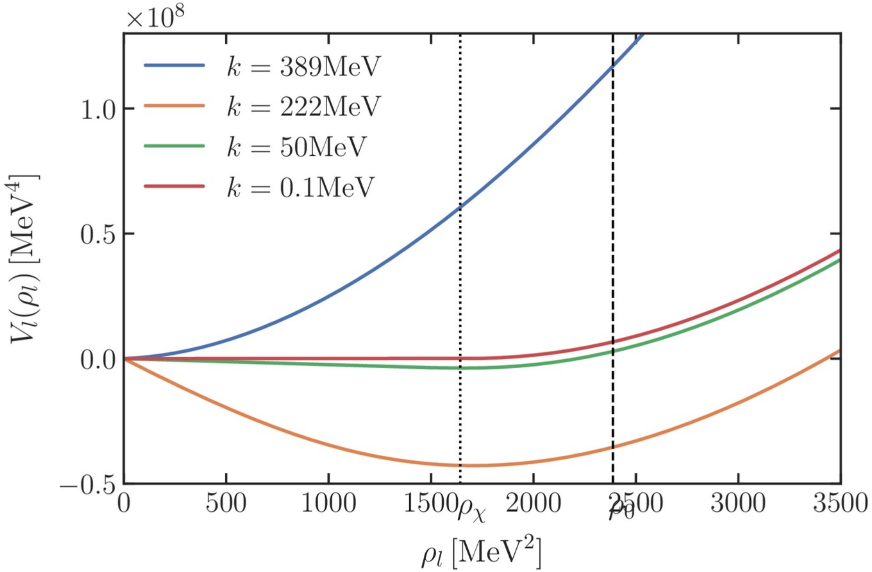
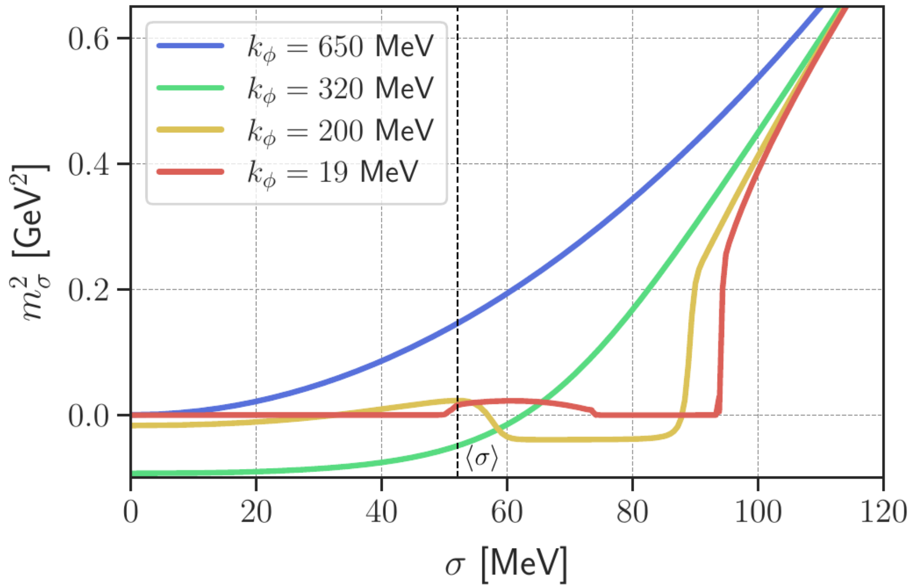
sensible RG-scale integration

Ihssen, Sattler, Wink (Phys.Rev.D 107 (2023) 11, 114009)



Full mesonic potential of QCD

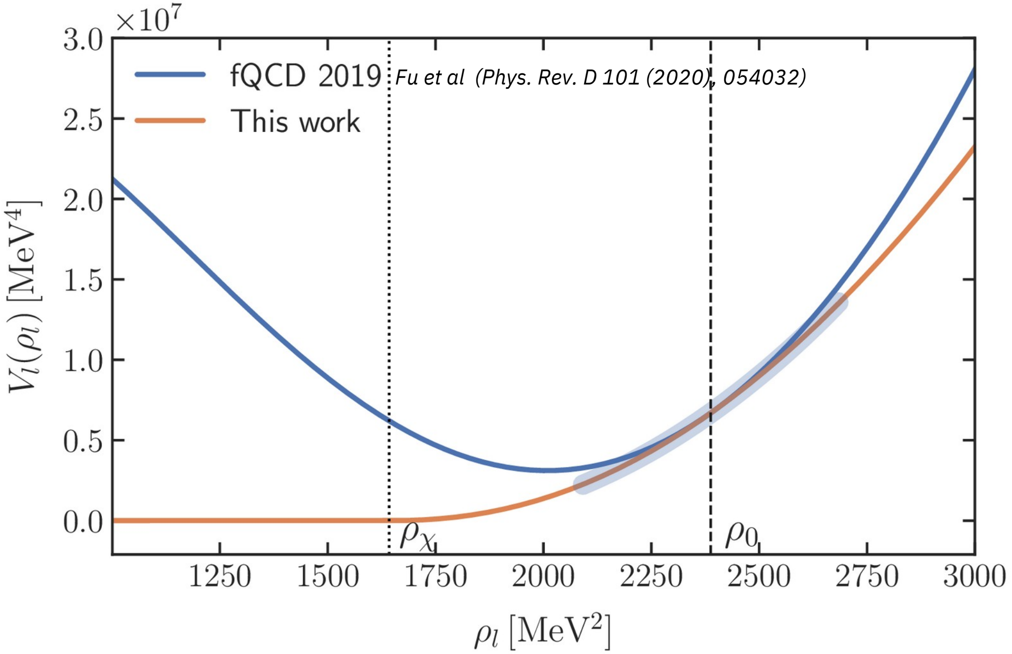


Ihssen, Pawlowski, Sattler, Wink [arXiv:2309.07335]

Hydro methods allow to access the full Potential.

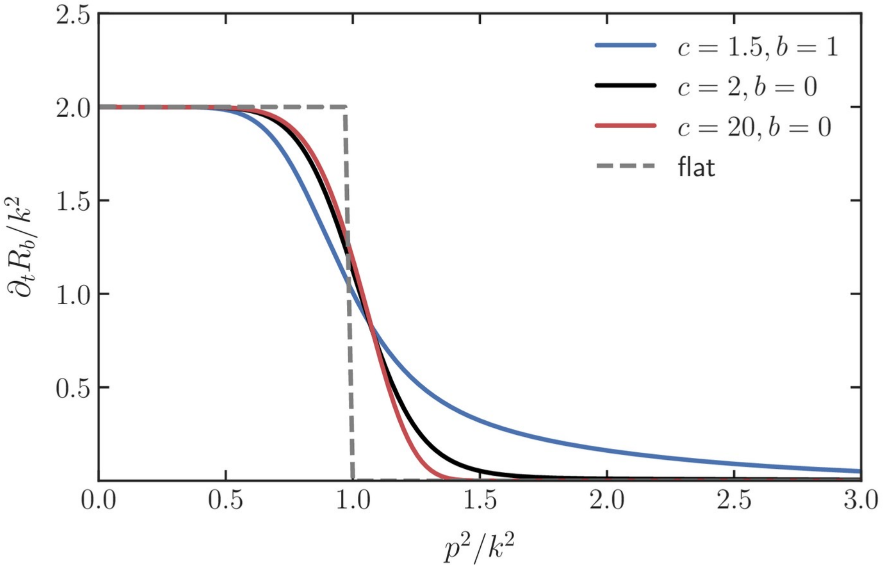
Important for phase transitions at high μ

Quantitative access to chiral limit!



Systematic errors I:
Regulator dependence

Easy regulator variation thanks to numerical framework:

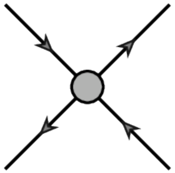


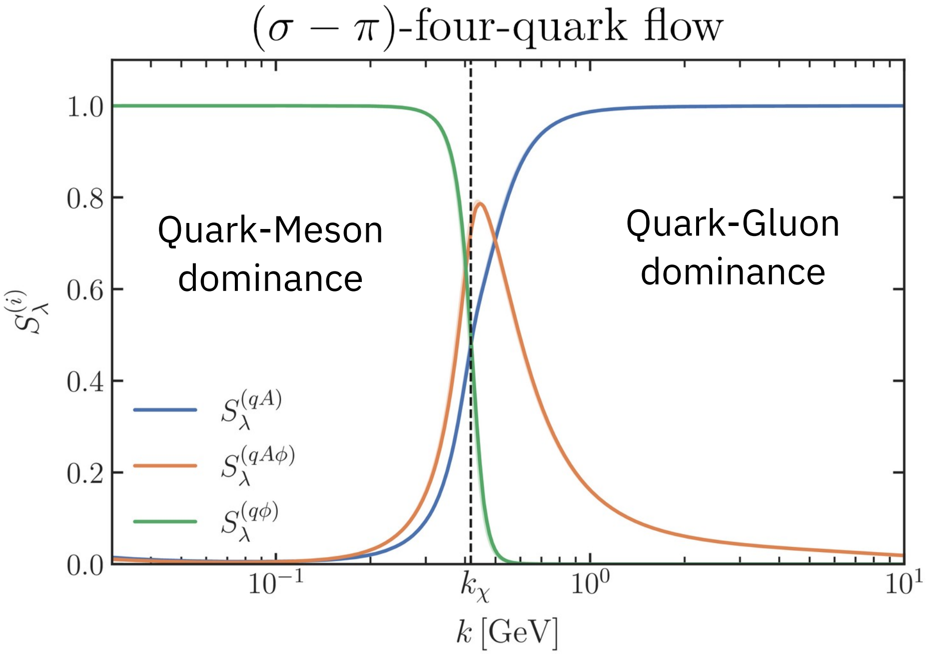
Observable	Value
$(f_K/f_\pi)_\chi$	$1.2168^{+0.0006}_{-0.0007}$
$f_{\pi,\chi}$ [MeV]	$93.2^{+3.5}_{-3.1}$
$m_{l,\chi}$ [MeV]	$311.6^{+0.3}_{-0.1}$
$m_{s,\chi}$ [MeV]	$446.7^{+0.3}_{-0.2}$
$m_{\sigma,\chi}$ [MeV]	$214.7^{+5.4}_{-9.3}$
$\sigma_{l,0,\chi}$ [MeV]	$67.1^{+1.2}_{-0.0}$

