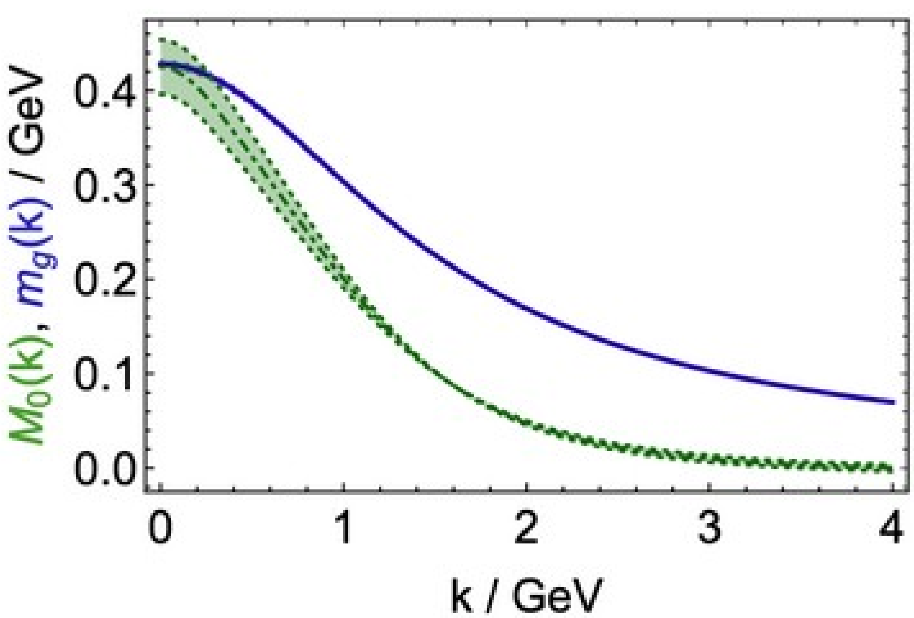
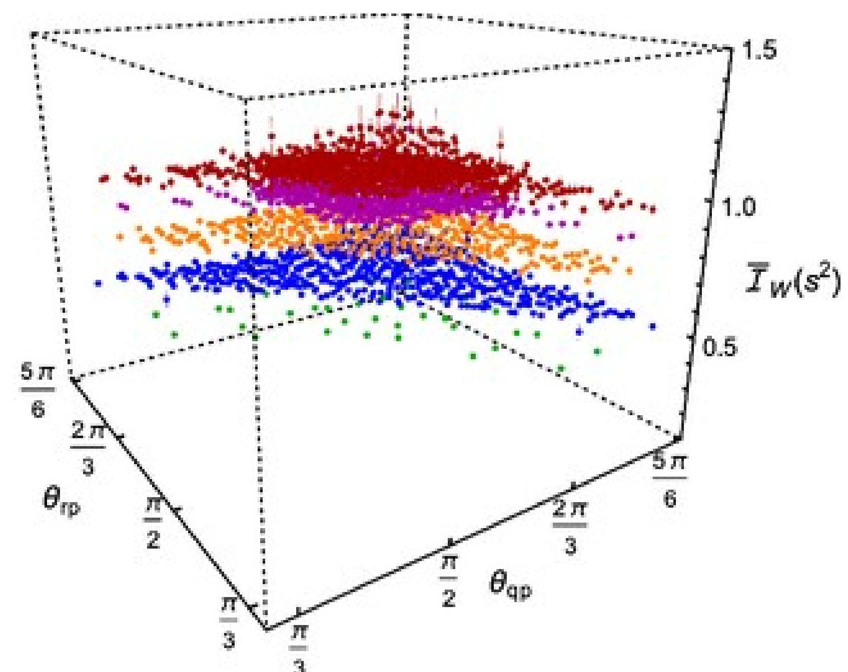
Complete analysis of the Landau-gauge 3-gluon vertex from lattice QCD

by J. Rodríguez-Quintero

Phys.Lett.B 838 (2023) 137737, Phys.Lett.B 841 (2023) 137906, Phys.Rev.D 110 (2024) 1, 014005

In collaboration with: A.C. Aguilar, F. De Soto, M.N. Ferreira, J. Papavassiliou, C. D. Roberts, F. Pinto-Gómez.





Valencia (Spain), QuantFunc2024, September 2th - 6th, 2024.

Motivation: The emergence of gluon mass

- **QCD** is characterized by two **emergent** phenomena: confinement and DGM, both tighly connected to the **running coupling**.
- RGI **gluon** and chiral-limit **quark** masses can be defined and found to be commesurate with each other and of the order of half of the proton mass.

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$

$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu,$$

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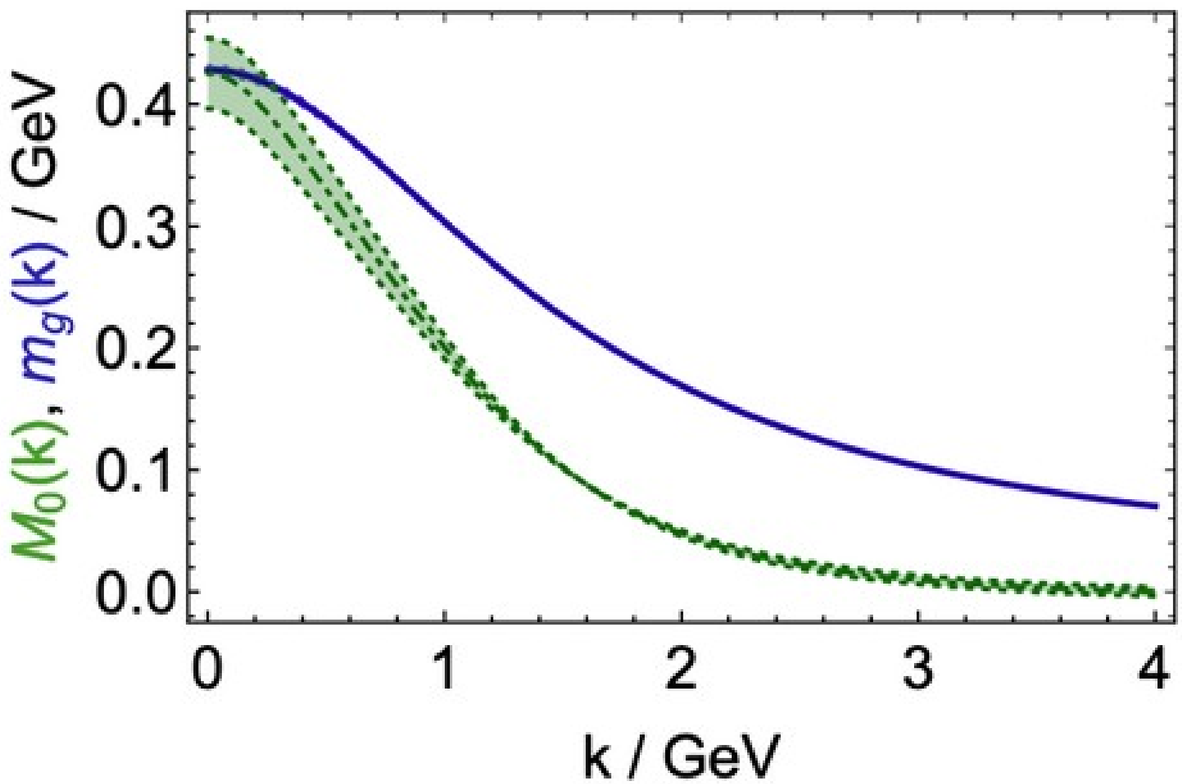
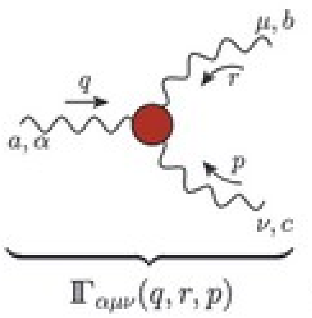


Motivation: The Schwinger mechanism

- The **3-gluon vertex**, triggered by the non-abelian nature of QCD,



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- The **3-gluon vertex**, triggered by the non-abelian nature of QCD, is a key ingredient for the gluon mass generation mechanism [**Schwinger mechanism**] .

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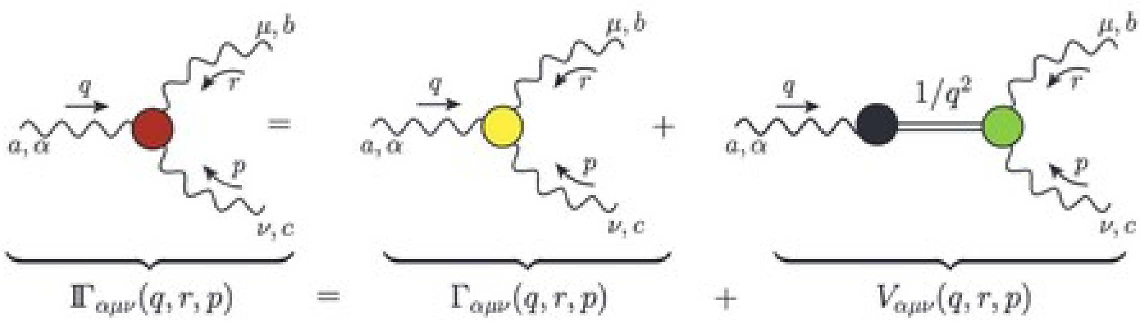

Longitudinal massless poles activating the **Schwinger mechanism**

J. M. Cornwall, Phys.Rev. D 26 (1982) 1453
Aguilar et al. Phys. Rev. D85 (2012) 014018.
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$$V_{\alpha\mu\nu}(q,r,p) = \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \cdots$$

$$\lim_{q\to 0} C_1(q,r,p) = 2(q\cdot r)\mathbb{C}(r^2) + \mathcal{O}(q^2)$$

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Displacement function

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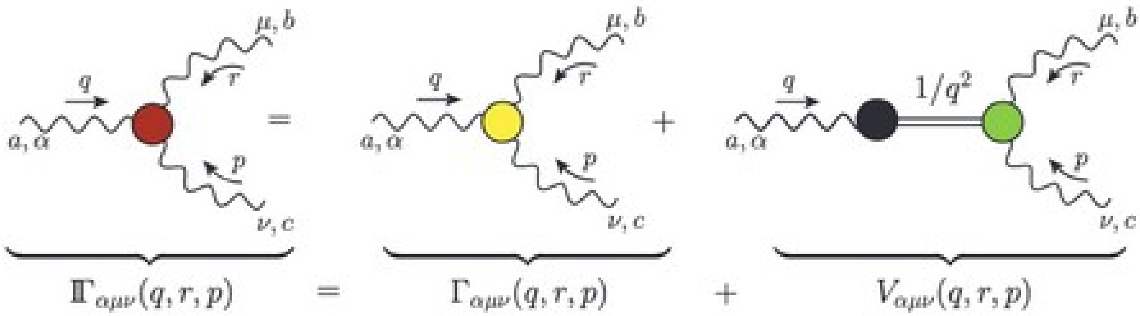

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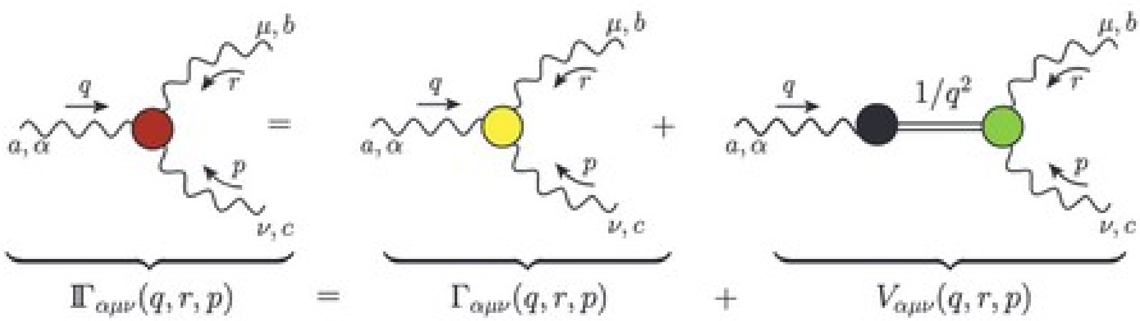

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Displacement function

Soft-gluon 3-gluon form factor
Gluon two-point function
Ghost-gluon finite RC
Ghost dressing

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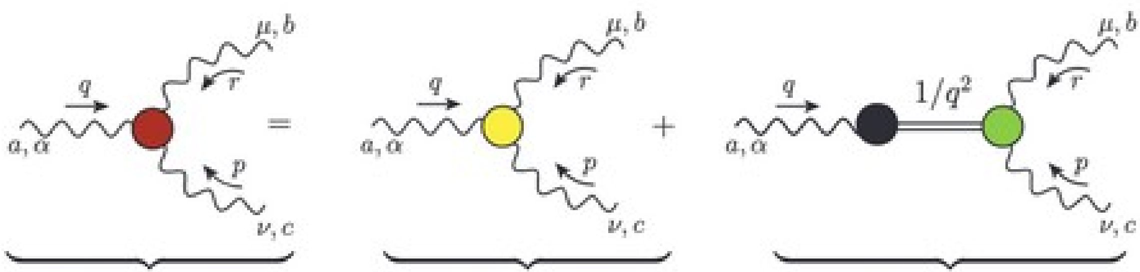

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$$\mathcal{W}_1(r^2) = \frac{g^2 C_A \widetilde{Z}_1}{6}\int_k \Delta(k^2)D(k^2)D(t^2)(r\cdot k)B_1(t,-k,-r)B_1(k,0,-k)\left[1-\frac{(r\cdot k)^2}{r^2k^2}\right]$$

$$\mathcal{W}_2(r^2) = -\frac{g^2 C_A \widetilde{Z}_1}{6}\int_k \Delta(k^2)\Delta(t^2)D(t^2)B_1(t,0,-t)\boxed{\mathcal{I}_{\mathcal{W}}(r^2,k^2,t^2)}$$

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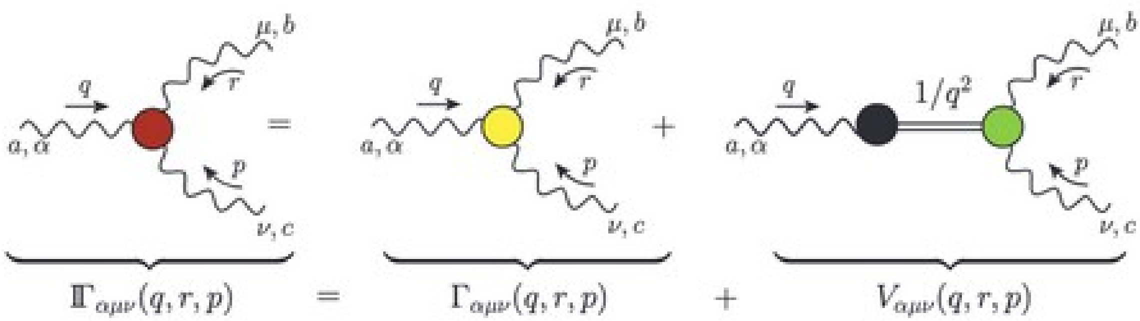

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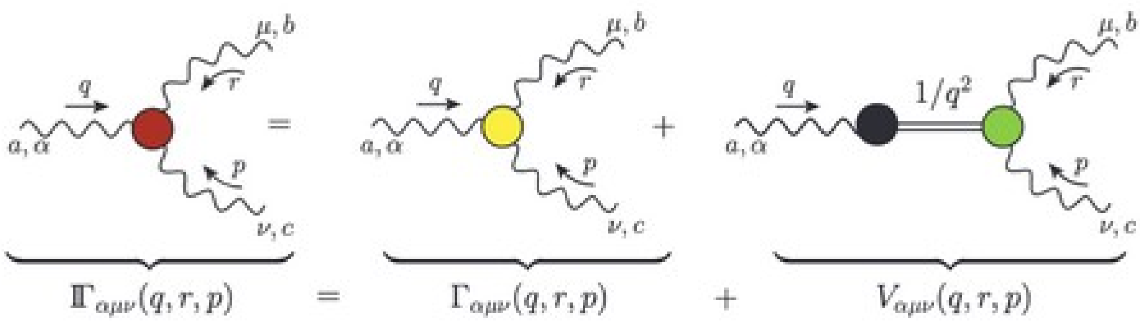

