Dynamical chiral symmetry breaking and quark-antiquark entanglement in the quark condensate

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Outline

1. Motivation
2. Definitions: entanglement entropies
3. The QCD vacuum, SχSB
4. Quark-gluon entanglement
5. Application - quark-antiquark entanglement
6. Conclusions & Perspectives

Work with David Rosa Jr and B. L. Costa (MSc dissertation)

Motivation

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- Brings in new concepts, such as entanglement and complexity
- Allows characterize matter phases of topological origin
- Opens new simulation paradigms, as analog and digital quantum computing

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Beyond QCD: quantum gravity, condensed matter, $\cdots$

High stakes and a lot of hype

Quantum Computing: a future perspective for scientific computing

Quantum Computing

Karl Jansen
Cairns, The XVIth Quark Confinement and
the Hadron Spectrum Conference, 19.8.2024

QC for high energy, nuclear and condensed matter physics

Leaving the era of 1+1 dimensional toy models

- real time simulation
  $\rightarrow$ **thermalization**
  $\rightarrow$ **scattering**
  $\rightarrow$ **quenching**
- various **spin models**
- thermal field theory
- ...

- real time simulation
  $\rightarrow$ string breaking, formation of bound states
- collisions, **scattering**
- **non-perturbative** renormalization
- ...

- simulations of **heavy ion collisions**
- **quark gluon plasma**
- **nuclear** physics
- multi-Higgs models
- conformal field theories
- ...

$\Rightarrow$ **Completely new insight in condensed matter and high energy physics!**

Entaglement

— State vector in bipartite Hilbert space: $|\Psi(A,B)\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_b$

— State operator ("density matrix"): $\hat{\rho}(A,B) = |\Psi(A,B)\rangle\langle\Psi(A,B)|$

— Since $\hat{\rho}(A,B)$ is pure state: $\hat{\rho}(A,B)^2 = \hat{\rho}(A,B)$

— Reduced density matrix: $\hat{\rho}(A) = \mathrm{Tr}_B\, \hat{\rho}(A,B)$

— If $\hat{\rho}(A)^2 \neq \hat{\rho}(A) \Rightarrow |\Psi(A,B)\rangle$ (or $\hat{\rho}_A$) is entangled

Measures of entanglement

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* In practice, not always possible to take $n \to 1$.
Use another representation of the log, maintaining n finite:
– distributional zeta-function method (Svaiter & Svaiter)

Most studies

Entanglement in real space:

— Entanglement between a spatial region ($V \sim R^d$) and its complement ($-V$)*

$$\rho_V = \mathrm{Tr}_{-V}\,\rho_{\rm vac} = \mathrm{Tr}_{-V}\,|\mathrm{vac}\rangle\langle\mathrm{vac}|$$

$$S(V) = \mathrm{Tr}_{-V}\,\rho_V \ln \rho_V = g_{d-1}(R)\,\Lambda^{d-1} + \cdots + g_1(R)\,\Lambda + g_0(R)\,\ln\Lambda + S_0(R)$$

$$\Lambda : \text{cutoff}, \quad g_n \sim R^n, \quad S_0 : \text{finite}$$

— A motivation: local measurements, spatial correlations readily accessible

— True, BUT finite resolution, measurements up to a momentum scale

— This talk: entanglement in momentum-space, entanglement in the QCD vacuum

— Our aim: get insight, gather knowledge on such "new observables" and perhaps understand better the QCD vacuum

* Casini & Huerta, J. Phys. A 42, 504007 (2009)

Entanglement in the QCD vaccum - SχSB

- QCD in light-quark sector: approximate $SU_L(3) \otimes SU_R(3)$ symmetry
- Limit of exact symmetry: the symmetry is spontaneously broken (SSB)
- A consequence of SSB: vacuum condensate (quark-mass generation)
- Condensate: quark-antiquark-gluon correlation (entanglement)

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How to quantify the quark-antiquark-gluon entanglement in SχSB vacuum?