Chiral limit observables

Sattler et al. (in preperation)

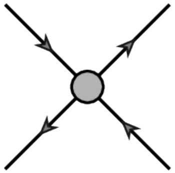
Systematic errors II: The LEGO® principle





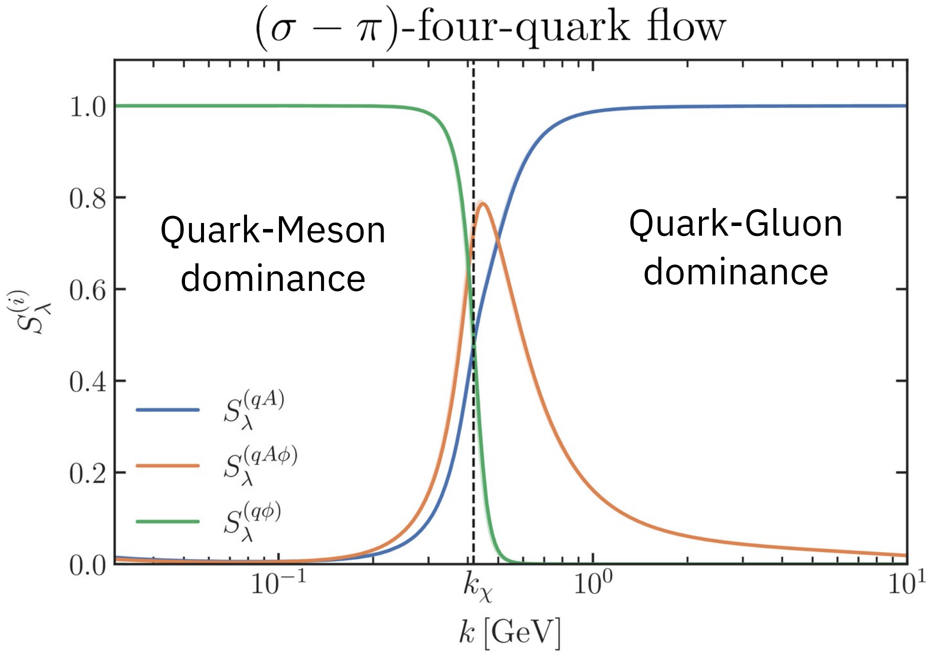
$$\mathcal{S}_\lambda^{(i)} = \frac{\left|\mathrm{Flow}_\lambda^{(i)}\right|}{\sqrt{\sum_j \left(\mathrm{Flow}_\lambda^{(j)}\right)^2}}\,, \qquad i,j = qA\,,\, q\phi\,,\, qA\phi$$

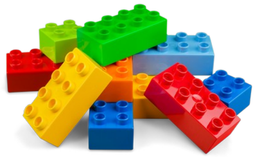
Systematic errors II: The LEGO® principle



Separate LEGO® blocks:

- Glue subsystem $\{\lambda_{\text{glue}}\} = \{\alpha_{A^3}\,,\, \alpha_{A^4}\,,\, \alpha_{c\bar{c}A}\}$
- Matter subsystem $\{\lambda_{\text{mat}}\} = \{h_\phi(\rho_0)\,,\, \lambda_{\phi,n}(\rho_0)\}$
- Interface blocks $\{\lambda_{\text{inter}}\} = \{\alpha_{l\bar{l}A}\}$





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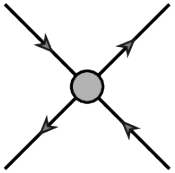

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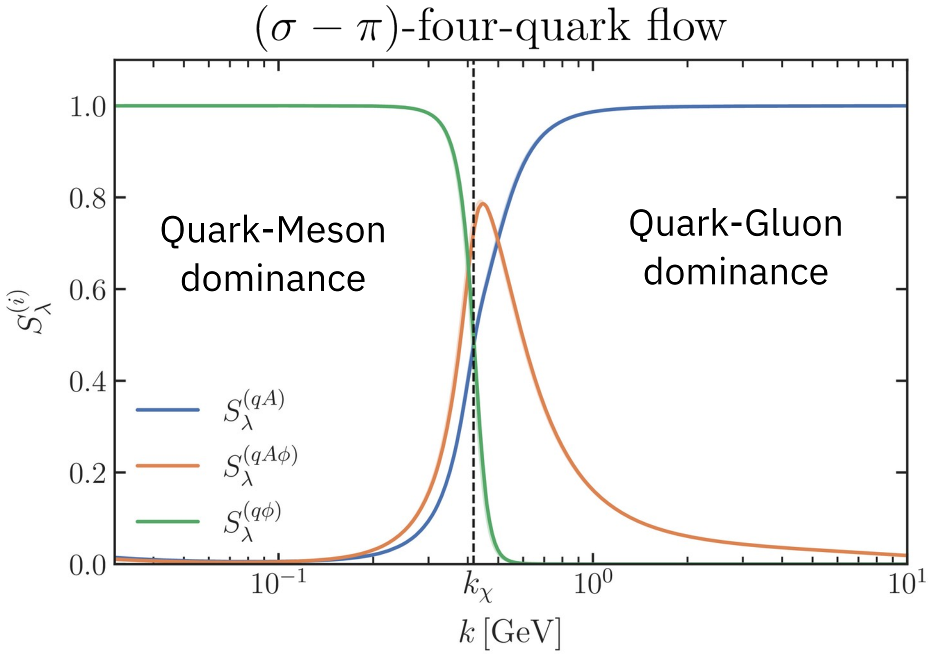
➔ Systematic error estimates from subsystems; preliminary estimate 10%.

➔ Low-energy effective theories.



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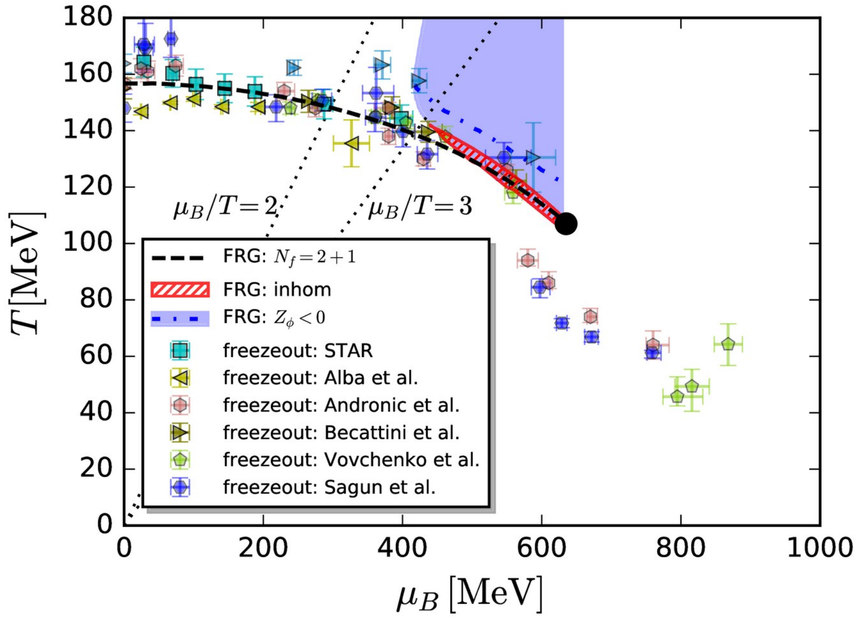
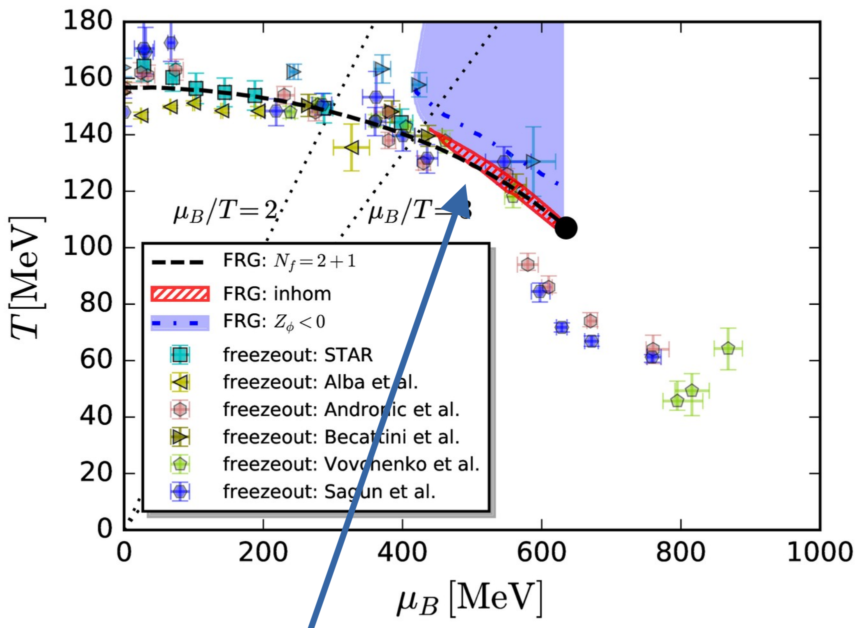


$\mu_B/T = 2$
$\mu_B/T = 3$
FRG: $N_f = 2+1$
FRG: inhom
FRG: $Z_\phi < 0$
freezeout: STAR
freezeout: Alba et al.
freezeout: Andronic et al.
freezeout: Becattini et al.
freezeout: Vovchenko et al.
freezeout: Sagun et al.
$T\,[\mathrm{MeV}]$
$\mu_B\,[\mathrm{MeV}]$