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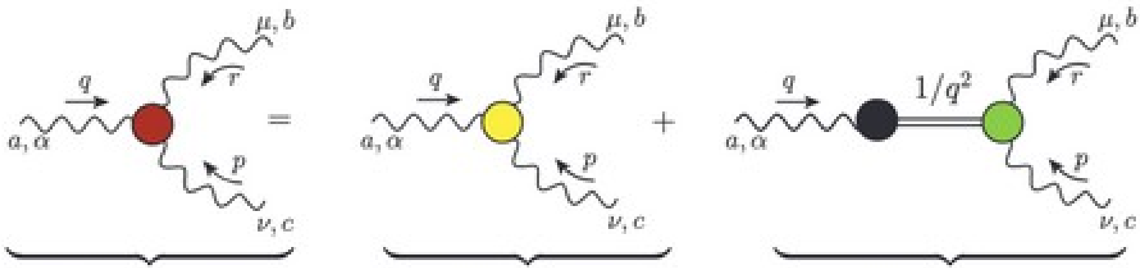

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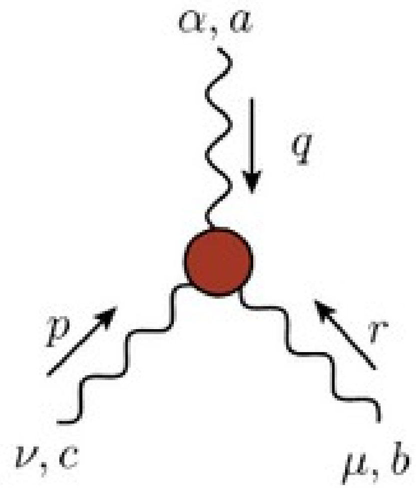
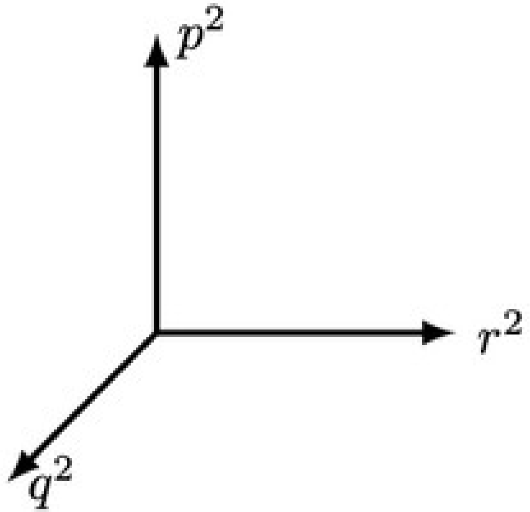
3-gluon vertex: Kinematics





A general kinematic configuration remains fully described by the three squared momenta and can be geometrically represented by the three cartesian coordinates:

$$(q^2, r^2, p^2)$$

With the the angles:

$$\cos\theta_{qr} = (p^2 - q^2 - r^2)/2\sqrt{q^2 r^2}$$

And equivalentely for θ_{rp}, θ_{pq}

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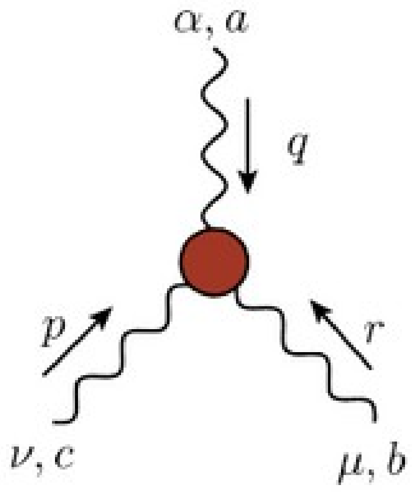

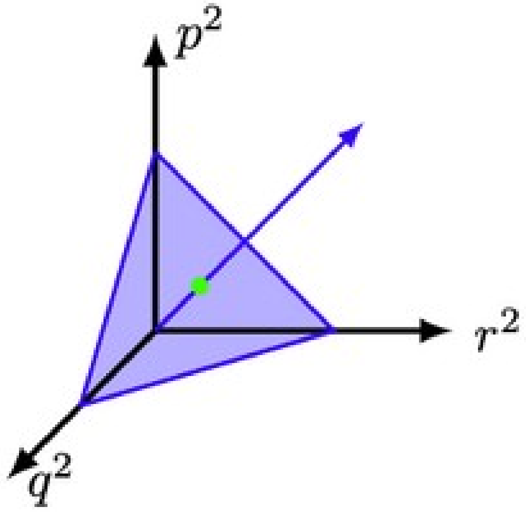

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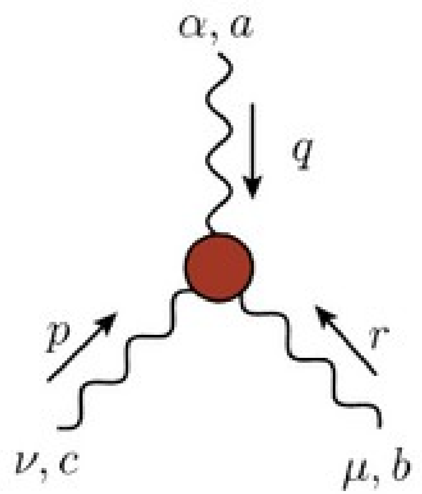

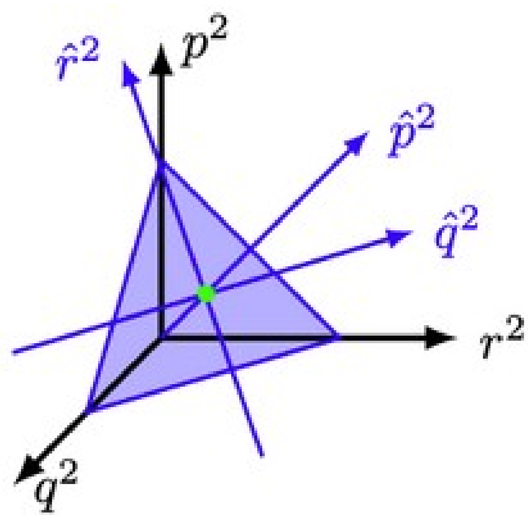

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$$(q^2, r^2, p^2) \longrightarrow (\hat{q}^2, \hat{r}^2, \hat{p}^2)$$

More symmetrically:

$$\hat{q}^2 = \left(r^2 - q^2\right) / \sqrt{2},$$

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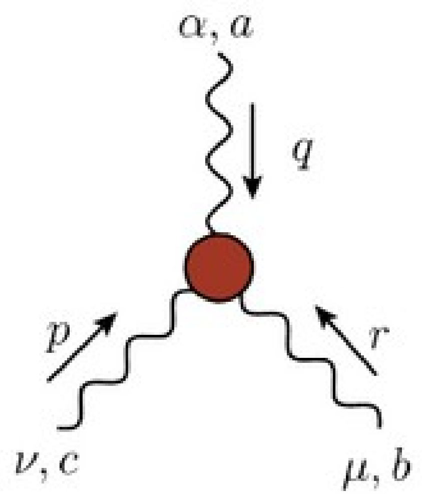

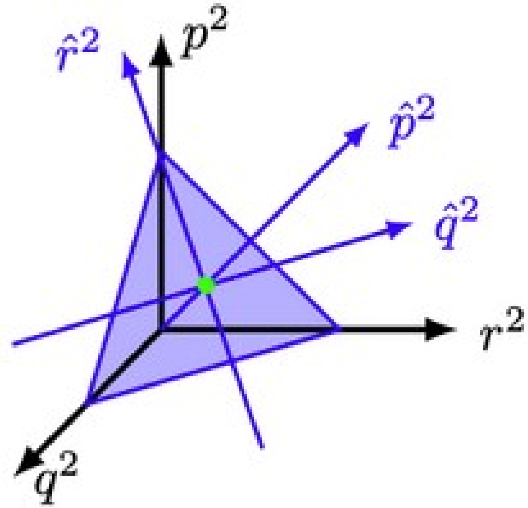

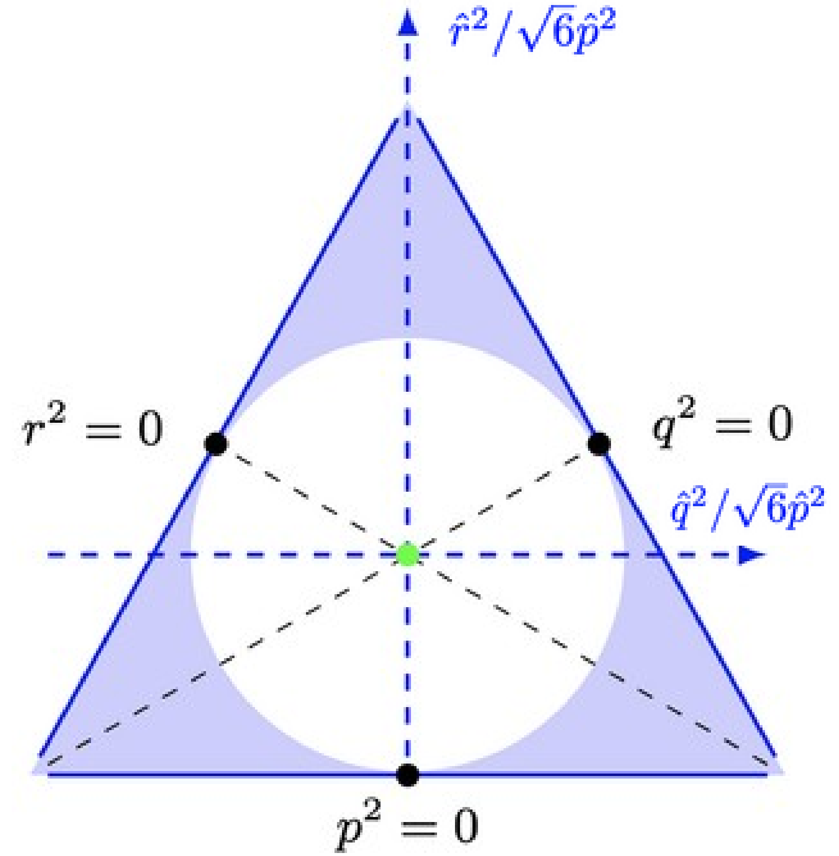

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Momentum conservation implies for the 3-g kinematic representations to lie on the incircle

$$\hat{q}^4 + \hat{r}^4 \leq \frac{1}{2}\hat{p}^4$$

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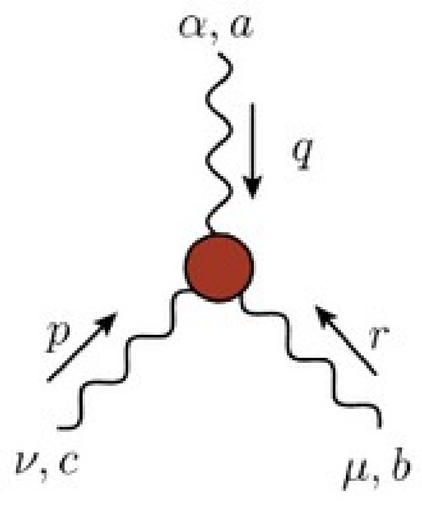