Entanglement in the SχSB vacuum

- Construct the SSB vacuum state vector $|\Omega\rangle$
- Compute the corresponding state operator $\hat{\rho} = |\Omega\rangle\langle\Omega|$
- Take partial trace over quark (or antiquark and gluon) d.o.f.
- Compute e.g. purity P, a Rényi entropy S_n, or entanglement entropy S

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An "easy" way to implement this in practice*:

1) start with thermal density matrix $\hat{\rho} = e^{-\beta \hat{H}_{\mathrm{QCD}}}, \quad \beta = 1/T$

2) compute partial traces and entropies

3) at the end, take $T \to 0$

* B. Andrade (IFT, 2020), M. Martins (IFT, 2021), PRD 106, 065024 (2022), PRD 107, 125014 (2023)

Lattice QCD (YM): Buividovich & Polikarpov (2008) $\cdots$ Rindlisbacher et al. (2022) $\cdots$

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Here: Coulomb gauge QCD Hamiltonian in continuum

QCD in Coulomb gauge*

$$H_{\mathrm{QCD}} = H_{\mathrm{q}} + H_{\mathrm{C}} + H_{\mathrm{T}}$$

$$H_{\mathrm{q}} = \int d^3x\, \psi^\dagger(\boldsymbol{x})\Big[\boldsymbol{\alpha}\cdot\big(-i\boldsymbol{\nabla} + g t^a \boldsymbol{A}^a(\boldsymbol{x})\big) + \beta m_0\Big]\psi(\boldsymbol{x})$$

$$H_{\mathrm{C}} = \frac{g^2}{2}\int d^3x \int d^3y\, J[A]^{-1}\rho^a(\boldsymbol{x}) J[A]\left[(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial})^{-1}(-\boldsymbol{\partial}^2)(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial})^{-1}\right]^{ab}(\boldsymbol{x},\boldsymbol{y})\,\rho^b(\boldsymbol{y})$$

$$H_{\mathrm{T}} = \frac{1}{2}\int d^3x\, \big(J^{-1}[A]\boldsymbol{\Pi}^a(\boldsymbol{x})\cdot J[A]\boldsymbol{\Pi}^a(\boldsymbol{x}) + \boldsymbol{B}^a(\boldsymbol{x})\cdot\boldsymbol{B}^a(\boldsymbol{x})\big)$$

$$J[A] = \mathrm{Det}(-\hat{\boldsymbol{D}}\cdot\boldsymbol{\partial}), \qquad \boldsymbol{\Pi}^a = -\boldsymbol{E}^a, \qquad \hat{\boldsymbol{D}} = \boldsymbol{\partial} + g\boldsymbol{A}$$

$$\rho^a(\boldsymbol{x}) = f^{abc}\boldsymbol{A}^b(\boldsymbol{x})\cdot\boldsymbol{\Pi}^c(\boldsymbol{x}) + \rho^a_m(\boldsymbol{x}), \qquad \rho^a_m(\boldsymbol{x}) = \psi^\dagger(\boldsymbol{x})t^a\psi(\boldsymbol{x})$$

* H. Reinhardt et al. Adv. High Energy Phys. 2018, 2312498

Variational approach*

Variational vacuum state:

$$|\Omega\rangle = \mathcal{N}\, e^{-\hat{\Phi}(\psi,\mathrm{A})}\, |0\rangle$$

$$\hat{\Phi}(\psi, A) = \int d^3x_1 d^3x_2 \left[\hat{\psi}^\dagger(\boldsymbol{x}_1) K(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{A})\, \hat{\psi}(\boldsymbol{x}_2) + \frac{1}{2}\omega^{ab}_{ij}(\boldsymbol{x}_1, \boldsymbol{x}_2) \hat{A}^a_i(\boldsymbol{x}_1) \hat{A}^b_j(\boldsymbol{x}_2)\right]$$

$$+ \frac{1}{3!}\int d^3x_1 d^3x_2 d^3x_3\, u^{abc}_{ijk}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)\, \hat{A}^a_i(\boldsymbol{x}_1) \hat{A}^b_j(\boldsymbol{x}_2) \hat{A}^c_j(\boldsymbol{x}_3)$$

$$+ \frac{1}{4!}\int d^3x_1 d^3x_2 d^3x_3 d^3x_4\, v^{abcd}_{ijkl}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4)\, \hat{A}^a_i(\boldsymbol{x}_1) \hat{A}^b_j(\boldsymbol{x}_2) \hat{A}^c_k(\boldsymbol{x}_3) \hat{A}^d_l(\boldsymbol{x}_4)$$