Moat regime at high μ
(possible inhomogeneous phase)

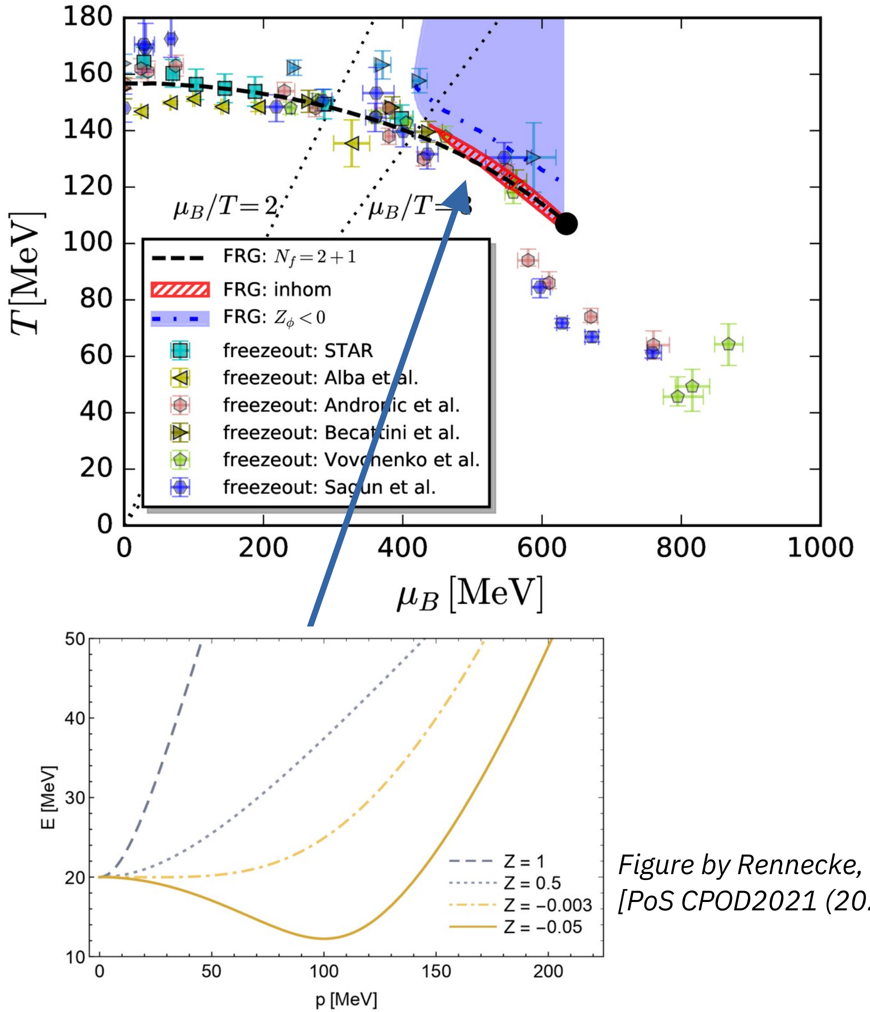


Figure by Rennecke, Pisarski [PoS CPOD2021 (2022) 016]

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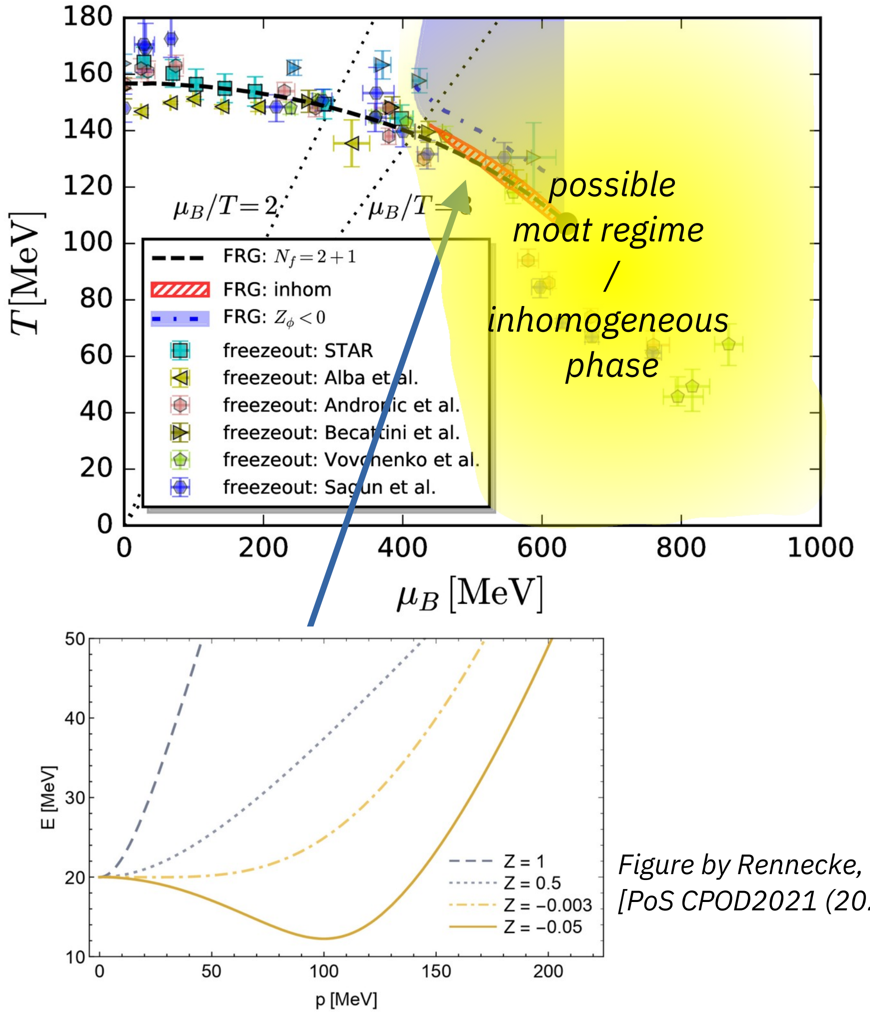

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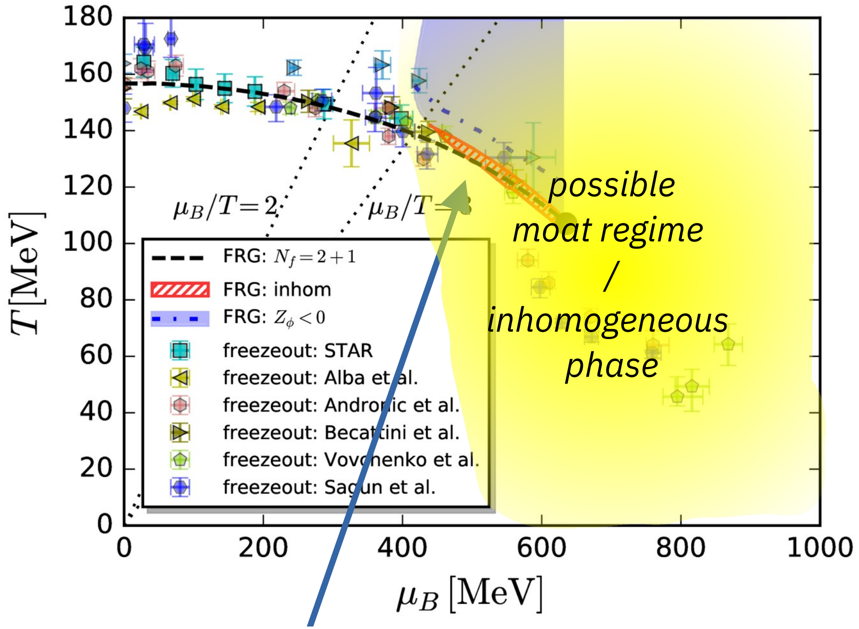
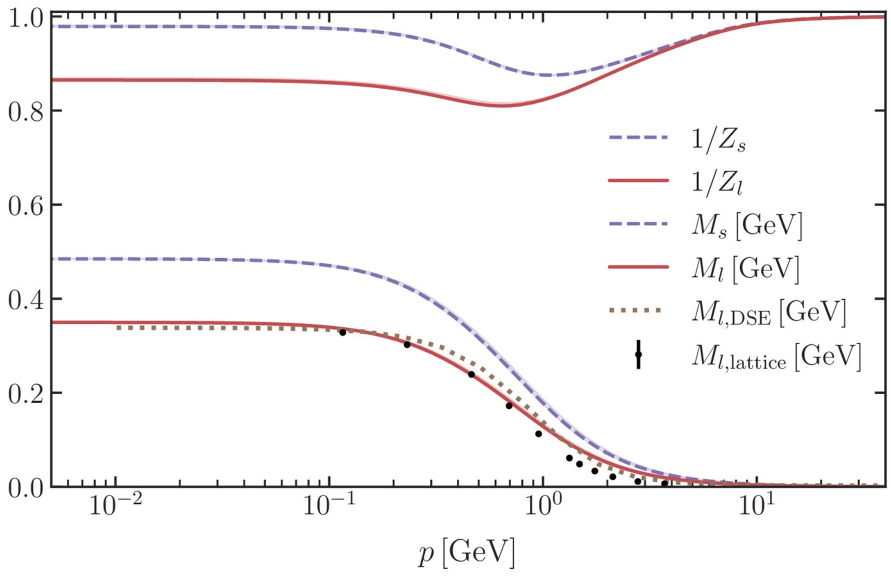


