Lattice gluon propagator in the center-symmetric Landau gauge

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Outline

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QCD phase diagram

- study of the phase diagram of QCD relevant e.g. for heavy ion experiments
- QCD has phase transition where quarks and gluons become deconfined for sufficiently high T
- Polyakov loop
  - order parameter for the confinement-deconfinement phase transition
  - $L = \langle L(\vec{x}) \rangle \;\propto\; e^{-F_q/T}$
  - Definition on the lattice:

$$L(\vec{x}) = \mathrm{Tr} \prod_{t=0}^{N_t-1} \mathcal{U}_4(\vec{x}, t)$$

  - $T < T_c : L = 0$ (center symmetry)
  - $T > T_c : L \neq 0$ (spontaneous breaking of center symmetry)

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Center symmetry

- Wilson gauge action is invariant under a center transformation
- temporal links on a hyperplane $x_4 = const$ multiplied by

$$z \in Z_3 = \{e^{-i2\pi/3}, 1, e^{i2\pi/3}\}$$

- Polyakov loop $L(\vec{x}) \to zL(\vec{x})$
- $T < T_c$
  - local P_L phase equally distributed among the three sectors

$$L = \langle L(\vec{x}) \rangle \approx 0$$

- $T > T_c$
  - Z_3 sectors not equally populated: $L \neq 0$

G. Endrődi, C. Gattringer, H.-P. Schadler, arXiv:1401.7228
C. Gattringer, A. Schmidt, JHEP **01**, 051 (2011)
C. Gattringer, Phys. Lett. **B 690**, 179 (2010)

F. M. Stokes, W. Kamleh, D. B. Leinweber, arXiv:1312.0991

Outline

Center-symmetric Landau gauge in the continuum

- Center-symmetric Landau gauge is defined by

$$D_\mu[\bar{A}_c](A_\mu - \bar{A}_{c,\mu}) = 0$$

- center symmetric background configuration:

$$\bar{A}_{c,\mu} = \frac{T}{g}\bar{r}_j t_j \delta_{\mu 0}$$

  - for $SU(2)$ we consider $j = 3$, with $\bar{r} = \pi$ and $t_3 = \sigma_3/2$
  - for $SU(3)$ we have $j = 3, 8$, with $\bar{r} = 4\pi/3, 0$ and $t_j = \lambda_j/2$

- background covariant derivative:

$$D_\mu[\bar{A}] = \partial_\mu - ig\left[\bar{A}_{c,\mu}, \ldots\right]$$

- final expression:

$$D_\mu[\bar{A}_c](A_\mu) = \partial_\mu A_\mu - ig\left[\bar{A}_0, A_0\right]$$

Gauge transformations and center transformations — on the lattice

- Periodic gauge transformations

$$g_0(n + L_\nu \hat{\nu}) = g_0(n)$$

$$U_\mu(n) \to g_0(n) U_\mu(n) g_0^\dagger(n + \hat{\mu}) \equiv U_\mu^{g_0}(n)$$

- center tranformations: periodic in the time direction but only modulo an element of the center of SU(3)

$$g(n + L_4 \hat{4}) = e^{\pm i 2\pi/3} g(n)$$

  - Wilson action invariant
  - but Polyakov loop changes:

$$P_L \to e^{\mp i \frac{2\pi}{3}} P_L$$

Lattice formulation

- Gauge fixing functional

$$F = \sum_{x,\mu} \text{Re Tr}\left[g_c^\dagger(\mu) g_0(x) U_\mu(x) g_0^\dagger(x+\hat{\mu}) \right]$$

where $g_c^\dagger(\mu) = g_c^\dagger(x+\mu) g_c(x)$

- $g_c^\dagger(\mu)$ can be written as

$$g_c^\dagger(\mu) = e^{iaT\bar{r}_j t_j \delta_{\mu 3}} = e^{iag\bar{A}_{c,\mu}\delta_{\mu 3}}$$

- $aT = 1/L_t$, where L_t is the number of points in the temporal direction.

