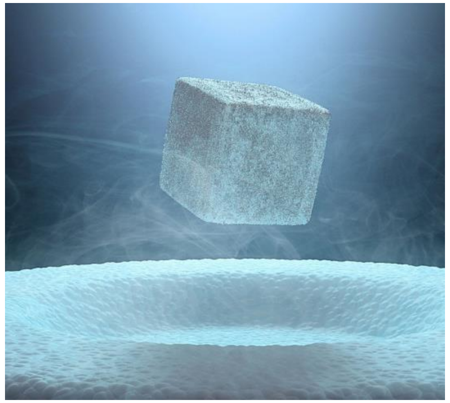
Quantum simulation/computation of non-Abelian gauge theories beyond 1D with finite resources

Alessio Celi

UAB
Universitat Autònoma de Barcelona

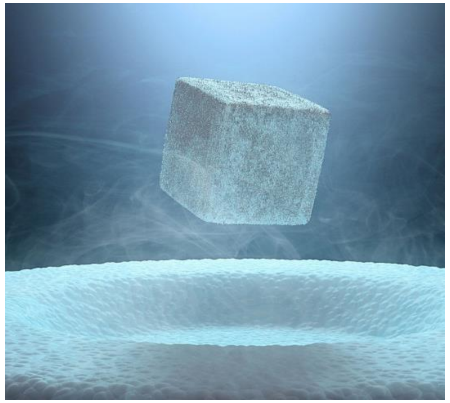
Quantum Atom Optics
Grup d'Òptica UAB
Dynamite
Gobierno de España
Ministerio de Ciencia, Innovación y Universidades
Agencia Estatal de Investigación
Quantum SPAIN
Plan de Recuperación, Transformación y Resiliencia
Generalitat de Catalunya
AGAUR Agència de Gestió d'Ajuts Universitaris i de Recerca

The quantum way



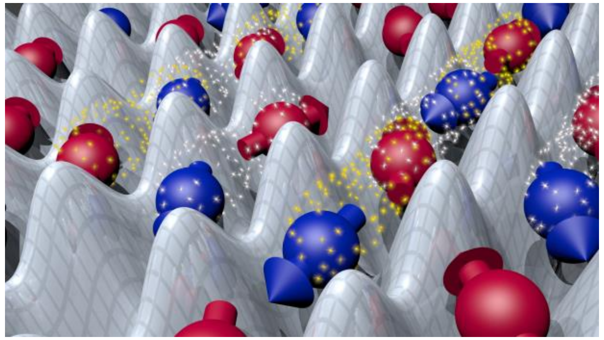
$$\hat{H} = \sum_{i} \sum_{\sigma\in\{\uparrow,\downarrow\}} \varepsilon_i \hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + \sum_{\langle i,j\rangle} \sum_{\sigma\in\{\uparrow,\downarrow\}} t_{ij}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{j,\sigma} + h.c.\right) + \sum_{\langle i,j\rangle} U_{ij}(\hat{n}_i - \tfrac{1}{2})(\hat{n}_j - \tfrac{1}{2})$$

The quantum way

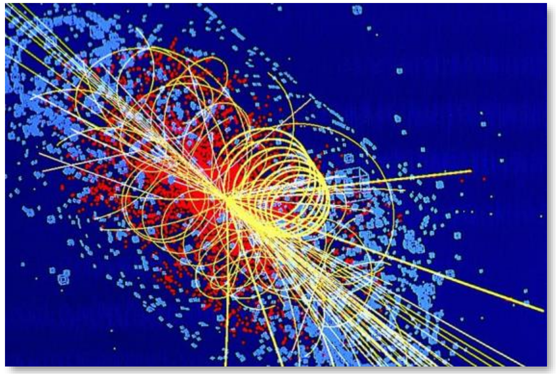


$$\hat{H} = \sum_i \sum_{\sigma \in \{\uparrow,\downarrow\}} \varepsilon_i \hat{c}^\dagger_{i,\sigma} \hat{c}_{i,\sigma} + \sum_{\langle i,j \rangle} \sum_{\sigma \in \{\uparrow,\downarrow\}} t_{ij} \left(\hat{c}^\dagger_{i,\sigma} \hat{c}_{j,\sigma} + h.c. \right) + \sum_{\langle i,j \rangle} U_{ij} (\hat{n}_i - \tfrac{1}{2})(\hat{n}_j - \tfrac{1}{2})$$

To simulate efficiently a quantum system you need a quantum processor

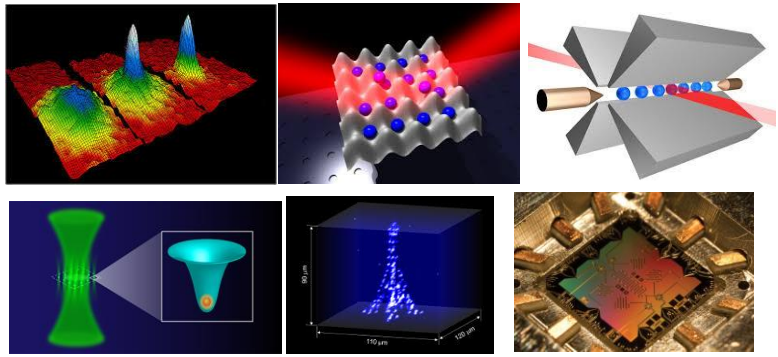


The quantum way for gauge theory!



$$H_{QCD} = -\sum_{s,\mu=x,y,z} J(\psi^\dagger_{s+\mu}\hat{U}_{s,\mu}\psi_s + H.c.) + g^2\hat{E}^2_{s,\mu} - \frac{1}{g^2}\sum_s(\hat{U}\hat{U}\hat{U}^\dagger\hat{U}^\dagger + H.c.)_s$$

To simulate efficiently
a quantum system you need a
quantum processor

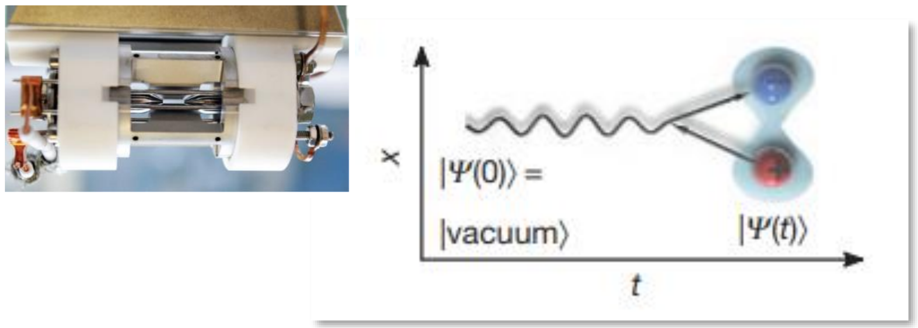


AMO experiments: from building blocks to complete gauge theories

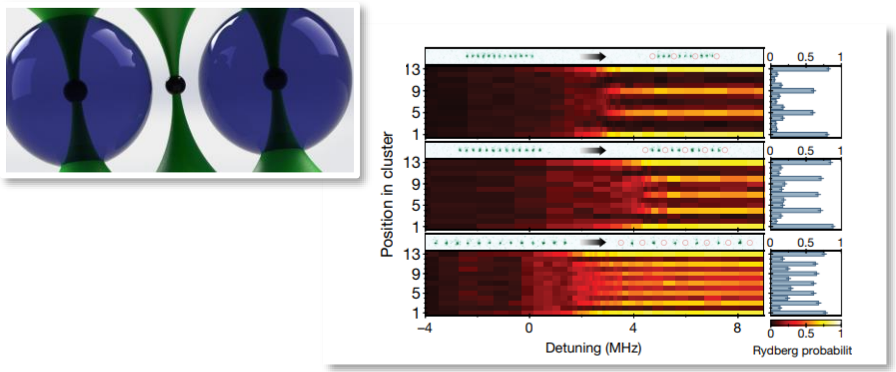
Innsbruck, Shanghai/Heidelberg, Chicago, Munich, Zurich, Heidelberg, Harvard, Palaiseau, Maryland…

The quantum way for gauge theory: experiments

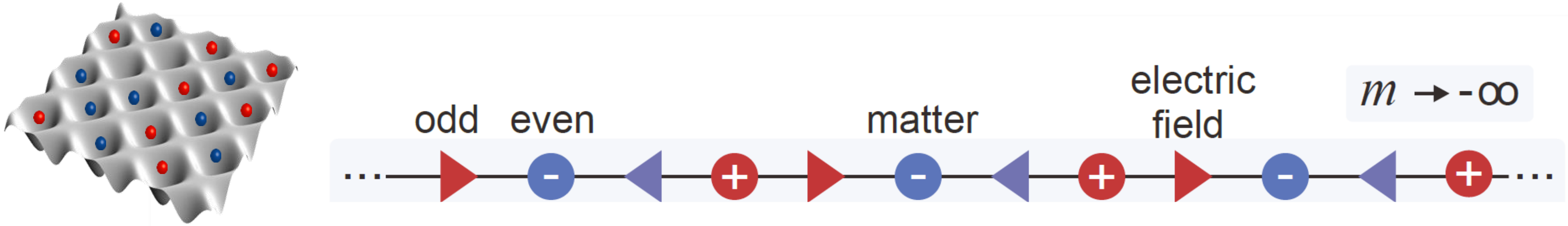
- QED in (1+1) dimensions in a lattice (Schwinger model)



E. A. Martinez *et al.*, *Nature* **534**, 516 (2016)
C. Kokail *et al.*, *Nature* **569**, 355 (2019)
N. H. Nguyen *et al.*, *PRX Quantum* **3**, 020324 (2022)



H. Bernien *et al.*, *Nature* **551**, 579 (2017)
F. M. Surace *et al.*, *Phys. Rev. X* **10**, 021041 (2020)

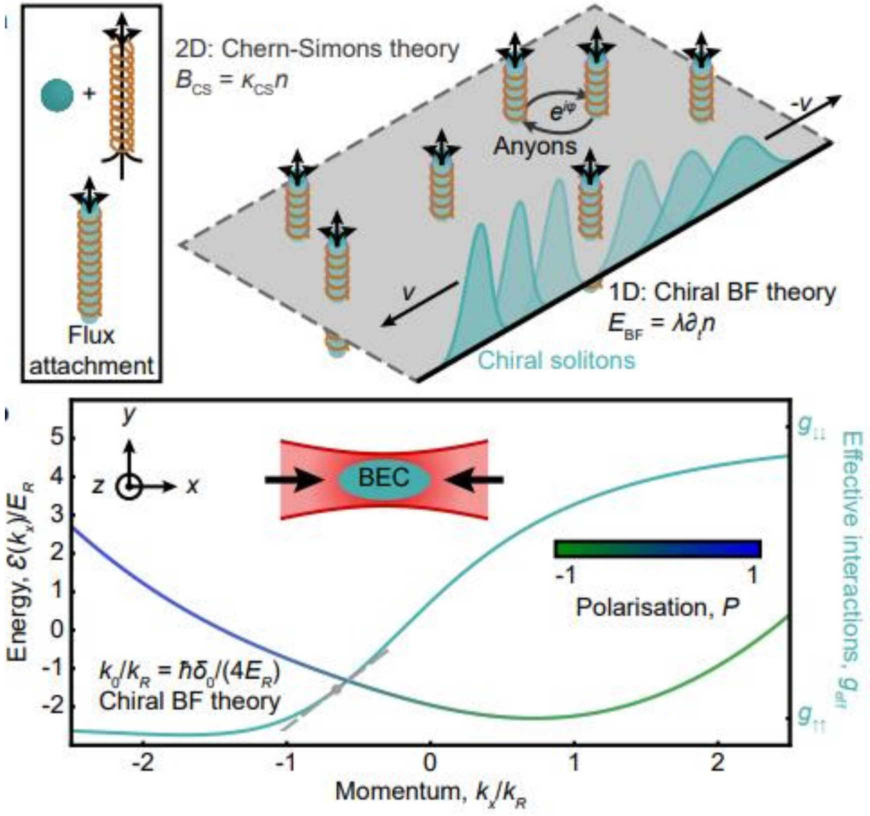


B. Yang *et al.*, *Nature* **587**, 392 (2020); Z.-Y. Zhou *et al.*, *Science* **377**, 311 (2022);
H. Wang *et al.*, arXiv:2210.17032

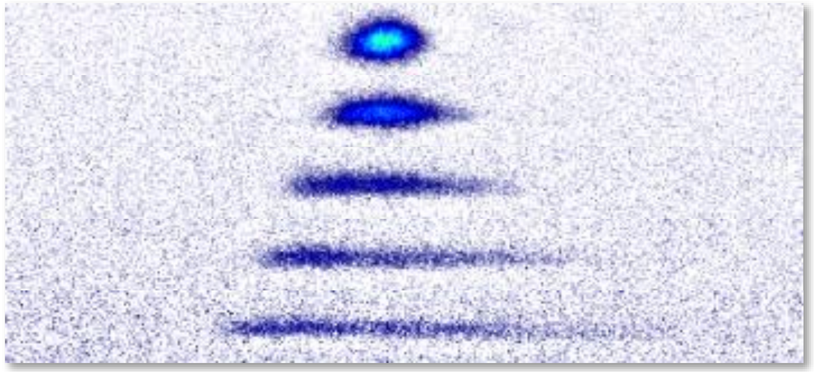
The quantum way for gauge theory: experiments

- QED in (1+1) dimensions in a lattice (Schwinger model)