Ansätze for K, ω, u, and v determine them by minimizing the vacuum energy (gap equations): $E_{\mathrm{vac}} = \langle\Omega|\hat{H}_{\mathrm{QCD}}|\Omega\rangle$

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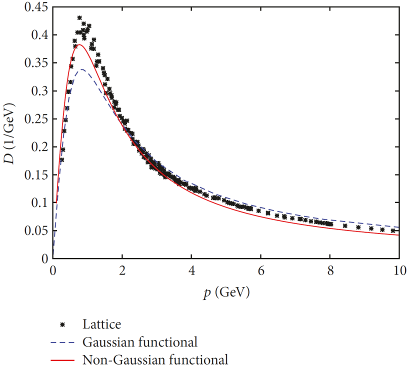
Variational approach - gluon propagator*



SU(2) static gluon propagator

$$D_{ij}^{ab}(\boldsymbol{x}-\boldsymbol{y}) = \langle\Omega|A_i^a(\boldsymbol{x})A_j^b(\boldsymbol{y})|\Omega\rangle$$

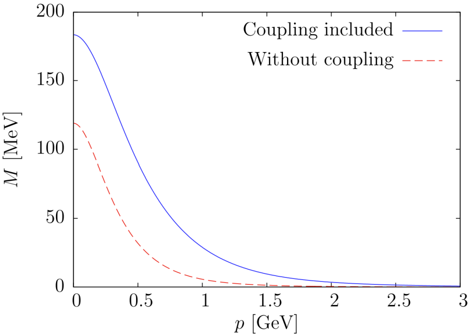
Well fitted by Gribov formula:

$$D(\boldsymbol{p}^2) = \frac{p^2}{p^4 + M^4}$$

$$M \simeq 880 \text{ MeV}$$

* H. Reinhardt et al. Adv. High Energy Phys. 2018, 2312498

Variational approach - quark propagator*



Coupling: gluons in the vacuum functional

Static quark propagator

$$S^{ij}_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{y}) = \frac{1}{2}\langle\Omega|[\psi^i_\alpha(\boldsymbol{x}), \psi^j_\beta(\boldsymbol{y})]|\Omega\rangle$$

Quark mass function:

$$S(\boldsymbol{p}) = \frac{Z(p)}{2\sqrt{\boldsymbol{p}^2 + M^2(\boldsymbol{p}^2)}}\left[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M(p^2)\right]$$

* H. Reinhardt et al. Adv. High Energy Phys. 2018, 2312498

Entanglement: analytical computation, insightful

Model: “Integrate out” the gluons

— Vacuum:

$$|\Omega\rangle = \mathcal{N}\, e^{-\hat{\Phi}(\psi)}|0\rangle \quad \text{with} \quad \hat{\Phi}(\psi) = \sum_{\boldsymbol{p},s} s\, \tan\theta(\boldsymbol{p})\, \hat{q}^\dagger(\boldsymbol{p},s)\hat{\bar{q}}^\dagger(-\boldsymbol{p},s)$$

— Quark field operator $\hat{\psi}_{cf}(\boldsymbol{x})$:

$$\hat{\psi}(\boldsymbol{x}) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{p},s} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \left[u_s(\boldsymbol{p})\, \hat{q}(\boldsymbol{p},s) + v_s(-p)\, \hat{\bar{q}}^\dagger(-\boldsymbol{p},s)\right]$$

$$= \frac{1}{\sqrt{V}} \sum_{\boldsymbol{p},s} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \left[U_s(\boldsymbol{p})\, \hat{Q}(\boldsymbol{p},s) + V_s(-\boldsymbol{p})\, \hat{\bar{Q}}^\dagger(-\boldsymbol{p},s)\right]$$

— Vacua $|0\rangle$ & $|\Omega\rangle$:

$$\hat{q}(\boldsymbol{p},s)|0\rangle = \hat{\bar{q}}^\dagger(\boldsymbol{p},s)|0\rangle = 0, \qquad \hat{Q}(\boldsymbol{p},s)|\Omega\rangle = \hat{\bar{Q}}_s^\dagger(\boldsymbol{p})|\Omega\rangle = 0$$