Lattice gauge fixing

- very similar to Landau gauge fixing
- infinitesimal gauge transformations $g(x) = 1 + i\omega(x)$
- first variation of the gauge fixing functional

$$\delta F = \frac{1}{2} \sum_x \mathrm{Tr}\left[i\omega(x) \Delta^\dagger(x) \right]$$

where

$$\Delta^\dagger(x) = \sum_\mu \left[U_\mu(x) g_c^\dagger(\mu) - g_c^\dagger(\mu) U_\mu(x - \hat{\mu}) - h.c. \right]$$

- $\delta F > 0$ for $i\omega(x) = \alpha\Delta(x)$, where $\alpha > 0$
- we use $g(x) = \exp\left(\alpha\Delta(x)/2\right)$ to approach a maximum of the gauge functional.
- also suitable for FFT acceleration

Continuum limit of $\Delta^\dagger(x)$

$$
\begin{aligned}
U_\mu(x) &= 1 + iagA_\mu(x+\hat{\mu}/2) - \frac{1}{2}a^2g^2\left(A_\mu(x+\hat{\mu}/2)\right)^2 \\
&\quad -\frac{i}{6}a^3g^3\left(A_\mu(x+\hat{\mu}/2)\right)^3 + \mathcal{O}(a^4) \\
U_\mu(x-\hat{\mu}) &= 1 + iagA_\mu(x-\hat{\mu}/2) - \frac{1}{2}a^2g^2\left(A_\mu(x-\hat{\mu}/2)\right)^2 \\
&\quad -\frac{i}{6}a^3g^3\left(A_\mu(x-\hat{\mu}/2)\right)^3 + \mathcal{O}(a^4) \\
A_\mu(x+\hat{\mu}/2) &= A_\mu(x) + \frac{a}{2}\partial_\mu A_\mu(x) + \frac{a^2}{8}\partial_\mu^2 A_\mu(x) + \mathcal{O}(a^3) \\
A_\mu(x-\hat{\mu}/2) &= A_\mu(x) - \frac{a}{2}\partial_\mu A_\mu(x) + \frac{a^2}{8}\partial_\mu^2 A_\mu(x) + \mathcal{O}(a^3) \\
g_c(\mu=3) &= 1 + iag\bar{A}_{c,3} - \frac{1}{2}a^2g^2\bar{A}_{c,3}^2 - \frac{i}{6}a^3g^3\bar{A}_{c,3}^3 + \mathcal{O}(a^4)
\end{aligned}
$$

Continuum limit of $\Delta^\dagger(x)$

- collecting the terms up to a^3

$$\Delta^\dagger(x) = 2ia^2 g \left(\partial_\mu A_\mu(x) - ig[\bar{A}_{c,3}, A_3]\right) + \mathcal{O}(a^4).$$

- at the end of the gauge fixing process, we will have

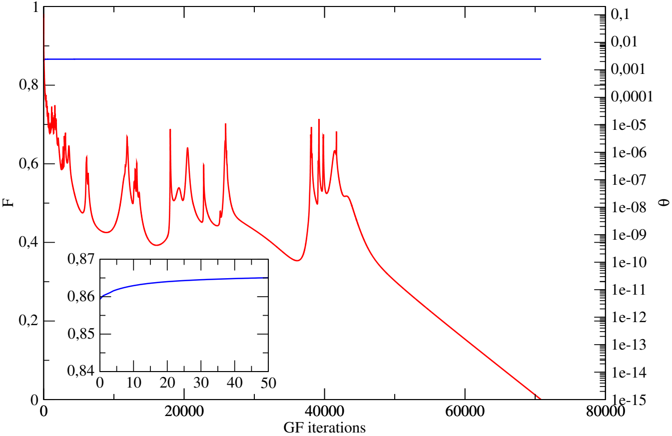
$$\partial_\mu A_\mu(x) - ig[\bar{A}_{c,3}, A_3] = 0 + \mathcal{O}(a^2).$$