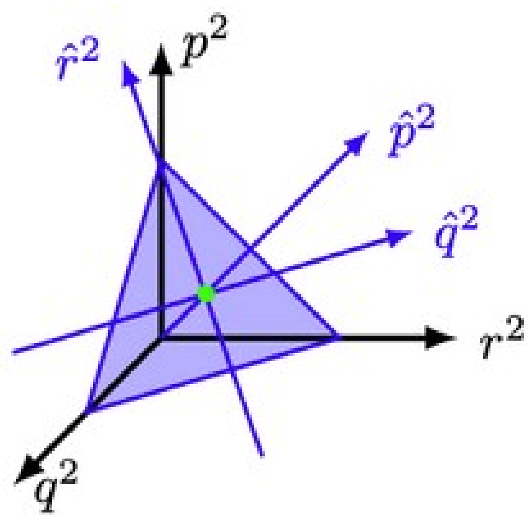

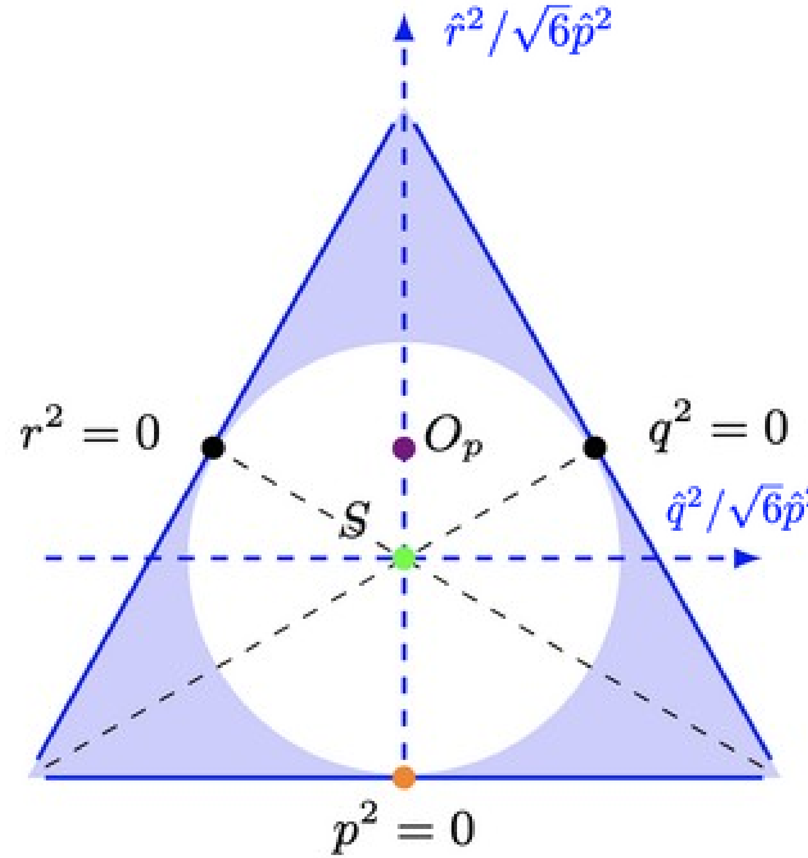

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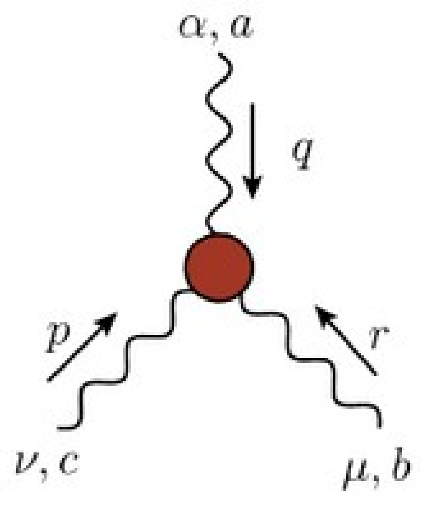
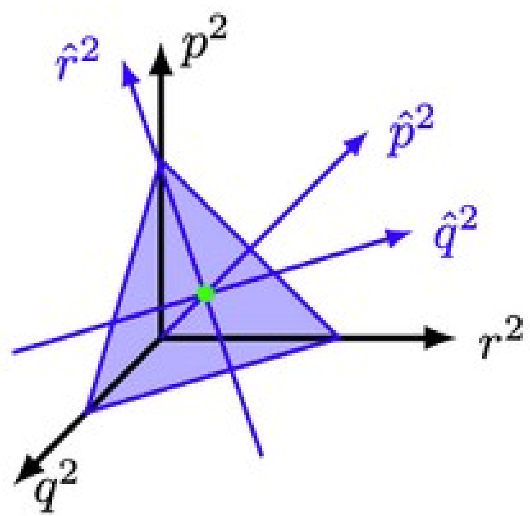
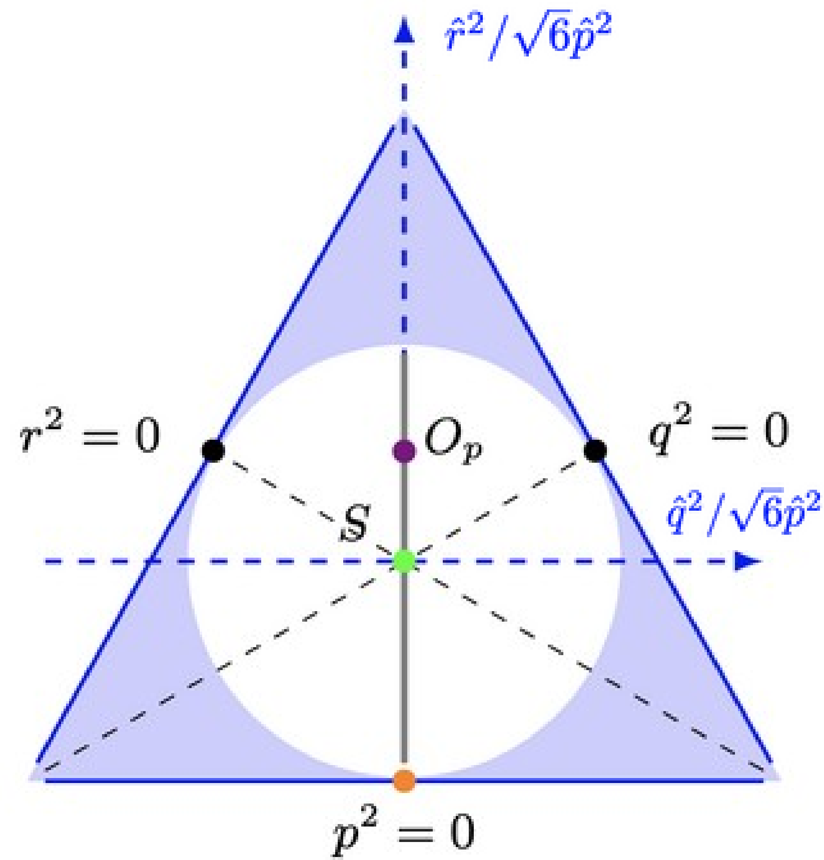
$$\hat{q}^4 + \hat{r}^4 \leq \frac{1}{2}\hat{p}^4$$

Some particular cases can be then displayed

3-gluon vertex: Kinematics







A general kinematic configuration remains fully described by the three squared momenta and can be geometrically represented by the three cartesian coordinates:

$$(q^2, r^2, p^2) \longrightarrow (\hat{q}^2, \hat{r}^2, \hat{p}^2)$$

More symmetrically:

$$\hat{q}^2 = \left(r^2 - q^2\right) / \sqrt{2},$$

$$\hat{r}^2 = \left(2p^2 - q^2 - r^2\right) / \sqrt{6}$$

$$\hat{p}^2 = \left(q^2 + r^2 + p^2\right) / \sqrt{3}$$

With the the angles:

$$\cos\theta_{qr} = (p^2 - q^2 - r^2)/2\sqrt{q^2 r^2}$$

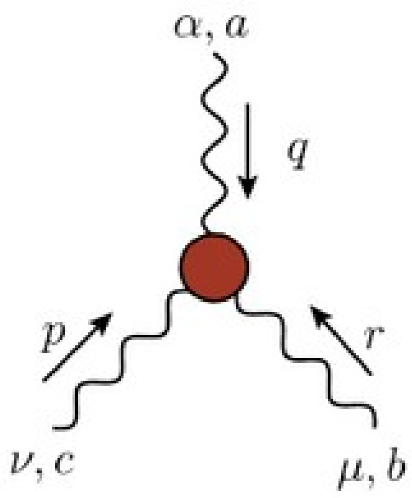
And equivalentely for θ_{rp}, θ_{pq}

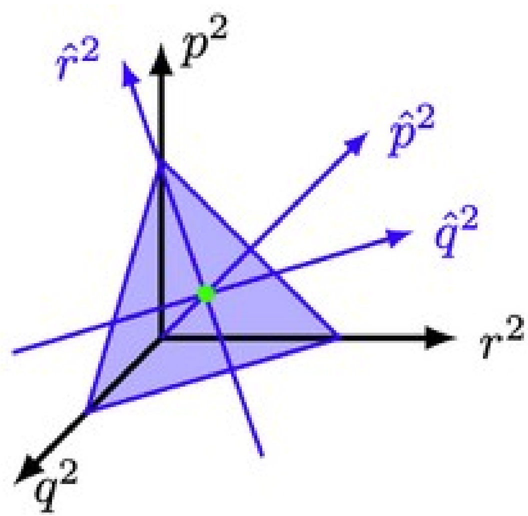
Momentum conservation implies for the 3-g kinematic representations to lie on the incircle

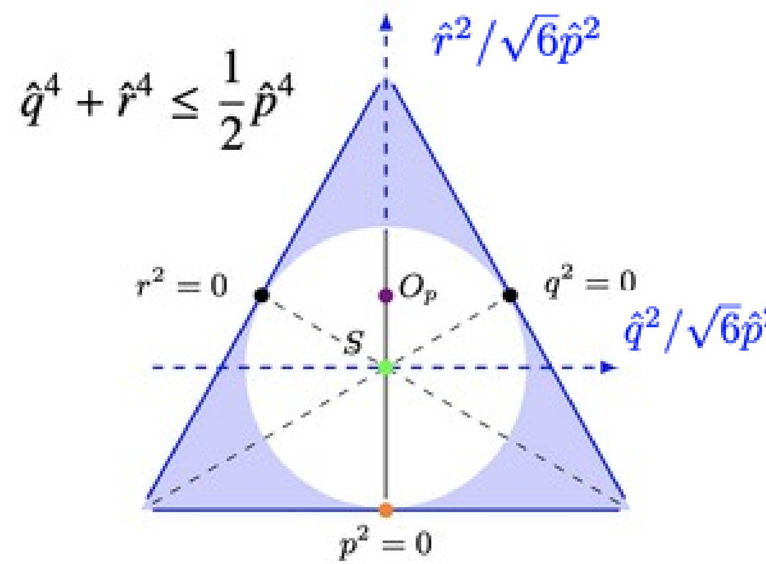
$$\hat{q}^4 + \hat{r}^4 \leq \frac{1}{2}\hat{p}^4$$

Some particular cases can be then displayed, particularly the **bisectoral line**.

3-gluon vertex: Kinematics







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$$(\hat{q}^2, \hat{r}^2, \hat{p}^2)$$

Also in terms of Bose-symmetric redefinitions of the kinematic variables:

$$s^2 = \frac{q^2 + r^2 + p^2}{2}$$

$$t^4 = \frac{(q^2 - r^2)^2 + (r^2 - p^2)^2 + (p^2 - q^2)^2}{3}$$

$$u^6 = (q^2 + r^2 - 2p^2)(r^2 + p^2 - 2q^2)(p^2 + q^2 - 2r^2)$$

G. Eichmann et al. Phys. Rev., D89:105014, 2014

3-gluon vertex: Lattice data and tensor basis

Capitalizing on lattice QCD, one can only access non-amputated Green's function:

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = \frac{1}{24} f^{abc} \langle \widetilde{A}^a_\alpha(q) \widetilde{A}^b_\mu(r) \widetilde{A}^c_\nu(p) \rangle$$

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$$\Delta(p^2) = \frac{1}{24} \delta^{ab} P_{\mu\nu}(p) \langle \widetilde{A}^a_\mu(p) \widetilde{A}^b_\mu(-p) \rangle$$

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$$\Gamma^{\alpha\mu\nu}(q,r,p) = \underbrace{\sum_{i=1}^{10} X_i(q^2,r^2,p^2)\,\ell_i^{\alpha\mu\nu}(q,r,p)}_{\text{10-d non-transverse subspace}} + \underbrace{\sum_{i=1}^{4} Y_i(q^2,r^2,p^2)\,t_i^{\alpha\mu\nu}(q,r,p)}_{\text{4-d transverse subspace}}$$

Ball-Chiu basis

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$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \underbrace{\sum_{i=1}^{4} \widetilde{\Gamma}_i(q^2,r^2,p^2) \widetilde{\lambda}_i^{\alpha\mu\nu}(q,r,p)}_{\text{4-d transverse subspace}}$$

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$$\underbrace{\begin{aligned}
\widetilde{\lambda}_1^{\alpha\mu\nu} &= P^\alpha_{\alpha'}(q) P^\mu_{\mu'}(r) P^\nu_{\nu'}(p) \left[\ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'} \right], \\
\widetilde{\lambda}_2^{\alpha\mu\nu} &= \frac{3}{2s^2} (q-r)^{\nu'} (r-p)^{\alpha'} (p-q)^{\mu'} P^\alpha_{\alpha'}(q) P^\mu_{\mu'}(r) P^\nu_{\nu'}(p), \\
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$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{4}\widetilde{\Gamma}_i(q^2,r^2,p^2)\widetilde{\lambda}_i^{\alpha\mu\nu}(q,r,p)$$

★ Thus:

$$\widetilde{\Gamma}_i(q^2,r^2,p^2) = \widetilde{\Gamma}_i(r^2,q^2,p^2) = \widetilde{\Gamma}_i(q^2,p^2,r^2)$$

The form factors can only depend on bose-symmetric combinations of the three momenta, as

$$\hat{p}^2 = \left(q^2+r^2+p^2\right)/\sqrt{3}$$

$$\underbrace{\begin{aligned}
\widetilde{\lambda}_1^{\alpha\mu\nu} &= P^{\alpha}_{\alpha'}(q)P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)\left[\ell_1^{\alpha'\mu'\nu'} + \ell_4^{\alpha'\mu'\nu'} + \ell_7^{\alpha'\mu'\nu'}\right],\\
\widetilde{\lambda}_2^{\alpha\mu\nu} &= \frac{3}{2s^2}(q-r)^{\nu'}(r-p)^{\alpha'}(p-q)^{\mu'}P^{\alpha}_{\alpha'}(q)P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p),\\
\widetilde{\lambda}_3^{\alpha\mu\nu} &= \frac{3}{2s^2}P^{\alpha}_{\alpha'}(q)P^{\mu}_{\mu'}(r)P^{\nu}_{\nu'}(p)\left[\ell_3^{\alpha'\mu'\nu'} + \ell_6^{\alpha'\mu'\nu'} + \ell_9^{\alpha'\mu'\nu'}\right],\\
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Ball-Chiu basis

An improved basis:

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{4} \widetilde{\Gamma}_i^*(q^2,r^2,p^2) \boxed{\widetilde{\lambda}^*_{i\alpha\mu\nu}(q,r,p)}$$

$$\boxed{\widetilde{\lambda}^*_{i\alpha\mu\nu}(q,r,p) = \widetilde{\lambda}_{i\alpha\mu\nu}(q,r,p) - \delta_{i3} \frac{3}{2} \widetilde{\lambda}_{1\alpha\mu\nu}(q,r,p)}$$

$$\underbrace{\begin{aligned}
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An improved basis:

$$\overline{\Gamma}_{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{4} \widetilde{\Gamma}^*_i(q^2,r^2,p^2) \widetilde{\lambda}^*_{i\alpha\mu\nu}(q,r,p)$$

$$\widetilde{\lambda}^*_{i\alpha\mu\nu}(q,r,p) = \widetilde{\lambda}_{i\alpha\mu\nu}(q,r,p) - \delta_{i3}\frac{3}{2}\widetilde{\lambda}_{1\alpha\mu\nu}(q,r,p)$$

$$\widetilde{\Gamma}^*_i(q^2,r^2,p^2) = \sum_{k=1}^{4} \left(\delta_{ik} + \frac{3}{2}\delta_{i1}\delta_{k3} \right) \widetilde{\Gamma}_k(q^2,r^2,p^2)$$

The only consequence is the borrowing of a a new piece by the tree-level form factor.

$$\underbrace{\begin{aligned}
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3-gluon vertex: Lattice data results

Specializing for the bisectoral case:

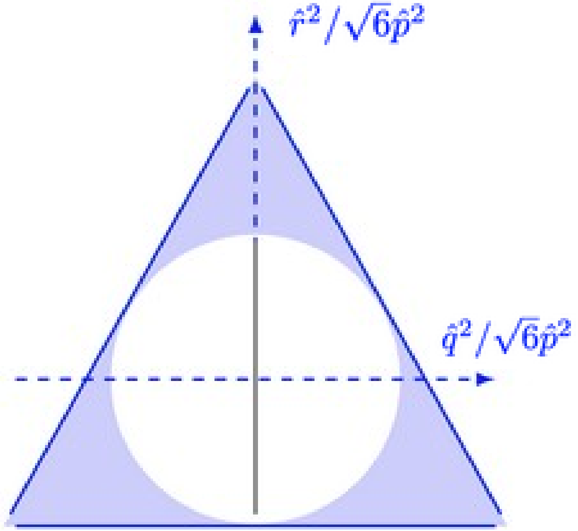
$$\lambda_i^{\alpha\mu\nu}(q,r,p) = \lim_{r^2\to q^2} \tilde{\lambda}_i^{\alpha\mu\nu}(q,r,p)$$

$$\lim_{r^2\to q^2} \lambda_4^{\alpha\mu\nu} = \sum_{i=1}^{3} f_i(z)\,\lambda_i^{\alpha\mu\nu}(q,r,p)$$

$$f_1(z) = \frac{9}{16} z(1-z) \qquad f_2(z) = \frac{9}{32} z - \frac{3}{8} \qquad f_3(z) = \frac{3}{8} z$$

$$z = p^2 / \left[q^2 + \frac{p^2}{2} \right]$$

$$\boxed{\overline{\Gamma}_i(q^2,q^2,p^2) = \widetilde{\Gamma}_i(q^2,q^2,p^2) + f_i(z)\,\widetilde{\Gamma}_4(q^2,q^2,p^2)}$$



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Specializing for the bisectoral case:

$$\lambda_i^{\alpha\mu\nu *}(q,r,p) = \lim_{r^2\to q^2} \tilde{\lambda}_i^{\alpha\mu\nu *}(q,r,p)$$

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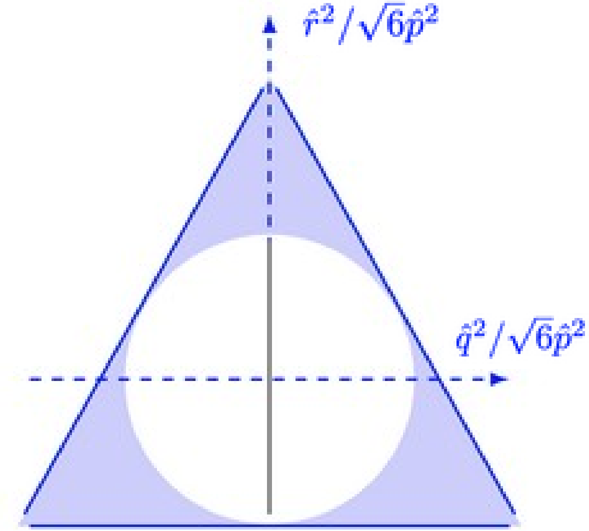
$$f_1^*(z) = \frac{9}{16} z\,(2 - z) \qquad f_2^*(z) = \frac{9}{32} z - \frac{3}{8} \qquad f_3^*(z) = \frac{3}{8} z$$

$$z = p^2 / \left[q^2 + \frac{p^2}{2} \right]$$

$$\overline{\Gamma}_i^*(q^2,q^2,p^2) = \widetilde{\Gamma}_i^*(q^2,q^2,p^2) + f_i^*(z)\, \widetilde{\Gamma}_4^*(q^2,q^2,p^2)$$



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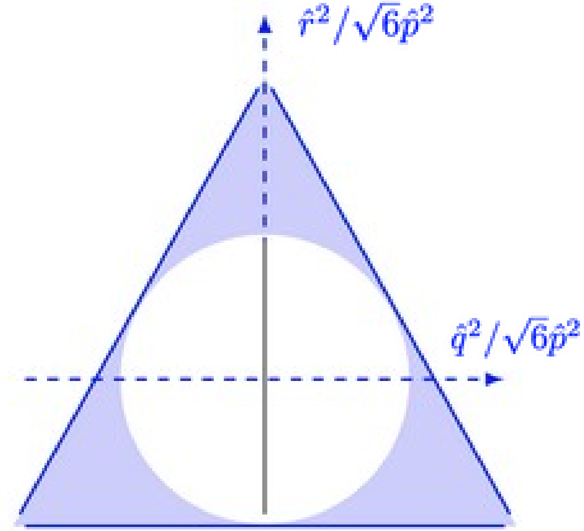

Exploiting the following lattice configurations

β	L^4/a^4	a (fm)	confs
5.6	32^4	0.236	2000
5.8	32^4	0.144	2000
6.0	32^4	0.096	2000
6.2	32^4	0.070	2000

To calculate the required 2- and 3-point Green's functions and project out the 3-g form factors.