Full momentum resolution of propagators



Lattice data: Cheng et al [Phys. Rev. D 104 (2021), 094509]
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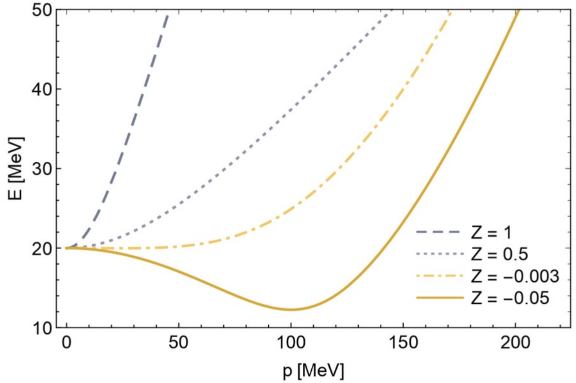

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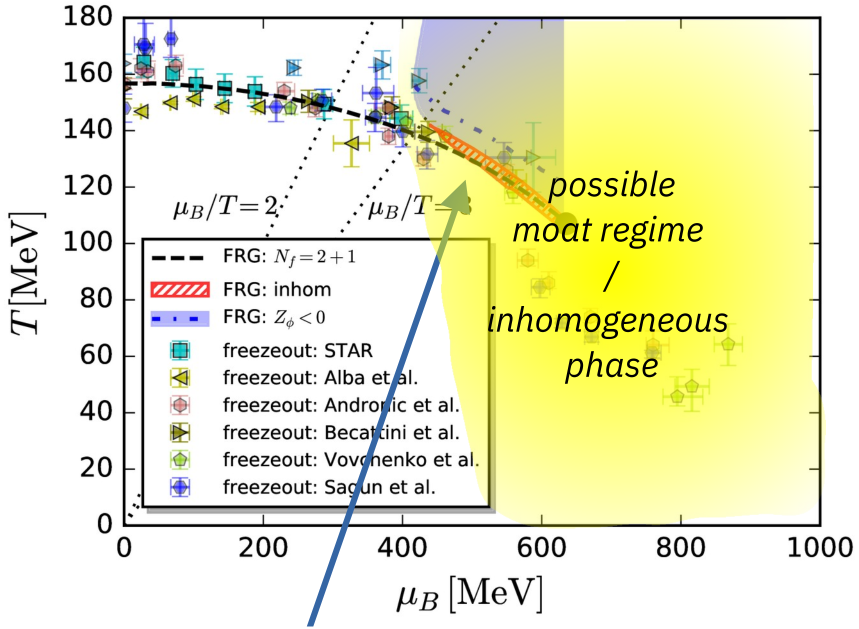

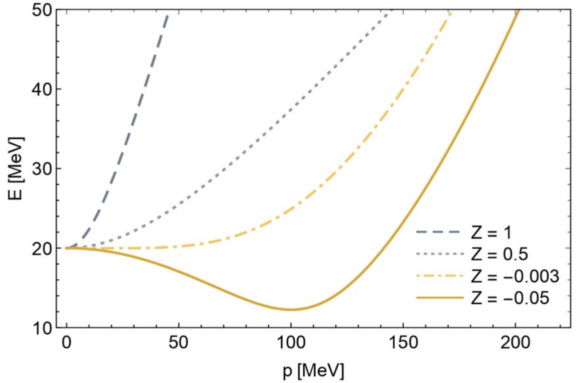

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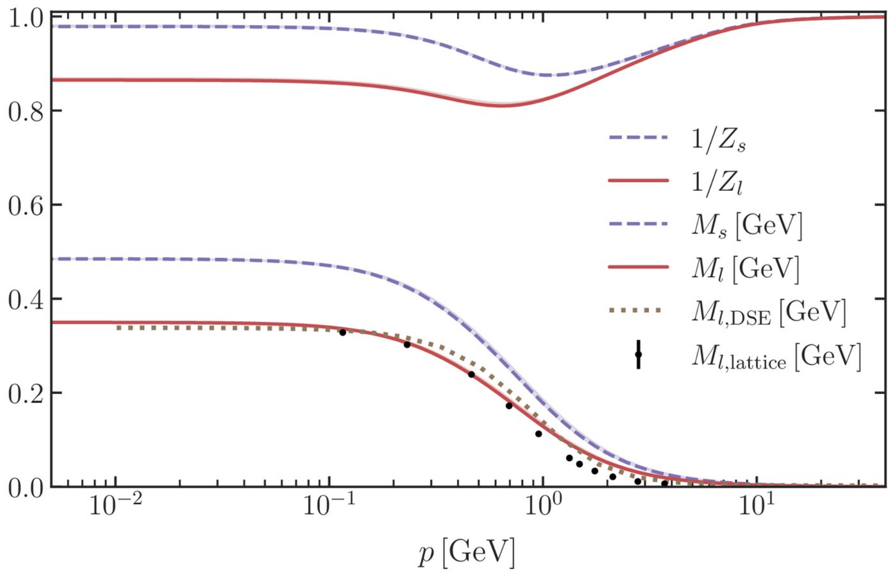

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Access to pole masses:

$$m_{\pi,\mathrm{vacuum}}^{(N_f=2)} = 139(12)\,\mathrm{MeV}$$

$$m_{\pi,\mathrm{vacuum}}^{(N_f=2+1)} = 138(9)\,\mathrm{MeV}$$

Conclusions

- **Motivation: Direct access to phase structure** of QCD through fRG
- **Quantitative Vacuum** results in agreement with Lattice & other functional approaches
- **Systematic error estimates**
- **Easily extendable setup**

Outlook

- Results at **finite (T,μ)** *(in progress)*
- More **momentum dependences** *(done in vacuum)*
- Rebosonisation of **further channels** *(in progress)*
- Increase **number of tensor structures** *(done in vacuum)*

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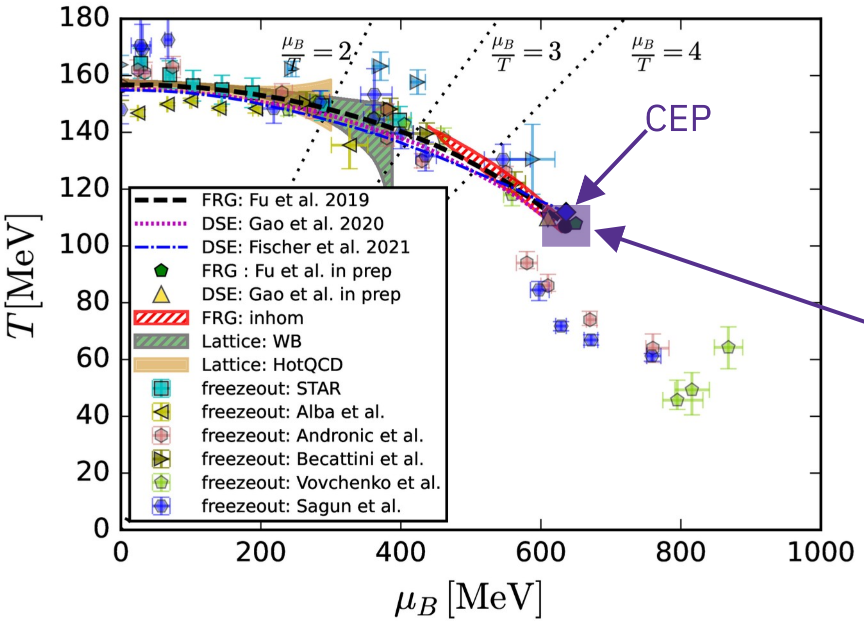
fQCD Collaboration:

Braun
Chen
Fu
Gao
Geissel
Huang
Ihssen
Lu
Pawlowski
Rennecke
Sattler
Schallmo
Stoll
Tan
Töpfel
Turnwald
Wen
Wessely
Wink
Yin
Zorbach

Thank you for your attention!

Backup slides

Mapping out the phase diagram



$$\langle \bar{q}\gamma_0 q \rangle$$

Fu, Pawlowski, Rennecke (Phys. Rev. D 101 (2020), 054032)
Gao, Pawlowski (Phys.Lett.B820(2021) 136584)
Gunkel, Fischer (Phys.Rev.D 104 (2021) 5, 054022)
Bellwied et al. (WB) (Phys.Lett.B 751 (2015) 559-564)
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Sattler et al. (in preperation)

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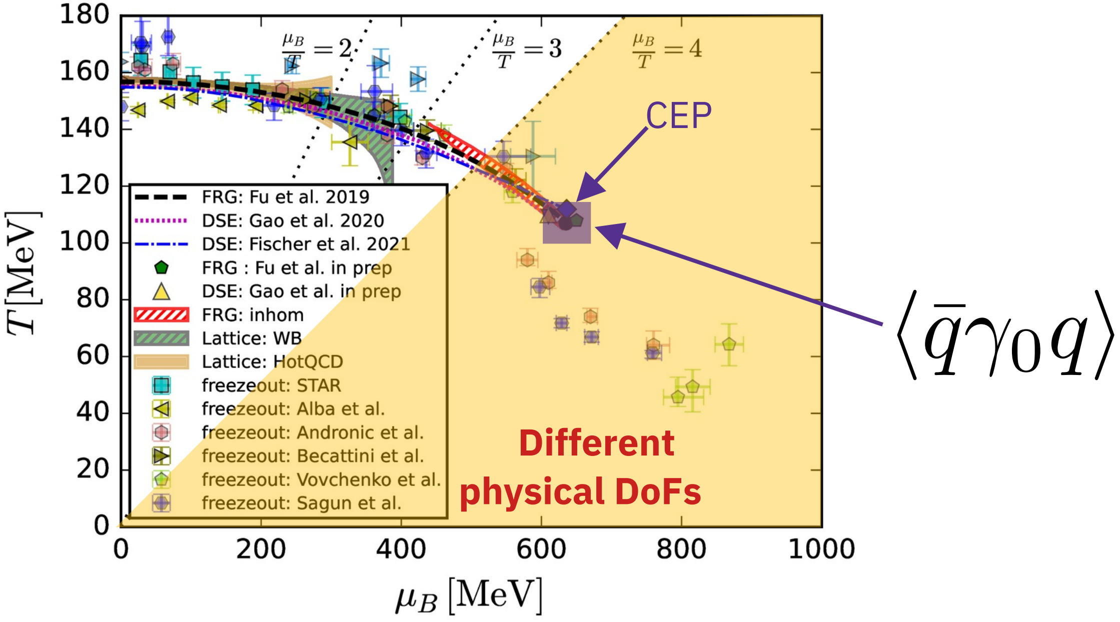