- $\Delta(x) = -\Delta^\dagger(x)$ has the correct continuum limit
- no linear corrections in the lattice spacing a
- gauge fixing process will be monitored by

$$\theta = \frac{1}{N_c V} \text{Re Tr} \left[\Delta(x)\Delta^\dagger(x)\right]$$

- in the Landau gauge we usually demand $\theta < 10^{-15}$

Typical gauge fixing run



Center invariance

- F is invariant under the particular center transformation

$$g(n) = e^{i\pi\frac{\lambda_4}{2}}\, e^{i\pi\frac{\lambda_1}{2}}\, e^{-i\frac{n_4}{L_4}\pi\left(\lambda_3+\frac{\lambda_8}{\sqrt{3}}\right)}$$

- it is periodic modulo a center element

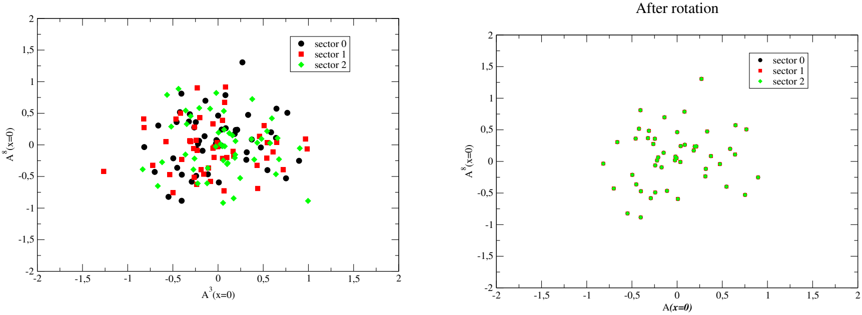
$$g(n+L_4\hat{4}) = e^{i\frac{2\pi}{3}}\, g(n)$$

- F is invariant, and U^g still maximizes F
- note that the Polyakov loop changes: $P_L \to e^{\mp i\frac{2\pi}{3}} P_L$

Outline

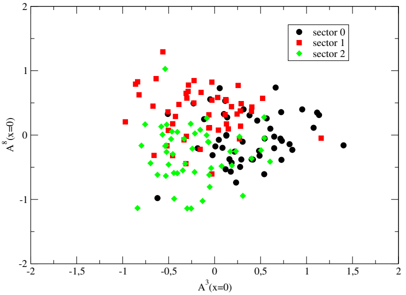
Plotting $(A_4^3(0), A_4^8(0))$ — below T_c

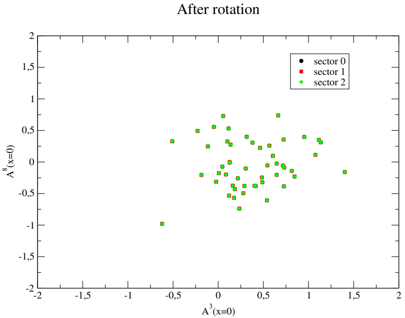
$64^3 \times 8$, T=243 MeV



Plotting $(A_4^3(0), A_4^8(0))$ — above T_c

$64^3 \times 6$, T=324 MeV





Predictions for the symmetric phase

- in the continuum: $\beta\langle gA_4^3(x)\rangle = \frac{4\pi}{3}$ that becomes $\langle agA_4^3(x)\rangle = \frac{4\pi}{3L_t}$

van Egmond, Reinosa, Phys.Rev.D 109 (2024) 3, 036002

van Egmond, Reinosa, Phys.Rev.D 106 (2022) 7, 074005

- on the lattice: $\langle agA_4^3(x)\rangle = -2\sin\left(\frac{2\pi}{3L_t}\right)$
- this can also be studied through the link average:

$$\frac{\langle U_4(x)\rangle}{\left(\det\langle U_4(x)\rangle\right)^{1/3}} = e^{-\frac{i}{L_4}\frac{4\pi}{3}\frac{\lambda^3}{2}}$$

