The complex structure of the Quark propagator with the spectral DSE

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QuantFunc2024 - Valencia

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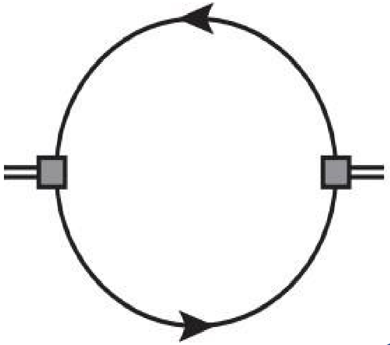
Real time correlators with spectral functional methods

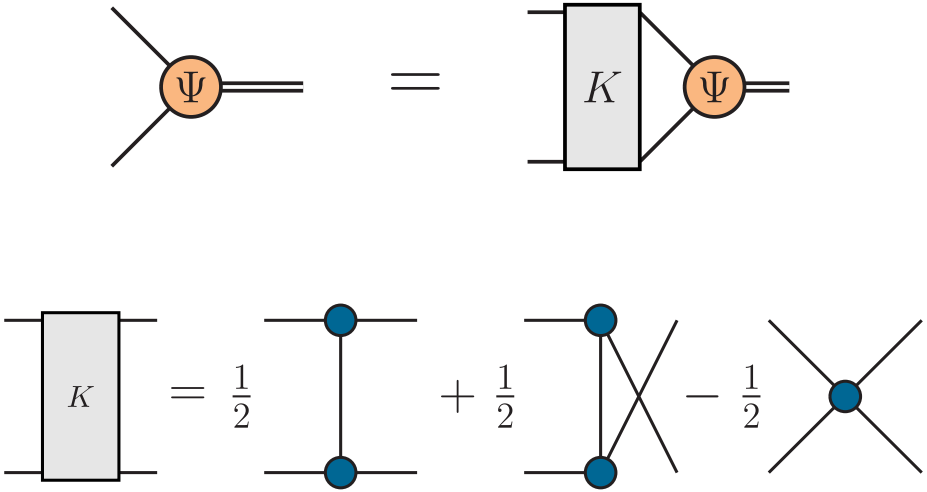
Transport coefficients

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

$$\sigma(\omega, \mathbf{p}) \propto \ \mathrm{Im}$$



Bound states



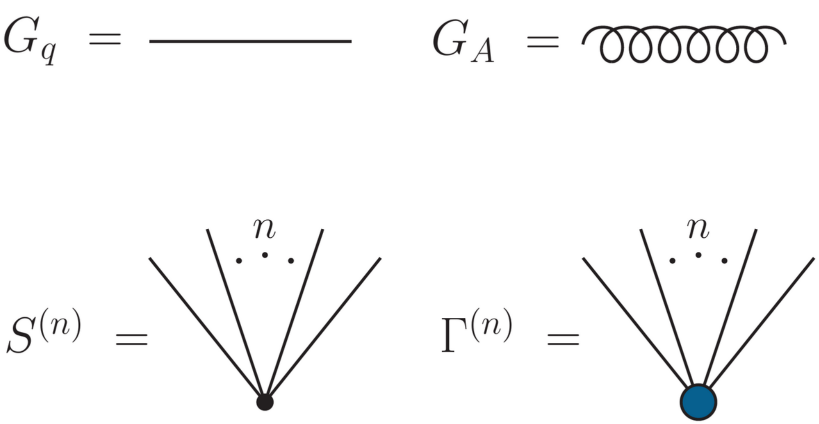
Need for real time correlation functions

The Quark spectral function

Need 2 input correlator

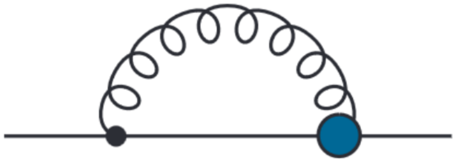
- Gluon Propagator
  - Get from reconstructed Spectral function
- Quark-Gluon Vertex
  - STI-Based causal construction

Notation



The spectral gap equation

$$G_A(q) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda \rho_A(\lambda)}{q^2 + \lambda^2}$$



$$G_q(p) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \frac{\rho_q(\lambda_q)}{\mathrm{i}\not{p} + \lambda_q}$$

The Quark spectral function

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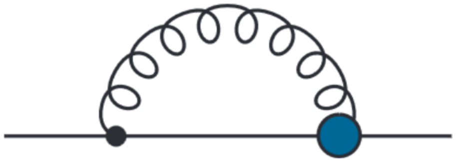

$$\propto g^2 \int_{\lambda_A, \lambda_q} \rho_A(\lambda_A) \rho_q(\lambda_q) \int_q \gamma_\mu \frac{\Pi_\perp^{\mu\nu}(q)}{(\mathrm{i}(\not{p} + \not{q}) + \lambda_q)(q^2 + \lambda_A^2)} \gamma_\nu$$

$$G_q(p) = \int_{-\infty}^\infty \frac{d\lambda}{2\pi} \frac{\rho_q(\lambda_q)}{\mathrm{i}\not{p} + \lambda_q}$$

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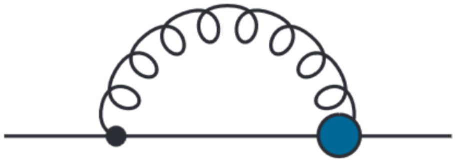


$$G_q(p) = \int_{-\infty}^\infty \frac{d\lambda}{2\pi} \frac{\rho_q(\lambda_q)}{\mathrm{i}\not{p} + \lambda_q}$$

- Loop integrals can be calculated in dimReg
- Access to the full complex plane

$$\propto g^2 \boxed{\int_{\lambda_A, \lambda_q} \rho_A(\lambda_A)\rho_q(\lambda_q)} \boxed{\int_q \gamma_\mu \frac{\Pi_\perp^{\mu\nu}(q)}{(\mathrm{i}(\not{p} + \not{q}) + \lambda_q)(q^2 + \lambda_A^2)} \gamma_\nu}$$

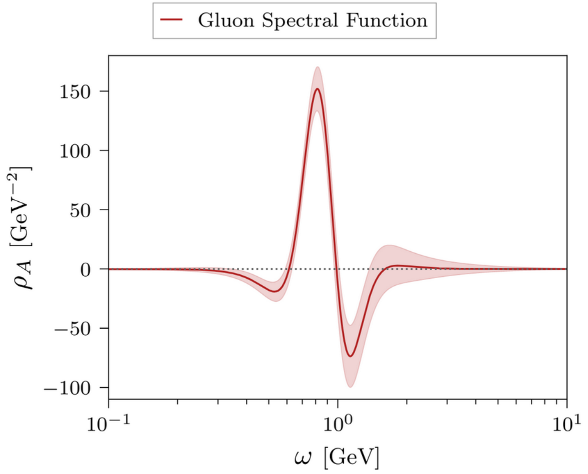
- But: additional (numerical) spectral integrals
- Spectral renormalisation for diverging diagrams

Horak, Pawlowski, Wink PRD 102 (2020) 125016

Horak, Pawlowski, Wink SciPost Phys. 15, 149 (2023)