3-gluon vertex: Lattice data results

Specializing for the bisectoral case:

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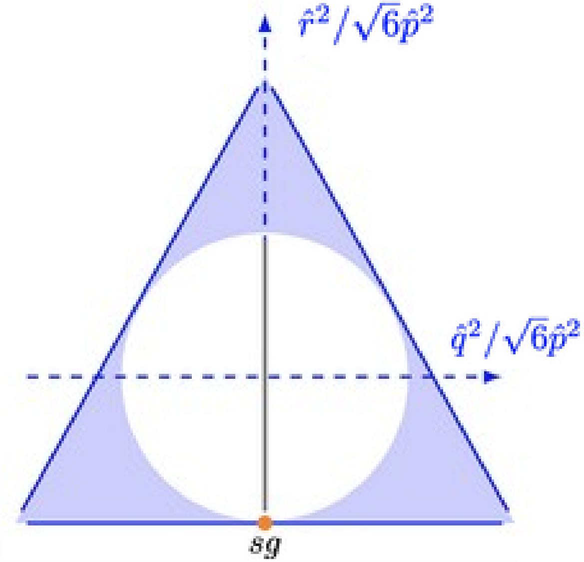
$$z = p^2 / \left[q^2 + \frac{p^2}{2} \right] \qquad \boxed{\overline{\Gamma}_i(q^2,q^2,p^2) = \widetilde{\Gamma}_i(q^2,q^2,p^2) + f_i(z)\, \widetilde{\Gamma}_4(q^2,q^2,p^2)}$$



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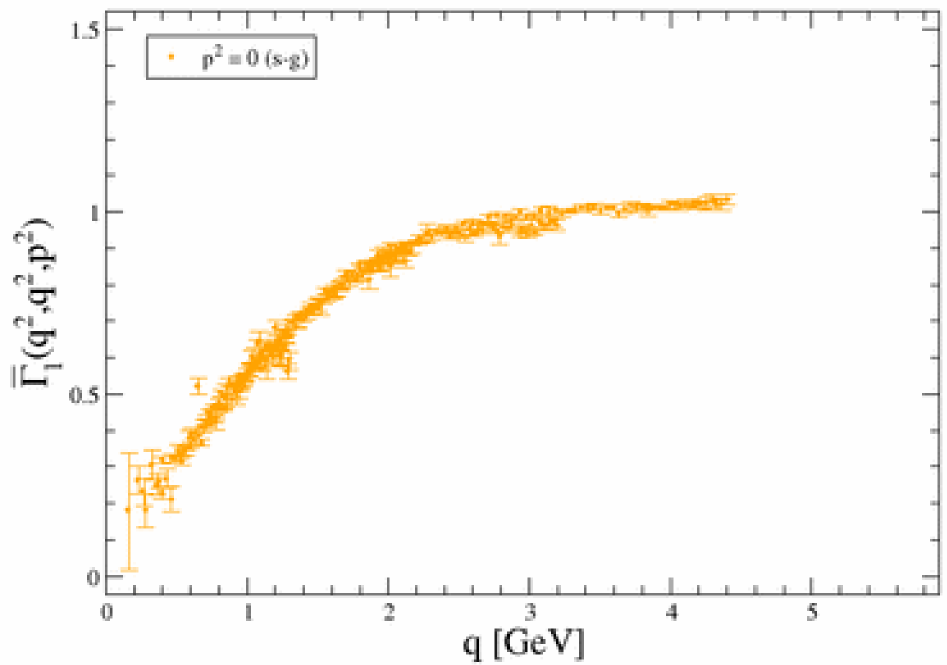


3-gluon vertex: Lattice data results

Specializing for the bisectoral case:

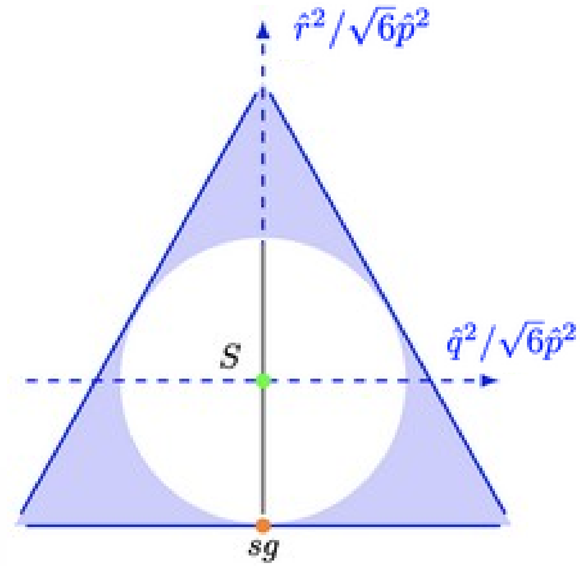
$$\lambda_i^{\alpha\mu\nu}(q,r,p) = \lim_{r^2\to q^2} \tilde{\lambda}_i^{\alpha\mu\nu}(q,r,p)$$

$$\lim_{r^2\to q^2} \lambda_4^{\alpha\mu\nu} = \sum_{i=1}^{3} f_i(z)\,\lambda_i^{\alpha\mu\nu}(q,r,p)$$

$$f_1(z) = \frac{9}{16}z(1-z) \qquad f_2(z) = \frac{9}{32}z - \frac{3}{8} \qquad f_3(z) = \frac{3}{8}z$$

$$z = p^2 \Big/ \left[q^2 + \frac{p^2}{2}\right]$$

$$\boxed{\overline{\Gamma}_i(q^2,q^2,p^2) = \widetilde{\Gamma}_i(q^2,q^2,p^2) + f_i(z)\,\widetilde{\Gamma}_4(q^2,q^2,p^2)}$$



Exploiting the following lattice configurations

β	L^4/a^4	a (fm)	confs
5.6	32^4	0.236	2000
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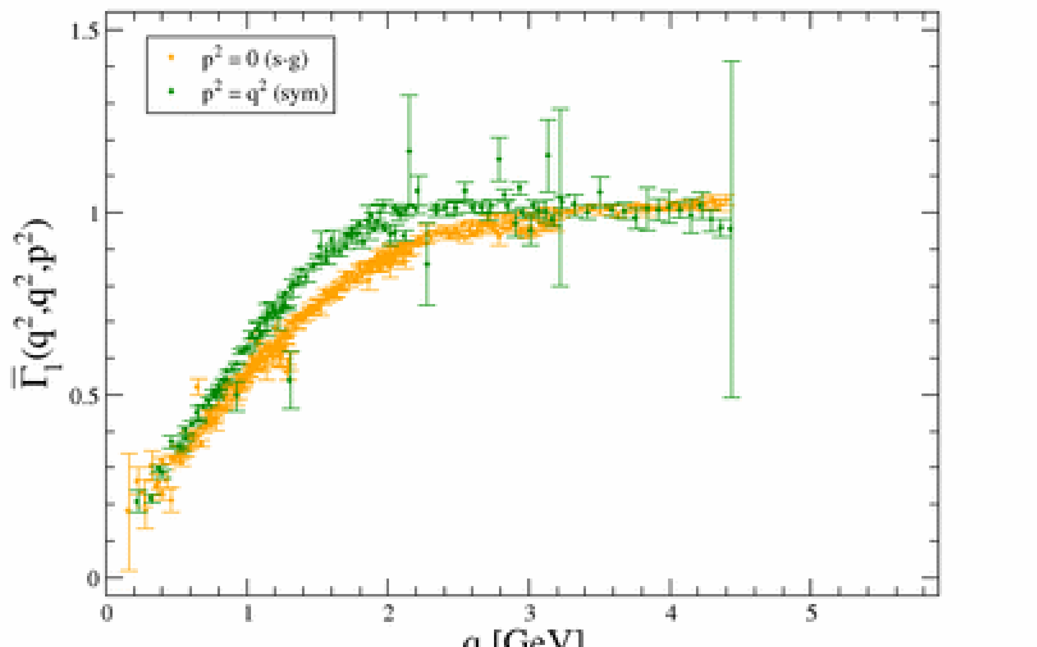

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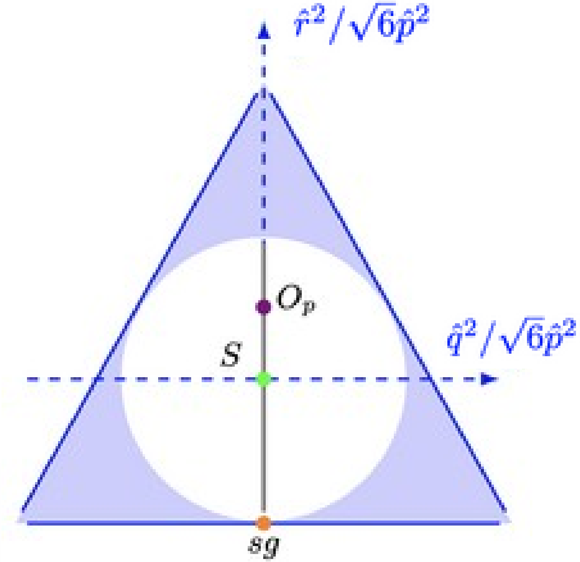

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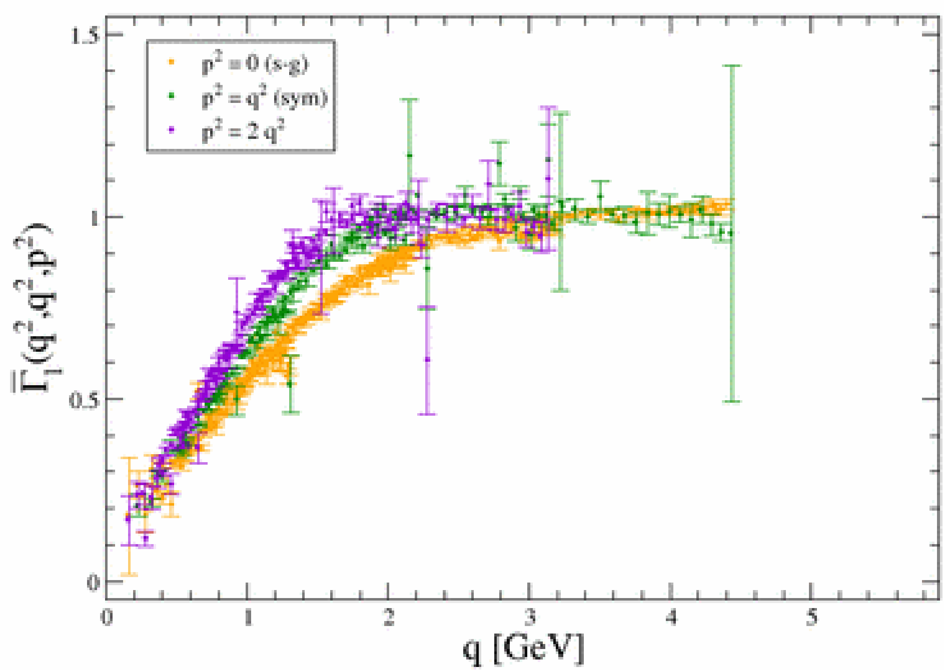

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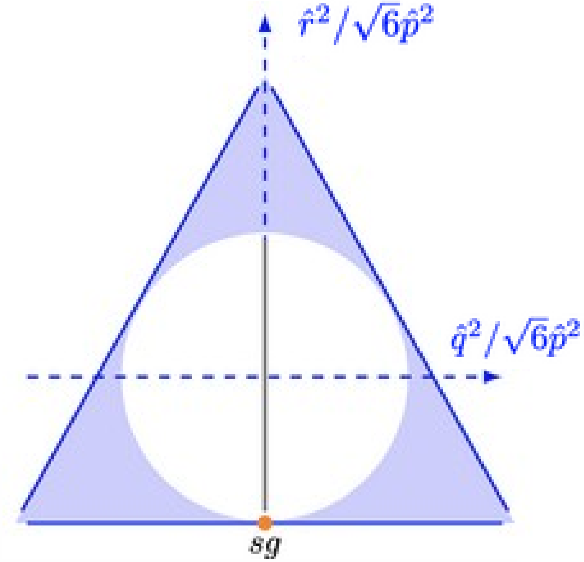

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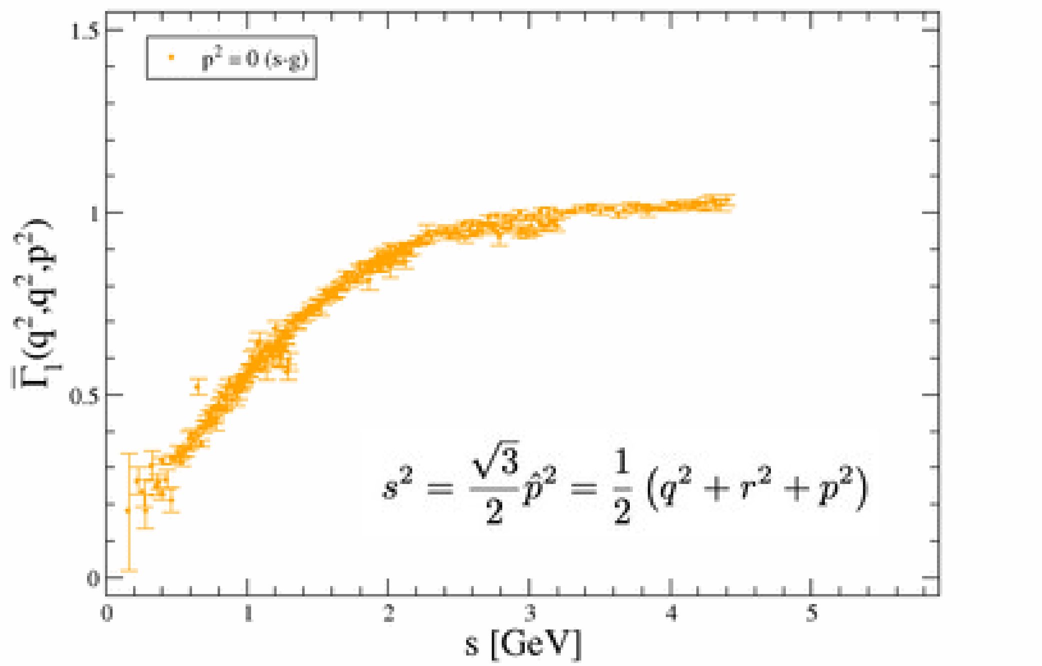

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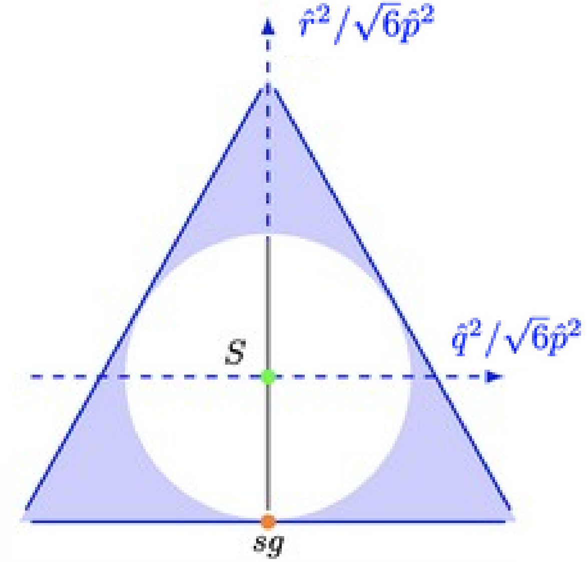