The spinors & chiral angle

$$u(\boldsymbol{p}, s) = \sqrt{\frac{E_p + m_0}{2E_p}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p + m_0} \end{pmatrix} \chi_s, \qquad v(\boldsymbol{p}, s) = \sqrt{\frac{E_p + m_0}{2E_p}} \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p + m_0} \\ 1 \end{pmatrix} \chi_s$$

$$U(\boldsymbol{p}, s) = \sqrt{\frac{\mathcal{E}(\boldsymbol{p}) + M(\boldsymbol{p})}{2\mathcal{E}(\boldsymbol{p})}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\mathcal{E}(\boldsymbol{p}) + M(\boldsymbol{p})} \end{pmatrix} \chi_s, \qquad V(\boldsymbol{p}, s) = \sqrt{\frac{\mathcal{E}(\boldsymbol{p}) + M(\boldsymbol{p})}{2\mathcal{E}(\boldsymbol{p})}} \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\mathcal{E}(\boldsymbol{p}) + M(\boldsymbol{p})} \\ 1 \end{pmatrix} \chi_s$$

$$E_p = \sqrt{\boldsymbol{p}^2 + m_0^2}, \qquad \mathcal{E}(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 + M^2(\boldsymbol{p})}$$

$$\sin 2\theta(\boldsymbol{p}) = \begin{cases} \dfrac{M(\boldsymbol{p})}{\mathcal{E}(\boldsymbol{p})} & \text{when } \; m_0 = 0 \\[2ex] \dfrac{M(\boldsymbol{p}) - m_0}{\mathcal{E}(\boldsymbol{p})} \left(\dfrac{p}{E_p}\right) & \text{when } \; m_0 \neq 0 \end{cases}$$

Compute traces: coherent states

—Eigenstates of the annihilation operators

Definition:

$$\hat{q}\,|z_q\rangle = z_q\,|z_q\rangle, \qquad |z_q\rangle = e^{z_q\,\hat{q}^\dagger}|0\rangle$$

$$\hat{\bar{q}}\,|z_{\bar{q}}\rangle = z_{\bar{q}}\,|z_{\bar{q}}\rangle, \qquad |z_{\bar{q}}\rangle = e^{z_{\bar{q}}\,\hat{\bar{q}}^\dagger}|0\rangle$$

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Completeness:

$$\int \mathcal{D}z_q^*\mathcal{D}z_q\,e^{-z_q^*\cdot z_q}\,|z_q\rangle\langle z_q^*| = 1, \qquad \int \mathcal{D}z_{\bar{q}}^*\mathcal{D}z_{\bar{q}}\,e^{-z_{\bar{q}}^*\cdot z_{\bar{q}}}\,|z_{\bar{q}}\rangle\langle z_{\bar{q}}^*| = 1$$

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Partial Trace:

$$\mathrm{Tr}_{\bar{q}}\,\hat{O}(\hat{q}^\dagger,\hat{q},\hat{\bar{q}}^\dagger,\hat{\bar{q}}) = \int \mathcal{D}z_{\bar{q}}^*\mathcal{D}z_{\bar{q}}\;e^{-z_{\bar{q}}^*\cdot z_{\bar{q}}}\;\langle -z_{\bar{q}}^*|\hat{O}(\hat{q}^\dagger,\hat{q},\hat{\bar{q}}^\dagger,\hat{\bar{q}})|z_{\bar{q}}\rangle$$

Reduced vacuum density matrix (quarks)

—Trace $\hat{\rho} = |\Omega\rangle\langle\Omega|$ over antiquarks

Reduced density operator (quarks):

$$\hat{\rho}_q = \mathrm{Tr}_{\bar{q}}\, |\Omega\rangle\langle\Omega| = \mathcal{N}_q \int Dz_{\bar{q}}^* Dz_{\bar{q}}\; e^{-z_{\bar{q}}^* \cdot z_{\bar{q}}} \,\langle -z_{\bar{q}}^* |\Omega\rangle\langle\Omega| z_{\bar{q}}\rangle$$

$$Dz_{\bar{q}}^* Dz_{\bar{q}} = \prod_{\boldsymbol{p},s} dz_{\bar{q}}^*(\boldsymbol{p},s) dz_{\bar{q}}(\boldsymbol{p},s), \quad z_{\bar{q}}^* \cdot z_{\bar{q}} = \sum_{\boldsymbol{p},s} z_{\bar{q}}^*(\boldsymbol{p},s) z_{\bar{q}}(\boldsymbol{p},s)$$