Input and Vertex construction 1

- Gluon Propagator: Reconstruction on $(2+1)$ flavour Lattice Data



Horak et al PRD 105 (2022), 3, 036014

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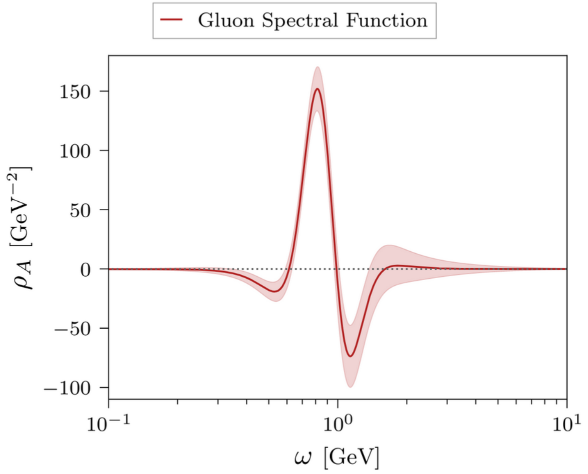
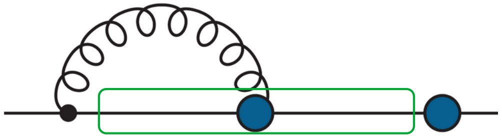
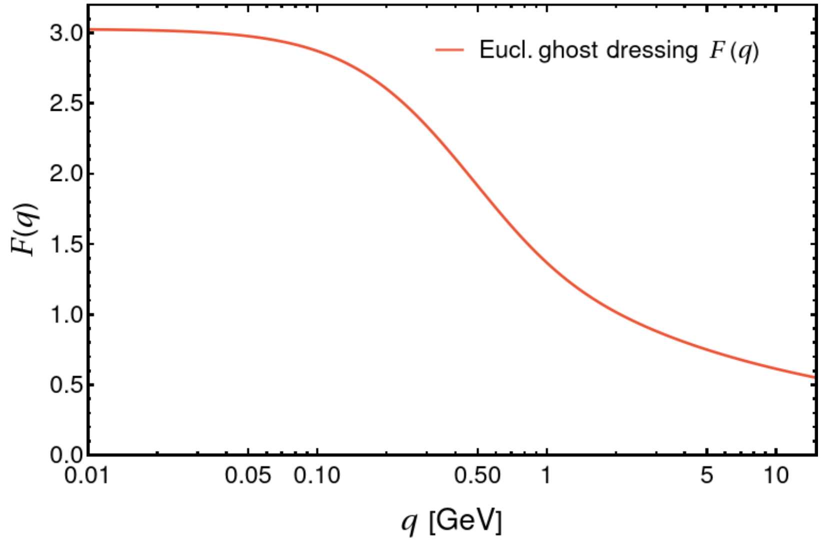


- Quark-gluon vertex: (spectral) STI construction



$$G_q(p+k)\Gamma^{\mu}_{\bar{q}qA}(p+k,p)G_q(p)$$

$$= g_s F(k^2) t^a \int_\lambda \rho(\lambda) \frac{1}{\mathrm{i}(\not{p}+\not{q})+\lambda}(-\mathrm{i}\gamma^\mu)\frac{1}{\mathrm{i}\not{p}+\lambda}$$



Horak et al PRD 105 (2022), 3, 036014

Pawlowski, Wessely in preparation

Horak, Papavassiliou, Pawlowski, Wink PRD 104 (2021), 074017

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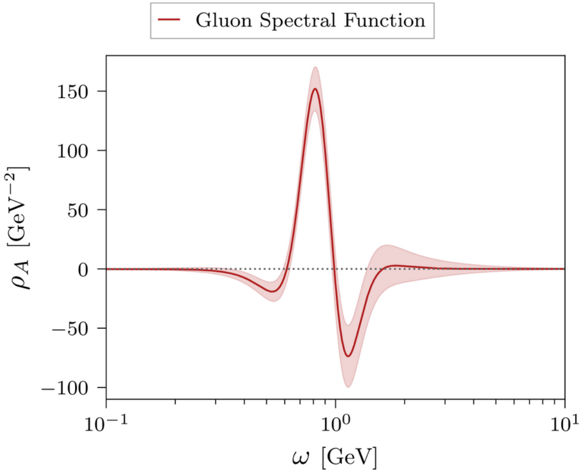
- Gluon Propagator: Reconstruction on $(2{+}1)$ flavour Lattice Data



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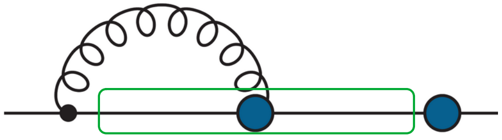


$$G_q(p+k)\Gamma^{\mu}_{\bar{q}qA}(p+k,p)G_q(p)$$

$$= g_s F(k^2) t^a \int_\lambda \rho(\lambda) \frac{1}{\mathrm{i}(\not{p}+\not{q})+\lambda}(-\mathrm{i}\gamma^\mu)\frac{1}{\mathrm{i}\not{p}+\lambda}$$

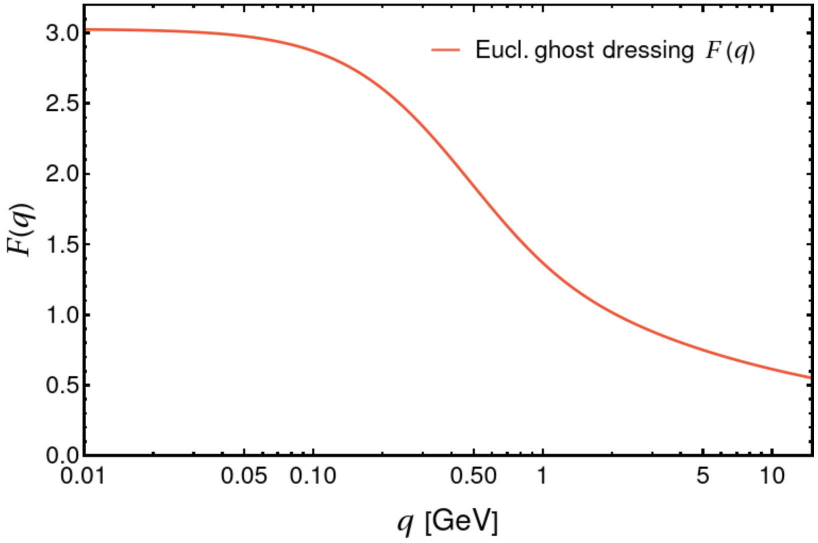
- For abelian versions (without ghostdressing) see

  Delbourgo, West J.Phys. A: Math. Gen. 10 1049 (1977)

  Jia, Pennington PRD 96 (2017) 3, 036021

- Or even simpler: (spectral) rainbow ladder:

$$\Gamma^{\mu}_{\bar{q}qA}(p+k,p) = g_s F(k^2) t^a \gamma^\mu$$



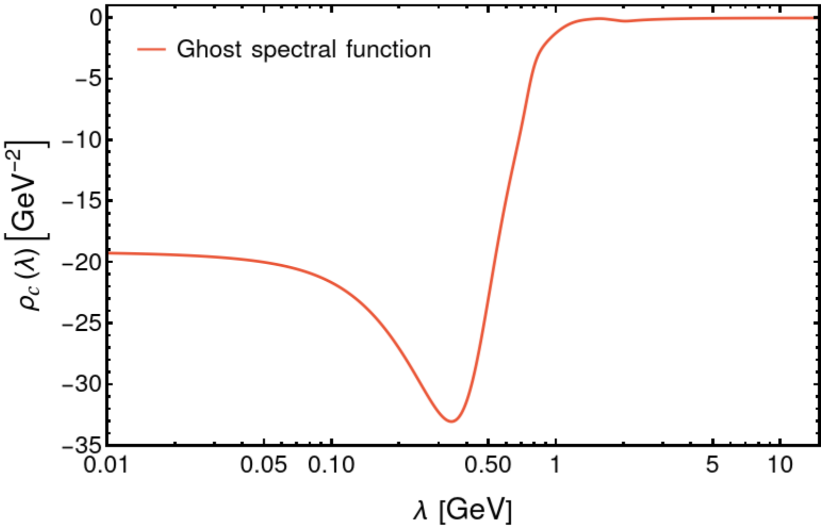
Horak et al PRD 105 (2022), 3, 036014

Horak, Papavassiliou, Pawlowski, Wink PRD 104 (2021), 074017

Input and Vertex construction 2

- Gluon and Ghost Propagator: Reconstruction on $(2+1)$ flavour Lattice Data

Horak et al PRD 105 (2022) 3, 036014



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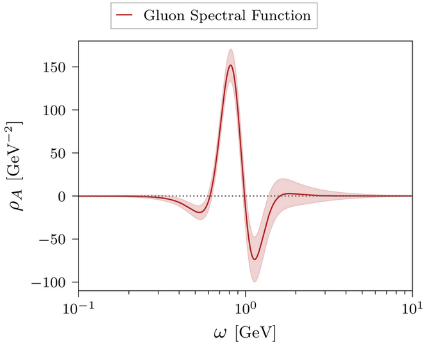
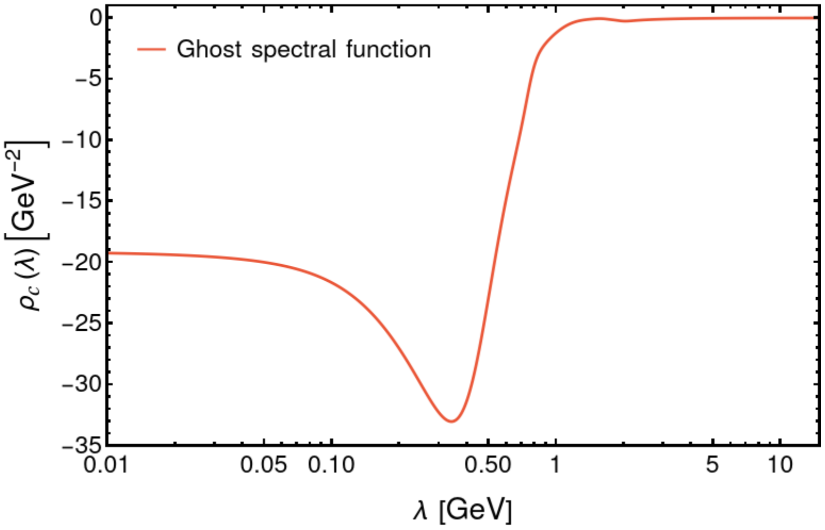
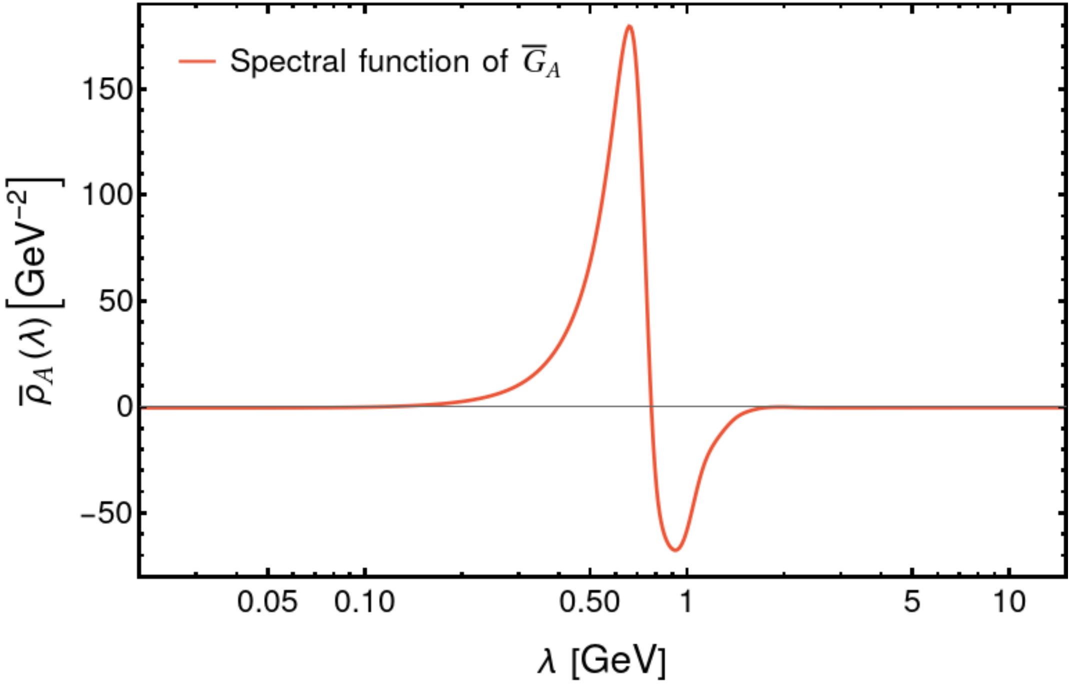


- In both Vertexmodels: Absorb Ghostdressing in Gluonpropagator for combined spectral representation

