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Fermion doubling in quantum cellular automata for quantum electrodynamics

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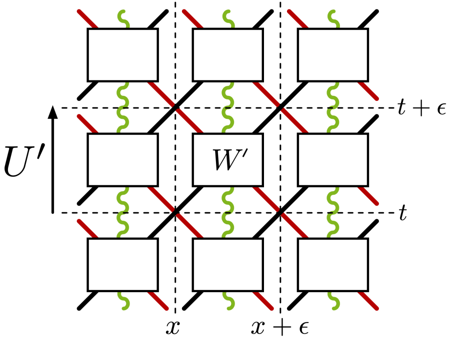
(1+1)D QCA for Quantum Electrodynamics (QED)

Introduction:

- Relativistic quantum simulation used for digital.
- Infinitely repeating quantum circuit simulating QED.

Description, 3 steps:

- Unitary evolution
  $\Rightarrow$ W' gate
- U(1) gauge symmetry
  $\Rightarrow$ gauge field on links
  $=$ green wiggly wires
- 1-particle sector
  $=$ Dirac Quantum Walk



Dirac QW in (1+1)D

- The state of (1+1)D QCA for QED at any time t in 1-particle sector is

$$|\psi(t)\rangle = \sum_x [\psi^+(x,t)\,|x+\rangle + \psi^-(x,t)\,|x-\rangle], \tag{1}$$

where $|x+\rangle, |x-\rangle$ are position eigenvectors with positive and negative chirality respectively.

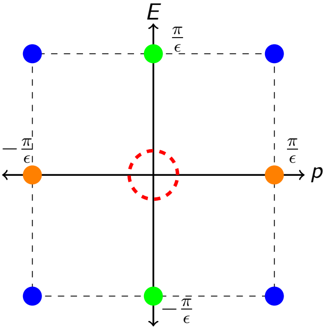
- After one step application of W, we obtain the evolution equations below

$$\begin{aligned} \psi^+(x,t+\epsilon) &= \cos(m\epsilon)\,\psi^+(x-\epsilon,t) - i\sin(m\epsilon)\,\psi^-(x,t) \\ \psi^-(x,t+\epsilon) &= \cos(m\epsilon)\,\psi^-(x+\epsilon,t) - i\sin(m\epsilon)\,\psi^+(x,t). \end{aligned} \tag{2}$$

- Fermions move wrt. their chiral charges, fermions with positive chirality move to right and fermions with negative chirality move to left.
- Fermions move in the speed of light with the probability of $\cos^2(m\epsilon)$, or they stay with the probability of $\sin^2(m\epsilon)$
- Notice that evolution equations are unitary and strictly local

Fermion-Doubling in Lattice Gauge Theories (LGT)

$$G_{LGT}((n,m),(k,l)) = \int_{-\pi/\epsilon}^{\pi/\epsilon} \frac{dEdp}{(2\pi)^2} e^{i\epsilon(-E(m-l)+p(n-k))}$$
$$\times(\frac{1}{\epsilon^2}(sin^2(E\epsilon) - sin^2(p\epsilon)) - m^2)^{-1} \begin{pmatrix} -sin(E\epsilon) + \frac{1}{\epsilon}sin(p\epsilon) & -m \\ -m & -sin(E\epsilon) - \frac{1}{\epsilon}sin(p\epsilon) \end{pmatrix}. \quad (3)$$



→ Only the red neighbourhood is physical!

→ Blue, green ,orange are infinite in continuum!

Figure: Energy momentum modes that contribute to G_{LGT} in the continuum limit

Fermion-Doubling Problem in the (1+1)D QED QCA

$$G_{QCA}((n,m),(k,l)) = \epsilon^2 \int_{-\pi/\epsilon}^{\pi/\epsilon} \frac{dEdp}{(2\pi)^2} e^{i\epsilon(-E(m-l)+p(n-k))}$$
$$\frac{1}{\mathcal{D}_1(E,p)} \begin{pmatrix} e^{iE\epsilon} - \cos(m\epsilon)e^{-ip\epsilon} & -i\sin(m\epsilon) \\ -i\sin(m\epsilon) & e^{iE\epsilon} - \cos(m\epsilon)e^{ip\epsilon} \end{pmatrix}. \qquad (4)$$

$$\mathcal{D}_1(E,p) = 1 + e^{-2iE\epsilon} - 2e^{-iE\epsilon}\cos(p\epsilon)\cos(m\epsilon). \qquad (5)$$

And it has a problematic **4-times degenerate** symmetry:

$$\mathcal{D}_1(E \pm \pi/\epsilon, p \pm \pi/\epsilon) = -\mathcal{D}_1(E,p). \qquad (6)$$

Because for $E\epsilon$ and $p\epsilon$ small, in the limit $\epsilon \to 0$, all 5 solutions are:

$$\propto \quad \mathcal{D}_1(E,p) = \epsilon^2(E^2 - p^2 - m^2) + \ldots \qquad (7)$$

Conclusion:

Again, contribution of high-frequency modes to the continuum.