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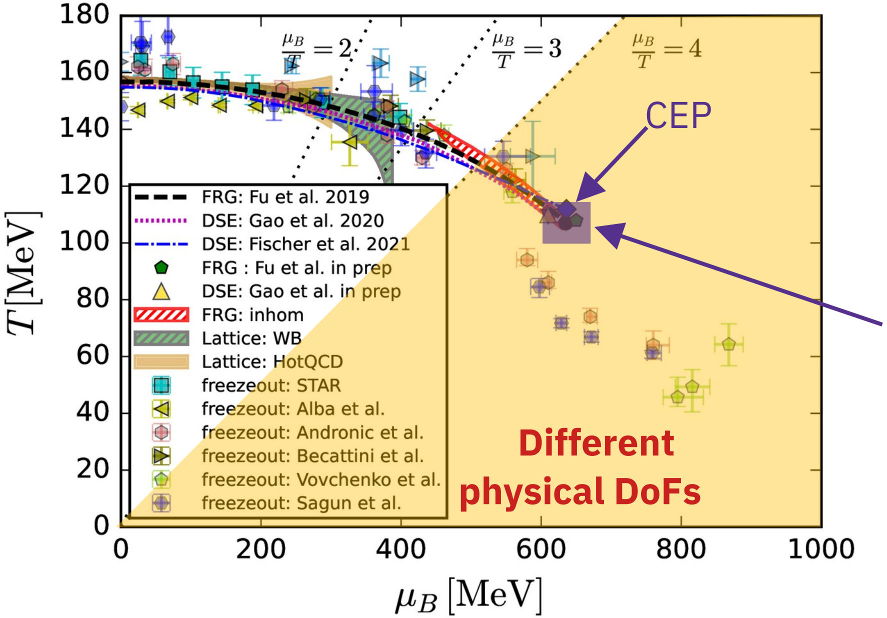
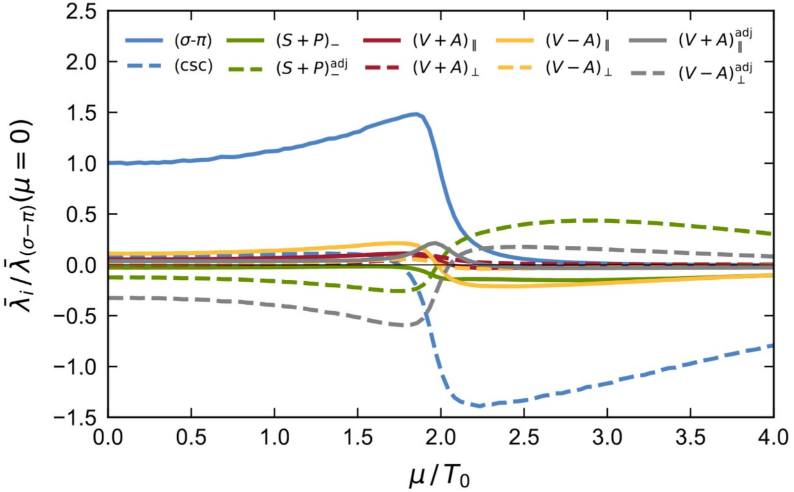


$$\langle \bar{q} \gamma_0 q \rangle$$

Other four-fermi channels



quark chemical potential

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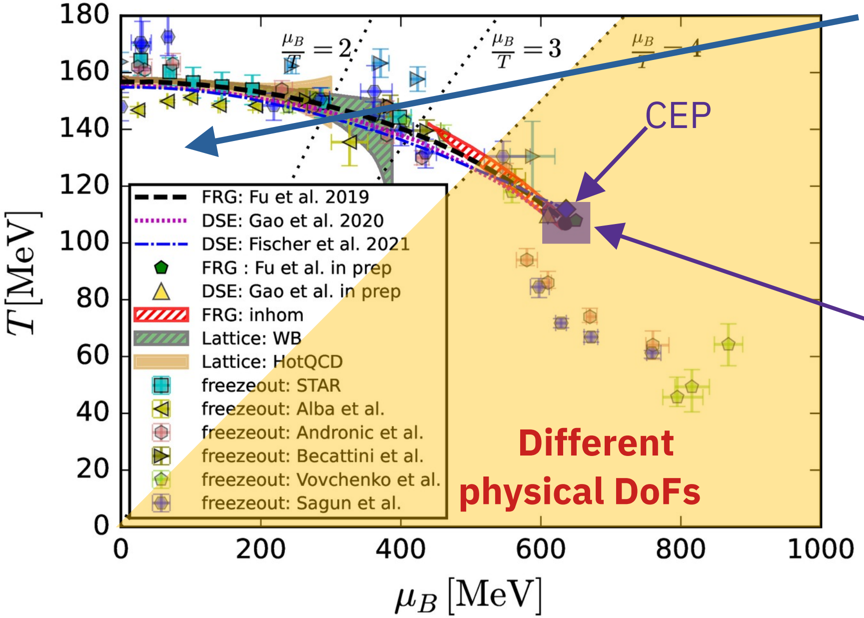
Braun, Leonhardt, Pospiech (Phys.Rev.D 101 (2020) 3, 036004)

Mapping out the phase diagram

$\langle \bar{q} q \rangle$



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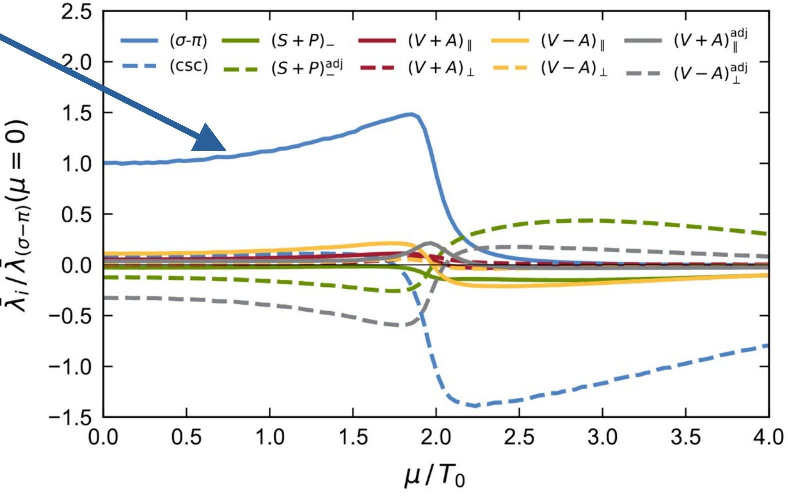
Other four-fermi channels



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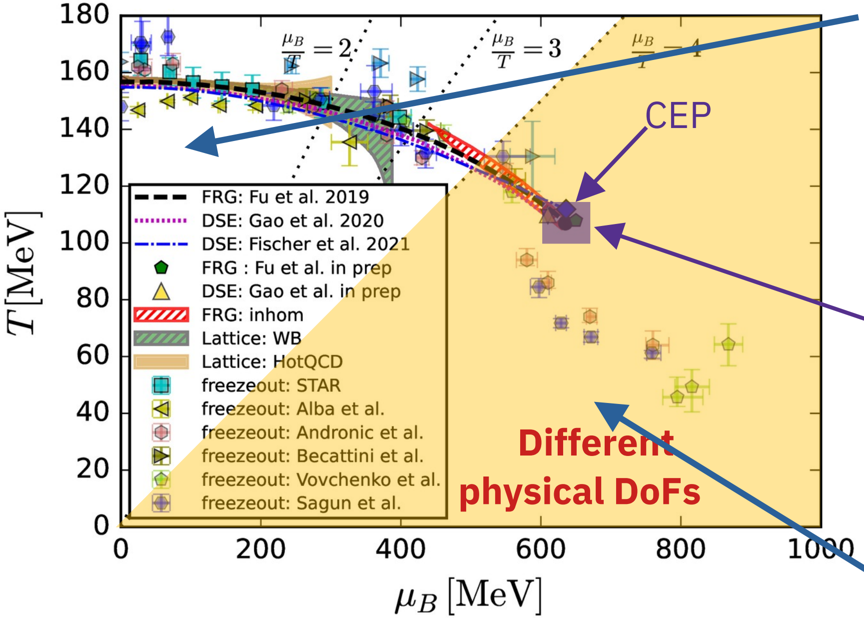

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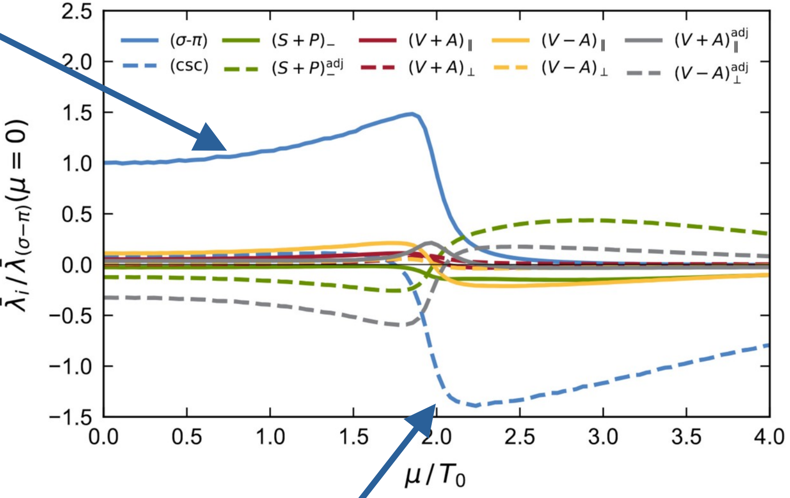