- Chiral BF theory in (1+1) dimensions in the continuum

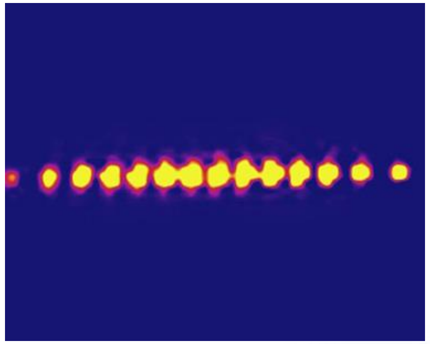
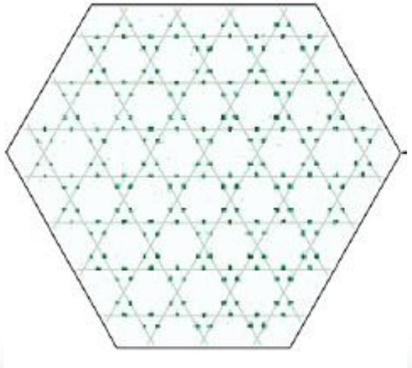
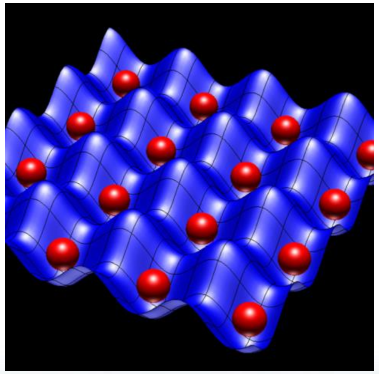
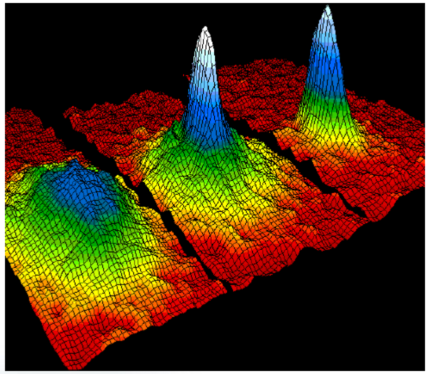


Self-generated electric field

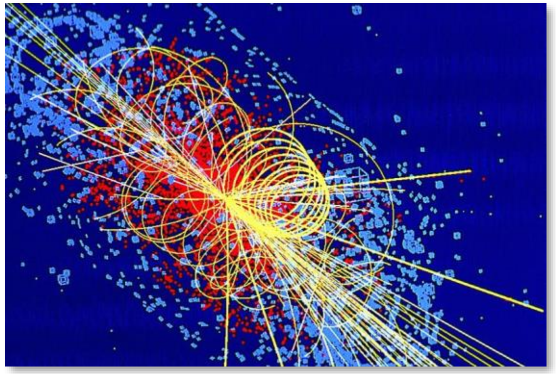


A. Frölian*, C. S. Chisholm*, E. Neri, C. R. Cabrera, R. Ramos, AC, and L. Tarruell, Nature **608**, 293 (2022)

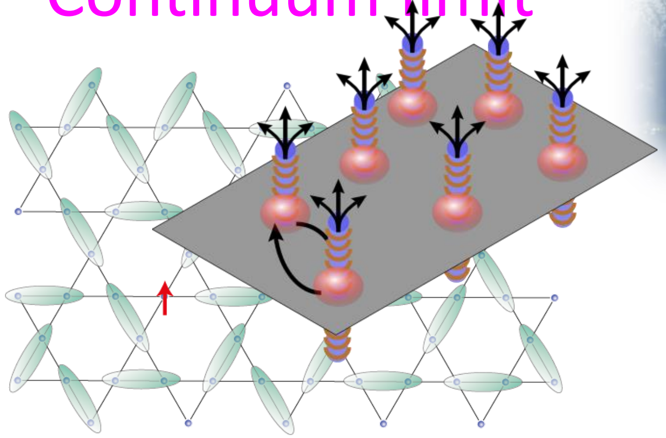
GAUGE THEORY & AMO

Real world: 2D&3D

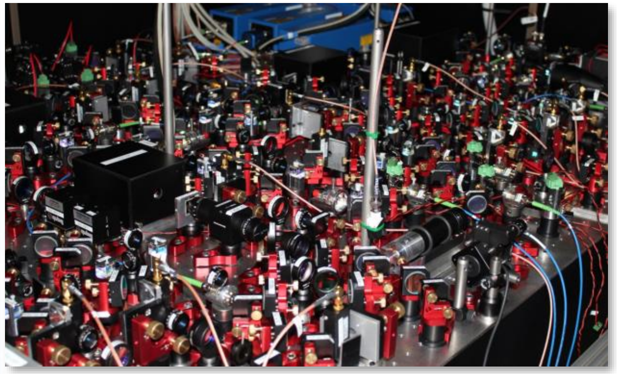


Continuum limit



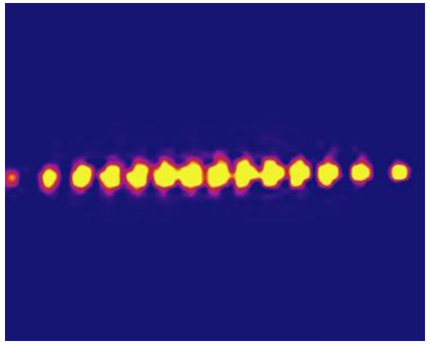
M.C. Bañuls *et al.*, *Eur. Phys. J. D* **74,** 165 (2020)
M. Dalmonte and S. Montangero, *Cont. Phys.* **57** 388 (2016)
E. Zohar, J.I. Cirac, and B Reznik, Rep. Prog. Phys. 79, 014401 (2015)

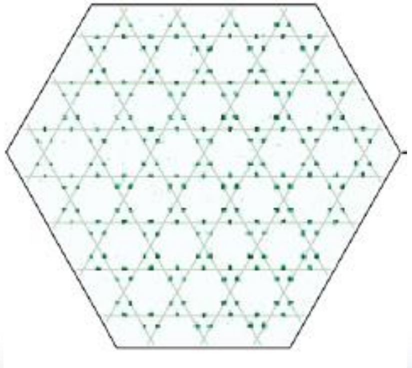
AMO experiments: 1D

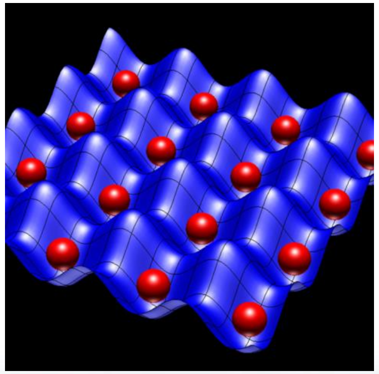


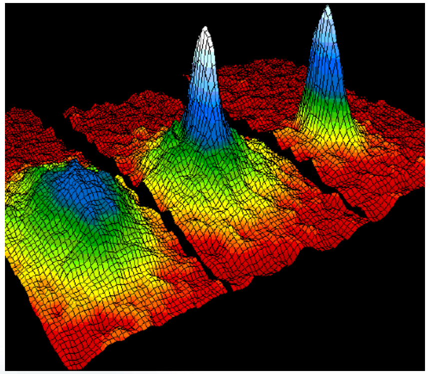
Innsbruck, Heidelberg, Chicago, Munich, Zurich, Paris, Harvard, **Barcelona…**

GAUGE THEORY & AMO

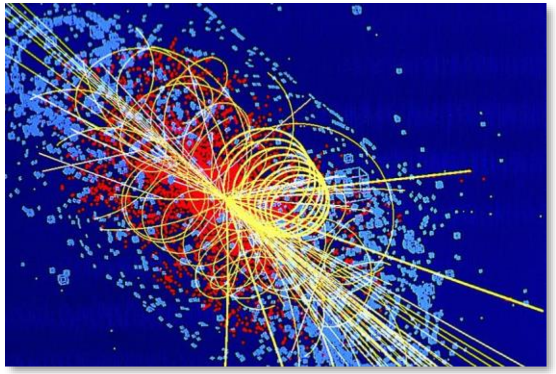






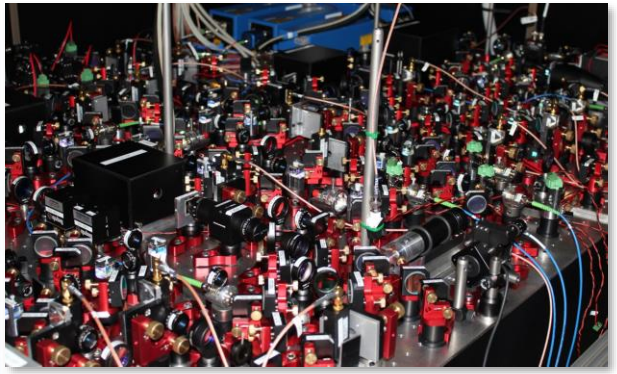


Real world: 2D&3D



Continuum limit

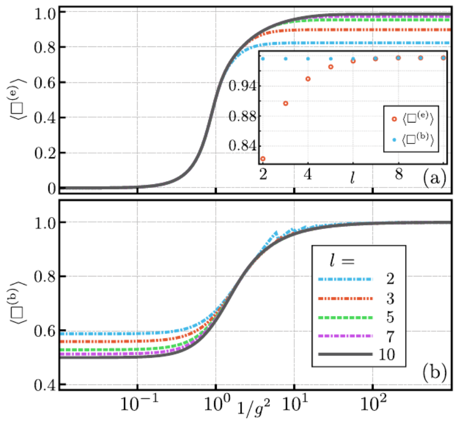
AMO experiments: 1D



Innsbruck, Heidelberg, Chicago, Munich, Zurich, Paris, Harvard, Barcelona…

M.C. Bañuls *et al.*, *Eur. Phys. J. D* **74,** 165 (2020)
M. Dalmonte and S. Montangero, *Cont. Phys.* **57** 388 (2016)
E. Zohar, J.I. Cirac, and B Reznik, Rep. Prog. Phys. 79, 014401 (2015)

Running of the coupling on a torus

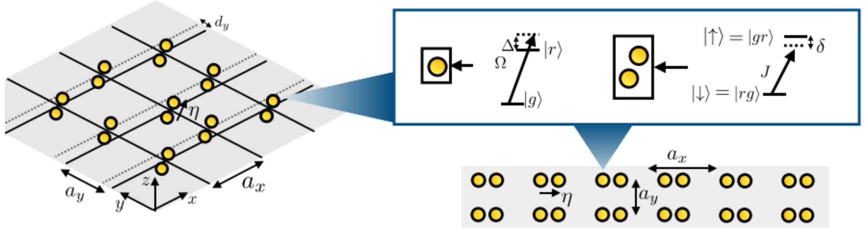


Encoding

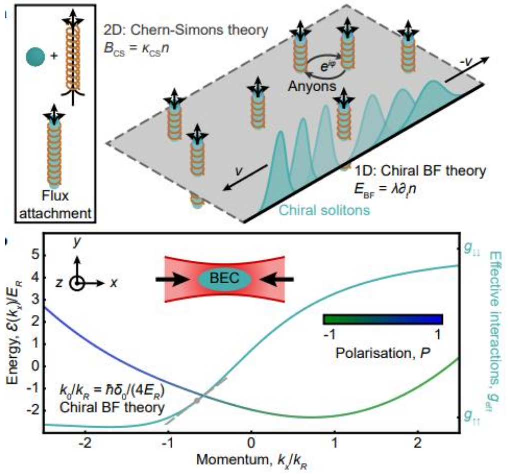
Dual Rokhsar-Kivelson Hamiltonian

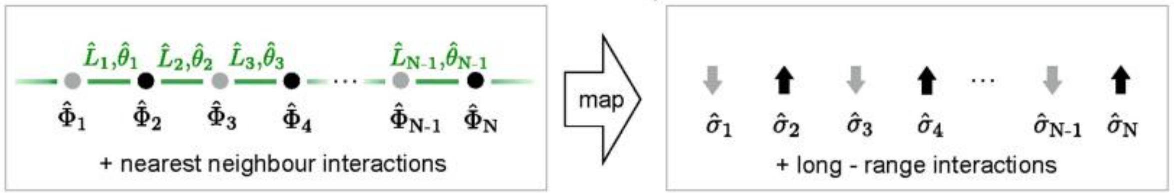
AC *et al.*, *PRX* **10**, 021057 (2020)

.