$$\bar{G}_A(p) = G_A(p)F(p)$$

Horak et al PRD 105 (2022) 3, 036014



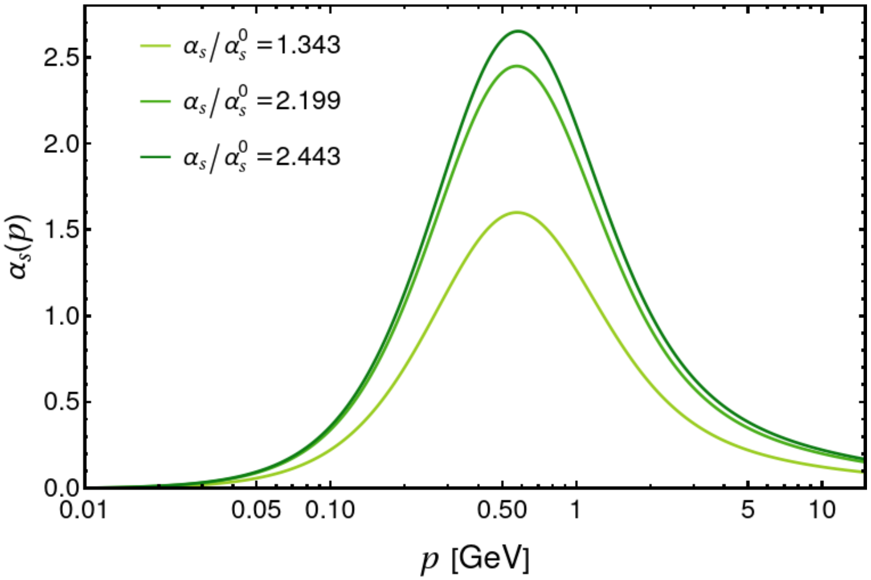


Pawlowski, Wessely in preparation

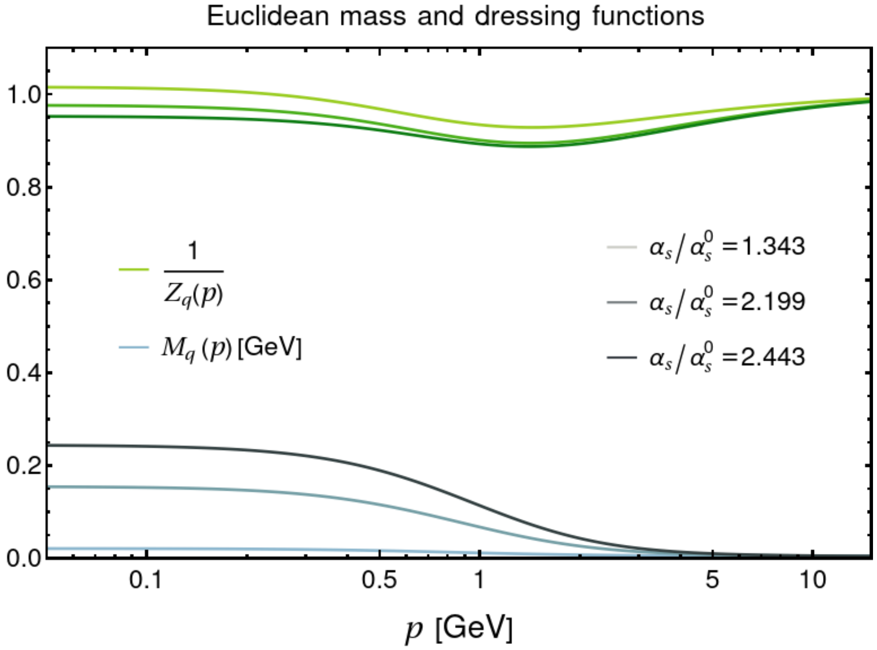
Chiral Symmetry breaking and the quark spectral Function 1

Simple model: need to enhance coupling to trigger "enough" $D_\chi SB$

$$\alpha_s(p) = \frac{1}{4\pi}\frac{(\lambda^{(1)}(p))^2}{Z_A(p)Z_q(p)^2}$$

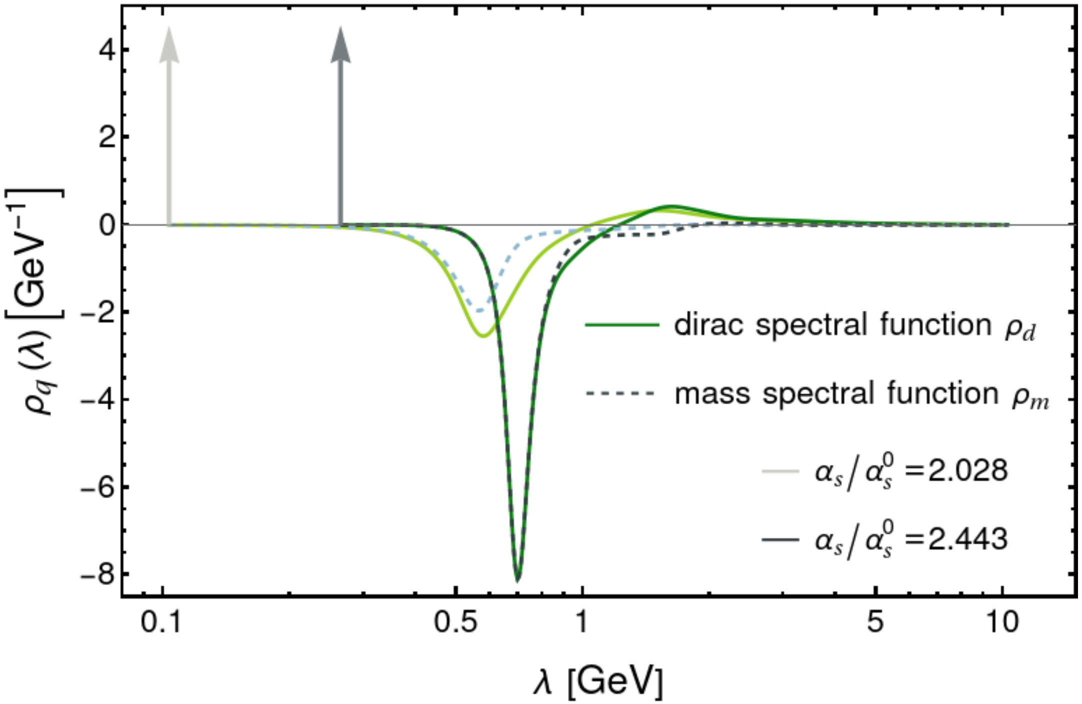


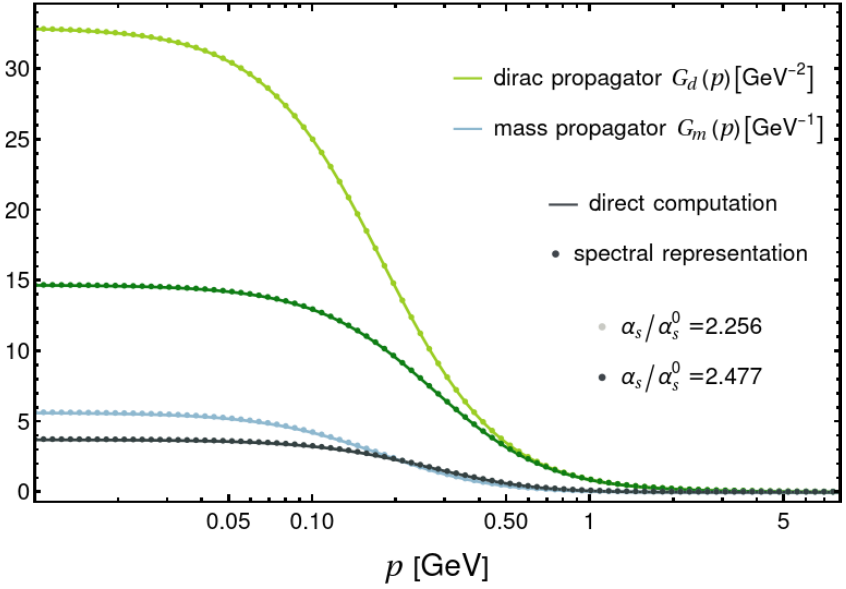
Here: m_l=2.7 MeV, $\alpha_s^0(\mu$=15GeV)=0.235



Pawlowski, Wessely in preparation

Chiral Symmetry breaking and the quark spectral Function 2





- Real pole at the edge of scattering continuum
- Spectral representation fullfilled withhin numerical accuracy

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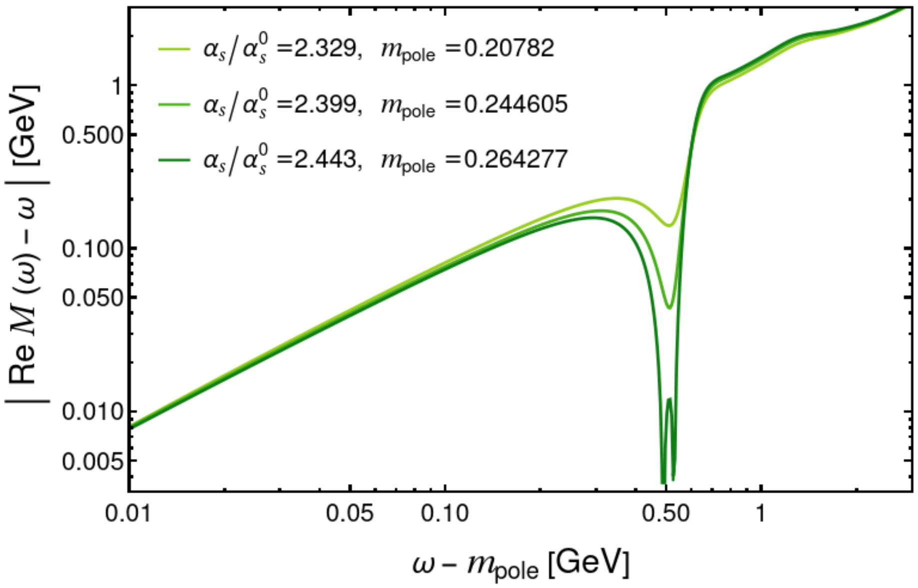
But ...