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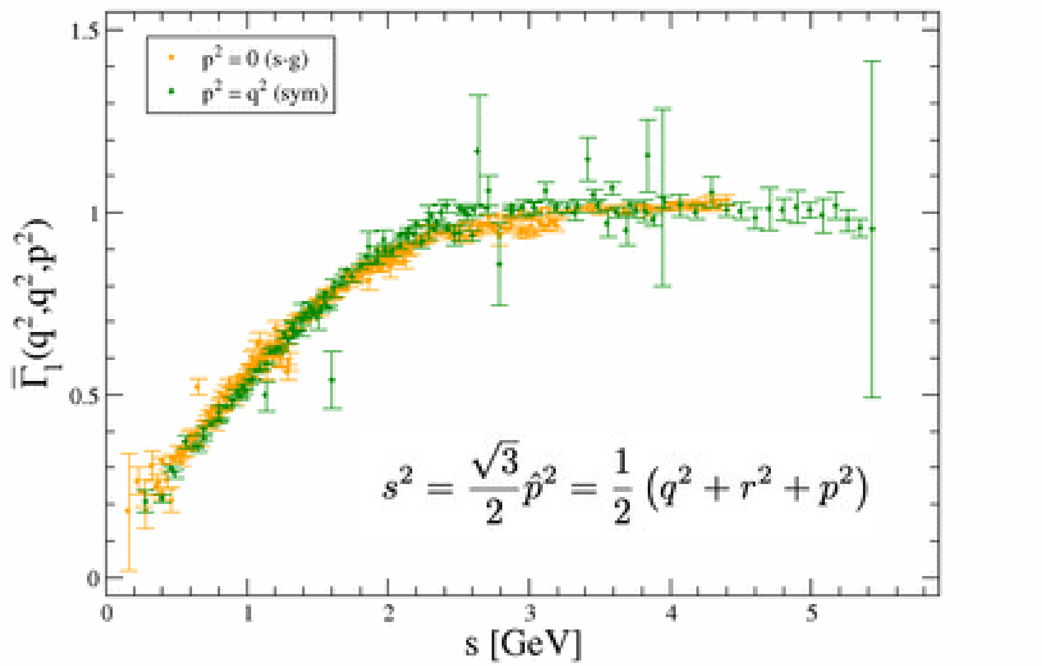

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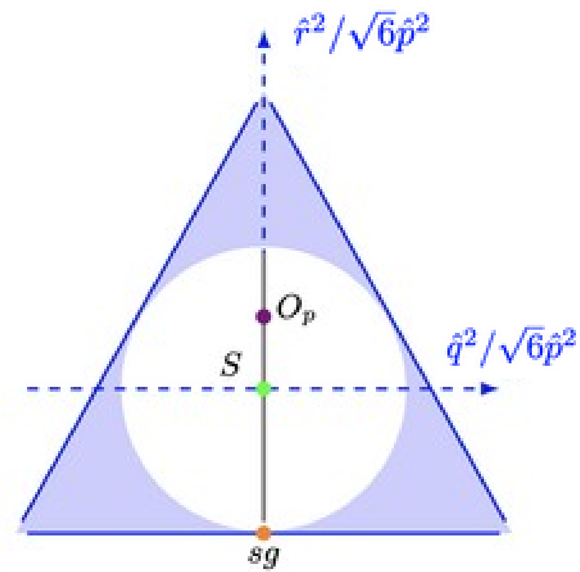

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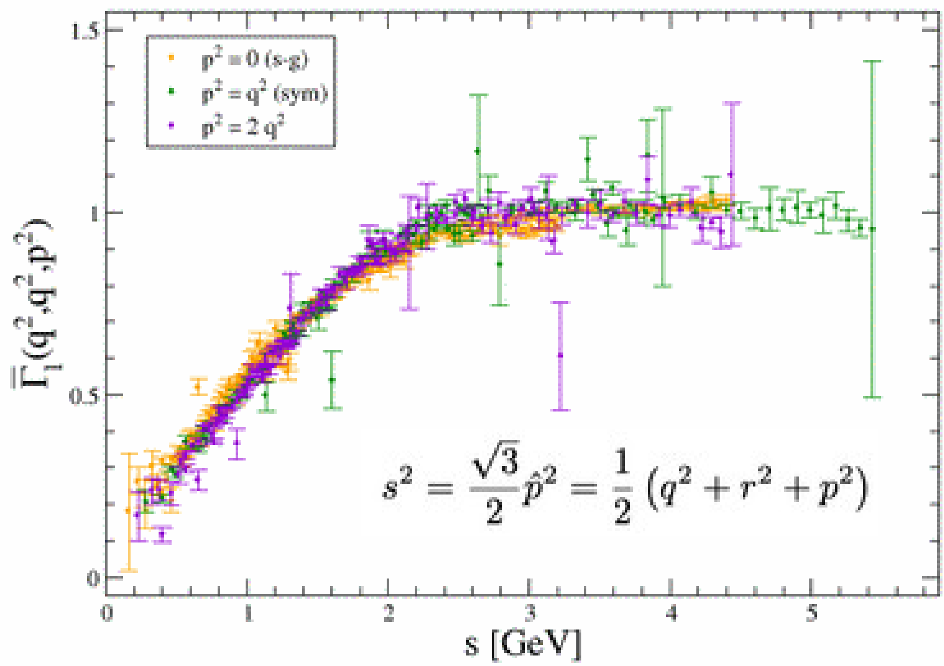

3-gluon vertex: Lattice data results

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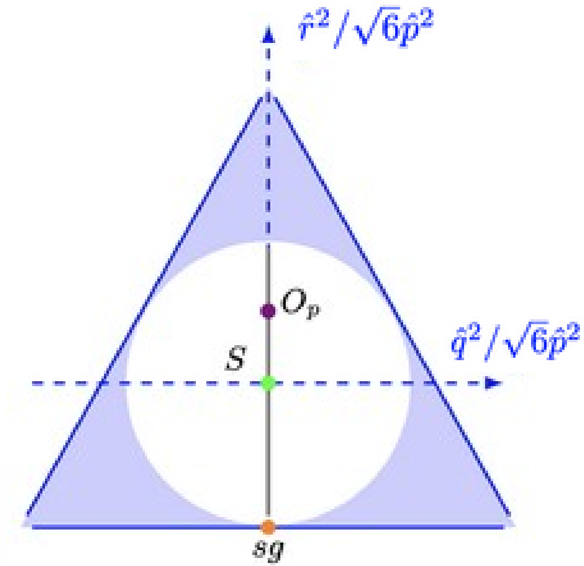

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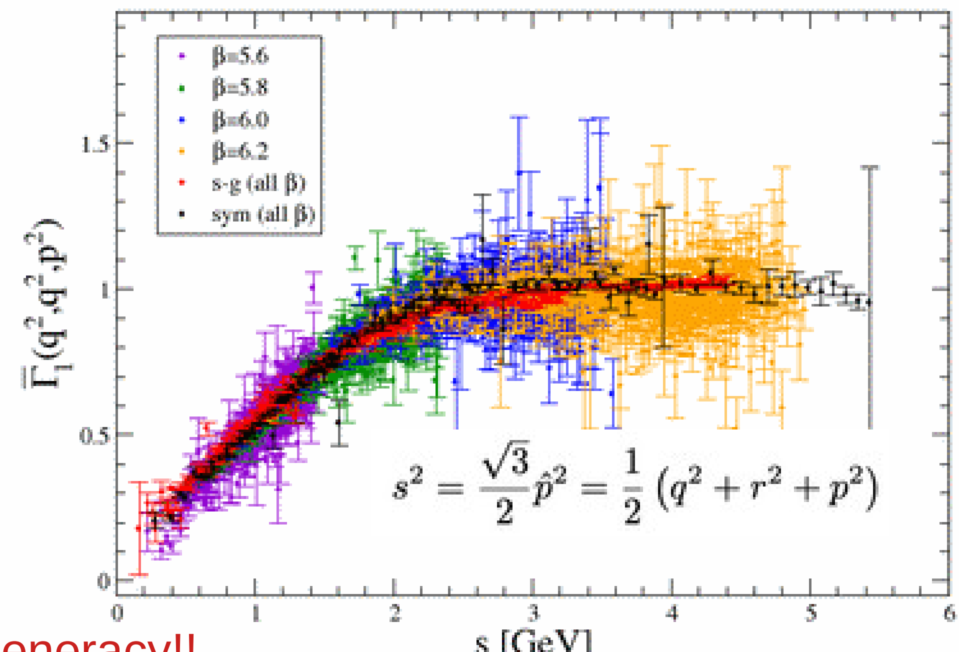

Planar degeneracy!!

3-gluon vertex: Lattice data results

Specializing for the bisectoral case:

$$\lambda_i^{\alpha\mu\nu}(q,r,p) = \lim_{r^2\to q^2} \widetilde{\lambda}_i^{\alpha\mu\nu}(q,r,p)$$

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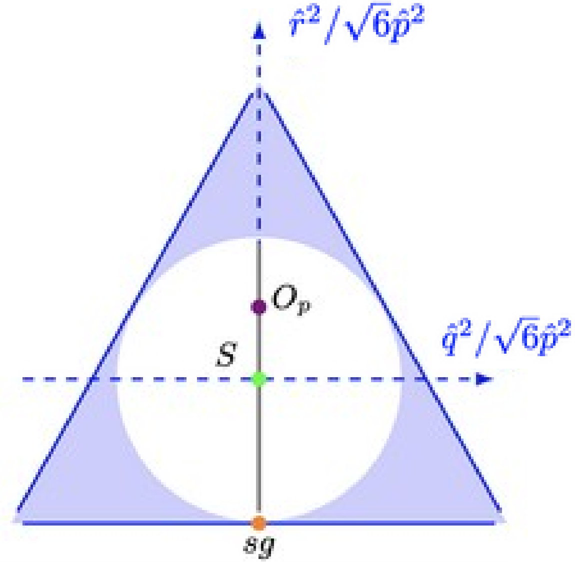
$$z = p^2 / \left[q^2 + \frac{p^2}{2}\right]$$

$$\overline{\Gamma}_i(q^2,q^2,p^2) = \widetilde{\Gamma}_i(q^2,q^2,p^2) + f_i(z)\,\widetilde{\Gamma}_4(q^2,q^2,p^2)$$



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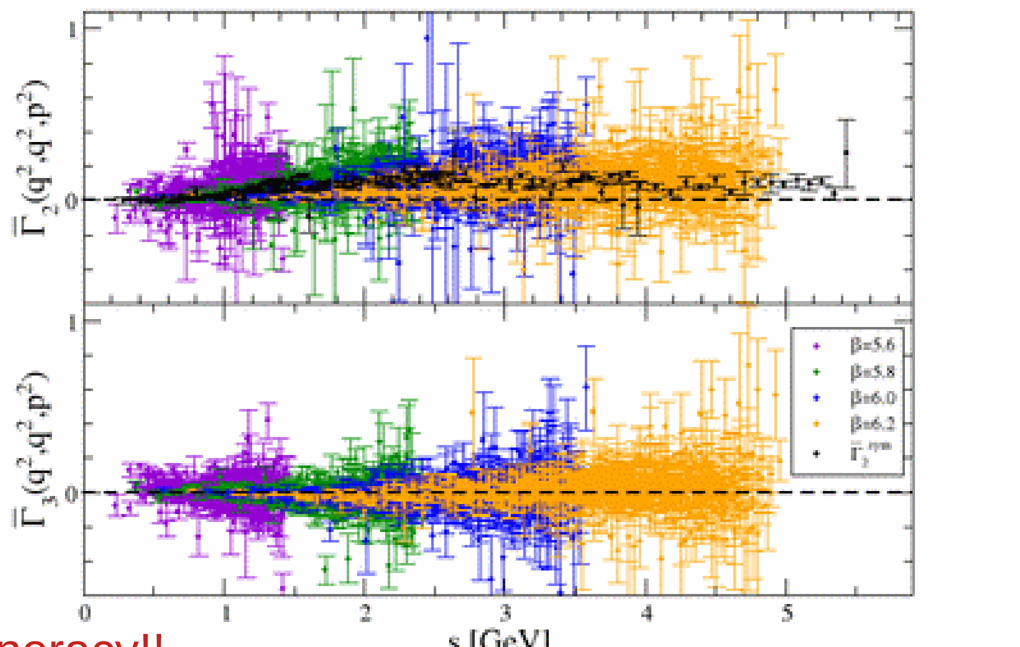

Planar degeneracy!!

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\overline{\Gamma}_{\alpha\mu\nu}(r,q,p) = \sum_{i=1}^{2} \overline{\Gamma}_i^{\rm sym}(q^2)\lambda_{i\alpha\mu\nu}(q,r,p) \quad \begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1(q^2,q^2,p^2) + \dfrac{1}{2}\overline{\Gamma}_3(q^2,q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,q^2,p^2) - \dfrac{3}{4}\overline{\Gamma}_3(q^2,q^2,p^2) \end{cases}$$

$$\lambda_{i\alpha\mu\nu}(q,r,p) = \lim_{p^2\to q^2} \overline{\lambda}_{i\alpha\mu\nu}(q,r,p)$$

i=1,2

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\overline{\Gamma}_{\alpha\mu\nu}(r,q,p) = \sum_{i=1}^{2} \overline{\Gamma}_i^{\rm sym}(q^2)\lambda_{i\alpha\mu\nu}(q,r,p) \quad \begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1(q^2,q^2,p^2) + \dfrac{1}{2}\overline{\Gamma}_3(q^2,q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,q^2,p^2) - \dfrac{3}{4}\overline{\Gamma}_3(q^2,q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1(q^2,q^2,p^2) + \frac{3}{2}\overline{\Gamma}_3(q^2,q^2,p^2)$$

$$\overline{\Gamma}_{\alpha\mu\nu}(r,q,p) = \overline{\Gamma}^{\rm sg}(q^2)\lambda_{1\alpha\mu\nu}(q,r,p)$$

$$\lambda_{i\alpha\mu\nu}(q,r,p) = \lim_{p^2\to q^2} \overline{\lambda}_{i\alpha\mu\nu}(q,r,p)$$

i=1,2

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\overline{\Gamma}_{\alpha\mu\nu}(r,q,p) = \sum_{i=1}^{2} \overline{\Gamma}_i^{\text{sym}}(q^2)\lambda_{i\alpha\mu\nu}(q,r,p) \left\{ \begin{array}{l} \overline{\Gamma}_1^{\text{sym}}(q^2) = \lim_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) - \overline{\Gamma}_3^*(q^2,p^2) \\ \overline{\Gamma}_2^{\text{sym}}(q^2) = \lim_{p^2\to q^2} \overline{\Gamma}_2^*(q^2,p^2) - \frac{3}{4}\overline{\Gamma}_3^*(q^2,p^2) \end{array} \right.$$

Specializing for the soft-gluon case:

$$\boxed{L_{\text{sg}}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

$$\overline{\Gamma}_{\alpha\mu\nu}(r,q,p) = \overline{\Gamma}^{\text{sg}}(q^2)\lambda_{1\alpha\mu\nu}(q,r,p)$$

$$\lambda_{i\alpha\mu\nu}(q,r,p) = \lim_{p^2\to q^2} \overline{\lambda}_{i\alpha\mu\nu}(q,r,p) = \lim_{p^2\to q^2} \overline{\lambda}^*_{i\alpha\mu\nu}(q,r,p)$$