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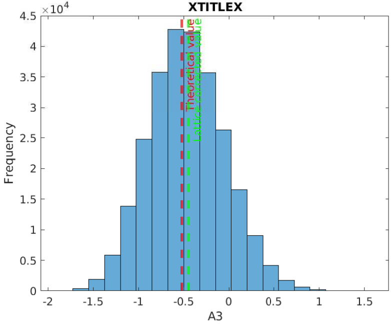
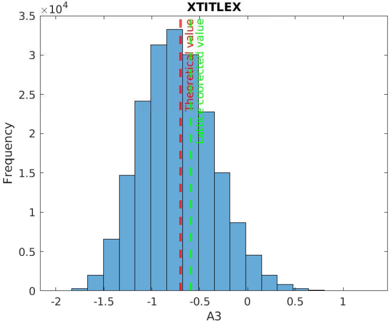
Histograms of $A_4^3(x)$ for one lattice configuration

$32^3 \times 8$, T=243 MeV

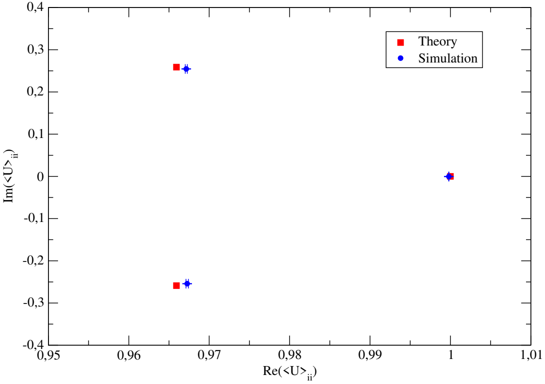


$32^3 \times 6$, T=324 MeV



Link average - below T_c

- $64^3 \times 8$, $\beta = 6.0$, T=243 MeV
- non-diagonal elements are zero within errors.



Link average - above T_c

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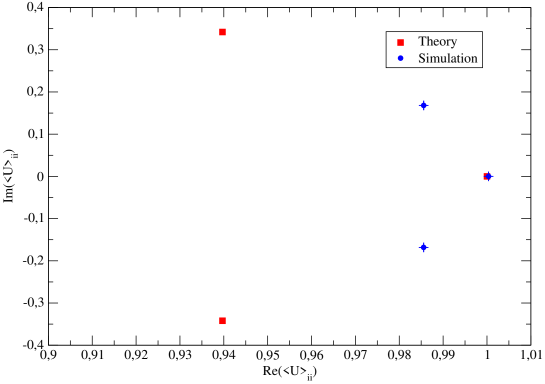
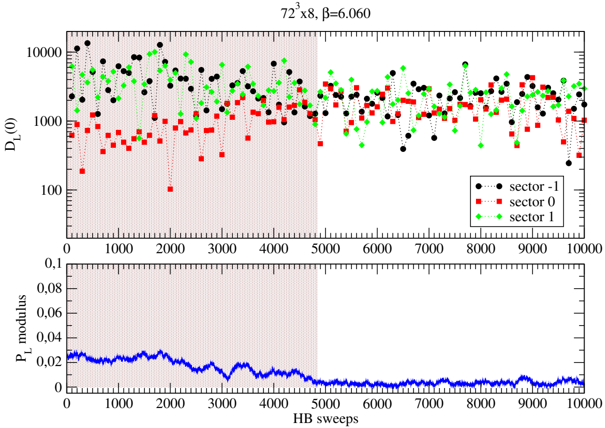
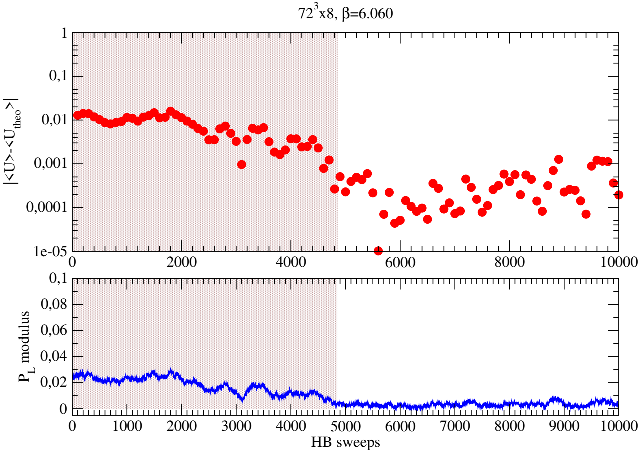