$D_\chi SB$ and CC-poles

- For large enhancements, analytic structure changes qualitatively
- In addition to the real pole, 2 pairs of CC-poles emerge



- Pole-structure determined by 0-crossings of the realpart of the inverse "universal" part of the quark propagator

$$g(\omega) = \frac{1}{M(\omega)^2 - \omega^2}$$

Pawlowski, Wessely in preparation

$D_\chi SB$ and CC-poles

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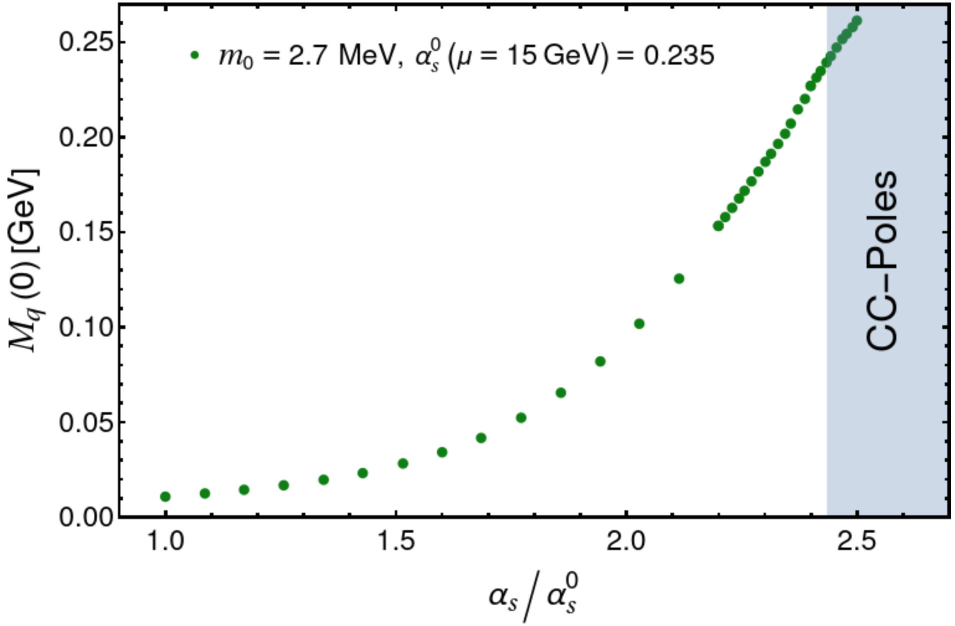

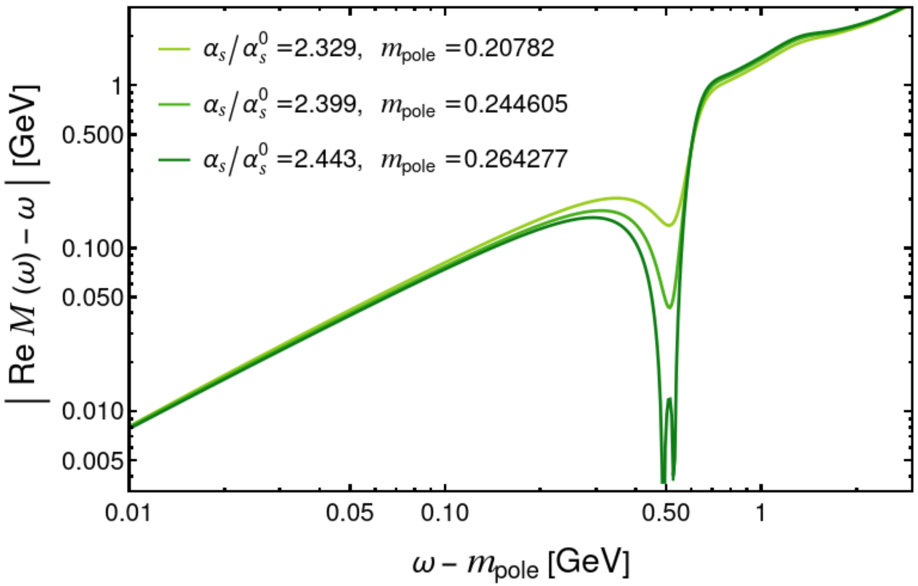

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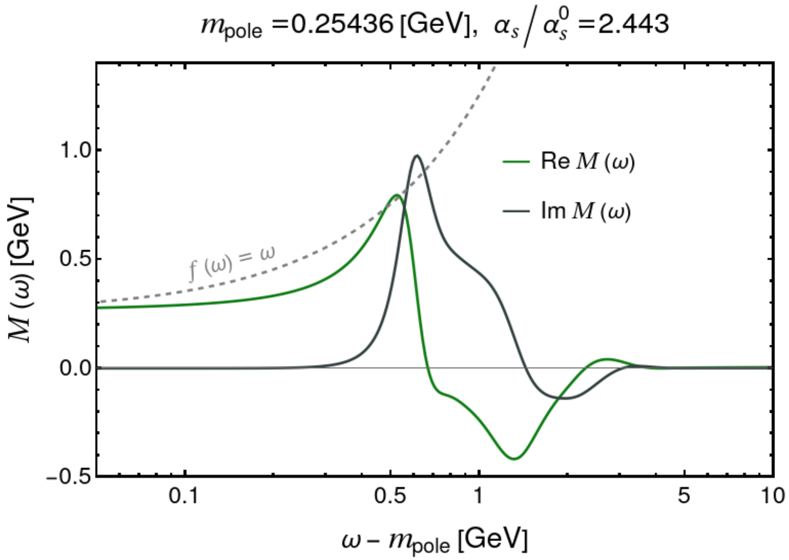
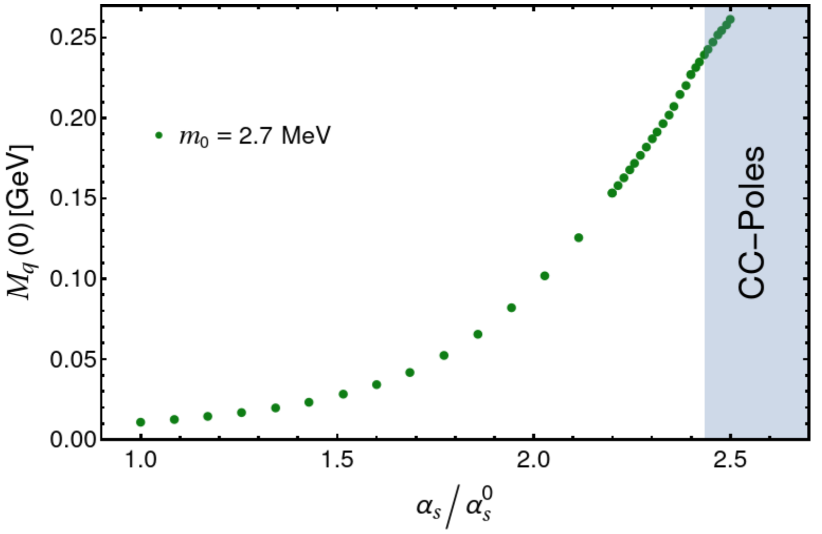
$DB_\chi S$ and CC-poles