Experimental demonstration of chiral BF theory



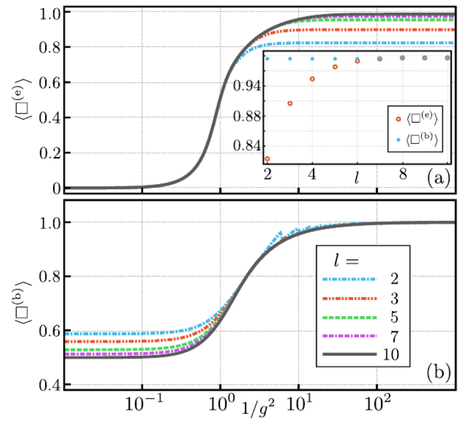


Schwinger model

E.A. Martinez *et al.*, Nature 534 (7608), 516-519 (2016)

C. Muschik *et al.*, New J. Phys. 19 (10), 103020 (2017)

Running of the coupling on a torus



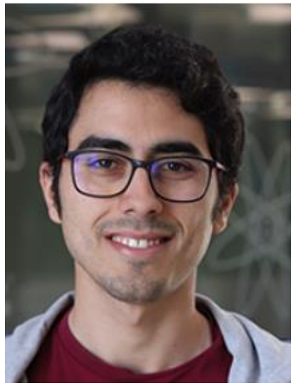
Encoding
+
Coupling
dependent
basis

- 2+1 QED

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

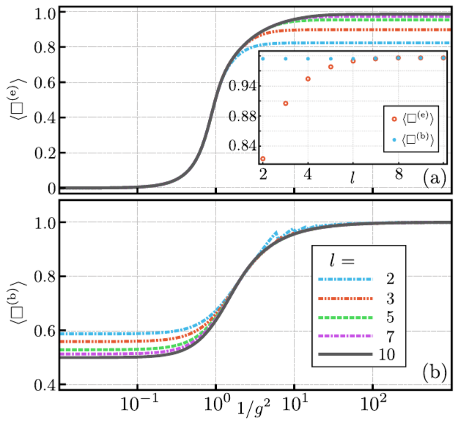
- 2+1 SU(2) Yang-Mills

P. Fontana, M. Miranda, and **AC**, *arXiv:2409.xxxx*



P. Fontana, Ph.D

M. Miranda, MSc



Running of the coupling on a torus

Encoding
+
Coupling dependent basis

- 2+1 QED

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

- 2+1 SU(2) Yang-Mills

P. Fontana, M. Miranda, and **AC**, *arXiv:2409.xxxx*

P. Fontana, Ph.D

M. Miranda, MSc

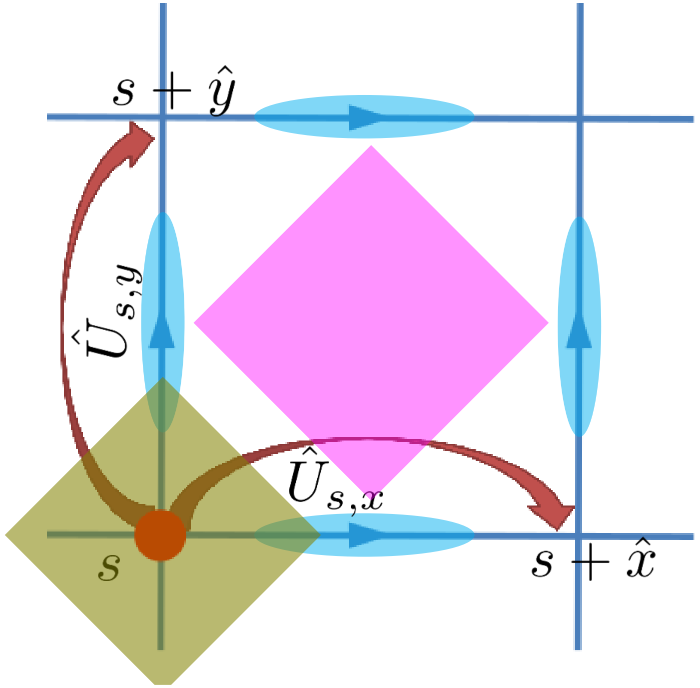
Hamiltonian lattice gauge theory

[Kogut,Susskind '75]

2D Maxwell: Dynamical gauge field = operator on electro-magnetic quanta

$$[\hat{E}_{s,\mu}, \hat{A}_{s,\mu}] = -i \longrightarrow [\hat{E}_{s,\mu}, \hat{U}_{s,\mu}] = -\hat{U}_{s,\mu} \qquad \hat{E}_{s,\mu} = (n+n_0)|n\rangle\langle n|, \;\; n \in \mathrm{Z}$$

Hilbert space: U(1) group algebra



Kinetic term preserves Gauss law

$$\hat{G} \equiv \hat{E}_{s,x} + \hat{E}_{s,y} - \hat{E}_{s-\hat{x},x} - \hat{E}_{s-\hat{y},y} - \hat{q}_s$$

$$[\hat{G}, \hat{H}] = 0, \qquad |\psi\rangle \in \{\text{Physical states}\} \iff \hat{G}|\psi\rangle = 0$$

$$e^{i\alpha_s \hat{G}}|\psi\rangle = |\psi\rangle, \forall \alpha_s$$

+ Electro-magnetic energy

$$\hat{H} = -t \sum_{s,\mu=x,y} \hat{c}^{\dagger}_{s+\mu}\hat{U}_{s,\mu}\hat{c}_s + H.c. + g^2 \hat{E}^2_{s,\mu} - \frac{1}{g^2}(\hat{U}\hat{U}\hat{U}^{\dagger}\hat{U}^{\dagger} + H.c.)$$

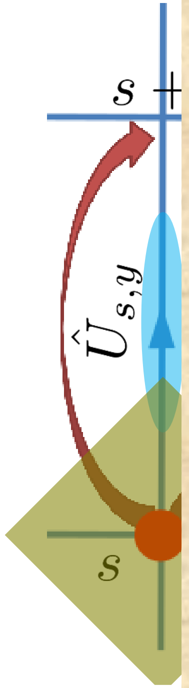
Hamiltonian lattice gauge theory

[Kogut,Susskind '75]

2D Maxwell: Dynamical gauge field = operator on electro-magnetic quanta

$[\hat{E}_{s,\mu},$ … $\mathbb{Z}$

…ebra



- The anisotropy btw time and space reflects in the opposite behavior with respect to the coupling
- *g>>1* Electric term dominates, strong coupling (as 1D)
- *g<<1* Magnetic term dominates, weak coupling, (perturbative) deconfinement
- Competition btw the two terms, characteristic feature D>1
- Consequence: Running coupling (dimensional transmutation)

$\hat{q}_s$

$= 0$

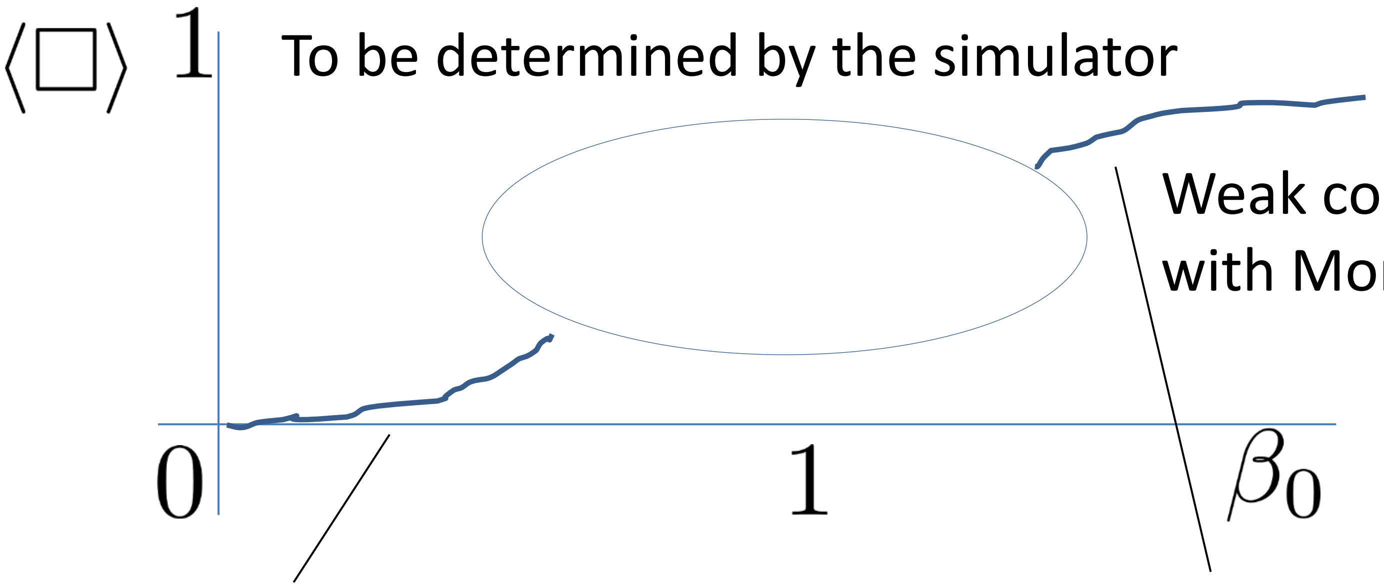
$= |\psi\rangle, \forall \alpha_s$

$$\hat{H} = -t \sum_{s,\mu=x,y} \hat{c}^{\dagger}_{s+\mu}\hat{U}_{s,\mu}\hat{c}_s + H.c. + g^2 \hat{E}^2_{s,\mu} - \frac{1}{g^2}(\hat{U}\hat{U}\hat{U}^{\dagger}\hat{U}^{\dagger} + H.c.)$$

Running coupling on a torus

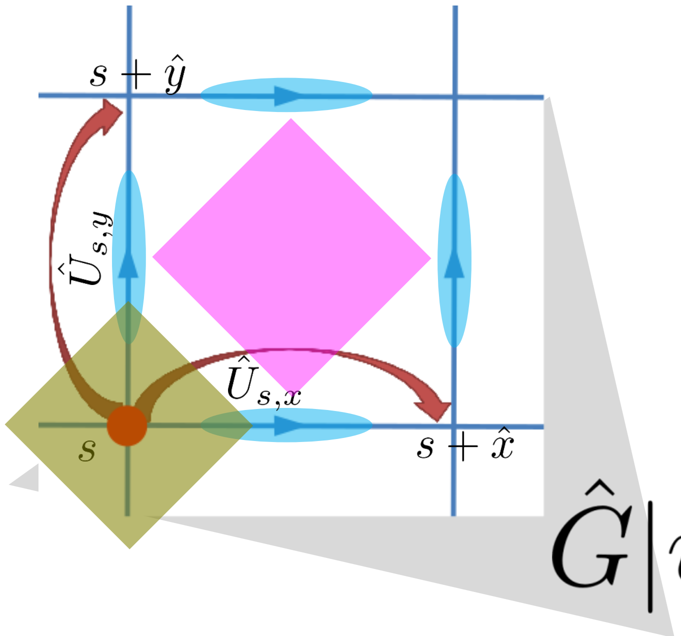
$$\beta_0 = \frac{1}{g^2} \qquad \langle \Box \rangle = \frac{\beta_R}{\beta_0}$$

Renormalization of the coupling [Creutz'80]

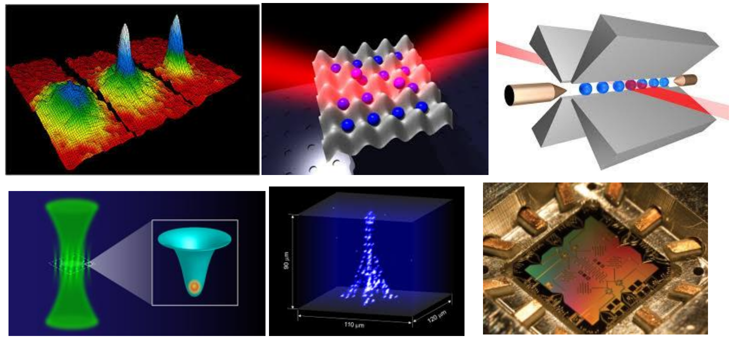


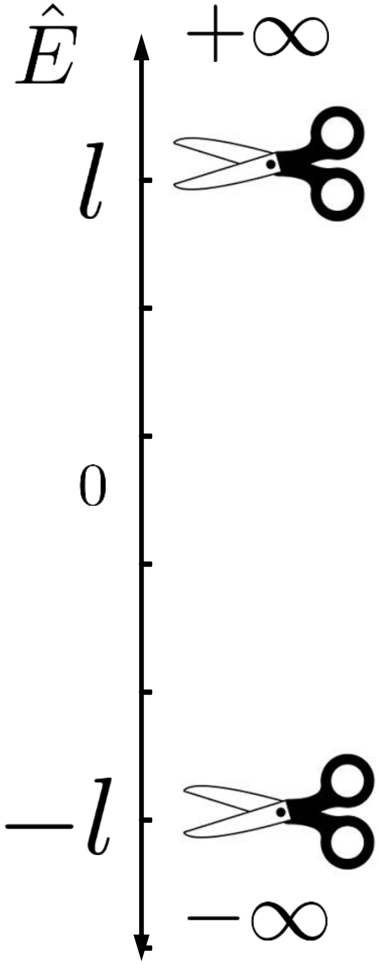
Alternative short-distance observables: G. Clemente, A. Crippa, K. Jansen, Phys. Rev. D **106**, 114511 (2022)

Running coupling with finite resources?