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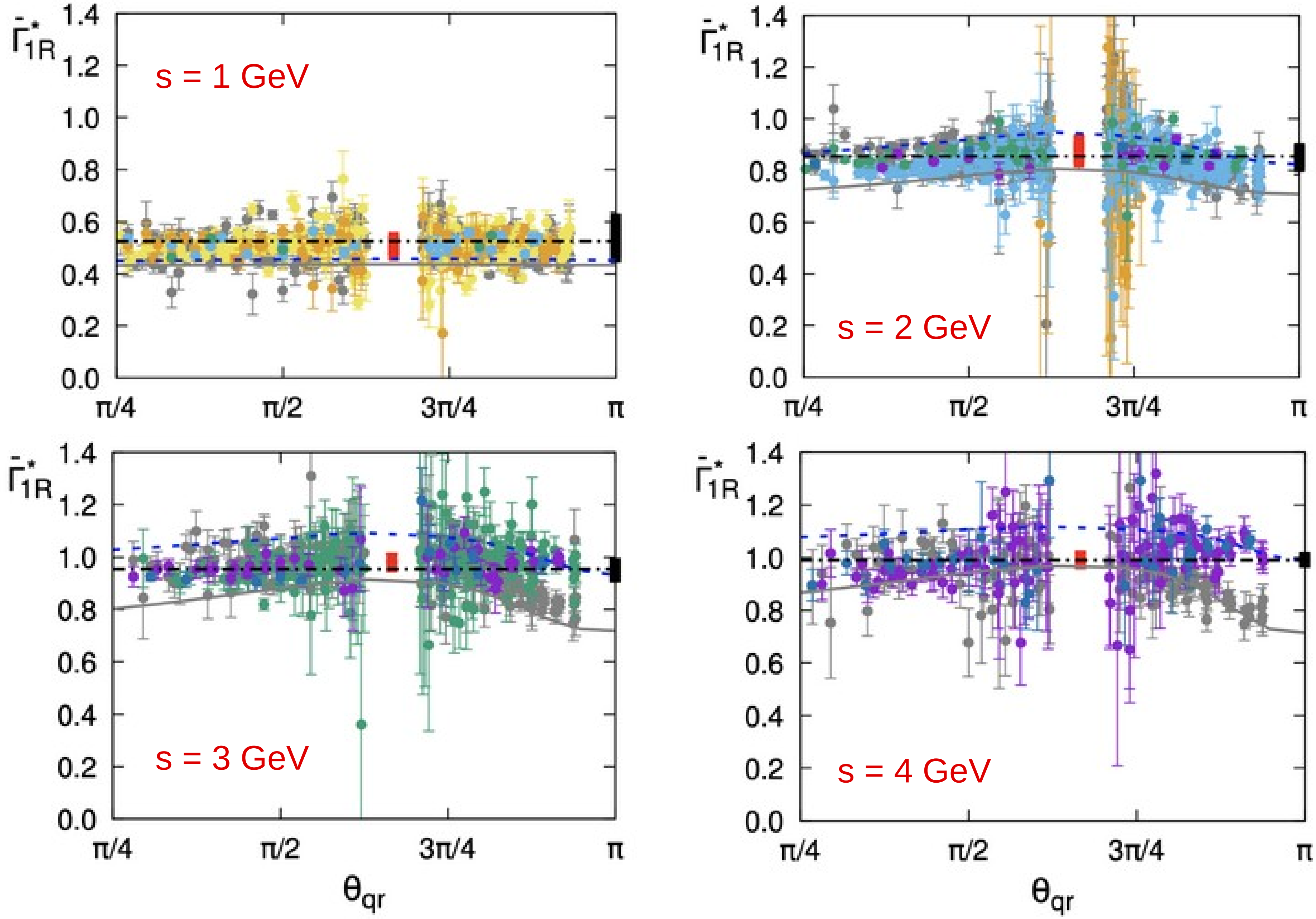
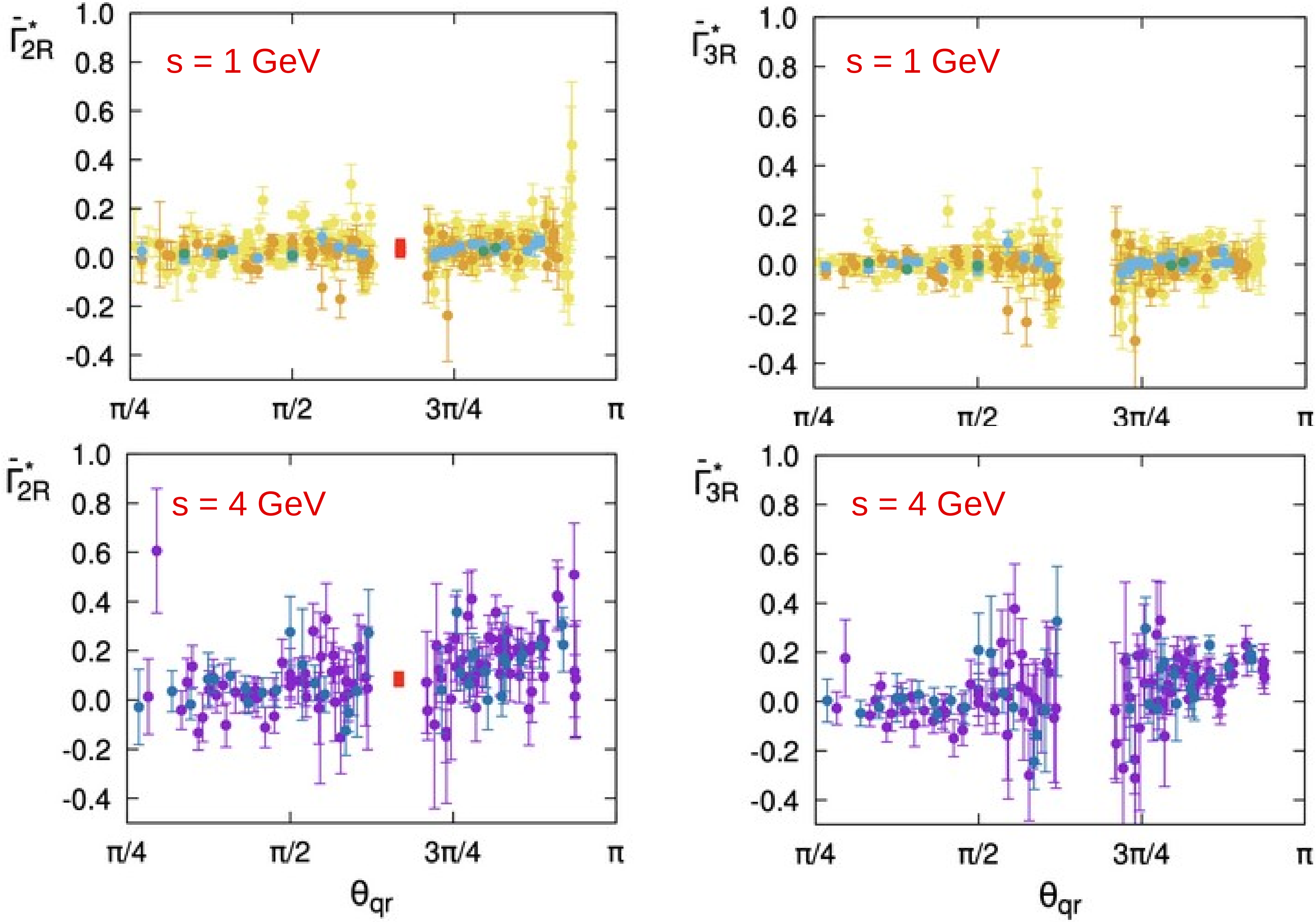
3-gluon vertex: planar degeneracy approximation



3-gluon vertex: planar degeneracy approximation



3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\text{sym}}(q^2) = \lim\limits_{p^2 \to q^2} \overline{\Gamma}_1^*(q^2, p^2) - \overline{\Gamma}_3^*(q^2, p^2) \\ \overline{\Gamma}_2^{\text{sym}}(q^2) = \lim\limits_{p^2 \to q^2} \overline{\Gamma}_2^*(q^2, p^2) - \frac{3}{4}\overline{\Gamma}_3^*(q^2, p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\text{sg}}(q^2)} = \lim_{p^2 \to 0} \overline{\Gamma}_1^*(q^2, p^2)$$

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\left\{\begin{aligned} \overline{\Gamma}_1^{\rm sym}(q^2) &= \lim_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) &= \lim_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{aligned}\right.$$

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$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

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Specializing for the soft-gluon case:

$$\boxed{L_{\text{sg}}(q^2)} = \lim_{p^2 \to 0} \overline{\Gamma}_1^{*}(q^2, p^2)$$

We have found that $\overline{\Gamma}_3^{*}$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^{*}(q^2, r^2, p^2) \approx \overline{\Gamma}_1^{*}(s^2, s^2, 0) \approx L_{\text{sg}}(s^2)$$

$$\overline{\Gamma}_2(q^2, r^2, p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3}, \frac{2s^2}{3}, \frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\text{sym}}\left(\frac{2s^2}{3}\right)$$

And then:

$$\overline{\Gamma}^{\alpha\mu\nu}(q, r, p) \approx L_{\text{sg}}\left(s^2\right) \widetilde{\lambda}_1^{\alpha\mu\nu}(q, r, p) + \overline{\Gamma}_2^{\text{sym}}\left(\frac{2s^2}{3}\right) \widetilde{\lambda}_2^{\alpha\mu\nu}(q, r, p)$$

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\text{sym}}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\text{sym}}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\text{sg}}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

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3-gluon vertex: planar degeneracy approximation

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$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q, r, p) \approx L_{\text{sg}}\left(s^2\right) \widetilde{\lambda}_1^{\alpha\mu\nu}(q, r, p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2) := \frac{1}{2}(q - r)^\nu \delta^{\alpha\mu} \overline{\Gamma}_{\alpha\mu\nu}(q, r, p)$$

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\text{sym}}(q^2) = \lim_{p^2 \to q^2} \overline{\Gamma}_1^{*}(q^2, p^2) \\ \overline{\Gamma}_2^{\text{sym}}(q^2) = \lim_{p^2 \to q^2} \overline{\Gamma}_2(q^2, p^2) \end{cases}$$

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$$\overline{\Gamma}_2(q^2, r^2, p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3}, \frac{2s^2}{3}, \frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\text{sym}}\left(\frac{2s^2}{3}\right)$$

And then:

$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q, r, p) \approx L_{\text{sg}}(s^2)\, \widetilde{\lambda}_1^{\alpha\mu\nu}(q, r, p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2, r^2, p^2) \approx \mathcal{I}_{\mathcal{W}}^{0}(q^2, r^2, p^2)\, L_{\text{sg}}(s^2)$$

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^*(q^2,r^2,p^2) \approx \overline{\Gamma}_1^*(s^2,s^2,0) \approx L_{\rm sg}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\rm sym}\left(\frac{2s^2}{3}\right)$$

And then:

$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\rm sg}\left(s^2\right) \widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\, L_{\rm sg}\left(s^2\right)$$

$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}\left(r^2-q^2-p^2\right)\left(q^2-r^2-p^2\right)\right]$$

3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^*(q^2,r^2,p^2) \approx \overline{\Gamma}_1^*(s^2,s^2,0) \approx L_{\rm sg}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\rm sym}\left(\frac{2s^2}{3}\right)$$

And then:

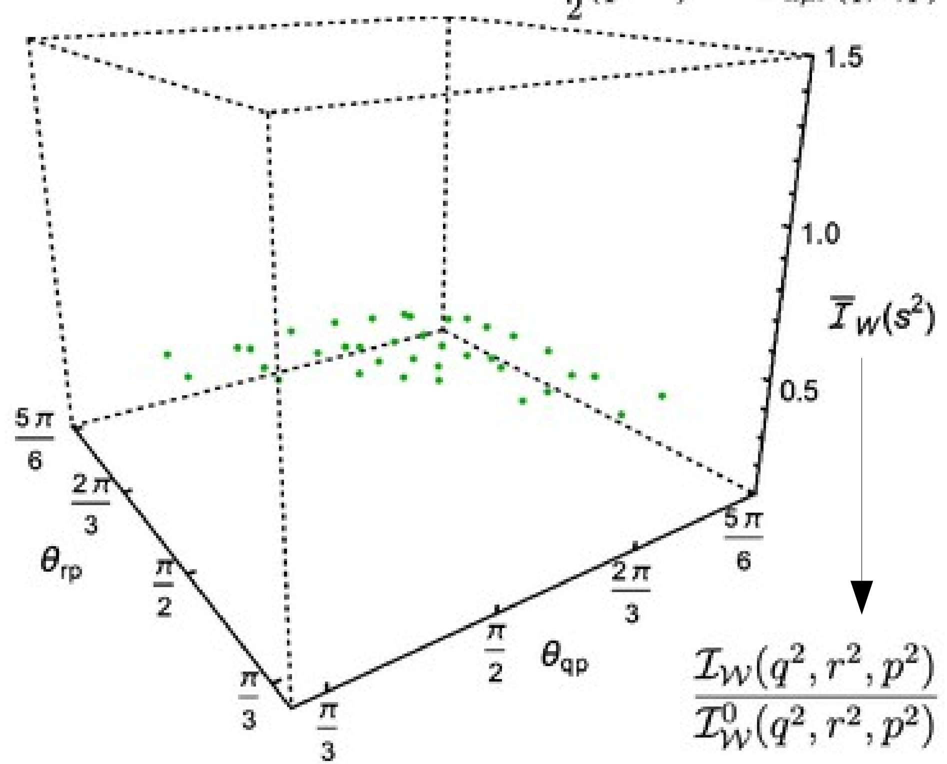
$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\rm sg}(s^2)\,\widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}_{\mathcal{W}}^0(q^2,r^2,p^2)\,L_{\rm sg}(s^2)$$

$$\mathcal{I}_{\mathcal{W}}^0(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2-(p^2-q^2-r^2)^2\right] \times \left[3q^2r^2-\frac{1}{4}(r^2-q^2-p^2)(q^2-r^2-p^2)\right]$$

Lattice direct calculation: $\frac{1}{2}(q-r)^\nu\delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^*(q^2,r^2,p^2) \approx \overline{\Gamma}_1^*(s^2,s^2,0) \approx L_{\rm sg}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\rm sym}\left(\frac{2s^2}{3}\right)$$

And then:

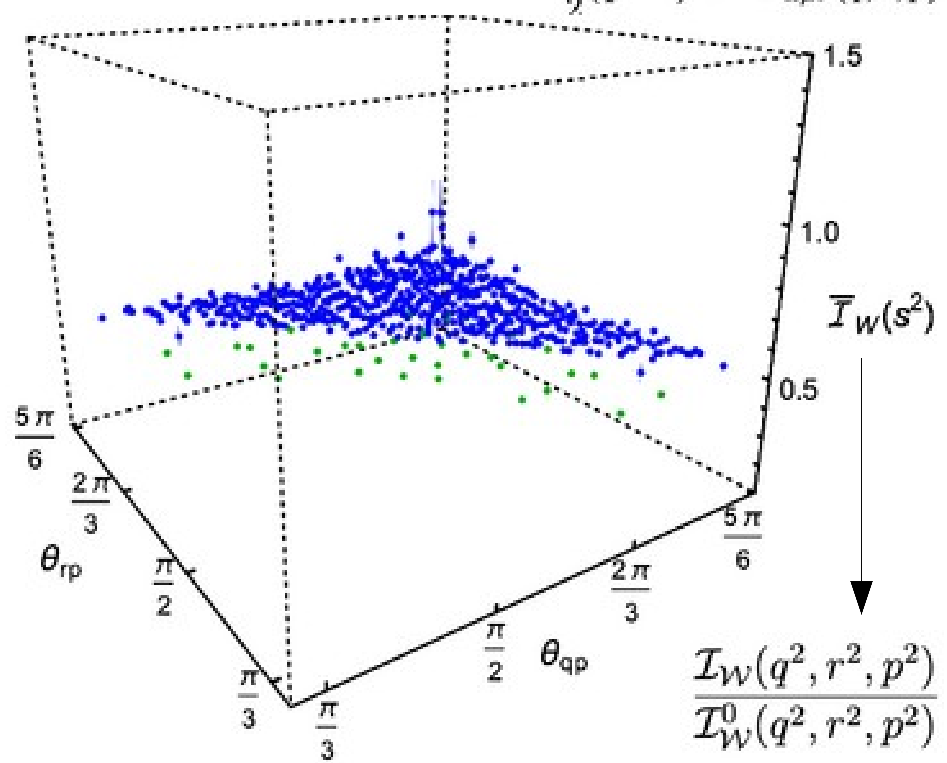
$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\rm sg}(s^2)\, \widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\, L_{\rm sg}(s^2)$$