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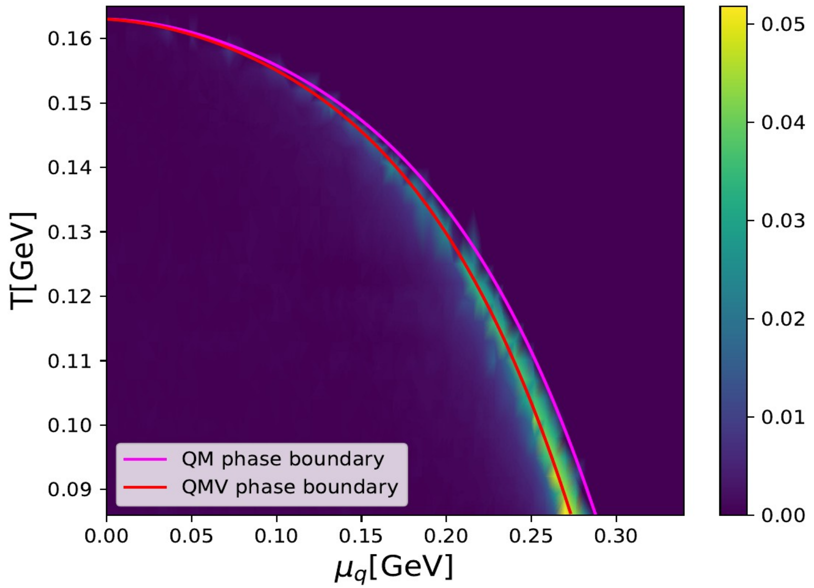
$\langle d^* d \rangle$

Inclusion of Density mode $\langle \bar{q}\gamma_0 q \rangle$ and Diquarks

Ihssen, Hendricks, Pawlowski, Sattler (in preparation)



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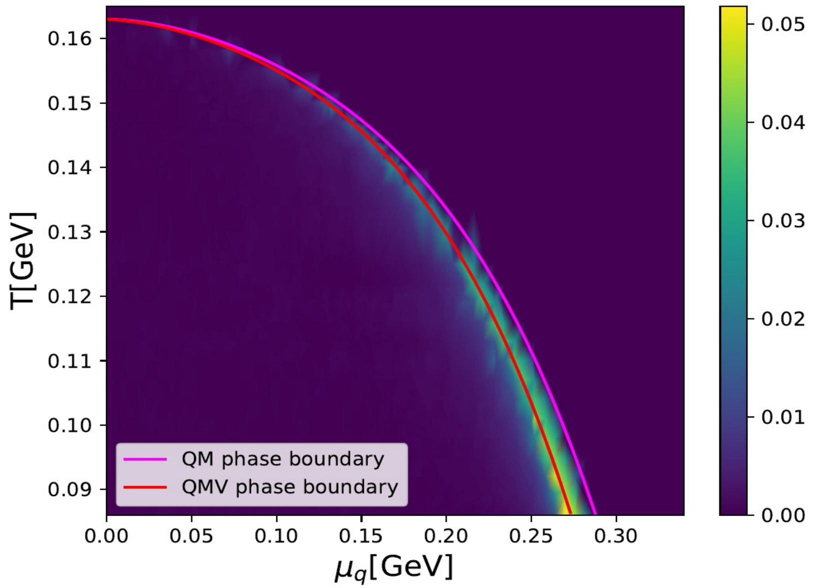


Full Nf = 2+1

Pawlowski, Sattler, Steck
(in preparation)

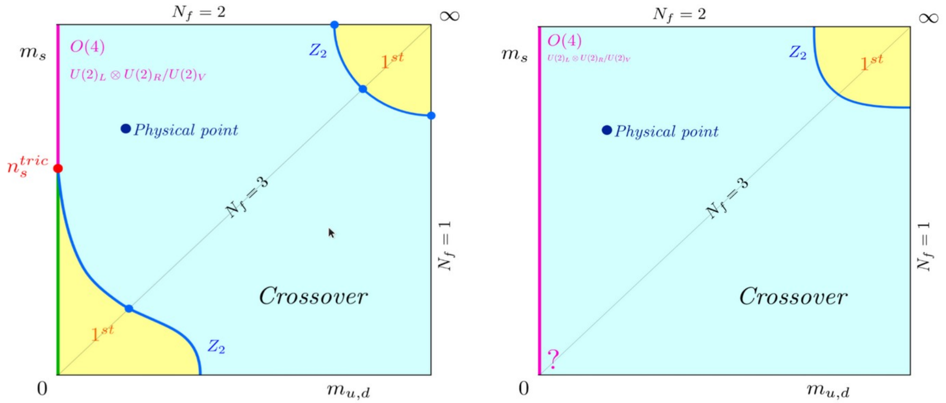
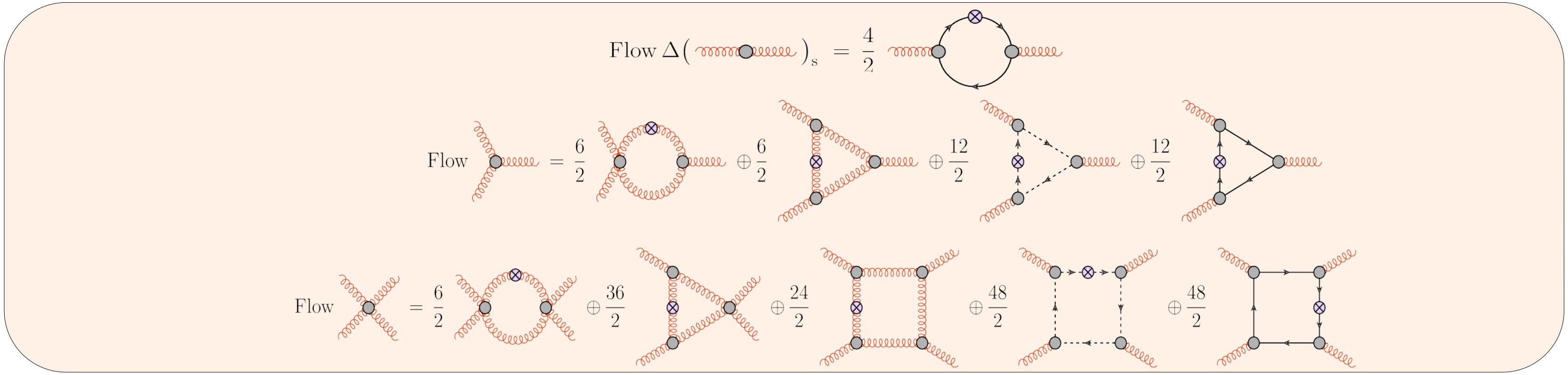
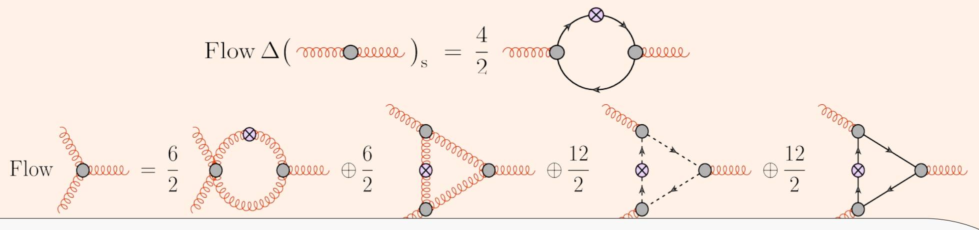


Figure from
Owe Philipsen (arXiv:2111.03590)