$$\hat{G}|\psi\rangle = 0$$





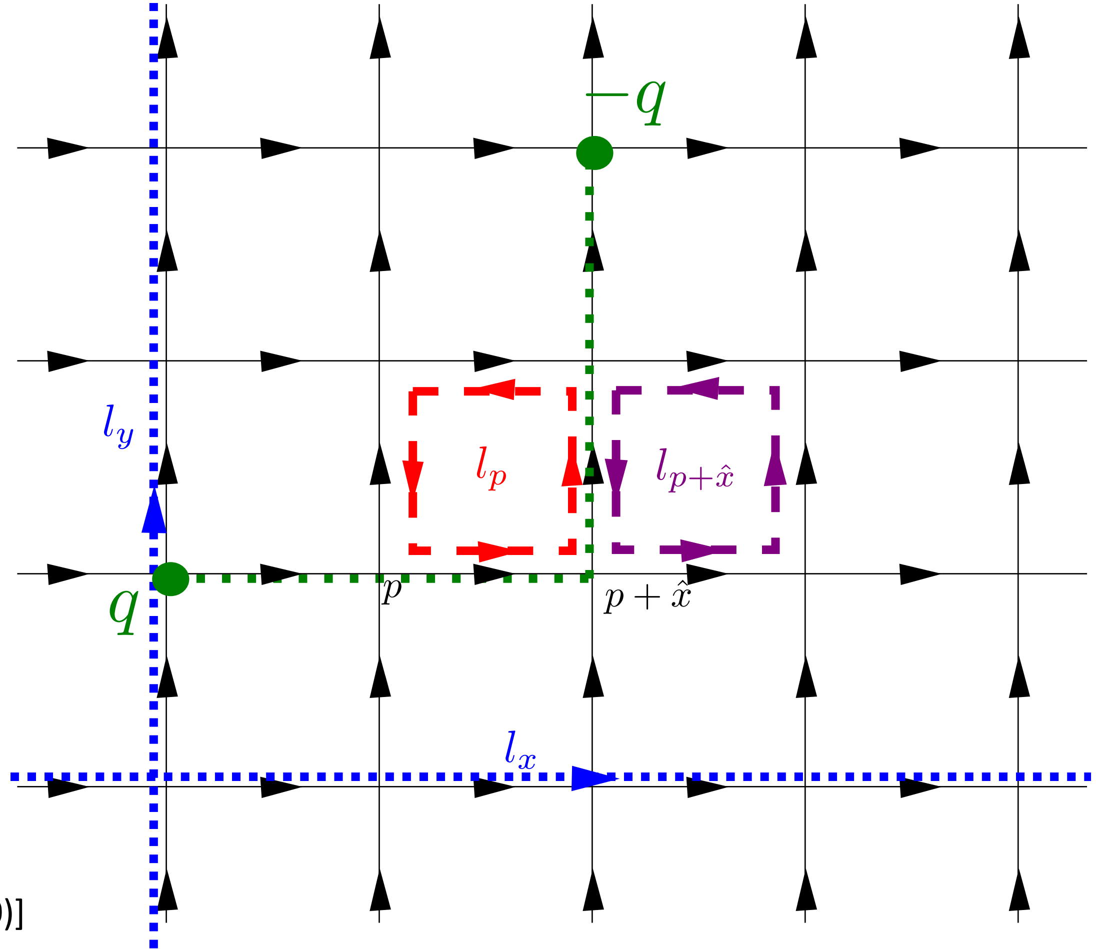
$$g \to 0 \Rightarrow l \to \infty$$

Electric-basis truncation inefficient

Encoding in a nutshell: 2+1 QED

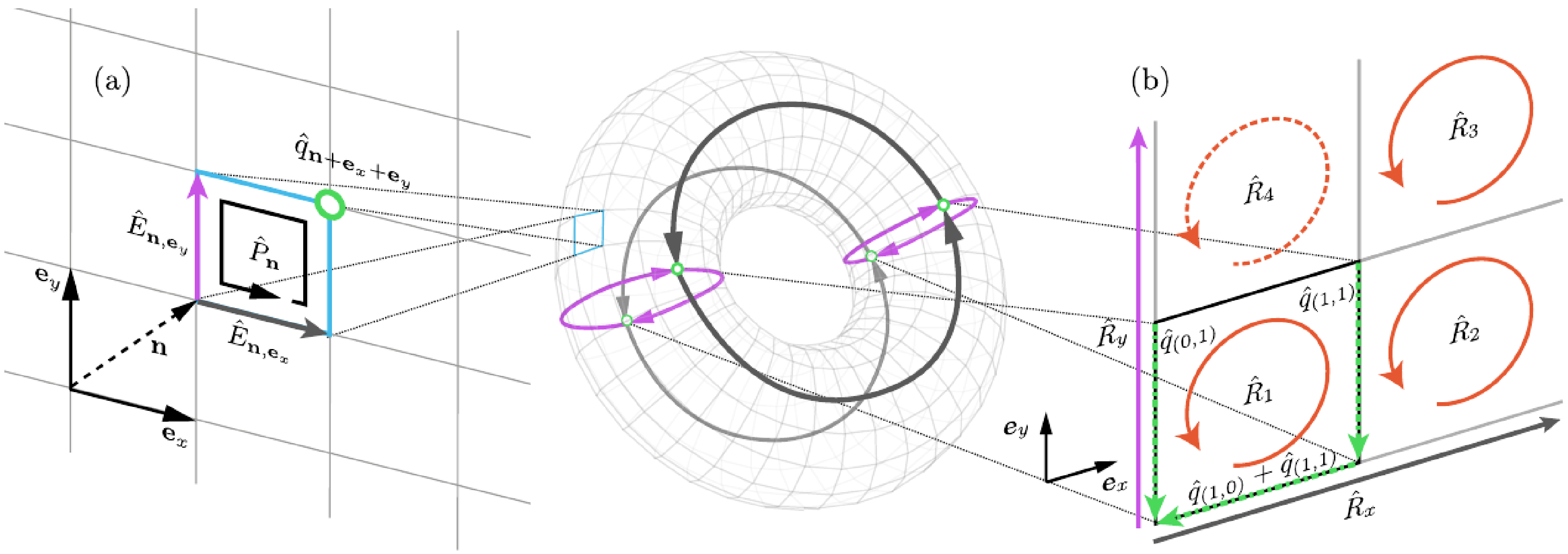
$$\nabla \cdot \mathbf{E} = n \qquad H = \sum_s \sum_{j=1}^{d} \frac{g^2}{2} E_{s,j}^2 + \frac{1}{2g} \sum_s \left(U U U^\dagger U^\dagger + \text{H.c.} \right) + \ldots$$

$$\mathbf{E} = \mathbf{E}_{\mathrm{S}} + \mathbf{E}_{\mathrm{T}} \qquad \nabla \cdot \mathbf{E}_{\mathrm{T}} = 0$$



Coulomb on the lattice [J. Bender and E. ZoharPhys. Rev. D 102, 114517 (2020)]

Running coupling of 2+1 QED on a minimal torus



$$\hat{H}^{(\mathrm{e})} = \hat{H}_E^{(\mathrm{e})} + \hat{H}_B^{(\mathrm{e})},$$

$$\hat{H}_E^{(\mathrm{e})} = 2g^2 \left[\hat{R}_1^2 + \hat{R}_2^2 + \hat{R}_3^2 - \hat{R}_2(\hat{R}_1 + \hat{R}_3) \right],$$

$$\hat{H}_B^{(\mathrm{e})} = -\frac{1}{2g^2a^2} \left[\hat{P}_1 + \hat{P}_2 + \hat{P}_3 + \hat{P}_1\hat{P}_2\hat{P}_3 + \mathrm{H.c.} \right]$$

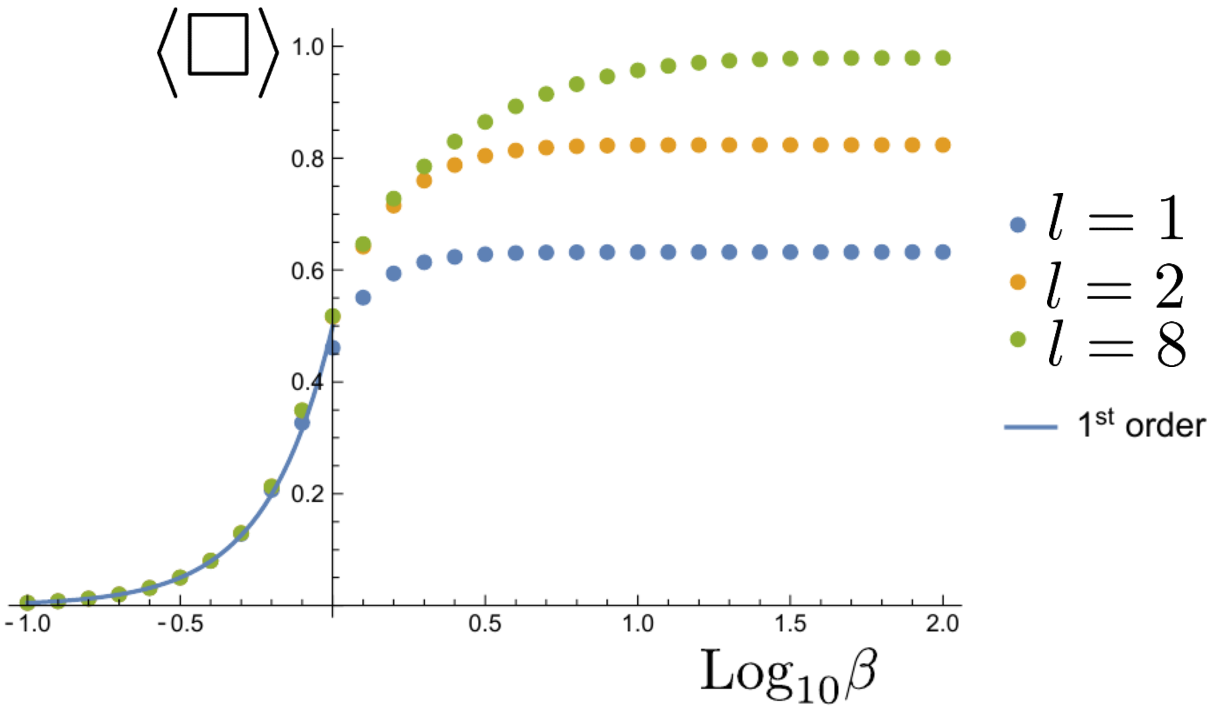
$$\boxed{\langle \square \rangle = -\frac{g^2a^2}{V} \langle \Psi_0 | \hat{H}_B | \Psi_0 \rangle}$$

J.F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

Running coupling of 2+1 QED on a minimal torus

Truncation in the electric basis: good at strong coupling

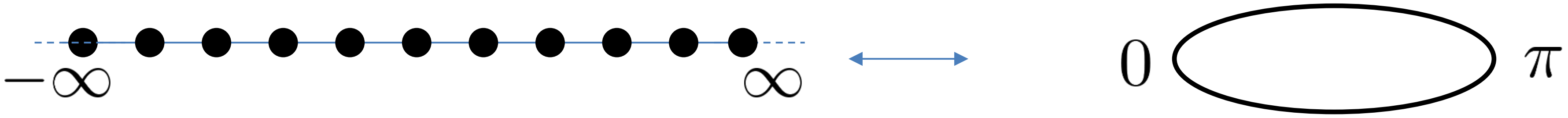
poor at weak coupling



Magnetic basis needed! Magnetic basis continuous

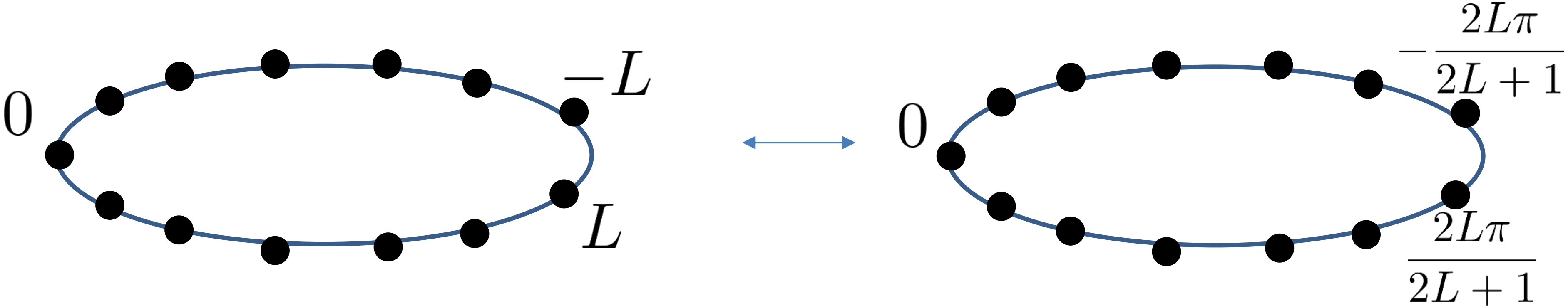
Efficient representation at *all couplings*