- Qualitative change in analytic structure before we reach the right amount of $DB_\chi S$

- Stable feature of approximation



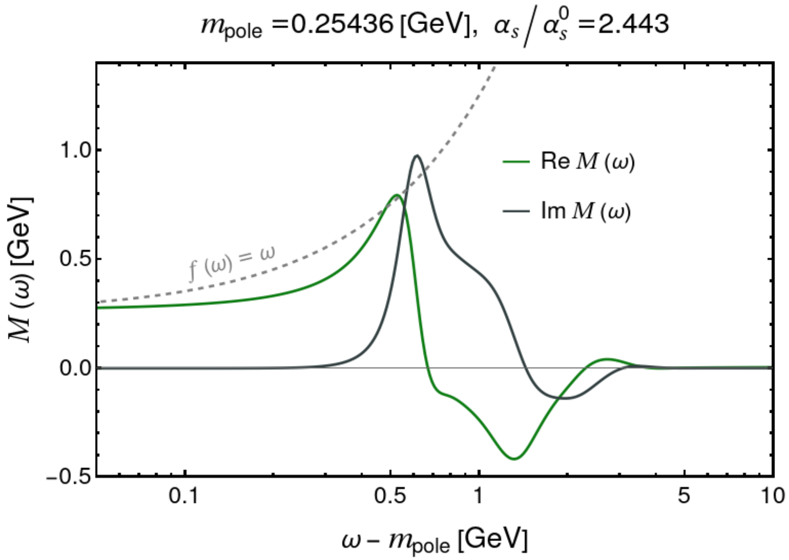


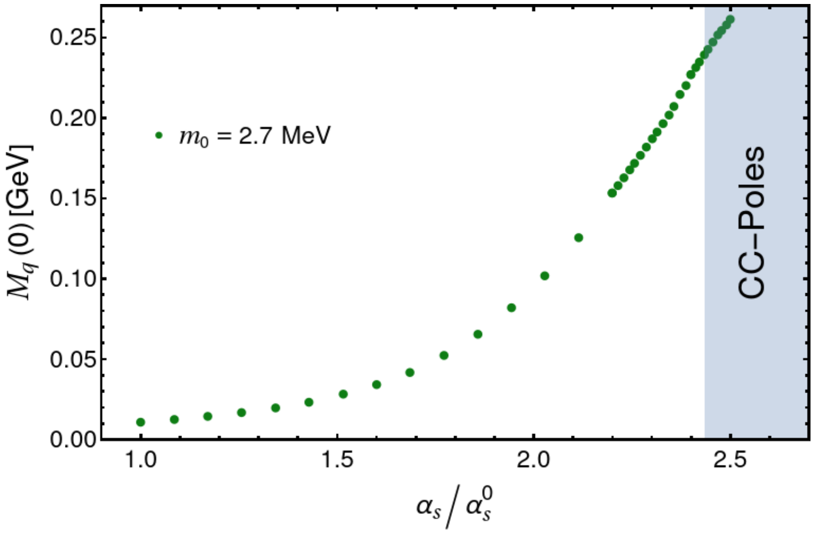
Pawlowski, Wessely in preparation

$DB_\chi S$ and CC-poles

- Qualitative change in analytic structure before we reach the right amount of $DB_\chi S$
- Stable feature of approximation
- But: Inclusion of other tensor structures is a way out
  - ➔ Smaller Enhancement
  - ➔ Different hirarchy of pole vs constituent mass

Fukushima, Horak, Pawlowski, Wink, Zelle arXiv:2308.16594





Pawlowski, Wessely in preparation

Conclusion & Outlook

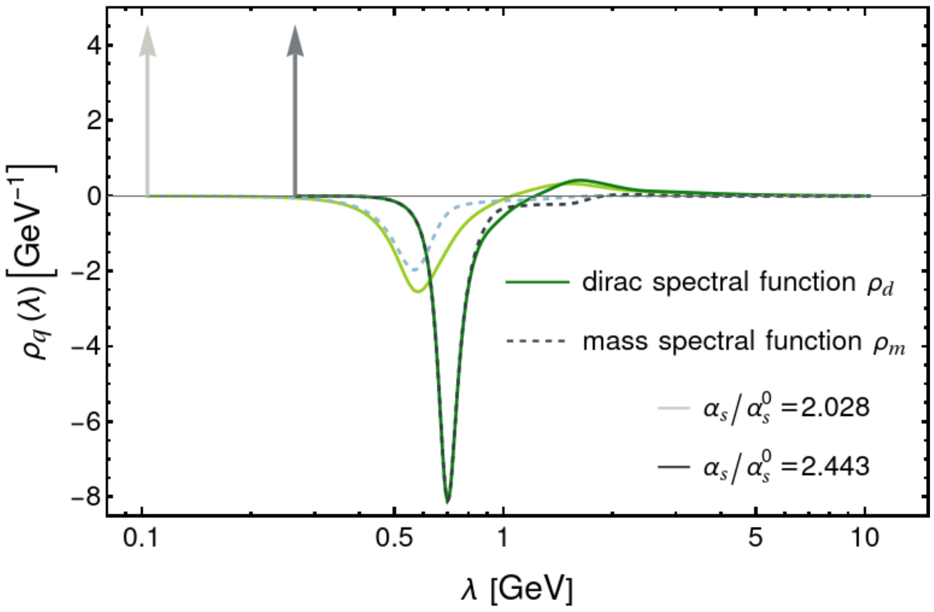
- Self-consistent solution of the spectral gap equation with a non-trivial vertex leads to a single real pole
- Additional CC-poles appear (relatively) far away from the real axis if the coupling is enhanced too much.

Todo:

→ Take full "spectral" STI-vertex into account

→ Monitor chiral symmetry breaking tensorstructures

→ Finite temperature quark spectral functions



Thanks for your attention!

Where functional Methods can help:

$$\mathscr{D}_s = \lim_{\omega \to 0} \frac{\sigma(\omega, \vec{p} = 0)}{\omega \chi_q \pi}$$

Kubo Formulas require infrared limits of realtime correlators

$$\sigma(\omega, \mathbf{p}) = \frac{1}{\pi} \int \mathrm{d}t \, e^{i\omega t} \int \mathrm{d}^3 x \, e^{i\mathbf{x}\mathbf{p}} \langle [J_i(t, \mathbf{x}), J_i(0,0)] \rangle$$

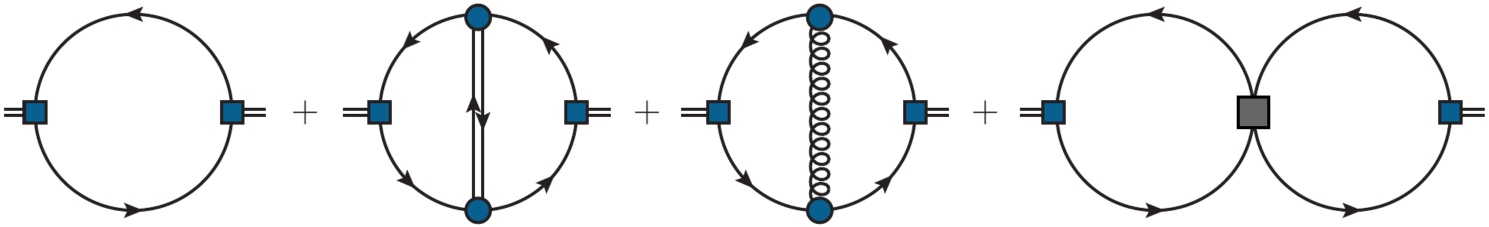
$$\text{with } J^\mu = \bar{\psi} \gamma^\mu \psi$$