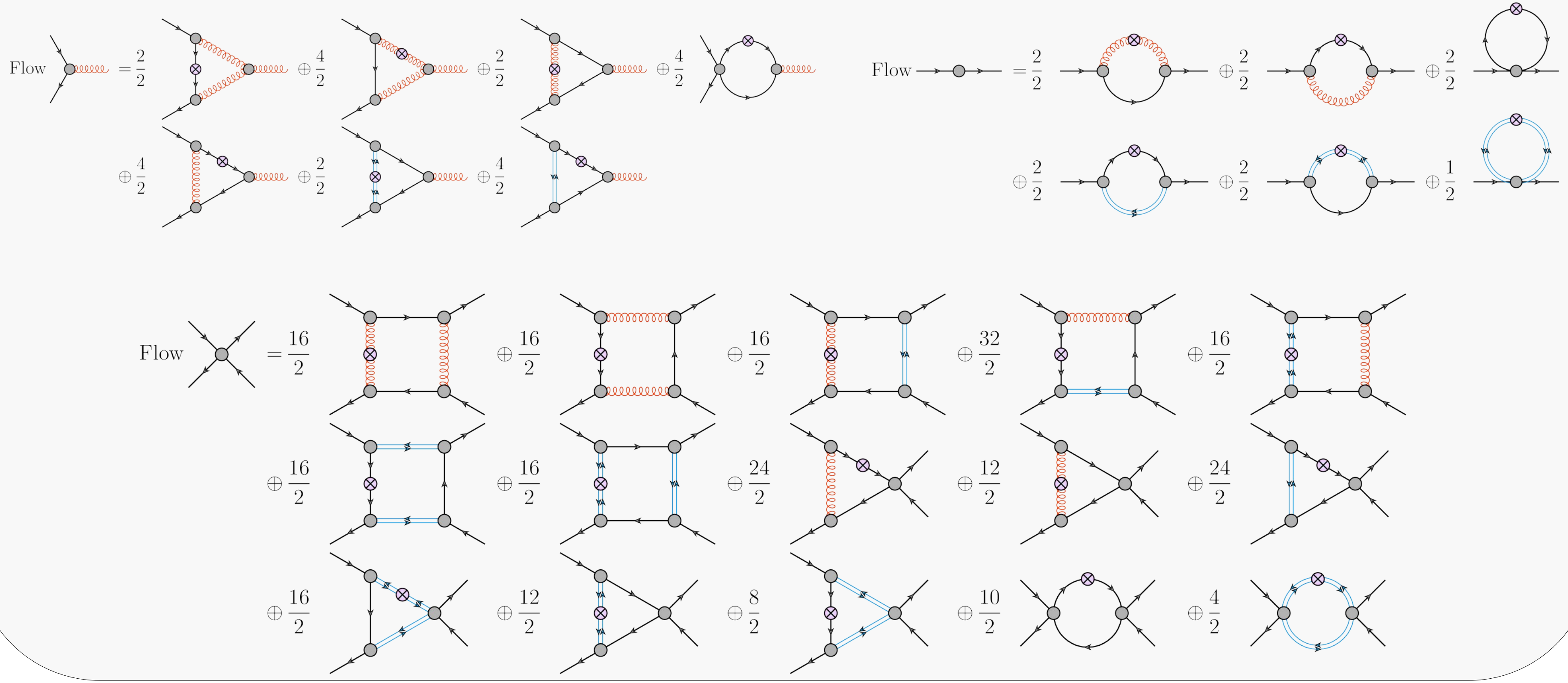
Gluons

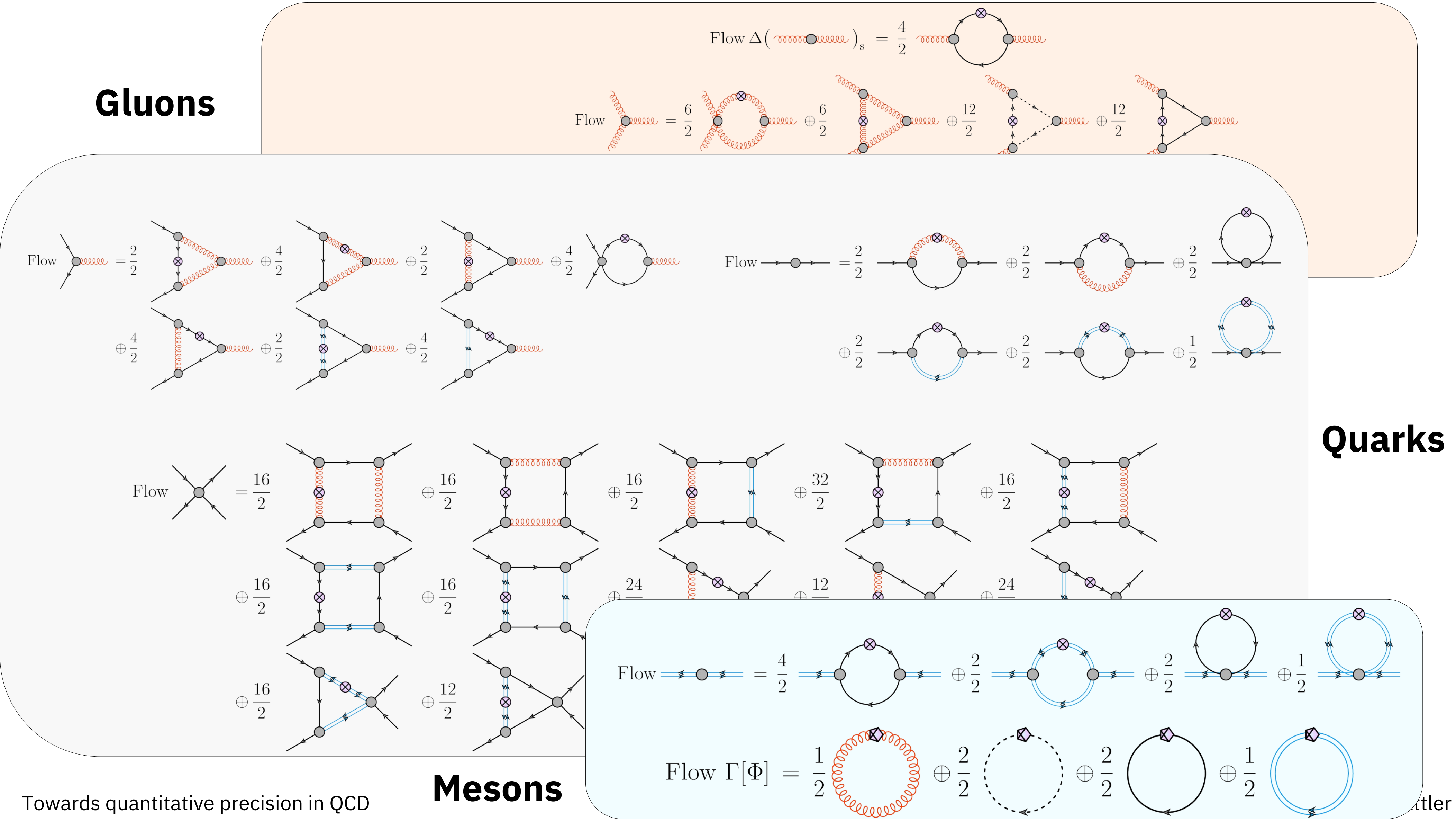


Gluons



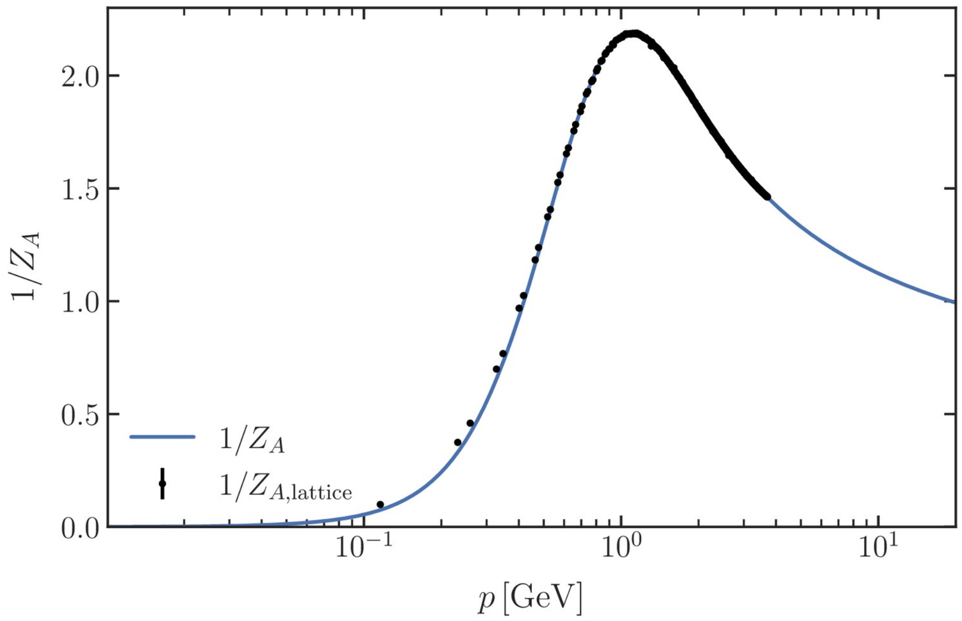
Quarks




Gluons
Quarks
Mesons
Flow Δ( )s = 4/2
Flow = 6/2 ⊕ 6/2 ⊕ 12/2 ⊕ 12/2
Flow = 2/2 ⊕ 4/2 ⊕ 2/2 ⊕ 4/2 ⊕ 4/2 ⊕ 2/2 ⊕ 4/2
Flow = 2/2 ⊕ 2/2 ⊕ 2/2 ⊕ 2/2 ⊕ 2/2 ⊕ 1/2
Flow = 16/2 ⊕ 16/2 ⊕ 16/2 ⊕ 32/2 ⊕ 16/2 ⊕ 16/2 ⊕ 16/2 ⊕ 24 ⊕ 12 ⊕ 24 ⊕ 16/2 ⊕ 12/2
Flow = 4/2 ⊕ 2/2 ⊕ 2/2 ⊕ 1/2
Flow Γ[Φ] = 1/2 ⊕ 2/2 ⊕ 2/2 ⊕ 1/2

Observables	Value	Parameter in $\Gamma_{\Lambda_{\rm UV}}$
$m_{\pi,\rm pol}$ [MeV]	138(9)	$c_{\sigma_l} = 4.67\,\text{GeV}^3$
f_K/f_π	1.1914	$\Delta m_{sl} = 134.2\,\text{MeV}$
$\alpha_{l\bar{l}A,\Lambda_{\rm UV}}$		$\alpha_{l\bar{l}A,\Lambda_{\rm UV}} = 0.227$
m_l [MeV]	350	$a = 0.0251 \quad b = 2\,\text{GeV}$
f_π [MeV]	$97.2^{+4.0}_{-2.2}$	———
m_s [MeV]	$485.0^{+0.0}_{-0.3}$	———
$m_{\pi,\rm cur}$ [MeV]	138	———
m_σ [MeV]	$388.1^{+0.0}_{-1.1}$	———
$\sigma_{0,l}$ [MeV]	$69.^{+1.2}_{-0.2}$	———

Results on physical point of QCD

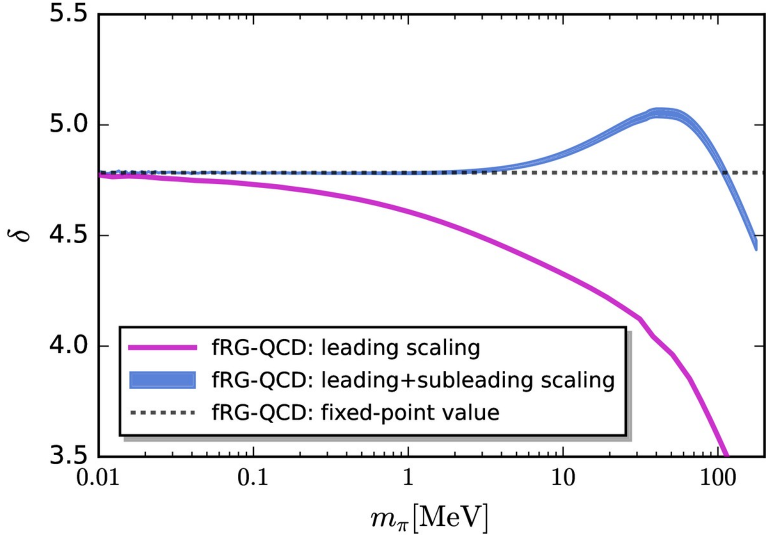


Lattice data from Boucaud et al.
[Phys.Rev.D 98 (2018) 11, 114515]