Electric and magnetic operator like position and momentum on the lattice



E To restore the symmetry ϕ

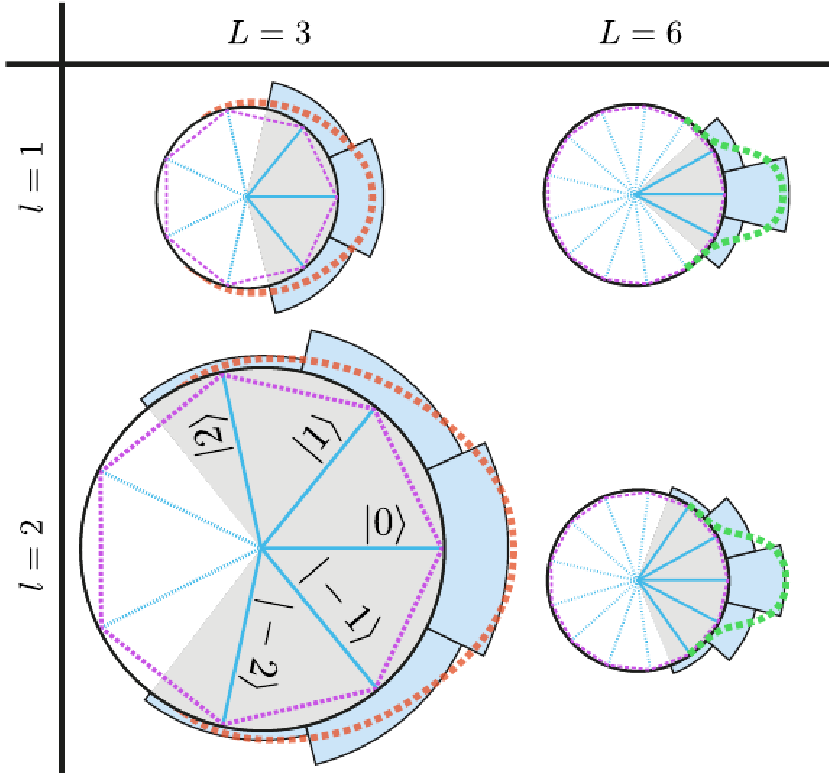
$$U(1) \sim Z_{2L+1}, L > l$$

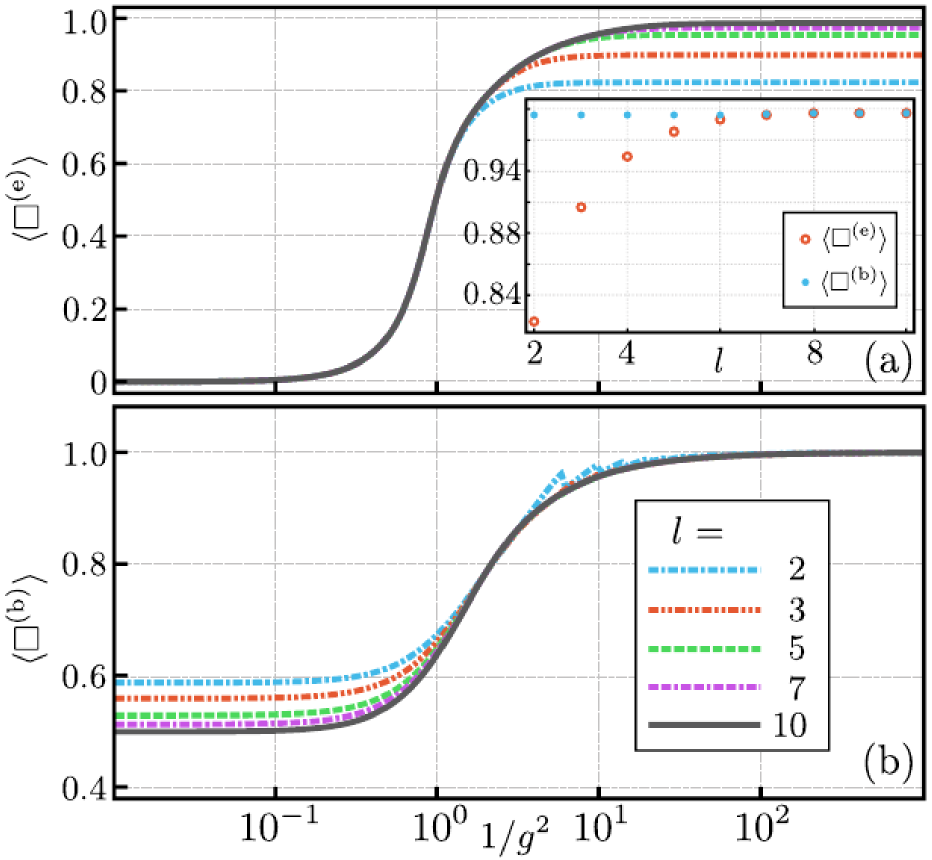


$\hat{H}^{(e)}(l)$ Interchanged by Fourier transformation $\hat{H}^{(b)}(l, L)$

Efficient representation at *all couplings*

$L = L(\beta)$ variational parameter for optimizing resources





Varying *L* allows to avoid freezing while minimizing the truncation

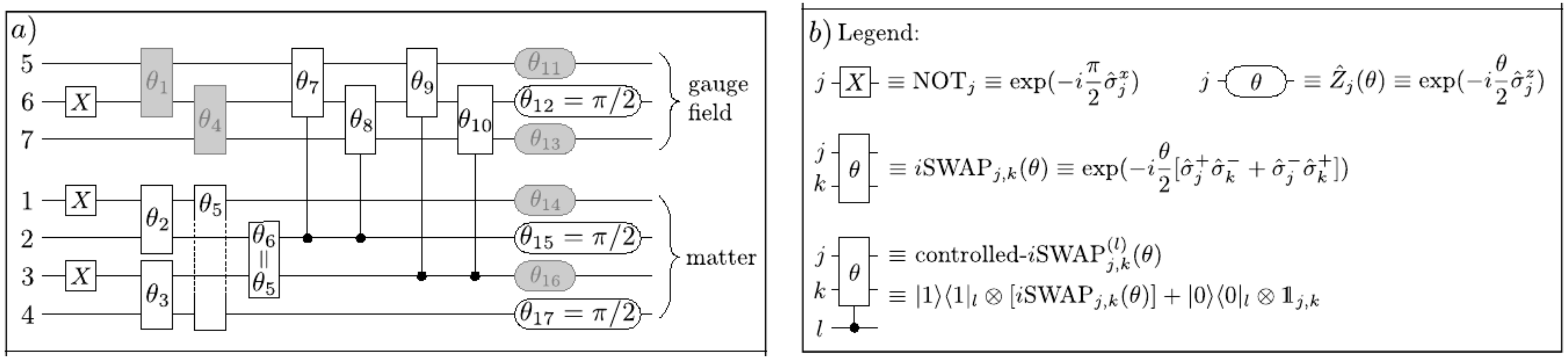
J.F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum 5, 393 (2021)

Variational simulations with trapped ions

Gauge dof: spin l = qudit ➡ $2l+1$ qubits (ions)

$$|-l+j\rangle = |\overbrace{0\ldots0}^{j}\,1\,\overbrace{0\ldots0}^{2l-j-1}\rangle \qquad \hat{E} = \frac{1}{2}\sum_{i=1}^{2l}\prod_{j=1}^{i}\sigma_j^z \qquad \hat{U} = \sum_{i=1}^{2l}\sigma_i^+\sigma_{i+1}^-$$

Variational preparation of the ground state convenient for long range interactions



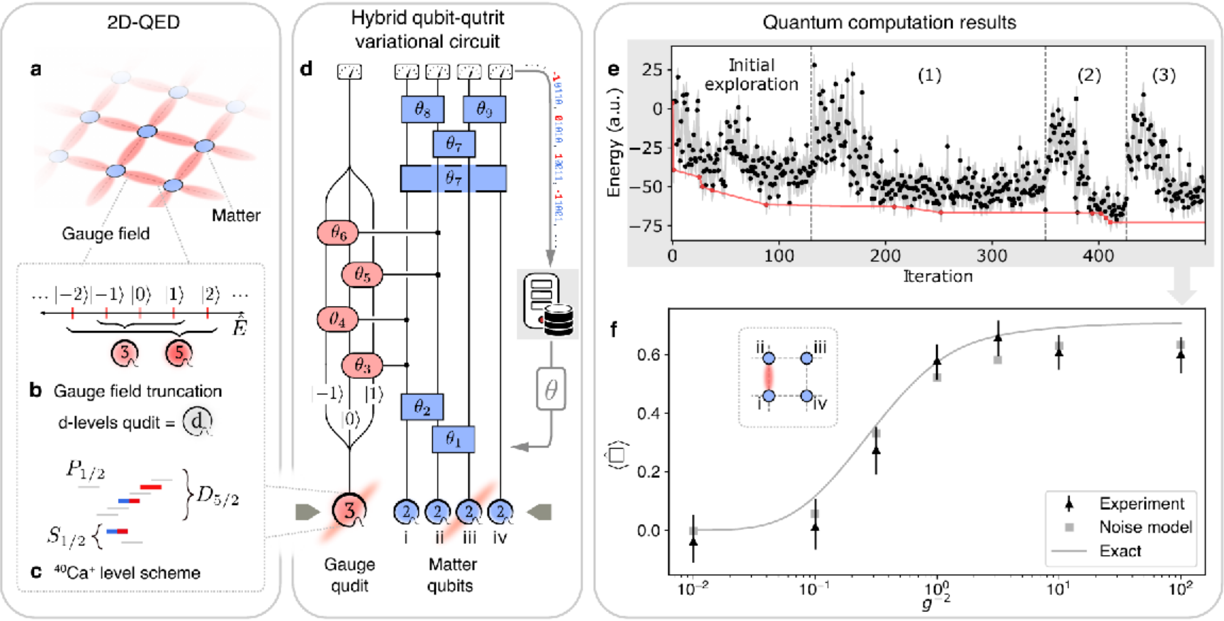
Feasible for moderate truncations in current trapped ions experiments!

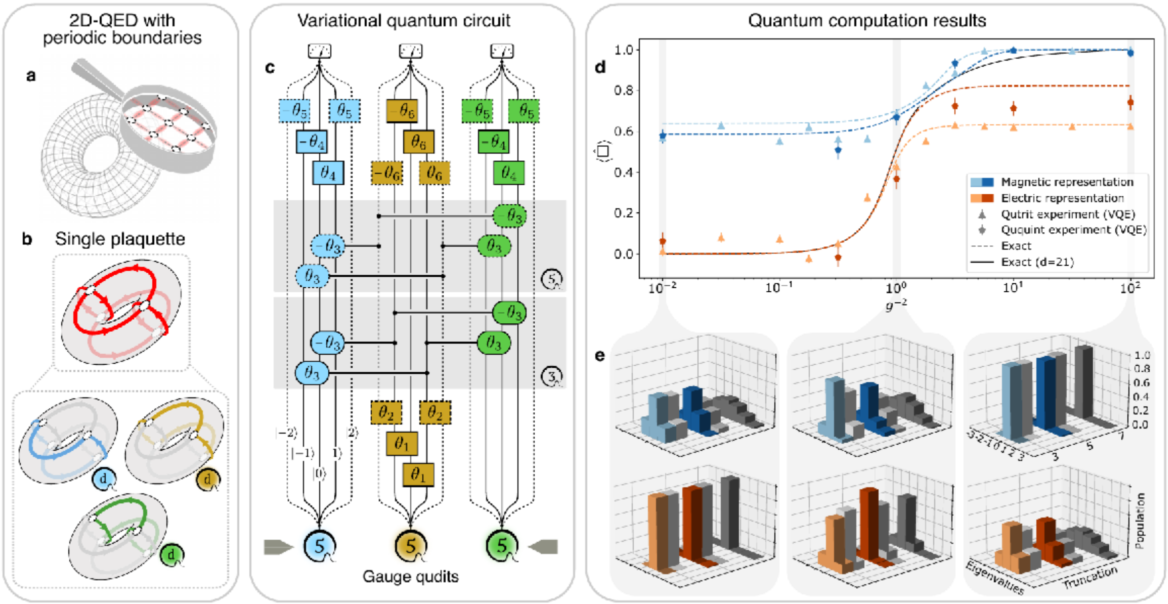
D Paulson, L Dellantonio, JF Haase, **AC**,...PRX Quantum 2 (3), 030334 (2021)

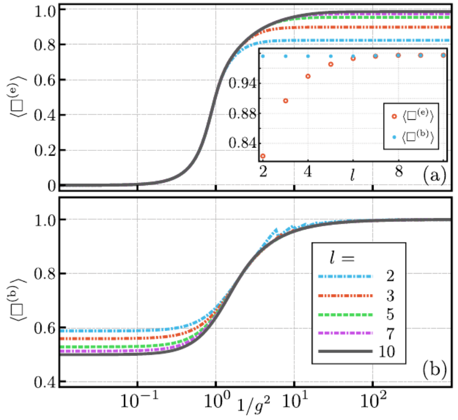
Simulating 2D lattice gauge theories on a qudit quantum computer

Michael Meth,[1] Jan F. Haase,[2,3,4] Jinglei Zhang,[2,3] Claire Edmunds,[1] Lukas Postler,[1] Andrew J. Jena,[2,3] Alex Steiner,[1] Luca Dellantonio,[2,3,5] Rainer Blatt,[1,6,7] Peter Zoller,[8,6] Thomas Monz,[1,7] Philipp Schindler,[1] Christine Muschik*,[2,3,9] and Martin Ringbauer*[1]

arXiv:2310.12110







Running of the coupling on a torus

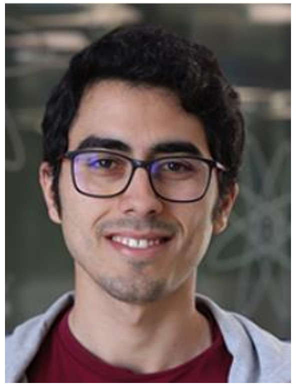
Encoding
+
Coupling dependent basis

- 2+1 QED

F. Haase*, L. Dellantonio*, **AC***, D. Paulson, A. Kan, K. Jansen, C.A. Muschik, Quantum **5**, 393 (2021)

- 2+1 SU(2) Yang-Mills

P. Fontana, M. Miranda, and **AC**, *arXiv:2409.xxxx*

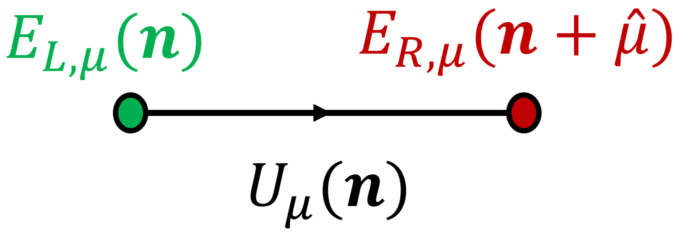


P. Fontana, Ph.D

P. Fontana, MSc

From $U(1)$ to $SU(N)$

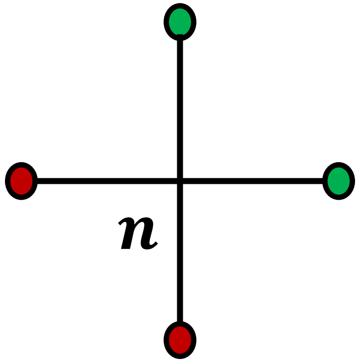
$$H = H_E + H_B = \frac{g^2}{2}\sum_{\boldsymbol{n},\hat{\mu}} \mathrm{Tr}\, E_\mu^2(\boldsymbol{n}) + \frac{1}{2g^2}\sum_P \mathrm{Tr}\left[2 - \left(U_P^\dagger + U_P\right)\right]$$



$$\left[E^a_{L,\mu}(\boldsymbol{n}), U_\nu(\boldsymbol{m})\right] = -\delta_{\mu\nu}\delta_{\boldsymbol{mn}} T^a U_\mu(\boldsymbol{n})$$