Simulating near T_c — standard Landau gauge



PJS, O. Oliveira, PRD **93** (2016) 114509

Simulating near T_c — center-symmetric Landau gauge



Outline

Gluon propagator

- for color indices 3 and 8, the propagator decomposition becomes the same as in the standard Landau gauge

$$D^{ab}_{\mu\nu}(\hat{q}) = \delta^{ab} \left(P^T_{\mu\nu} D_T(q_4, \vec{q}) + P^L_{\mu\nu} D_L(q_4, \vec{q}) \right)$$

$$\begin{aligned} D^{aa}_{ii}(q) &= \frac{2}{V} \left\langle \mathrm{Tr} \left[A_i(\hat{q}) A^\dagger_i(\hat{q}) \right] \right\rangle = \delta^{aa} \left(P^T_{ii} D^T + P^L_{ii} D^L \right) \\ D^{aa}_{44}(q) &= \frac{2}{V} \left\langle \mathrm{Tr} \left[A_4(\hat{q}) A^\dagger_4(\hat{q}) \right] \right\rangle = \delta^{aa} \left(P^T_{44} D^T + P^L_{44} D^L \right) \end{aligned}$$

- theoretical prediction: $D^{33} = D^{88}$ in the symmetric phase

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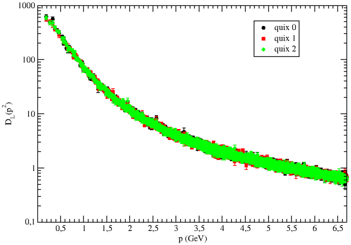
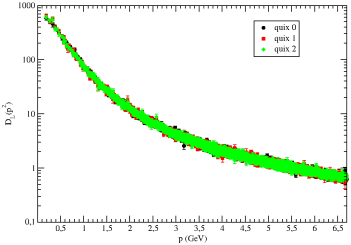
Longitudinal gluon propagator — below T_c

$64^3 \times 8$, T=243 MeV

$a = 3$



$a = 8$



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Transverse gluon propagator — below T_c

$64^3 \times 8$, T=243 MeV

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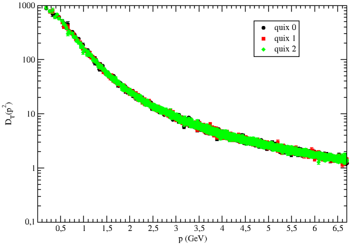