Spectral Functional Methods:

- Access to realtime correlations and finite chemical potentials

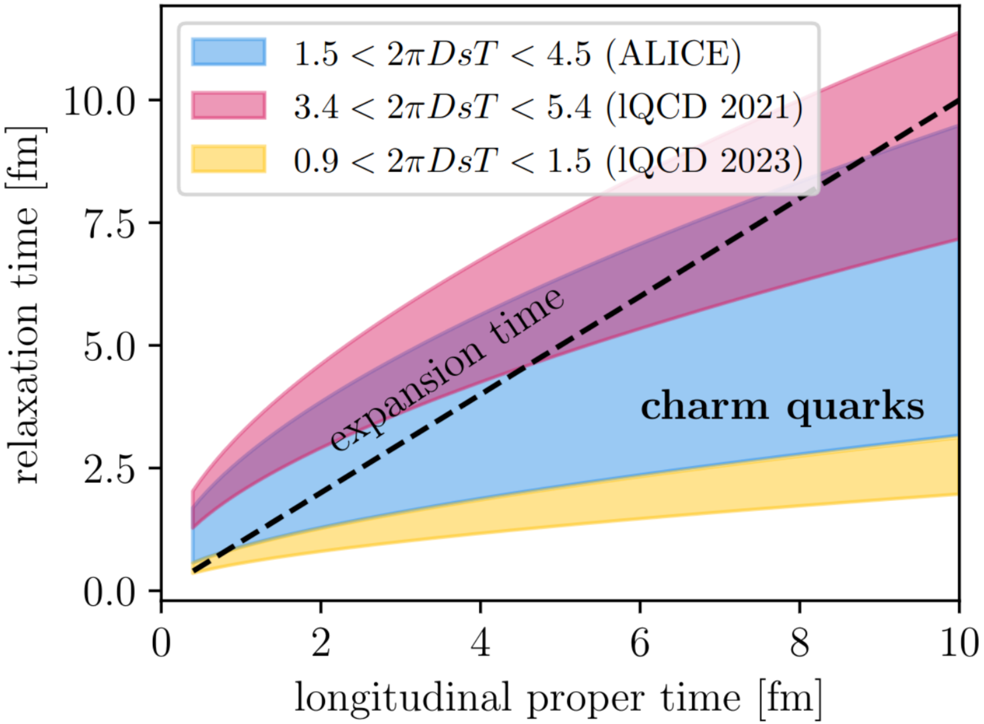
$\rightarrow$ Direct computation

$$\langle J_\mu(x) J_\nu(0) \rangle \propto$$

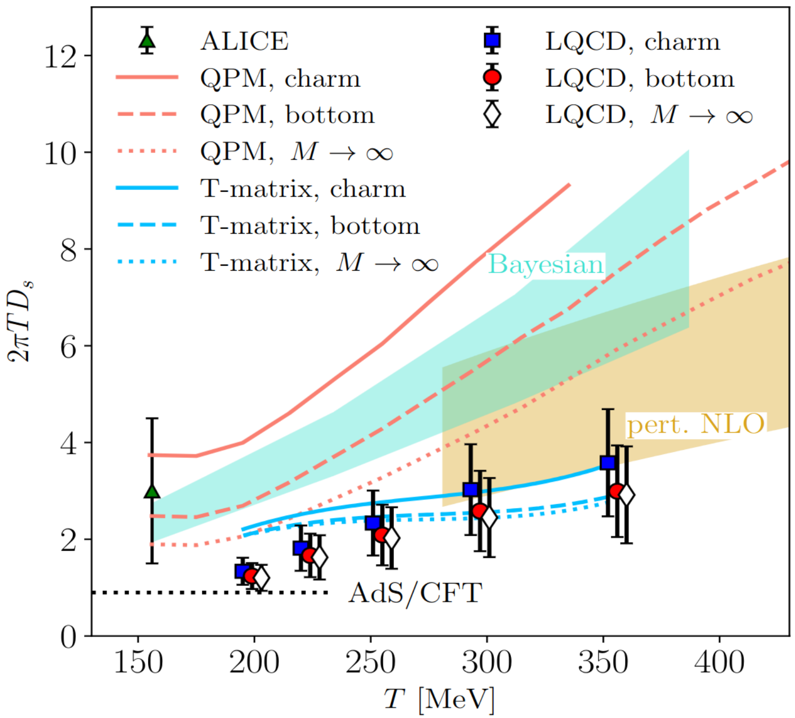


Need quark propagator in real time

Heavy Quark Diffusion, What We Know:

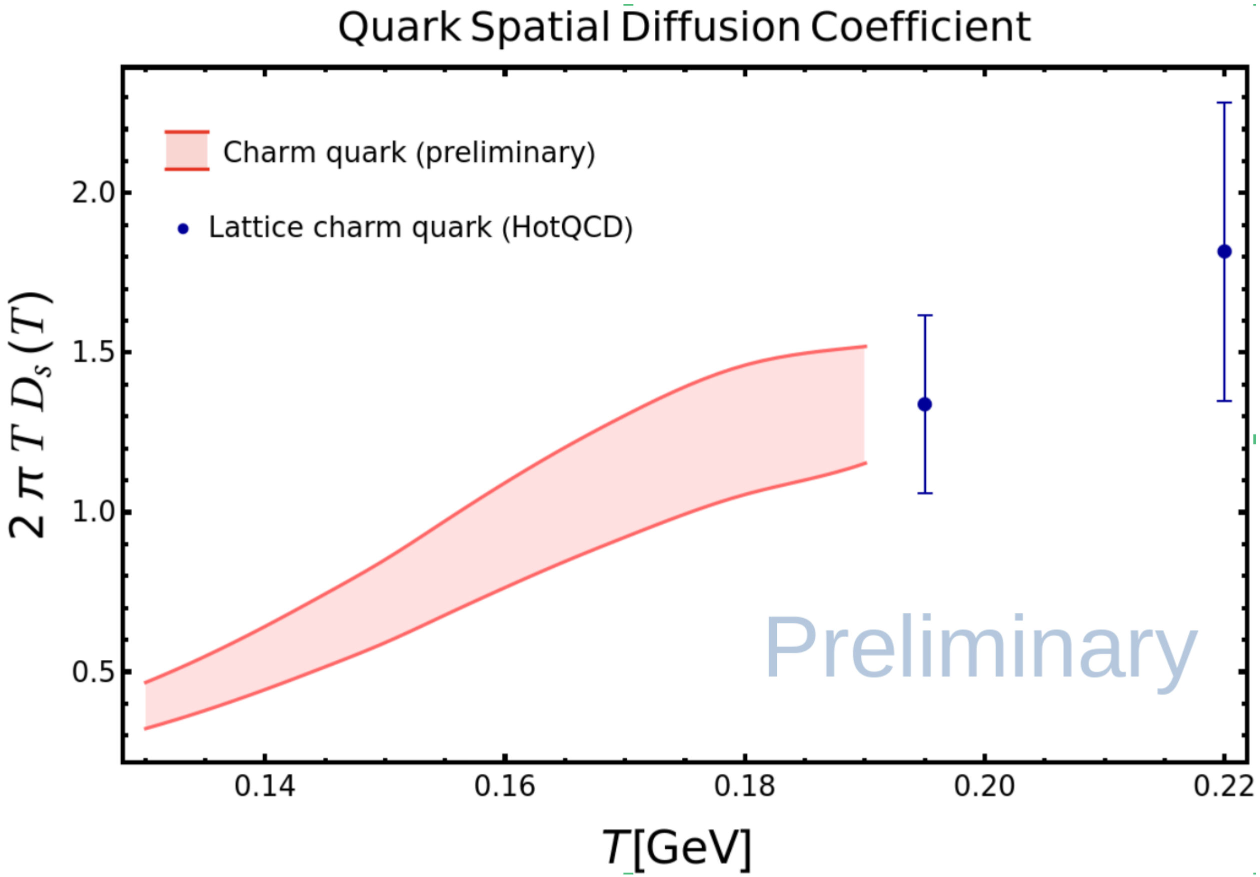


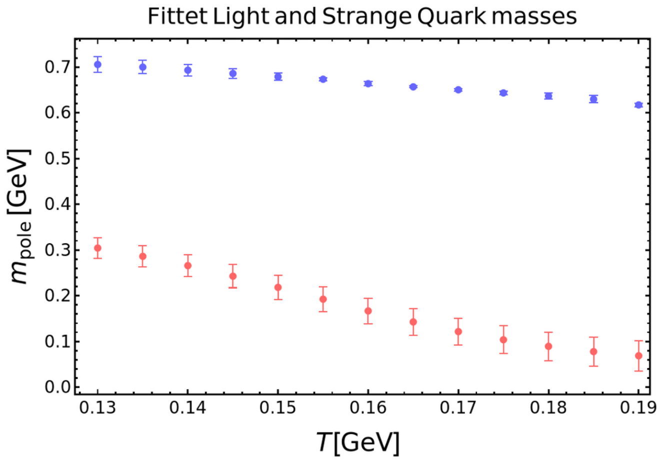
Capellino et al. arXiv:2312.10125



HotQCD Phys.Rev.Lett. 132 (2024) 5, 051902

First Results





Input masses and effective coupling fittet to euclidean DSE Data,

Lu, Gao, Liu, Pawlowski
arXiv:2310.18383

Quark Number suszeptibility from

Borsányi et al. Phys. Rev. D 92, 114505

Normalisation fixed at lattice results

HotQCD Phys.Rev.Lett. 132 (2024) 5, 051902

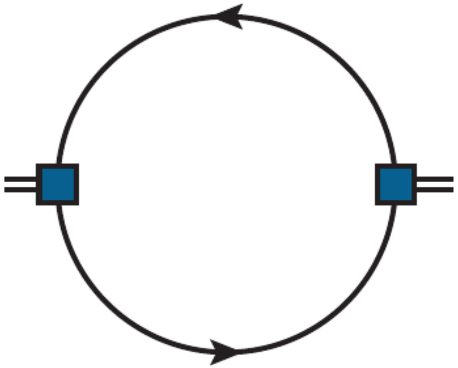
Outlook

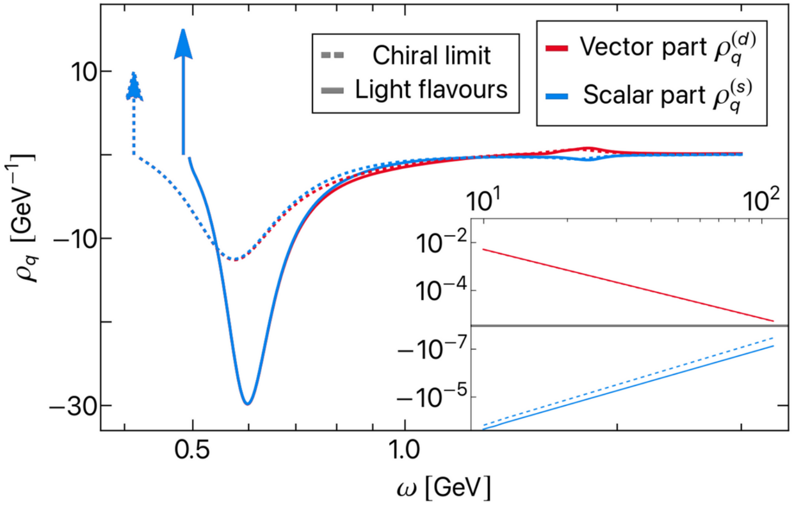
- Quark diffusion coefficient for all flavours
- Quark contributions to the shear viscosity
- Electric conductivity
- Estimate of relaxation times
- At finite chemical potential

- Quark gluon vertex
  - Momentum/frequency dependencies
  - Additional tensor structures
- Fully self consistent solution without eucl. Input
  -> systematic error analysis
- Computation of further (subleading) diagrams
  -> gluon and pion exchange

Utilize spectral functions

J.Horak, J. Pawlowski, N. WinkSciPost Phys. 15, 149 (2023)





$$\propto \int_q G(q)G(p-q)$$

$$G(p^2) = \int_{-\infty}^{\infty} \frac{d\lambda\lambda}{2\pi} \frac{\rho(\lambda)}{\lambda^2 + p^2}$$

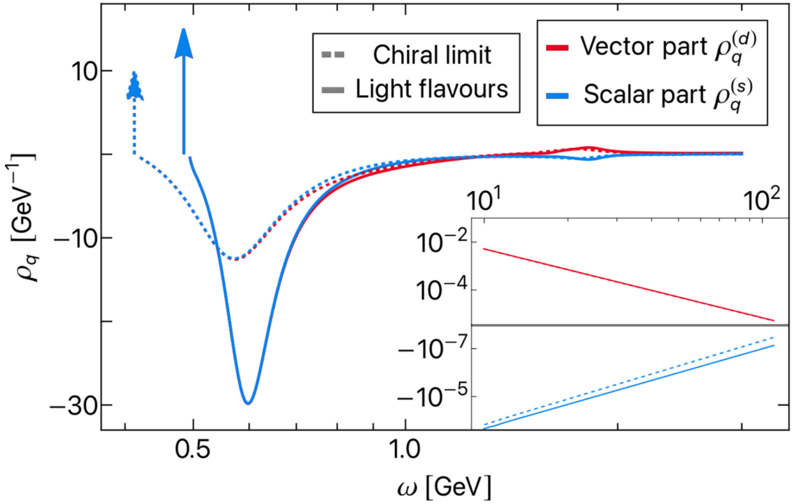
$$= \int_{\lambda_1,\lambda_2} \rho(\lambda_1)\rho(\lambda_2) \boxed{\int_q \frac{1}{(\lambda_1^2 + q^2)(\lambda_2^2 + (p-q)^2)}}$$

Spectral functional methods

$$G(p^2) = \int_{-\infty}^{\infty} \frac{d\lambda\lambda}{2\pi} \frac{\rho(\lambda)}{\lambda^2 + p^2}$$

$$\boxed{\int_{\lambda_q,\lambda_A} \rho_A(\lambda_1)\rho_q(\lambda_2) \int_q \frac{1}{(\lambda_q + i\gamma_\mu q^\mu)(\lambda_A^2 + (p-q)^2)}}$$

$$G^{-1}_{q\bar{q}}(p^2) \longrightarrow \boxed{\rho_q(\lambda)}$$



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