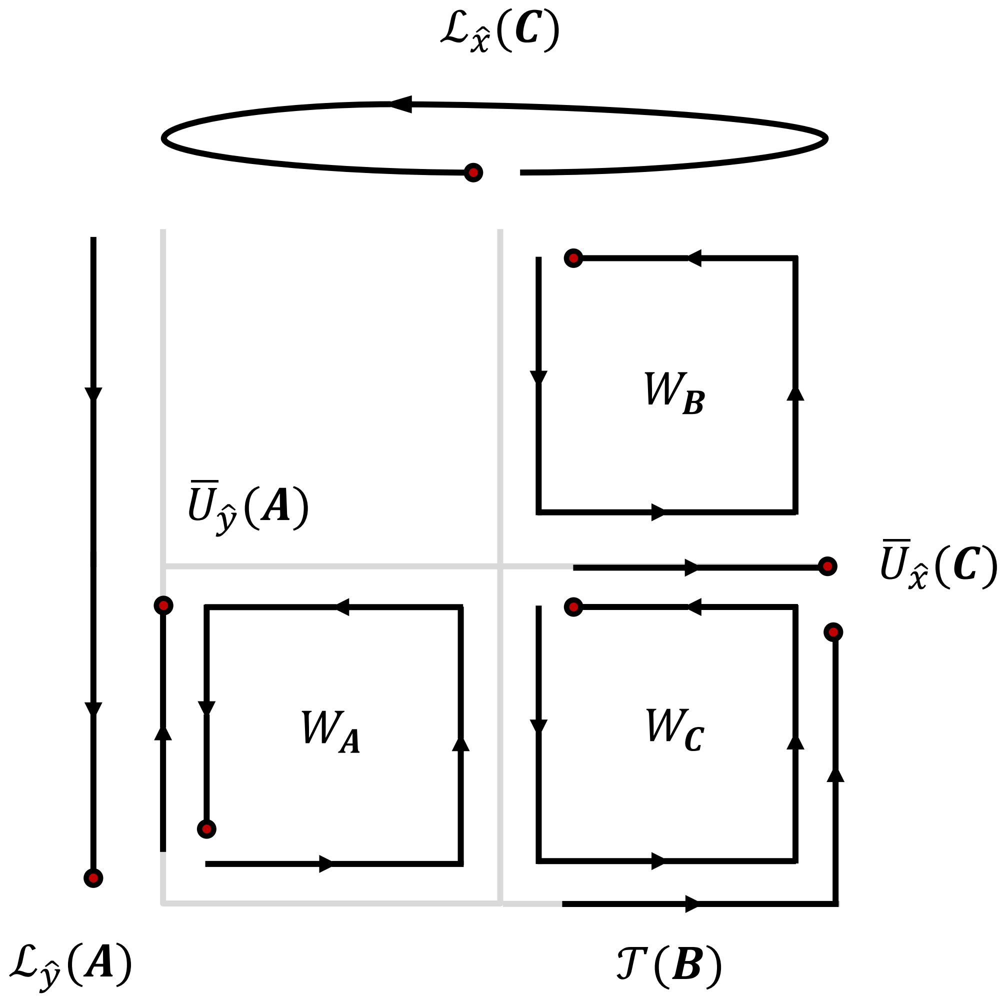
$$\left[E^a_{R,\mu}(\boldsymbol{n}+\hat{\mu}), U_\nu(\boldsymbol{m})\right] = \delta_{\mu\nu}\delta_{\boldsymbol{mn}} U_\mu(\boldsymbol{n})\, T^a$$

$$T^a, \qquad a = 1, \dots, N^2 - 1$$



$$\mathcal{G}^a(\boldsymbol{n}) = \sum_\mu \left[E^a_{L,\mu}(\boldsymbol{n}) + E^a_{R,\mu}(\boldsymbol{n})\right] = 0$$

Encoding by canonical transformations



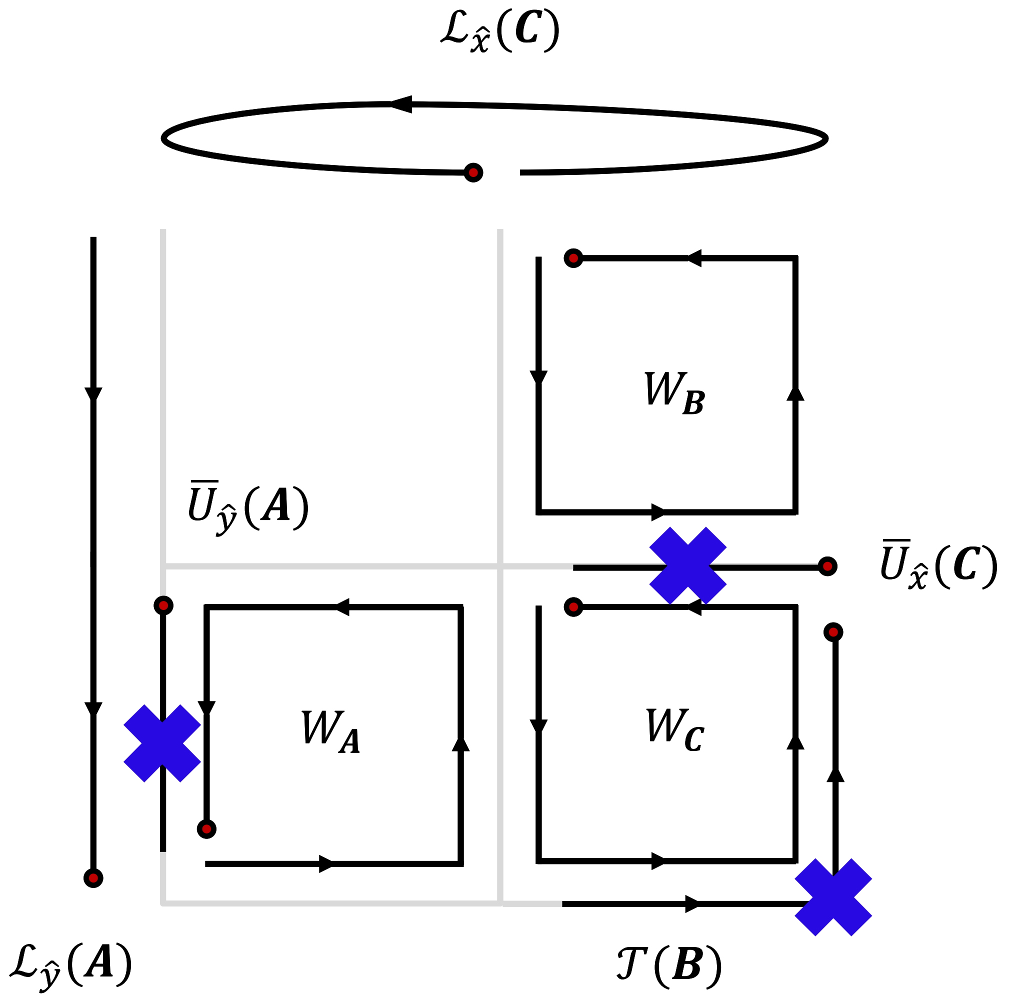
8 links $(E_{L,\mu}(\boldsymbol{n}), U_{\mu}(\boldsymbol{n}))$

- 3 plaquettes $(\mathcal{E}_{\boldsymbol{n}}, W_{\boldsymbol{n}})$
- 2 large loops $(\mathcal{L}_{\hat{\mu}}, , U_{\hat{\mu}})$
- 3 open strings $(\mathcal{T}(\boldsymbol{n}), U(\boldsymbol{n}))$

Loops $\bigoplus$ Strings

Open boundaries conditions: M. Mathur and T. P. Sreeraj, Phys. Rev. D 92, 125018 (2015)

Encoding by canonical transformations



8 links $(E_{L,\mu}(\boldsymbol{n}), U_\mu(\boldsymbol{n}))$

- 3 plaquettes $(\mathcal{E}_{\boldsymbol{n}}, W_{\boldsymbol{n}})$
- 2 large loops $\mathcal{L}_{\hat{\mu}}$
- 3 open strings

Loops $\oplus$ ~~Strings~~

Solve Gauss law

Only physical variables left!

Open boundaries conditions: M. Mathur and T. P. Sreeraj, Phys. Rev. D 92, 125018 (2015)

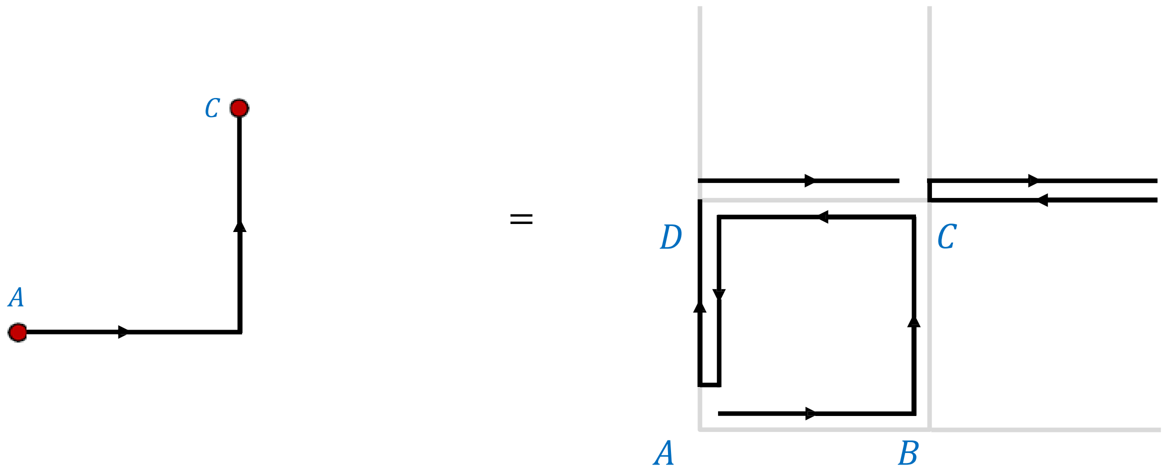
Encoded (Dual) Hamiltonian

$$H = H_E(\mathcal{E}, \mathcal{L}, W) + H_B(W)$$

$$H_B(W) = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \mathrm{Tr}\left[2 - \left(W_{\boldsymbol{n}}^\dagger + W_{\boldsymbol{n}}\right)\right]$$

$$H_E(\mathcal{E}, \mathcal{L}, W) = H_{E,\mathrm{local}} + H_{E,\mathrm{non-local}}$$

$$H_{E,\mathrm{non-local}} \ni \mathcal{E}^a(\boldsymbol{n}) \mathcal{R}_{ab}(\mathcal{P}) \mathcal{E}^b(\boldsymbol{m})$$



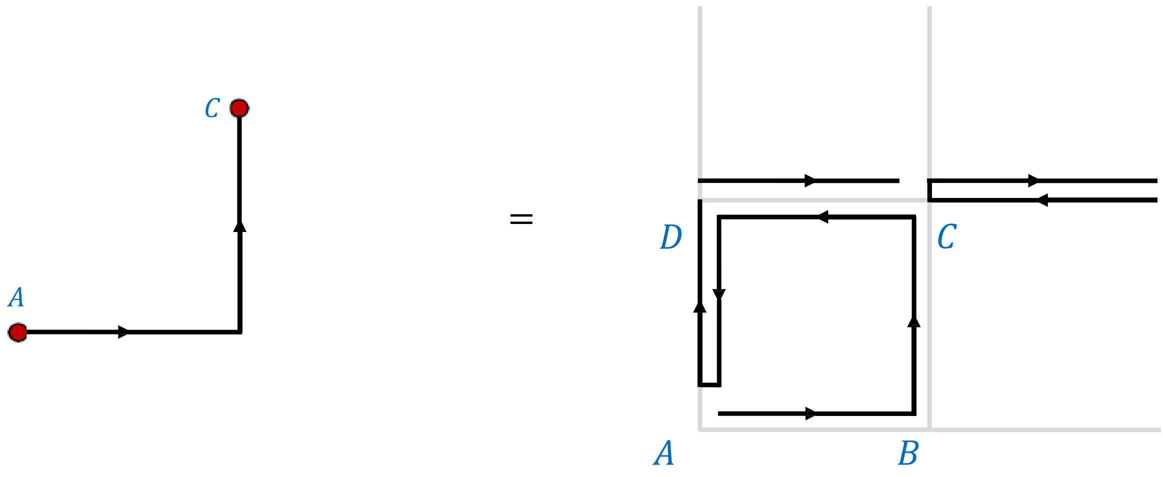
Encoded (Dual) Hamiltonian

$$H = H_E(\mathcal{E}, \mathcal{L}, W) + H_B(W)$$

$$H_B(W) = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \mathrm{Tr}\left[2 - \left(W_{\boldsymbol{n}}^{\dagger} + W_{\boldsymbol{n}}\right)\right]$$

$$H_E(\mathcal{E}, \mathcal{L}, W) = H_{E,\mathrm{local}} + H_{E,\mathrm{non-local}}$$

$$H_{E,\mathrm{non-local}} \ni \mathcal{E}^a(\boldsymbol{n}) \mathcal{R}_{ab}(\mathcal{P}) \mathcal{E}^b(\boldsymbol{m})$$