Solution for the (1+1)D QED QCA: Momentum Space

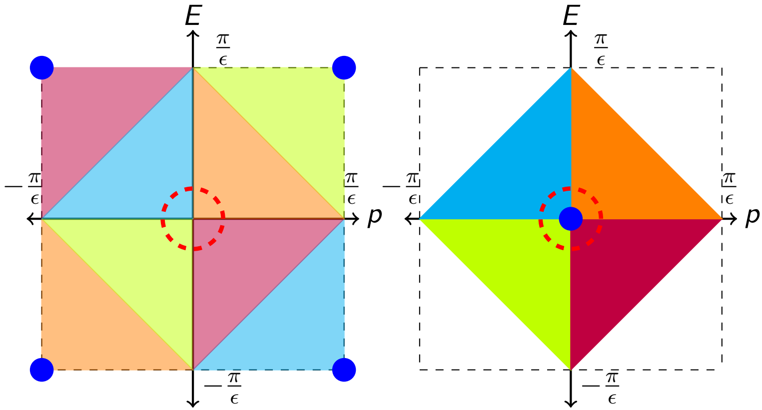


Figure: **On the left**, symmetries of $\mathcal{D}_1(E, p)$ due to its periodicity (which imply a contribution of high-frequency neighborhoods to the continuum limit). **On the right**, proposal of our two-layer solution.

Solution for the (1+1)D QED QCA: Real Space

Two layers in the Brillouin zone
= two sublattices in real/direct space:

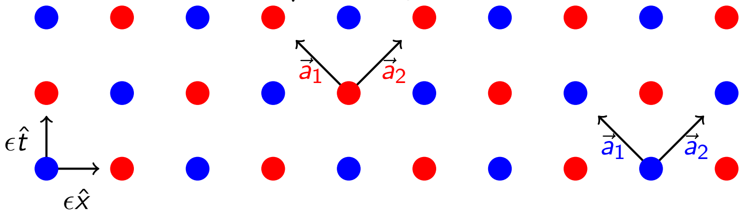


Figure: $\Gamma_r^2 \cup \Gamma_b^2$

- Due to Poincare-Hopf theorem, total chiral charges of red and blue fermions are zero separately.
- We simulate Dirac particles in QCA for QED, but this solution of fermion doubling allows us to consider Weyl particles through the flavour interaction.

Fermion-Doubling Problem for the (3+1)D QED QCA

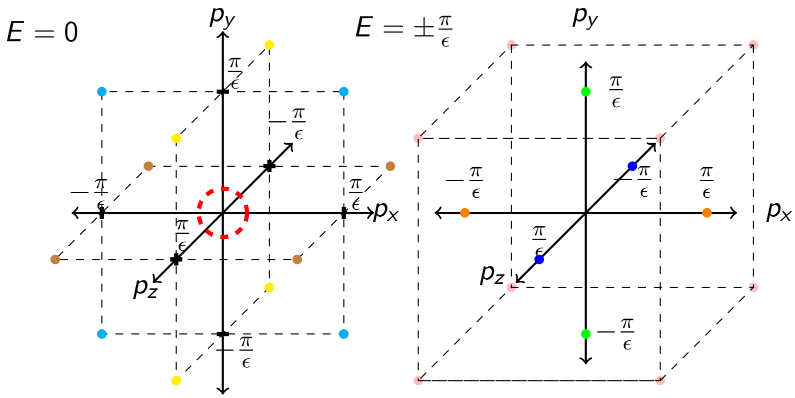


Figure: Degenerate lattice solutions of continuum-limit physics in (3+1)D QED QCA

Solution for the (3+1)D QED QCA (Momentum Space)

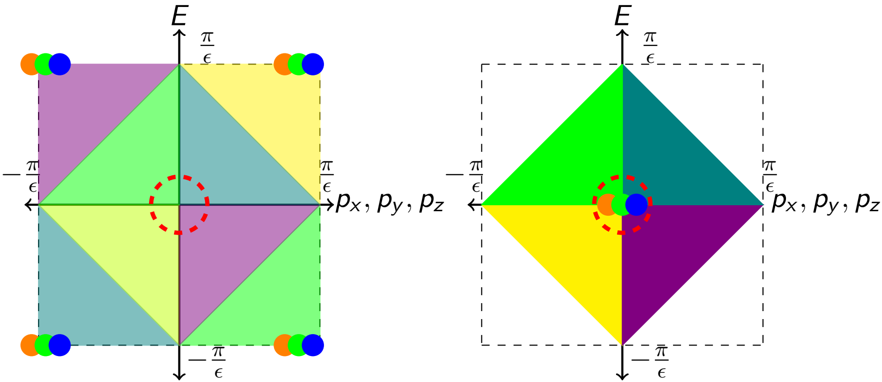


Figure: **On the left**, symmetries of $\mathcal{D}_3(E, p_x, p_y, p_z)$ due to its periodicity (which imply a contribution of high-frequency neighborhoods to the continuum limit, represented on the hyperplanes, $E - p_x$, $E - p_y$, $E - p_z$). **On the right**, proposal of our eight-layer solution.

Advantages of QCA over LGT

- QCA has less problematic fermion doubling, number of flavours needed is 2^{d-1}, whereas in LGT number of needed flavours is 2^d.
- The solution of fermion doubling in QCA does not break chirality unlike the techniques used in LGT.
- Existence of Weyl particles in QCA can be done via interactions of sub-lattices.
- QCA has unitary evolution operator unlike (1+1) dimensional LGTs.