$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}(r^2-q^2-p^2)(q^2-r^2-p^2)\right]$$

Lattice direct calculation: $\frac{1}{2}(q-r)^\nu \delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\text{sym}}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\text{sym}}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\text{sg}}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^*(q^2,r^2,p^2) \approx \overline{\Gamma}_1^*(s^2,s^2,0) \approx L_{\text{sg}}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\text{sym}}\left(\frac{2s^2}{3}\right)$$

And then:

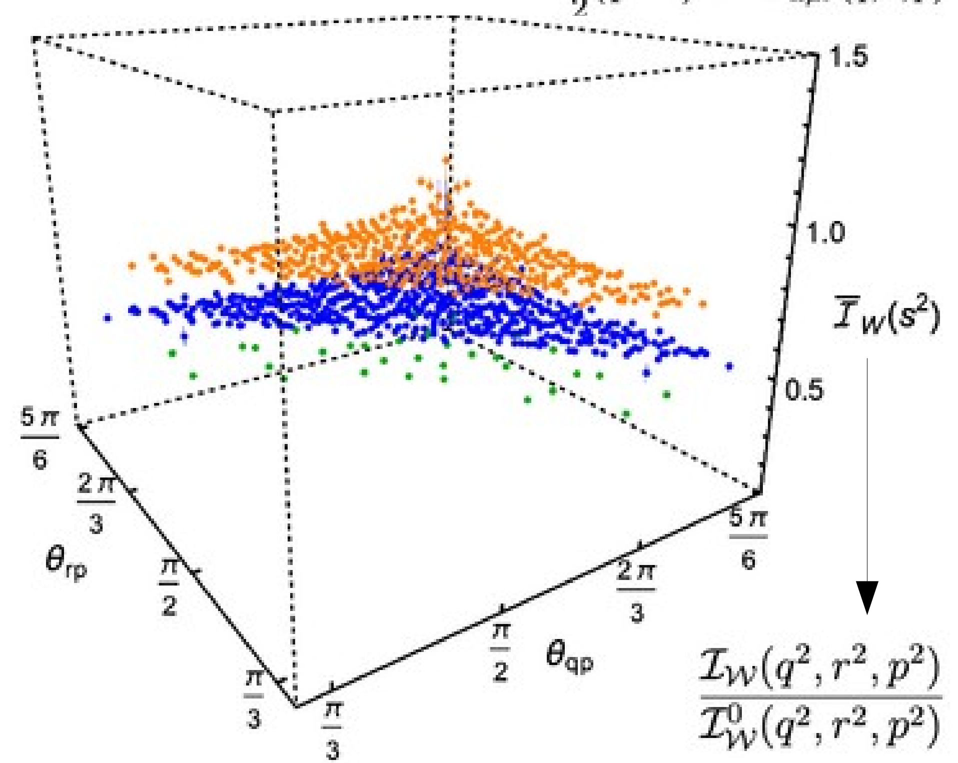
$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\text{sg}}(s^2)\,\widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)}$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\, L_{\text{sg}}(s^2)$$

$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}(r^2-q^2-p^2)(q^2-r^2-p^2)\right]$$

Lattice direct calculation: $\frac{1}{2}(q-r)^\nu \delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\left\{\begin{array}{l} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^{*}(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{array}\right.$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^{*}(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^{*}$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^{*}(q^2,r^2,p^2) \approx \overline{\Gamma}_1^{*}(s^2,s^2,0) \approx L_{\rm sg}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\rm sym}\left(\frac{2s^2}{3}\right)$$

And then:

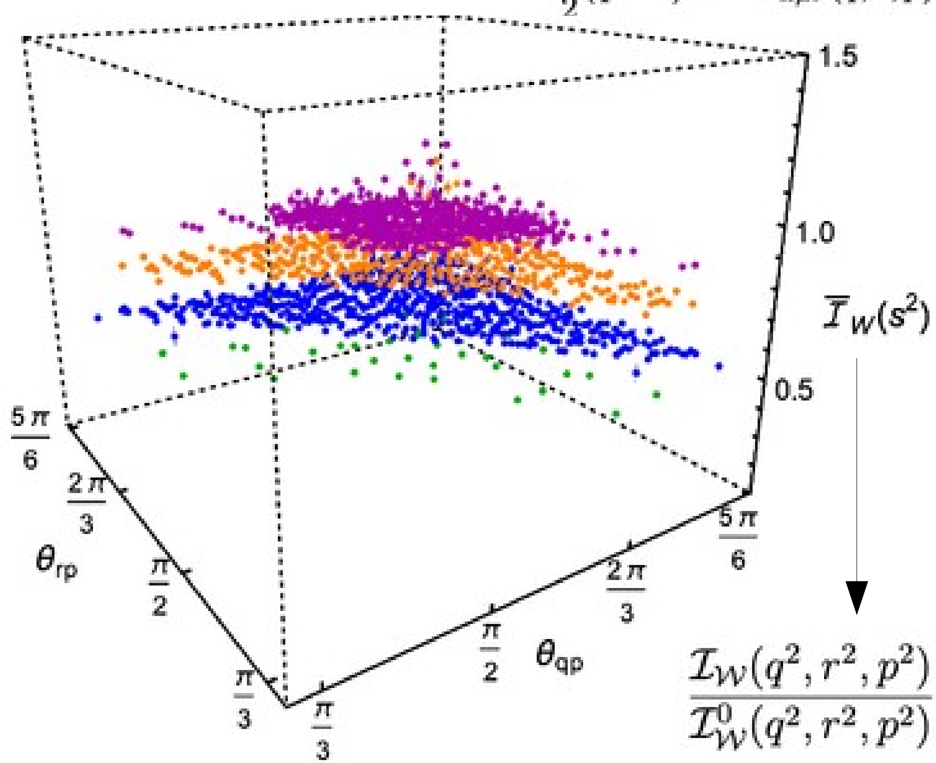
$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\rm sg}(s^2)\, \widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\, L_{\rm sg}(s^2)$$

$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}(r^2-q^2-p^2)(q^2-r^2-p^2)\right]$$

Lattice direct calculation: $\frac{1}{2}(q-r)^\nu\delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



3-gluon vertex: planar degeneracy approximation

Specializing for the symmetric case:

$$\begin{cases} \overline{\Gamma}_1^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_1^*(q^2,p^2) \\ \overline{\Gamma}_2^{\rm sym}(q^2) = \lim\limits_{p^2\to q^2} \overline{\Gamma}_2(q^2,p^2) \end{cases}$$

Specializing for the soft-gluon case:

$$\boxed{L_{\rm sg}(q^2)} = \lim_{p^2\to 0} \overline{\Gamma}_1^*(q^2,p^2)$$

We have found that $\overline{\Gamma}_3^*$ is compatible with zero except near the soft-gluon limit and at large momenta; preliminary exploration also indicates that $\widetilde{\Gamma}_4$ also is. Thus, the **planar degeneracy approximation** for the 3-gluon vertex implies:

$$\widetilde{\Gamma}_1^*(q^2,r^2,p^2) \approx \overline{\Gamma}_1^*(s^2,s^2,0) \approx L_{\rm sg}(s^2)$$

$$\overline{\Gamma}_2(q^2,r^2,p^2) \approx \widetilde{\Gamma}_2\left(\frac{2s^2}{3},\frac{2s^2}{3},\frac{2s^2}{3}\right) \approx \overline{\Gamma}_2^{\rm sym}\left(\frac{2s^2}{3}\right)$$

And then:

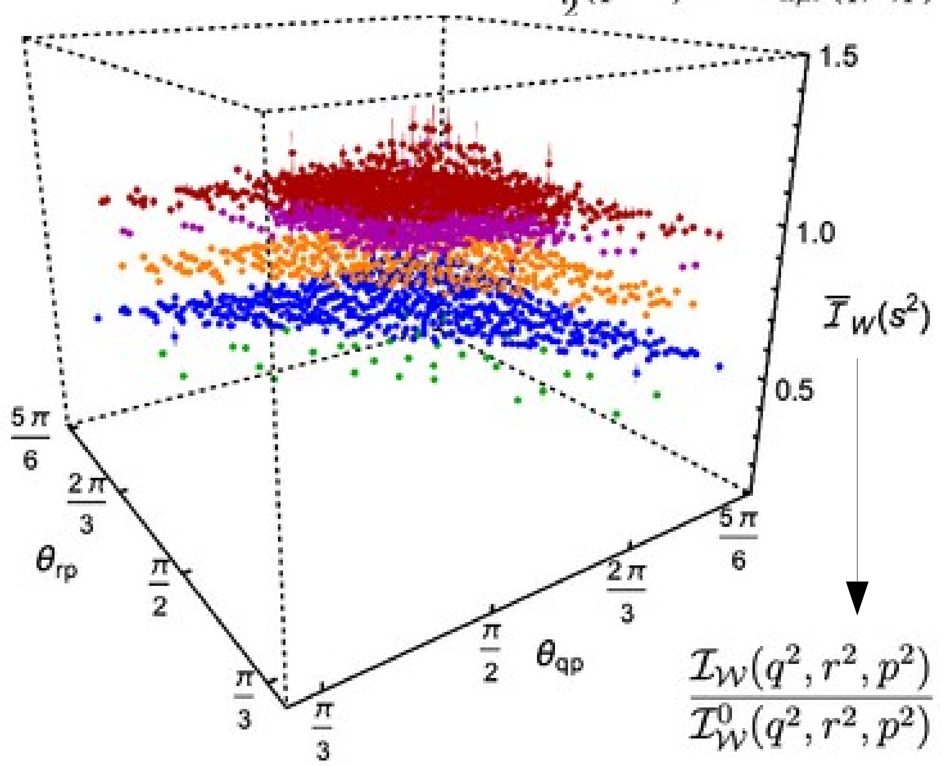
$$\boxed{\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{\rm sg}\left(s^2\right)\widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)}$$

Concerning the kernel for the displacement function:

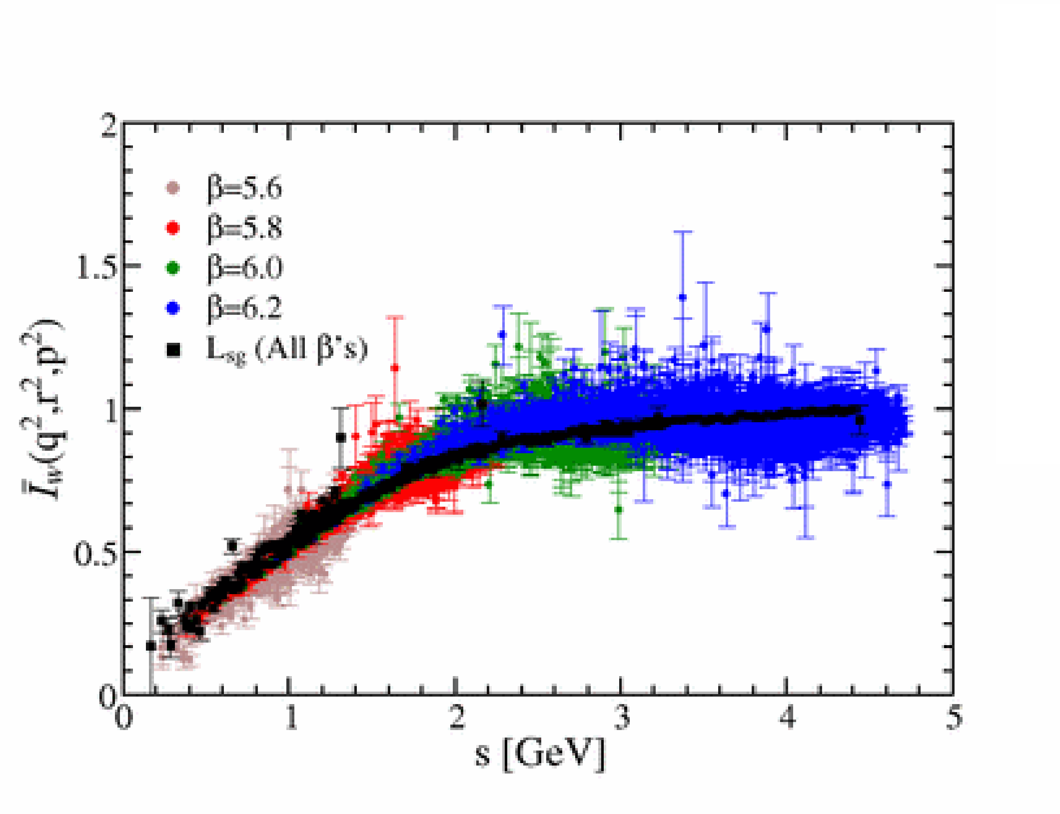
$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}_{\mathcal{W}}^0(q^2,r^2,p^2)\,L_{\rm sg}\left(s^2\right)$$

$$\mathcal{I}_{\mathcal{W}}^0(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}\left(r^2-q^2-p^2\right)\left(q^2-r^2-p^2\right)\right]$$

Lattice direct calculation: $\frac{1}{2}(q-r)^\nu \delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



3-gluon vertex: planar degeneracy approximation



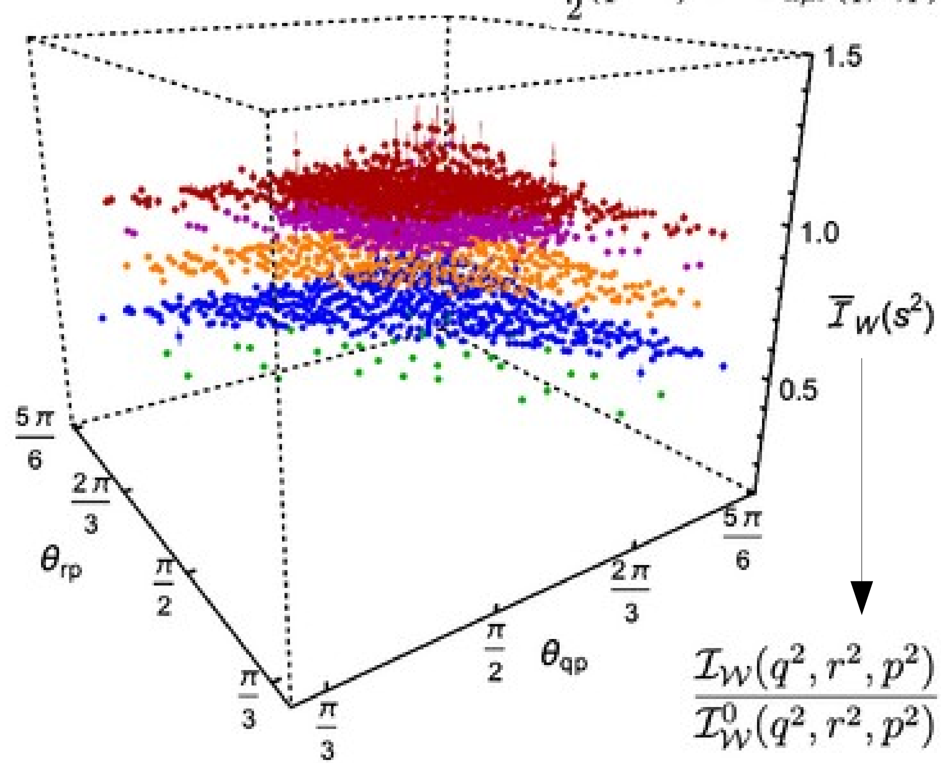
$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{sg}\left(s^2\right)\widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\,L_{sg}\left(s^2\right)$$