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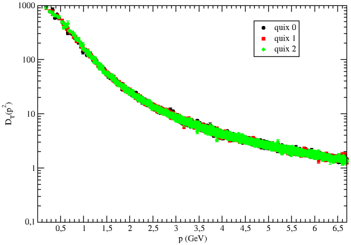
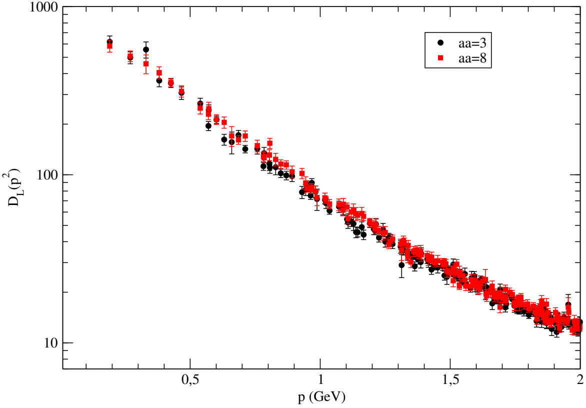
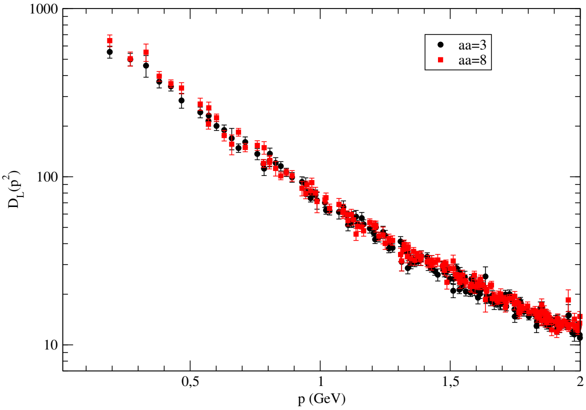


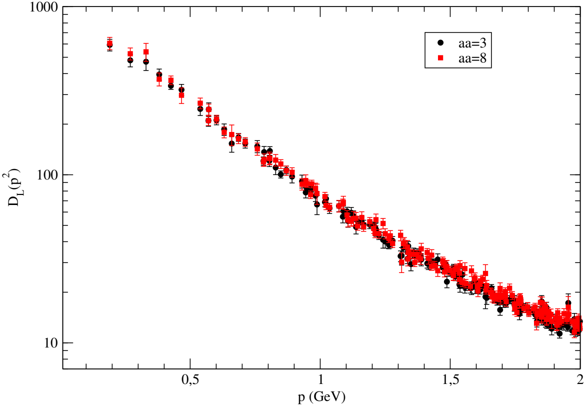
Longitudinal gluon propagator — 0 CTs, below T_c



Longitudinal gluon propagator — 1 CTs, below T_c



Longitudinal gluon propagator — 2 CTs, below T_c



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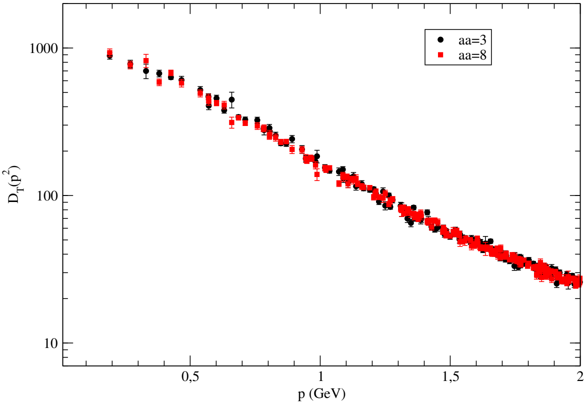
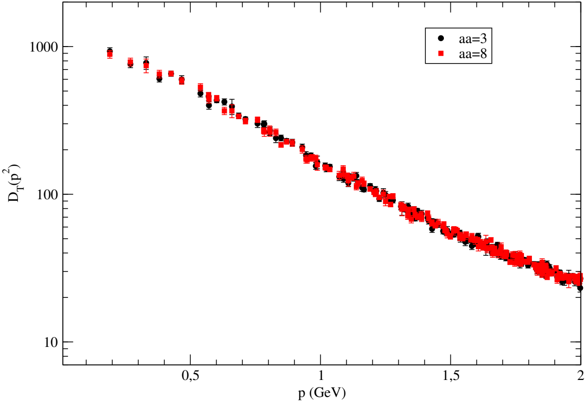