A comparison to lattice

Soft modes in hot QCD matter

*Braun, Chen, Fu, Gao, Huang, Ihssen, Pawlowski, Rennecke, **Sattler**, Tan, Wen, Yin (arXiv:2310.19853)*



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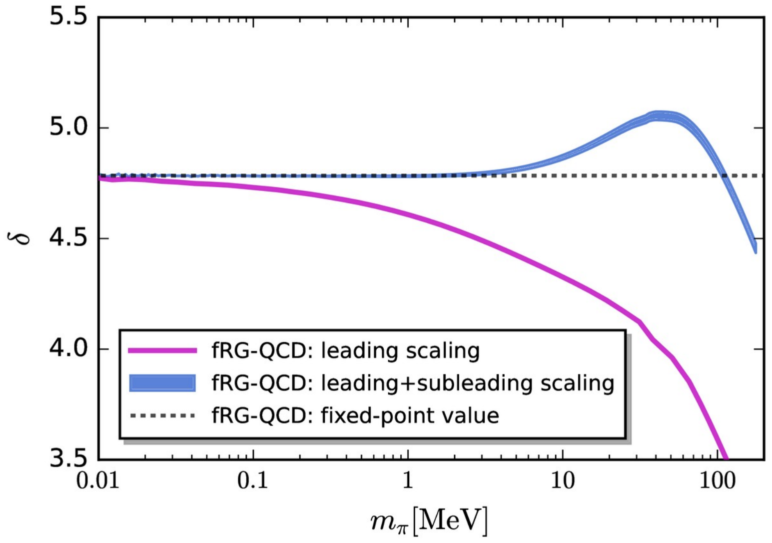


Columbia Plot

*Pawlowski, **Sattler**, Steck*
(in preparation)

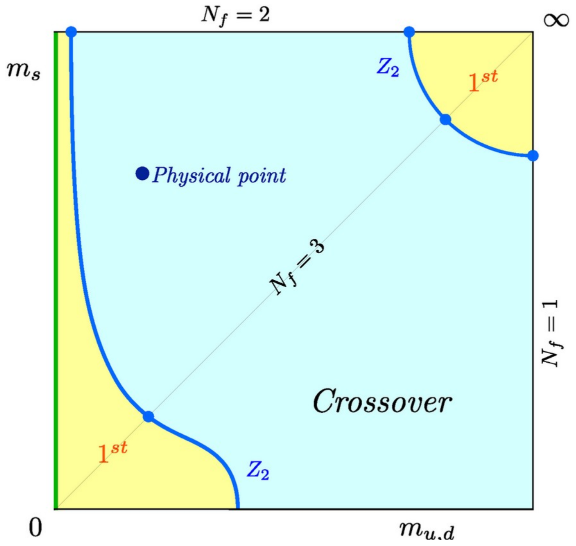
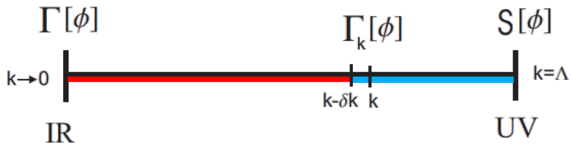


Figure from Owe Philipsen (arXiv:2111.03590)

Wilsonian approach:

Integrate out momentum shells



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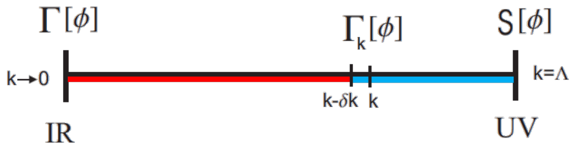

$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi]+\int d^d x J^a(x)\varphi_a(x)}$$

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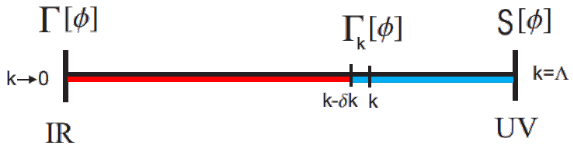

$$Z_k[J] = \int [D\varphi]_k e^{-S[\varphi]+\int d^dx J^a(x)\varphi_a(x)}$$

Introduce mass-like **"Regulator"**

$$[D\varphi]_k = [D\varphi]_{\mathrm{ren}}\, e^{-\frac{1}{2}\int d^dx \varphi_a(x) R_k^{ab}(x)\varphi_b(x)}$$

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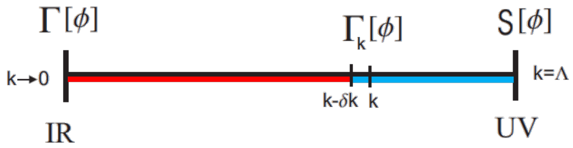

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Obtain Flow equation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\sum_{\mathrm{a,b}} \int \frac{d^d p}{(2\pi)^d} G^{(2)}_{ab}[\phi](p)\partial_t R_k^{ab}(p)$$

Infinite Tower of Functional equations

$$\partial_t \Gamma[\bar\phi] = \frac{1}{2}\mathrm{Tr}\, G_k\, \partial_t R_k\,,$$

$$\partial_t \Gamma^{(1)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\, \Gamma_k^{(3)} \left(G_k\, \partial_t R_k\, G_k\right),$$

$$\partial_t \Gamma^{(2)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\left[\Gamma_k^{(4)} - 2\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\right]\left(G_k\, \partial_t R_k\, G_k\right),$$

$$\partial_t \Gamma^{(3)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\left[\Gamma_k^{(5)} - 6\,\Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} + 6\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\right]\left(G_k\, \partial_t R_k\, G_k\right),$$

$$\begin{aligned}\partial_t \Gamma^{(4)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\Big[&\Gamma_k^{(6)} - 8\,\Gamma_k^{(5)}\, G_k\, \Gamma_k^{(3)} - 6\,\Gamma_k^{(4)}\, G_k\, \Gamma_k^{(4)} + 18\,\Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \\ &+ 12\,\Gamma_k^{(3)}\, G_k\, \Gamma_k^{(4)}\, G_k\, \Gamma_k^{(3)} - 24\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)}\, G_k\, \Gamma_k^{(3)} \cdot G_k\, \Gamma_k^{(3)}\Big]\left(G_k\, \partial_t R_k\, G_k\right),\end{aligned}$$

$$\vdots \qquad\qquad \vdots$$

Infinite Tower of Functional equations

$$\partial_t \Gamma[\bar\phi] = \frac{1}{2}\mathrm{Tr}\, G_k\, \partial_t R_k\,,$$

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$$\partial_t \Gamma^{(2)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\left[\Gamma^{(4)}_k - 2\,\Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k\right]\left(G_k\, \partial_t R_k\, G_k\right),$$

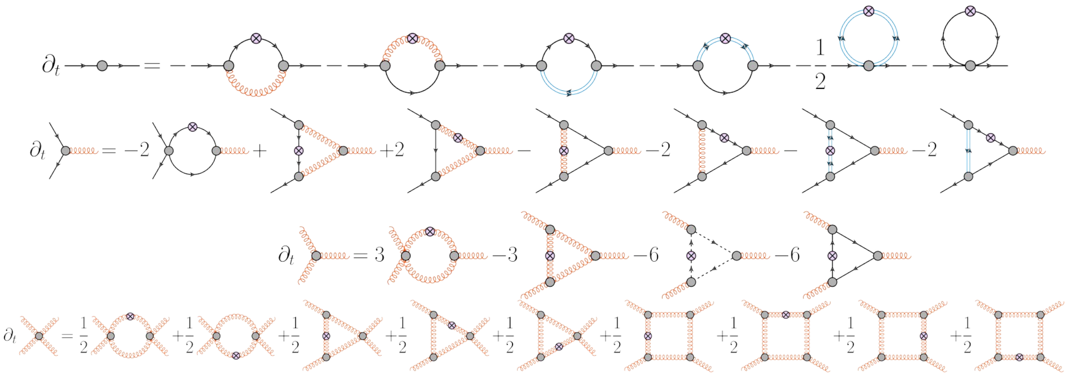
$$\partial_t \Gamma^{(3)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\left[\Gamma^{(5)}_k - 6\,\Gamma^{(4)}_k\, G_k\, \Gamma^{(3)}_k + 6\,\Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k\right]\left(G_k\, \partial_t R_k\, G_k\right),$$

$$\partial_t \Gamma^{(4)}[\bar\phi] = -\frac{1}{2}\mathrm{Tr}\Big[\Gamma^{(6)}_k - 8\,\Gamma^{(5)}_k\, G_k\, \Gamma^{(3)}_k - 6\,\Gamma^{(4)}_k\, G_k\, \Gamma^{(4)}_k + 18\,\Gamma^{(4)}_k\, G_k\, \Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k$$
$$+ 12\,\Gamma^{(3)}_k\, G_k\, \Gamma^{(4)}_k\, G_k\, \Gamma^{(3)}_k - 24\, G_k\, \Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k\, G_k\, \Gamma^{(3)}_k \cdot G_k\, \Gamma^{(3)}_k\Big]\left(G_k\, \partial_t R_k\, G_k\right),$$

⋮ ⋮

=

Infinite Tower of Diagrams



⋮ ⋮ ⋮ ⋮