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Matrix element of $\hat{\rho}_q$:

$$\rho_q(z_q^*, z_q') \equiv \langle z_q^* | \hat{\rho}_q | z_q' \rangle = \mathcal{N}_q\, \exp\left[\sum_{\boldsymbol{p},s} \tan^2\theta(\boldsymbol{p}) z_q^*(\boldsymbol{p},s) z_q'(\boldsymbol{p},s)\right]$$

$$\mathcal{N}_q^{-1} = \exp\left[2N_c N_f \sum_{\boldsymbol{p}} \log\left(1 + \tan^2\theta(\boldsymbol{p})\right)\right]$$

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Entropies

Rényi entropies:

$$S_n = \frac{1}{1-n} \ln \, \mathrm{Tr}_q \, \hat{\rho}_q^n$$
$$= \frac{2N_c N_f}{1-n} V \int \frac{d^3p}{(2\pi)^3} \left[\ln(1+\tan^{2n}\theta(p)) - \, n \log(1+\tan^2\theta(p))\right]$$

Entanglement entropy:

$$S_E = \lim_{n\to 1} S_n$$
$$= -2N_c N_f \, V \int \frac{d^3p}{(2\pi)^3} \left[\ln(1+\tan^2\theta(p)) - \frac{\tan^2\theta(p)\ln(\tan^2\theta(p))}{1+\tan^2\theta(p)}\right]$$

First look - NJL model

NJL Hamiltonian:

$$\hat{H} = \int d^3x \, \left\{ \hat{\psi}^\dagger(\boldsymbol{x})(-i\boldsymbol{\alpha} \cdot \boldsymbol{\partial})\hat{\psi}(\boldsymbol{x}) + G\left[(\bar{\psi}(\boldsymbol{x})\psi(\boldsymbol{x}))^2 + (\psi(\bar{\boldsymbol{x}})i\gamma_5\tau^a\psi(\boldsymbol{x}))^2 \right] \right\}$$

Chiral angle: $\tan 2\theta(p) = \frac{M}{p}$

Minimize the vacuum energy: gap equation

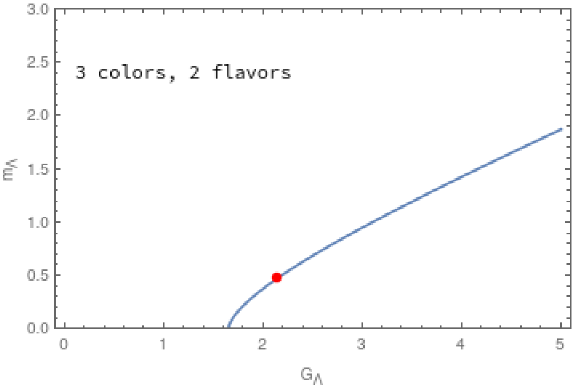
$$p \tan 2\theta(\mathbf{p}) = 4GN_cN_f \int \frac{d^3p}{(2\pi)^3} \sin 2\theta(\mathbf{p}) = \frac{2GN_cN_f}{\pi^2} \int_0^\Lambda dp p^2 \frac{M}{\sqrt{p^2 + M^2}} = M$$

Condensate:

$$\langle \bar{\psi}\psi \rangle = -2N_cN_f \int \frac{d^3p}{(2\pi)^3} \sin 2\theta(\mathbf{p}) = -2N_cN_f \int \frac{d^3p}{(2\pi)^3} \frac{M}{\sqrt{\mathbf{p}^2 + M^2}}.$$

First look - NJL model

$$m_\Lambda = \frac{M}{\Lambda} \qquad G_\Lambda = G\Lambda^2$$



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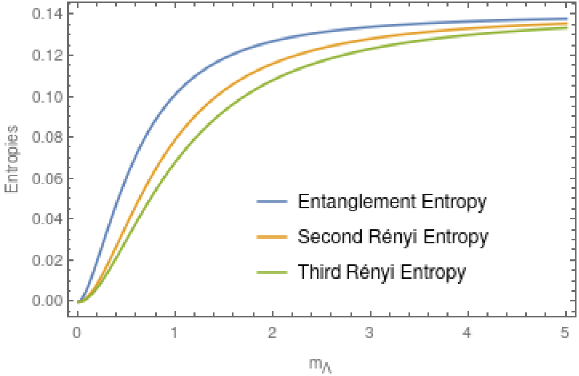