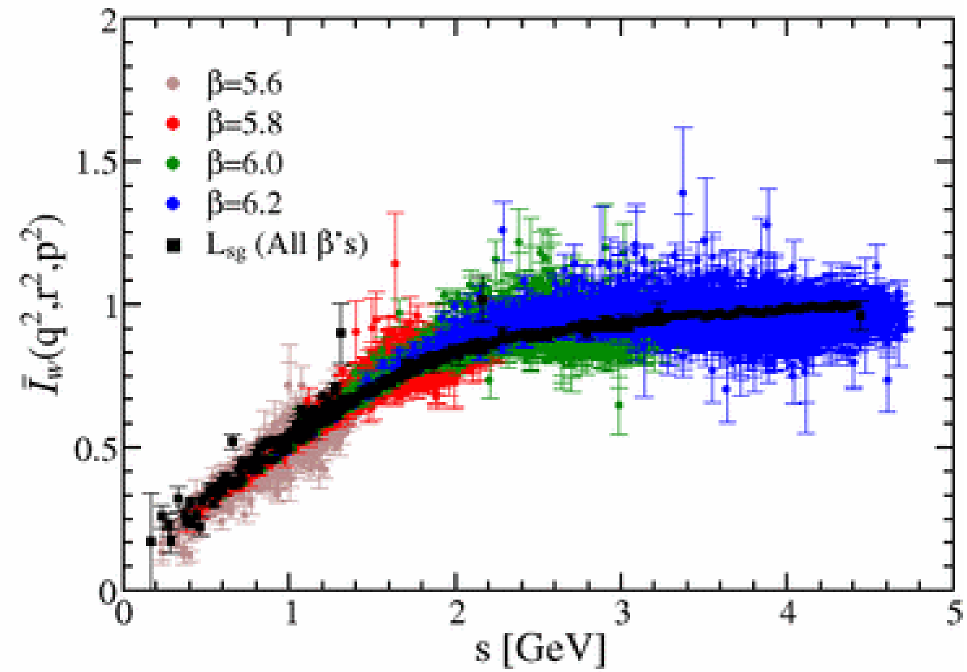
$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}\left(r^2-q^2-p^2\right)\left(q^2-r^2-p^2\right)\right]$$

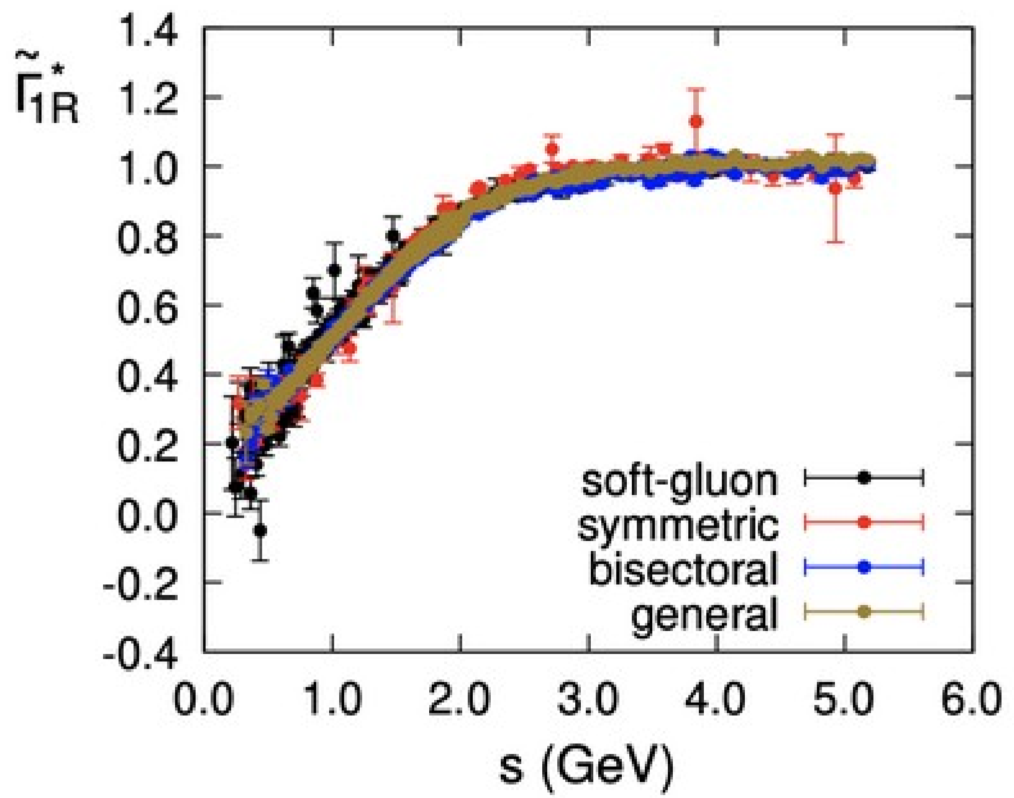
Lattice direct calculation: $\frac{1}{2}(q-r)^\nu\delta^{\alpha\mu}\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)$



$$\frac{\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2)}{\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)}$$

3-gluon vertex: planar degeneracy approximation





$$\overline{\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{sg}(s^2)\,\widetilde{\lambda}_1^{\alpha\mu\nu}(q,r,p)$$

Concerning the kernel for the displacement function:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) \approx \mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2)\,L_{sg}(s^2)$$

$$\mathcal{I}^0_{\mathcal{W}}(q^2,r^2,p^2) = \frac{1}{2p^2q^2r^2}\left[4q^2r^2 - (p^2-q^2-r^2)^2\right] \times \left[3q^2r^2 - \frac{1}{4}(r^2-q^2-p^2)(q^2-r^2-p^2)\right]$$

Averaging over all kinematic configurations sharing the same s^2, a smoothly behaving curve fully agreeing with L_{sg} is left, both strongly supporting **planar degeneracy** and greatly improving the statistics for the determination of this latter form factor.

3-gluon vertex: renormalization and coupling

Assuming multiplicative renormalizability

$$\Delta_R(q^2) = \lim_{a\to 0} Z_A^{-1}(a)\Delta(q^2;a)$$

$$\mathcal{G}_{R\alpha\mu\nu}(q,r,p) = \lim_{a\to 0} Z_A^{-3/2}(a)\mathcal{G}_{\alpha\mu\nu}(q,r,p;a)$$

Only one further renormalization constant needs to be introduced for all the form factors of the 1PI 3-gluon vertex

$$\widetilde{\Gamma}^*_{iR}(q^2,r^2,p^2) = \lim_{a\to 0} Z_3(a)\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a)$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)}$$

$$g_R = \lim_{a\to 0} Z_A^{3/2}(a) Z_3^{-1}(a)\, g(a)$$

3-gluon vertex: renormalization and coupling

Assuming multiplicative renormalizability and applying **MOM** prescription

$$\Delta_R(q^2) = \lim_{a\to 0} Z_A^{-1}(a)\Delta(q^2;a) \longrightarrow \boxed{Z_A(a^2) = \zeta^2\Delta(\zeta^2;a)}$$

$$\mathcal{G}_{R\alpha\mu\nu}(q,r,p) = \lim_{a\to 0} Z_A^{-3/2}(a)\mathcal{G}_{\alpha\mu\nu}(q,r,p;a)$$

Only one further renormalization constant needs to be introduced for all the form factors of the 1PI 3-gluon vertex

$$\widetilde{\Gamma}^*_{iR}(q^2,r^2,p^2) = \lim_{a\to 0} Z_3(a)\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a) \longrightarrow \boxed{\lim_{a\to 0} Z_3^{\mathrm{k}(\zeta)}(a)\, \widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)} = 1}$$

$$\mathrm{k}(\zeta) \equiv \{\zeta^2, \theta_{qp}, \theta_{rp}\}$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)}$$

$$g_R = \lim_{a\to 0} Z_A^{3/2}(a)Z_3^{-1}(a)\, g(a)$$

3-gluon vertex: renormalization and coupling

Assuming multiplicative renormalizability and applying **MOM** prescription

$$\Delta_R(q^2) = \lim_{a\to 0} Z_A^{-1}(a)\Delta(q^2;a) \longrightarrow Z_A(a^2) = \zeta^2\Delta(\zeta^2;a)$$

$$\mathcal{G}_{R\alpha\mu\nu}(q,r,p) = \lim_{a\to 0} Z_A^{-3/2}(a)\mathcal{G}_{\alpha\mu\nu}(q,r,p;a)$$

Only one further renormalization constant needs to be introduced for all the form factors of the 1PI 3-gluon vertex

$$\widetilde{\Gamma}^*_{iR}(q^2,r^2,p^2) = \lim_{a\to 0} Z_3(a)\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a) \longrightarrow \lim_{a\to 0} Z_3^{\mathrm{k}(\zeta)}(a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)} = 1$$

$$\mathrm{k}(\zeta) \equiv \{\zeta^2,\theta_{qp},\theta_{rp}\}$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)}$$

$$g_R = \lim_{a\to 0} Z_A^{3/2}(a) Z_3^{-1}(a)\, g(a)$$

$$\left\{\begin{aligned} \widetilde{\Gamma}^*_{iR\mathrm{k}(\zeta)}(q^2,r^2,p^2) &= \lim_{a\to 0} \frac{\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a)}{\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}} \\ g_{R\mathrm{k}}(\zeta^2) &= \lim_{a\to 0} \zeta^3\Delta^{3/2}(\zeta^2;a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}\, g(a) \end{aligned}\right.$$

3-gluon vertex: renormalization and coupling

Assuming multiplicative renormalizability and applying **MOM** prescription

$$\Delta_R(q^2) = \lim_{a\to 0} Z_A^{-1}(a)\Delta(q^2;a) \longrightarrow Z_A(a^2) = \zeta^2\Delta(\zeta^2;a)$$

$$\mathcal{G}_{R\alpha\mu\nu}(q,r,p) = \lim_{a\to 0} Z_A^{-3/2}(a)\mathcal{G}_{\alpha\mu\nu}(q,r,p;a)$$

Only one further renormalization constant needs to be introduced for all the form factors of the 1PI 3-gluon vertex

$$\widetilde{\Gamma}^*_{iR}(q^2,r^2,p^2) = \lim_{a\to 0} Z_3(a)\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a) \longrightarrow \lim_{a\to 0} Z_3^{\mathrm{k}(\zeta)}(a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)} = 1$$

$$\mathrm{k}(\zeta) \equiv \{\zeta^2,\theta_{qp},\theta_{rp}\}$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)}$$

$$g_R = \lim_{a\to 0} Z_A^{3/2}(a)Z_3^{-1}(a)\,g(a)$$

$$\left\{\begin{aligned} &\widetilde{\Gamma}^*_{iR\mathrm{k}(\zeta)}(q^2,r^2,p^2) = \lim_{a\to 0}\frac{\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a)}{\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}} \\ &g_{R\mathrm{k}}(\zeta^2) = \lim_{a\to 0}\zeta^3\Delta^{3/2}(\zeta^2;a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}\,g(a)\end{aligned}\right.$$

$$g_{R\mathrm{k}}(\zeta^2) = \frac{\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}}{\overline{\Gamma}^{\mathrm{sg}}(\zeta^2;a)}\,g_{R\mathrm{s}}(\zeta^2)$$

3-gluon vertex: renormalization and coupling

Assuming multiplicative renormalizability and applying **MOM** prescription

$$\Delta_R(q^2) = \lim_{a\to 0} Z_A^{-1}(a)\Delta(q^2;a) \quad\longrightarrow\quad Z_A(a^2) = \zeta^2\Delta(\zeta^2;a)$$

$$\mathcal{G}_{R\alpha\mu\nu}(q,r,p) = \lim_{a\to 0} Z_A^{-3/2}(a)\mathcal{G}_{\alpha\mu\nu}(q,r,p;a)$$

Only one further renormalization constant needs to be introduced for all the form factors of the 1PI 3-gluon vertex

$$\widetilde{\Gamma}^*_{iR}(q^2,r^2,p^2) = \lim_{a\to 0} Z_3(a)\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a) \quad\longrightarrow\quad \lim_{a\to 0} Z_3^{\mathrm{k}(\zeta)}(a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)} = 1$$

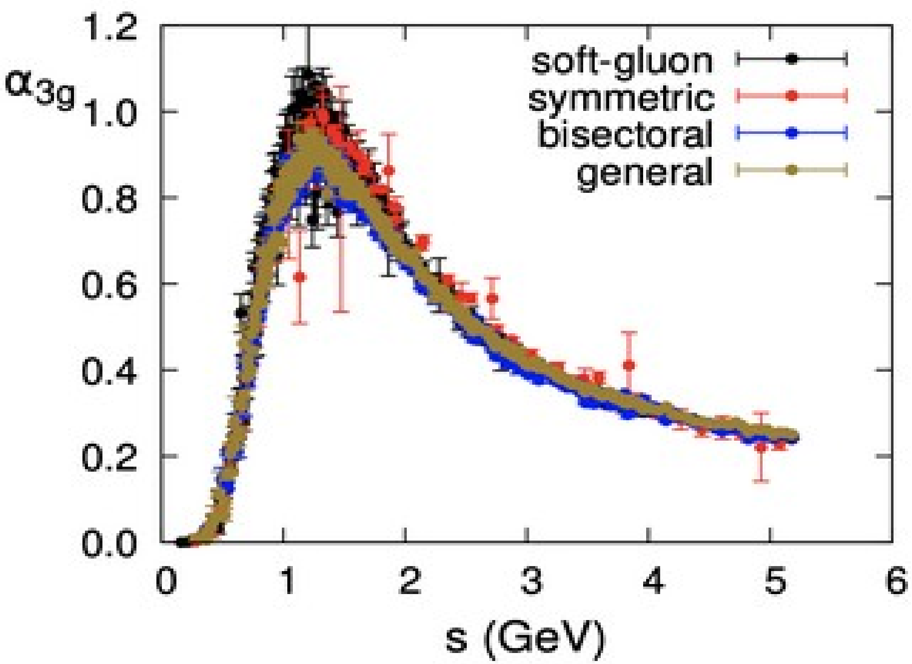
$$\mathrm{k}(\zeta) \equiv \{\zeta^2,\theta_{qp},\theta_{rp}\}$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\overline{\Gamma}_{\alpha\mu\nu}(q,r,p)\Delta(q^2)\Delta(r^2)\Delta(p^2)}$$

$$g_R = \lim_{a\to 0} Z_A^{3/2}(a)Z_3^{-1}(a)\,g(a)$$

$$\left\{\begin{aligned} \widetilde{\Gamma}^*_{iR\mathrm{k}(\zeta)}(q^2,r^2,p^2) &= \lim_{a\to 0}\frac{\widetilde{\Gamma}^*_i(q^2,r^2,p^2;a)}{\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}} \\ g_{R\mathrm{k}}(\zeta^2) &= \lim_{a\to 0}\zeta^3\Delta^{3/2}(\zeta^2;a)\,\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}\,g(a) \end{aligned}\right.$$

$$g_{R\mathrm{k}}(\zeta^2) = \frac{\widetilde{\Gamma}^*_1(q^2,r^2,p^2;a)\Big|_{\mathrm{k}(\zeta)}}{\overline{\Gamma}^{\mathrm{sg}}(\zeta^2;a)}\,g_{R\mathrm{s}}(\zeta^2)$$



Summary

- The **3-gluon vertex**, triggered by the non-perturbative nature of QCD, contains a key ingredient for the activation of **Schwinger mechanism,** responsible for the gluon mass generation. Such an ingredient is made crucially manifest by analysing the **STId** involving the 3-gluon vertex, through the so-called **displacement function**.

- To perform this analysis, the required piece can be directly accessed from lattice QCD calculations: the **transversely projected 3-gluon vertex.**

- We have expanded the **trasversely projected 3-gluon vertex** by using a basis for which any of its elements satisfies the **Bose symmetry**, thus obtaining form factors that can only depend on **Bose-symmetric** combinations of momenta. Such form factors, particularly the one behaving as the tree-level one, are seen to depend basically on $s^2 = \frac{1}{2}\left(q^2 + r^2 + p^2\right)$ and nothing else. We called this property **planar degeneracy**.

- Owing to **planar degeneracy**, the transverselly projected 3-gluon vertex can be well and easily approximated, and applied to deliver a compact expression of the kernel involved in the computation of the **displacement function.** A direct lattice calculation supportss the approximation.

- To the extent that **planar degeneracy** works as a reliable approximation, a unique definition of the **3-gluon QCD coupling** can be made, irrespectively of any particular kinematic configuration.