$$H = H_{\mathrm{local}} + H_{\mathrm{non-local}}$$

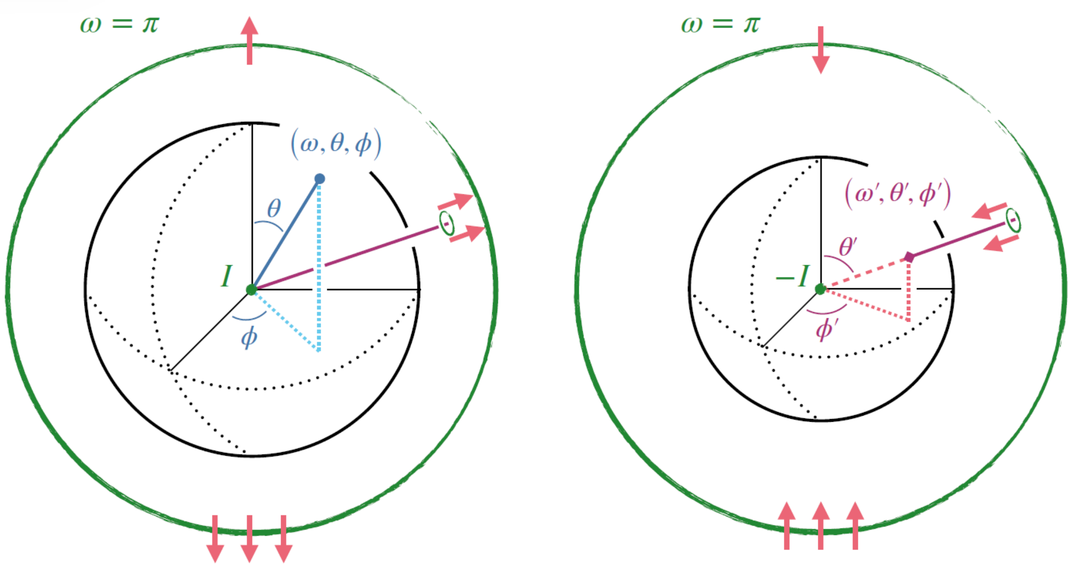
$$H_{\mathrm{local}} = \sum_{\boldsymbol{loops}} H_{1\mathrm{plaquette}}$$

Efficient representation at *all couplings*

$H_{\text{1plaquette}}(g)$ ⟷ g -dependent basis of eigenstates

SU(2)

$(\omega,\theta,\phi) \leftrightarrow S^3$



$$H_{\text{1plaquette}}(g_i) = V_B(\omega) + 2g_i^2\hat{\mathcal{E}}^2$$

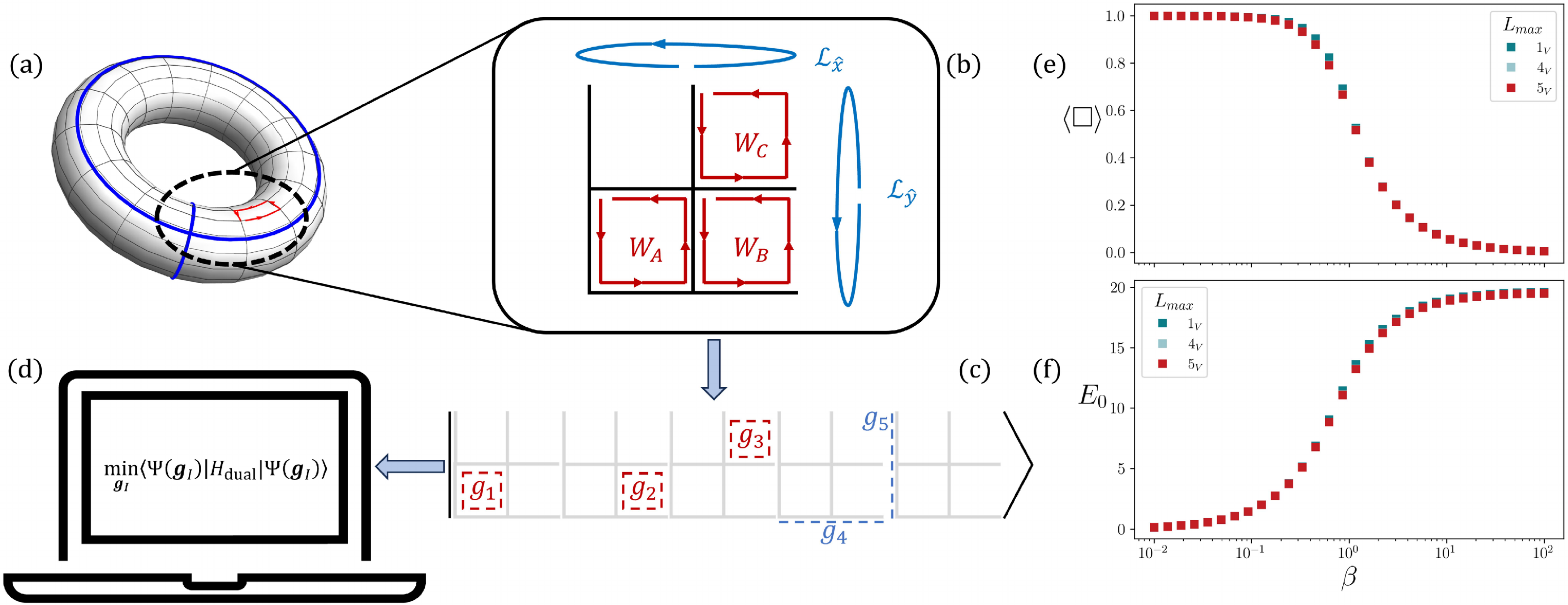
$$V_B(\omega) \equiv \frac{2}{g^2}\left(1-\cos\frac{\omega}{2}\right)$$

$$\hat{\mathcal{E}}^2 = \frac{\mathbf{L}^2}{\sin^2\frac{\omega}{2}} - 4\left[\partial_\omega^2 + \cot\frac{\omega}{2}\partial_\omega\right]$$

Adapted from I. D'Andrea *et al.*, , *Phys. Rev. D 109, 074501 (2024)*

Truncation and g_i independent for each loop: variational parameters!

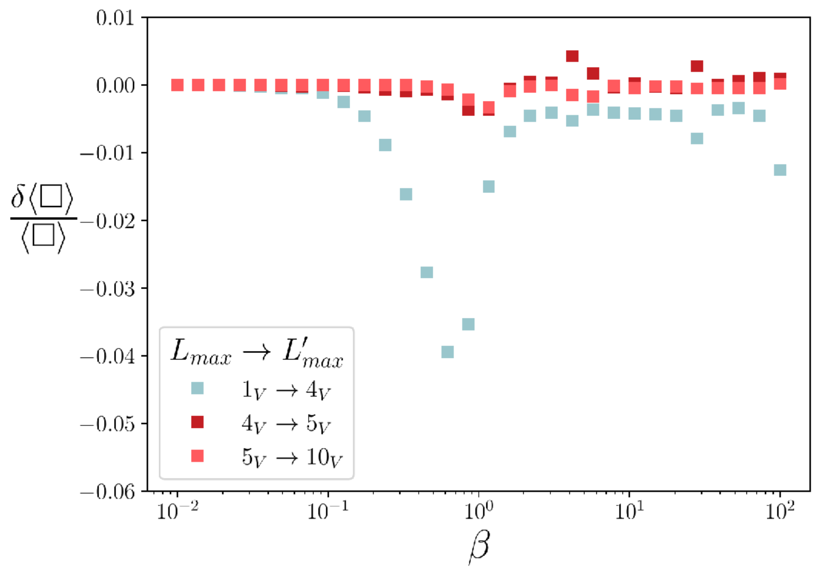
Running of the coupling: scheme



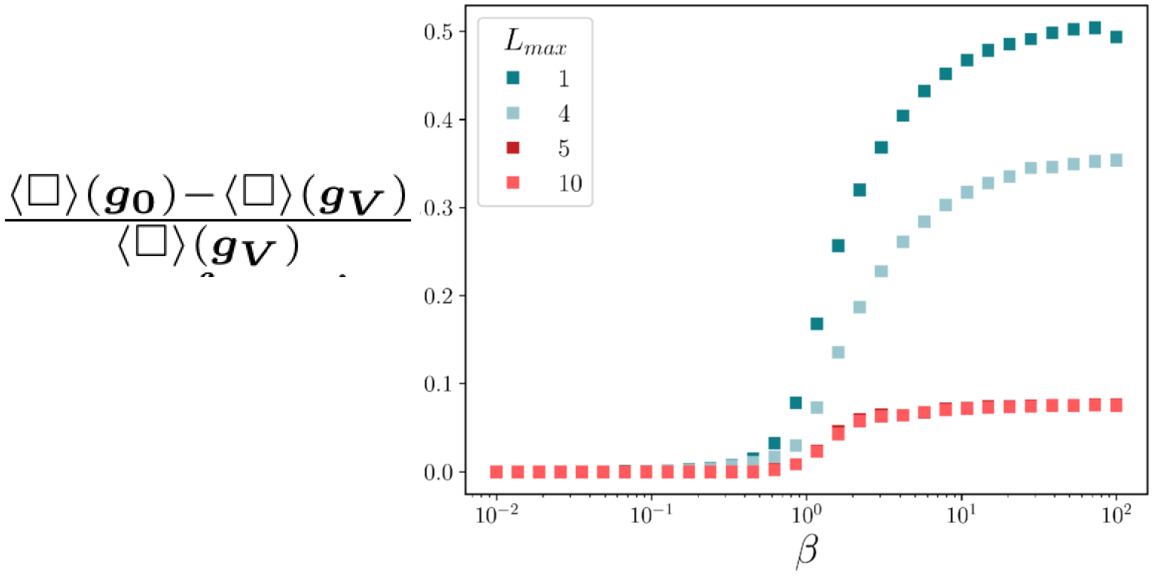
Variational optimization of states and Hamiltonian representation

Running of the coupling: results

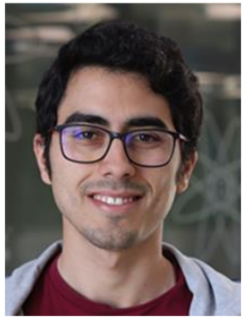
- % precision with 5 states per plaquette



- Basis' optimization > convenient
  for sharp truncations at weak coupling



Conclusions and Outlook



P. Fontana

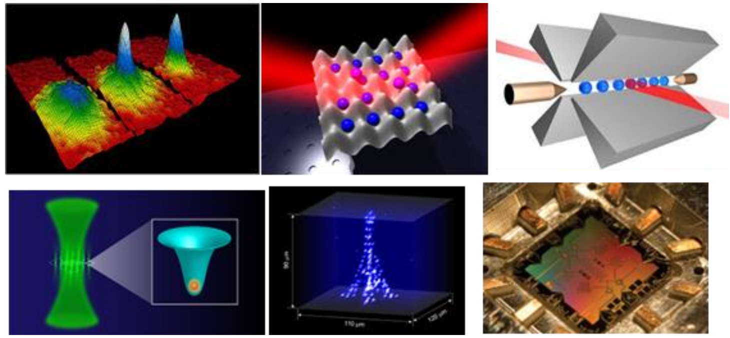
M. Mirand,

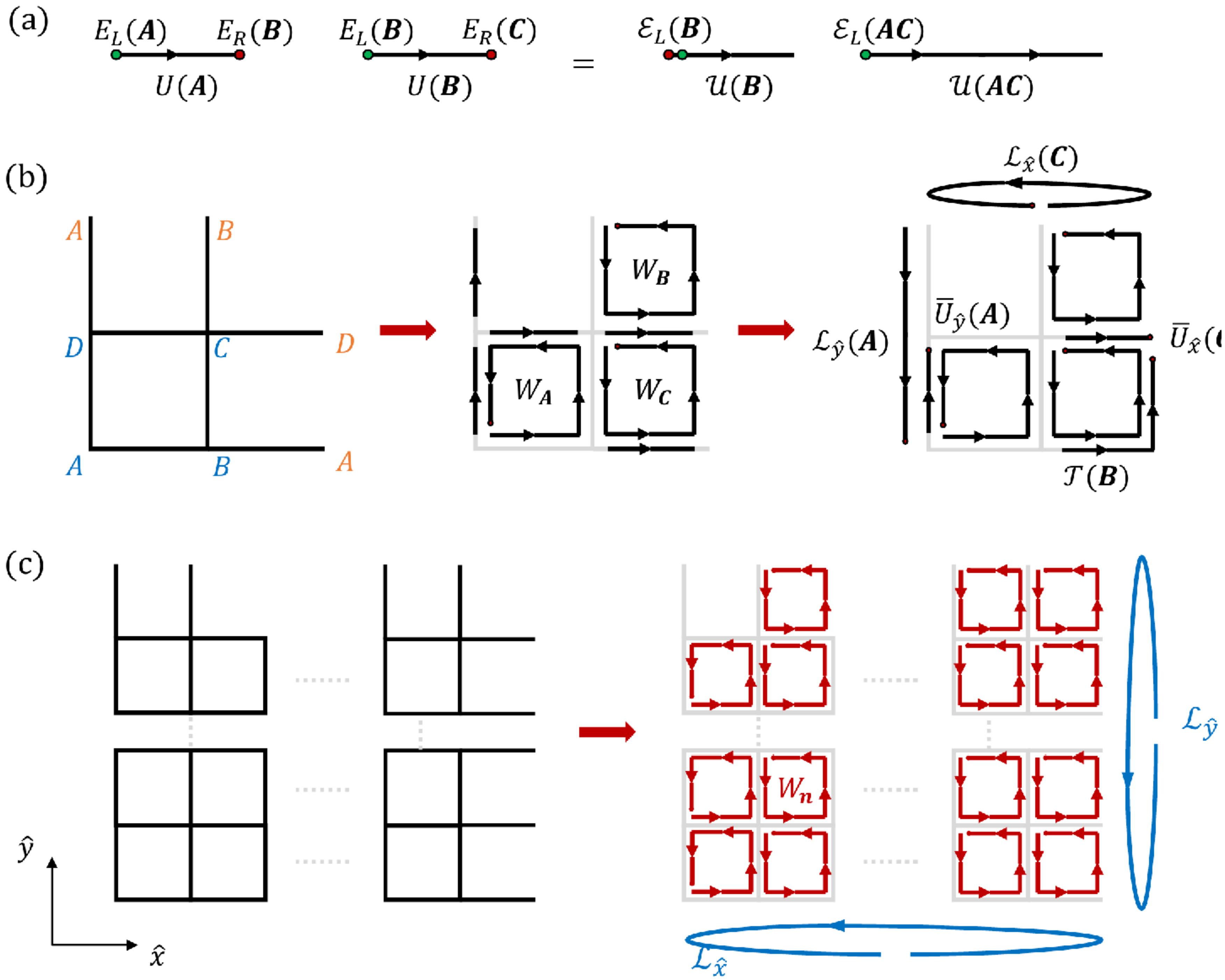
- **Formulation matters!**

β	Standard truncation (electric basis)	Reformulation (interpolating basis)	Variational (interpolating basis)
0.01	1	1	1
1	2744	64	64
100	>2744	125	1