Model - QCD Coulomb gauge Hamiltonian

$$\hat{H} = \int d^3x \; \hat{\psi}^\dagger(\boldsymbol{x})(-i\boldsymbol{\alpha}\cdot\boldsymbol{\partial} + m_0)\hat{\psi}(\boldsymbol{x}) - \frac{1}{2}\int d^3x d^3y \; \rho_m^a(\boldsymbol{x}) \, V_c(|\boldsymbol{x}-\boldsymbol{y}|) \, \rho_m^a(\boldsymbol{y})$$

$$+ \frac{1}{2}\int d^3x d^3y \; J_i^a(\boldsymbol{x}) \, D^{ij}(\boldsymbol{x}-\boldsymbol{y}) \, J_j^a(\boldsymbol{y})$$

$$J_i^a(\boldsymbol{x}) = \hat{\psi}^\dagger(\boldsymbol{x}) t^a \gamma_i \hat{\psi}(\boldsymbol{x}), \qquad D^{ij}(\boldsymbol{x}-\boldsymbol{y}) = \left(\delta^{ij} - \frac{\partial^i \partial^j}{\partial^2}\right) D(|\boldsymbol{x}-\boldsymbol{y}|)$$

A choice of potentials: fit to lattice data of hyperfine splitting of light hadron masses
(LLanes-Estrada et al. Phys. Rev. C, vol. 70, p. 035202, 2004)

$$V_c = V_l + V_s = \frac{8\pi\sigma}{p^4} + \frac{4\pi\alpha(p)}{p^2}, \qquad \alpha(p) = \frac{4\pi Z}{\left[\beta \log(c + p^2/\Lambda_{QCD}^2)\right]^{3/2}}$$

$$D(p) = \frac{-4\pi\alpha_T}{(p^2 + m_g^2/4)\ln^{1.42}(\tau + p^2/m_g^2)}$$

$$Z = 5.94, \quad c = 40.68, \quad \beta = 10.08, \quad m_g = 550\,\mathrm{MeV}, \quad \alpha_T = 0.5, \quad \tau = 1.05$$

Gap equation

One needs to find $\theta(\boldsymbol{p})$ or, equivalently, $M(\boldsymbol{p})$:

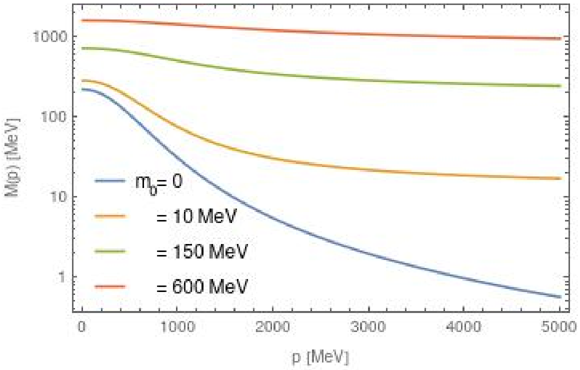
Minimize the vacuum energy $\langle\Omega|\hat{H}|\Omega\rangle$

$$M(\boldsymbol{p}) = m_0 + \frac{2}{3}\int \frac{d^3k}{(2\pi)^3}\Big[F_1(\boldsymbol{p},\boldsymbol{k})V_c(|\boldsymbol{p}-\boldsymbol{k}|) + 2G_1(\boldsymbol{p},\boldsymbol{k})D(|\boldsymbol{p}-\boldsymbol{k}|)\Big]$$

$$F_1(\boldsymbol{p},\boldsymbol{k}) = \frac{M(\boldsymbol{k})}{\mathcal{E}(\boldsymbol{k})} - \frac{M(\boldsymbol{p})}{\mathcal{E}(\boldsymbol{k})}\frac{k}{p}\hat{p}\cdot\hat{k}$$

$$G_1(\boldsymbol{p},\boldsymbol{k}) = \frac{M(\boldsymbol{k})}{\mathcal{E}(\boldsymbol{k})} + \frac{M(\boldsymbol{p})}{\mathcal{E}(\boldsymbol{k})}\frac{k}{p}\frac{(\boldsymbol{p}\cdot\boldsymbol{k}-p^2)(\boldsymbol{p}\cdot\boldsymbol{k}-k^2)}{pk|\boldsymbol{p}-\boldsymbol{k}|^2}$$