Transverse gluon propagator — 1 CTs, below T_c



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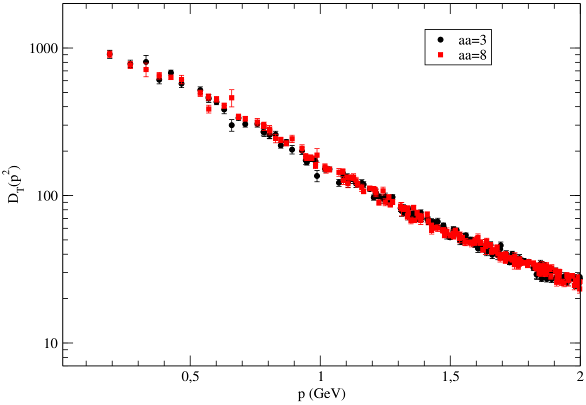

Longitudinal gluon propagator — above T_c

$64^3 \times 6$, T=324 MeV

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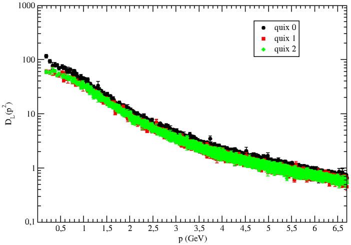

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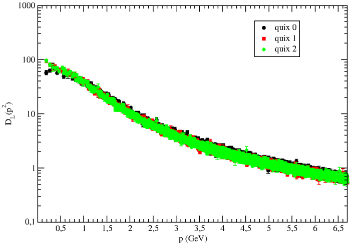

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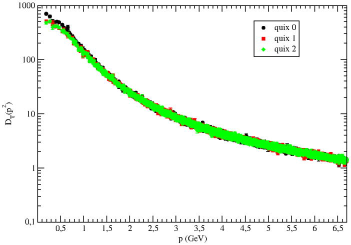

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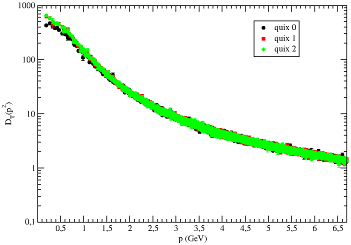
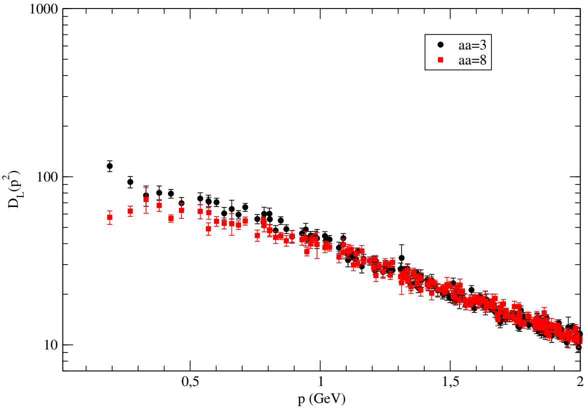
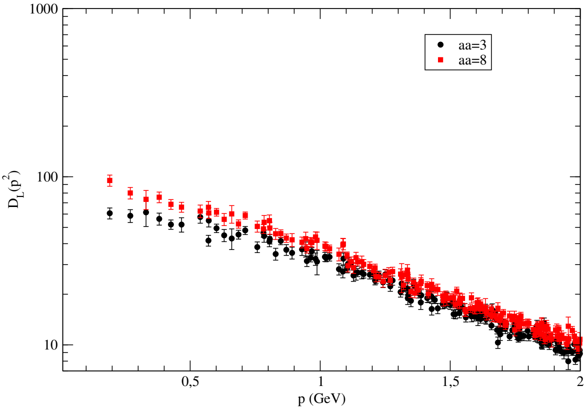