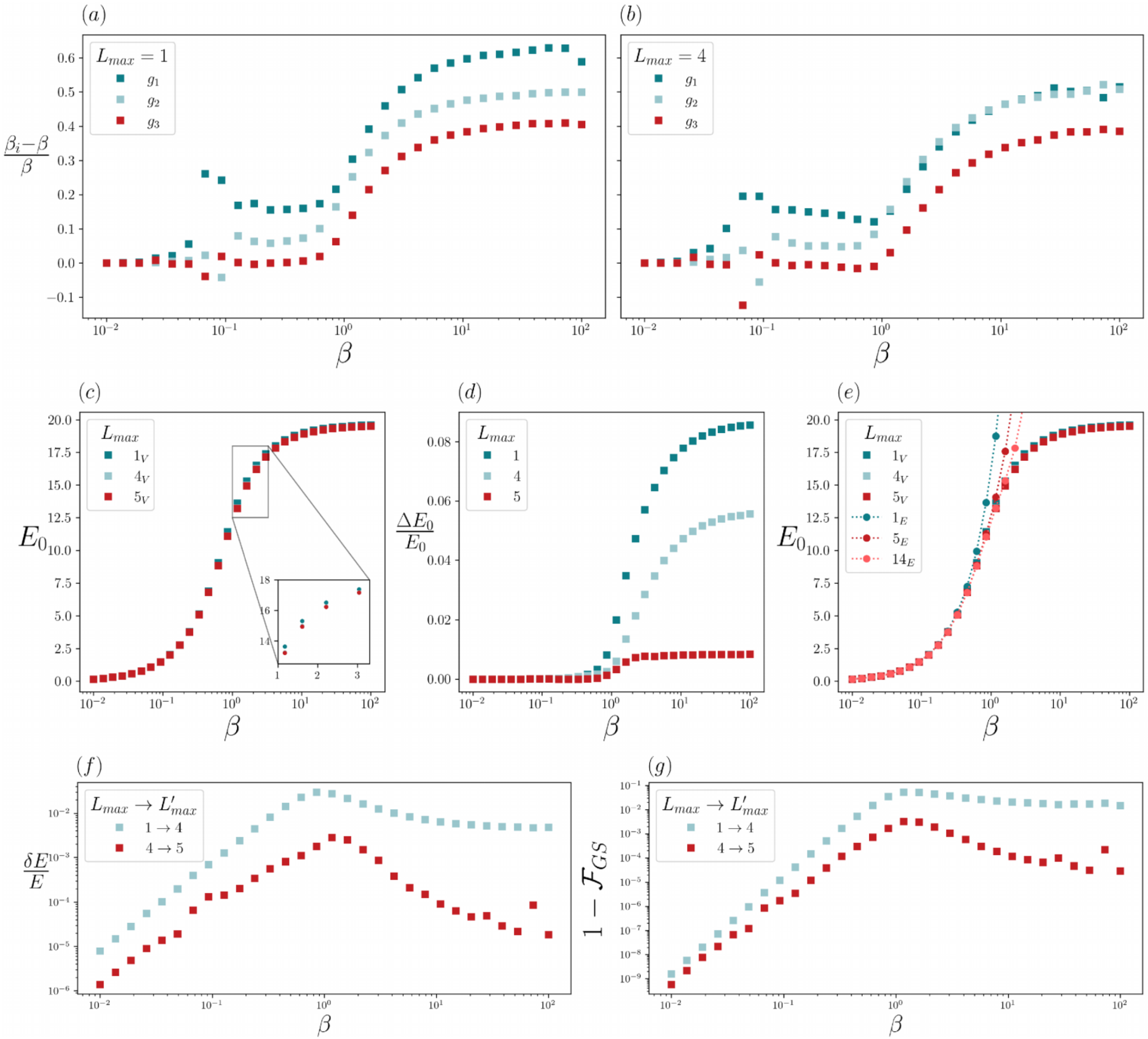
Total # of states to achieve <1% infidelity in the ground state of SU(2) YM on minimal torus

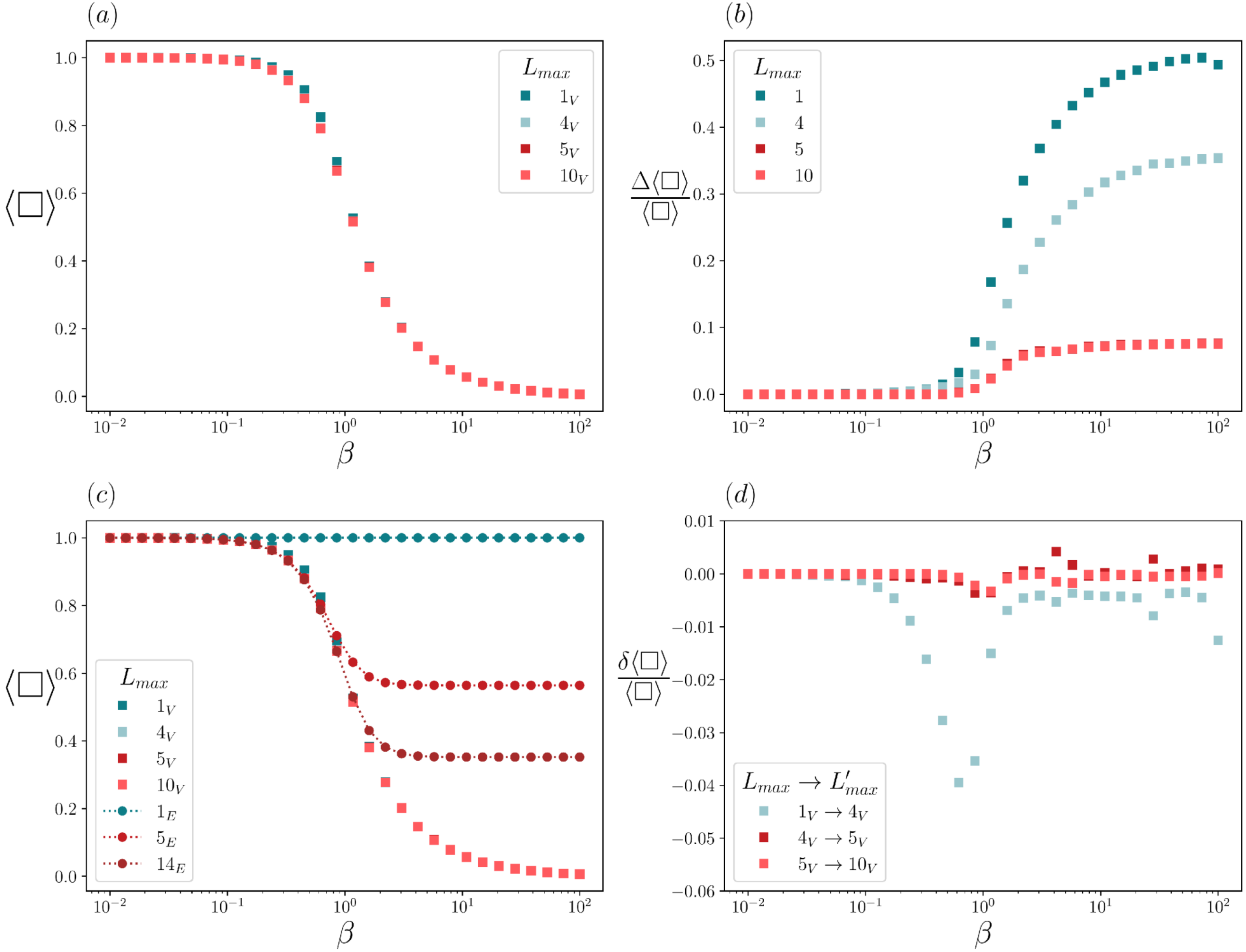
To be tested on larger tori, with matter, with SU(3), in 3D…





(a)
$E_L(\boldsymbol{A})$
$E_R(\boldsymbol{B})$
$U(\boldsymbol{A})$
$E_L(\boldsymbol{B})$
$E_R(\boldsymbol{C})$
$U(\boldsymbol{B})$
=
$\mathcal{E}_L(\boldsymbol{B})$
$\mathcal{U}(\boldsymbol{B})$
$\mathcal{E}_L(\boldsymbol{AC})$
$\mathcal{U}(\boldsymbol{AC})$
(b)
A
B
D
C
D
A
B
A
$W_{\boldsymbol{B}}$
$W_{\boldsymbol{A}}$
$W_{\boldsymbol{C}}$
$\mathcal{L}_{\hat{x}}(\boldsymbol{C})$
$\bar{U}_{\hat{y}}(\boldsymbol{A})$
$\mathcal{L}_{\hat{y}}(\boldsymbol{A})$
$\mathcal{T}(\boldsymbol{B})$
(c)
$\hat{y}$
$\hat{x}$
$W_{\boldsymbol{n}}$
$\mathcal{L}_{\hat{y}}$
$\mathcal{L}_{\hat{x}}$

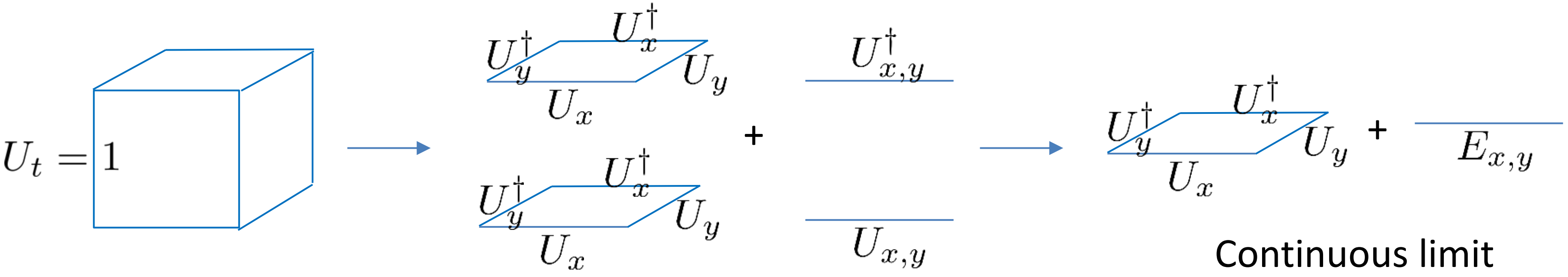

(a)
(b)
$L_{max} = 1$
$L_{max} = 4$
g_1
g_2
g_3
$\frac{\beta_i - \beta}{\beta}$
β
(c)
(d)
(e)
E_0
$\frac{\Delta E_0}{E_0}$
L_{max}
1_V
4_V
5_V
1_E
5_E
14_E
(f)
(g)
$L_{max} \rightarrow L'_{max}$
$1 \rightarrow 4$
$4 \rightarrow 5$
$\frac{\delta E}{E}$
$1 - \mathcal{F}_{GS}$



(a)
(b)
(c)
(d)
L_{max}
1_V
4_V
5_V
10_V
1_E
5_E
14_E
1
4
5
10
$L_{max} \to L'_{max}$
$1_V \to 4_V$
$4_V \to 5_V$
$5_V \to 10_V$
$\langle \square \rangle$
$\frac{\Delta \langle \square \rangle}{\langle \square \rangle}$
$\frac{\delta \langle \square \rangle}{\langle \square \rangle}$
β

From Lagrangian to Hamiltonian Lattice Formulation

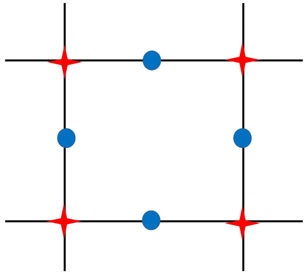
Convenient gauge $A_0=0$ + $\Delta t \to 0$ [Creutz 77]



Continuous limit

$B^2 + E^2$

Path Integral $\longrightarrow$ Operator approach



Matter d.o.f. on sites

Gauge d.o.f. on sites

E operators act on links

U operators act on links and matter

Gauge invariance $\longrightarrow$ Gauss law $(\nabla \cdot E = 4\,\pi\,\rho)$