Mass function



Generic features:

1. SSB dressing in $M(p^2)$ large in the infrared (low momenta)
2. Relative SSB dressing, $M(p^2) - m_0$, increases with m_0
3. BUT $(M(p \sim 0) - m_0)/m_0$ gets smaller for heavier quarks.

Entanglement entropy: first lesson

1) In the chiral limit, the EE density is finite, because*:

$$M(p) \sim \frac{1}{p^2\,(\ln p^2/\Lambda^2_{QCD})^{-\gamma}}, \qquad \gamma = 0.84$$

2) Away from the chiral limit, the EE runs, because:

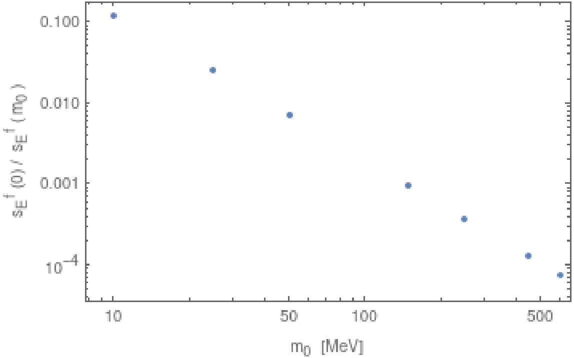
$$M(p^2) \sim \frac{\hat{m}}{(\ln p^2/\Lambda^2_{\mathrm{QCD}})^{\gamma}}$$

* Burgio et al. PRD 86, 014506, lattice QCD in Coulomb gauge

Are these model independent features of QCD?

Entanglement entropy: m_0 dependence

$$s_R^f(m_0) = \frac{s_E^f(0)}{s_0^f(m_0)} \leftarrow \begin{cases} s_E^f(0) & \text{chiral limit} \\ s_0^f(m_0) & \text{explicit chiral breaking} \end{cases}$$



$S_0^f(m_0)$ computed up to $p_{\max} = 2.5 \times 10^5$ MeV

Entanglement entropy: second lesson

1) EE grows with explicit symmetry breaking ($m_0 \neq 0$), because

$M(p^2)$ increases due to m_0, not due to the interaction

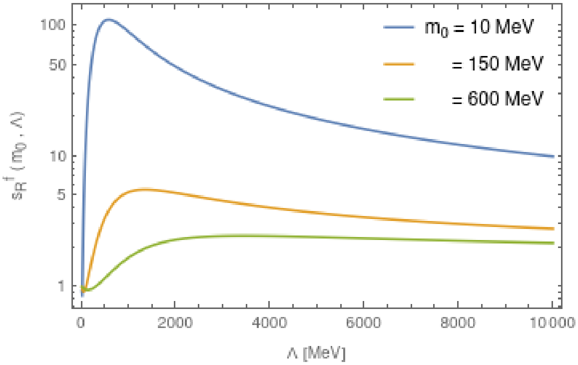
2) Artifact due to definition of $|\Omega\rangle$: $|0\rangle$ includes m_0

For a better assessment of SSB on EE, ratio of EE with and without interaction

Entanglement entropy: dressing effect

Integral for EE up to $p = \Lambda$ and make ratio:

$$s^f_R(m_0,\Lambda) = \frac{s^f_E(m_0,\Lambda)}{s^f_0(m_0,\Lambda)} \leftarrow \begin{cases} s^f_E(m_0,\Lambda) : \text{with interaction} \\ s^f_0(m_0,\Lambda) : \text{no interaction} \end{cases}$$



Entanglement entropy: third lesson

— EE reveals a clear signal of the QCD chiral SSB

— EE integrand reveals quark-antiquark entangling momentum-scale

— Entanglement strength is strongly suppressed by explicit χSB

Conclusions and Perspectives

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7. Comparisons with lattice QCD simulations, when available

Thank you

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