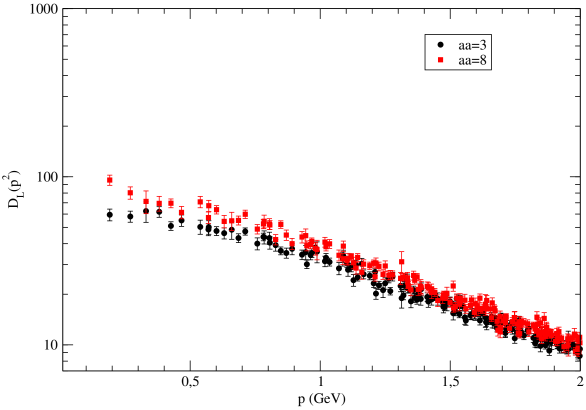
Longitudinal gluon propagator — 0 CTs, above T_c



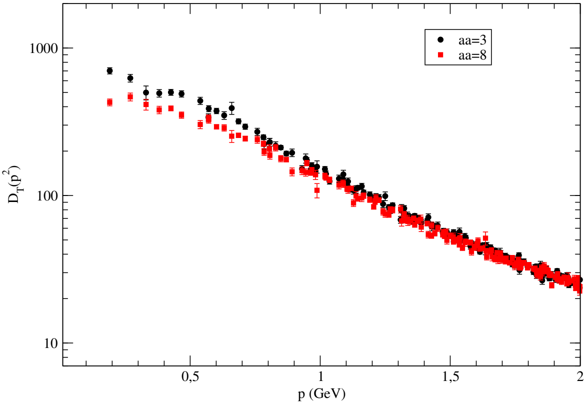
Longitudinal gluon propagator — 1 CTs, above T_c



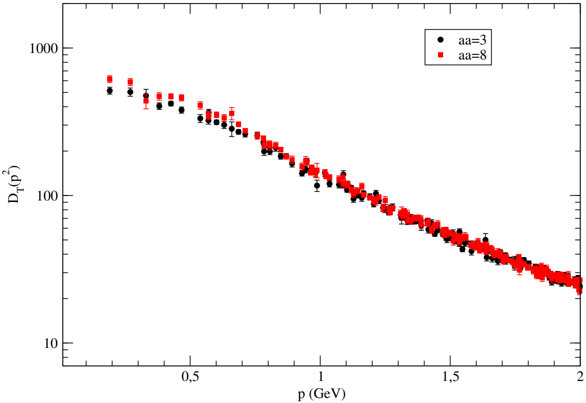
Longitudinal gluon propagator — 2 CTs, above T_c



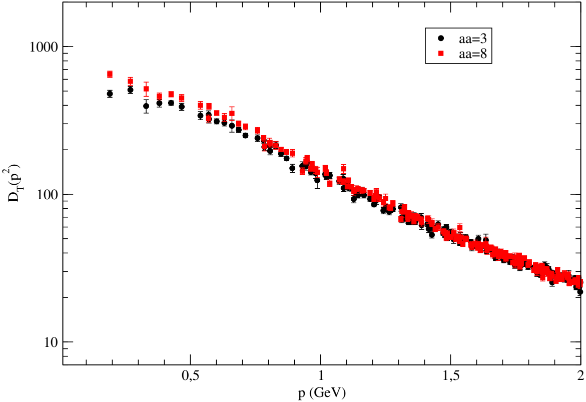
Transverse gluon propagator — 0 CTs, above T_c



Transverse gluon propagator — 1 CTs, above T_c



Transverse gluon propagator — 2 CTs, above T_c



Conclusions and Outlook

- Center-symmetric Landau gauge
  - lattice implementation — first results
- Main continuum properties seem to hold on the lattice
  - $\beta g \langle A_4^3 \rangle$
  - $D^{33} = D^{88}$ in the symmetric phase
- Outlook:
  - increase statistics
  - other temperatures
  - higher-order correlation functions
  - SU(2)
  - dynamical configurations

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