The four-gluon vertex in collinear and soft kinematics

Leonardo Rodrigues dos Santos

In collaboration with:
A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou,
F. Pinto-Gómez and J. Rodríguez-Quintero

Based on: Eur. Phys. J. C 84, 676 (2024) and arXiv:2408.06135 [hep-ph] (2024)

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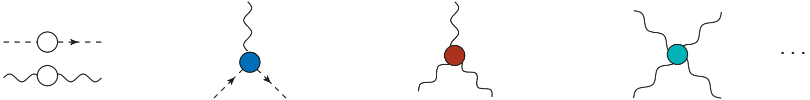
Conselho Nacional de Desenvolvimento Científico e Tecnológico

Introduction and Motivation

- Nonperturbative QCD from correlation functions: *Basic building blocks*

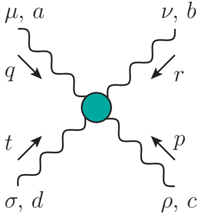


- Closely connected to other Green's functions, *e.g.*, SDEs, BSEs,...

- Basic challenge: numerical determinations of these functions
  - Continuum and discrete approaches

- **Four-gluon vertex**: Nonperturbative regime is relatively poorly known

- Immense technical complexity: tensor structure, dynamics, kinematics



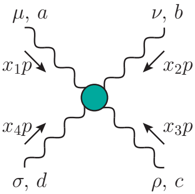
$$-ig^2\mathbb{\Gamma}^{abcd}_{\mu\nu\rho\sigma}(q,r,p,t)$$

Goal: Four-gluon vertex through SDEs

D. Binosi, D. Ibañez, and J. Papavassiliou, J. High Energy Phys. 09, 059 (2014)
A. K. Cyrol, M. Q. Huber, and L. von Smekal, Eur. Phys. J. C75, 102 (2015)
M. Q. Huber, Phys. Rev. D 101 114009 (2020)
M. Colaço, O. Oliveira, and P. J. Silva, Phys. Rev. D 109, 074502 (2024)
N. Barrios, P. De Fabritiis, and M. Peláez, Phys. Rev. D 109, L091502 (2024)

Collinear and soft kinematic configurations

- General kinematics $\rightarrow$ *very complicated!*
- We focus on two families of kinematic configurations



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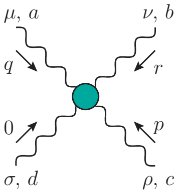


$$-ig^2\mathbb{I}^{abcd}_{\mu\nu\rho\sigma}(q,r,p,0)$$

Collinear kinematics

All momenta are parallel, with a single momenta scale, p.

$$(q,r,p,t) \rightarrow (x_1p, x_2p, x_3p, x_4p)$$

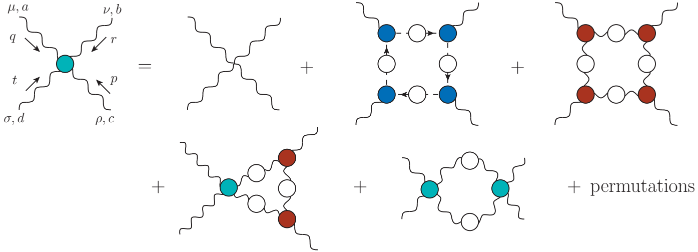
Soft kinematics

One momentum $t \rightarrow 0$ with arbitrary q, r and p

$$(q,r,p,t) \rightarrow (q,r,p,0)$$

Schwinger-Dyson 4PI

- SDE/EOM from the 4PI effective action at the four-loop level[1]



J. Berges, n-PI effective action techniques for gauge theories, Phys. Rev. D 70, 105010 (2004)
M. E. Carrington and Y. Guo, Phys. Rev. D 83, 016006 (2011)

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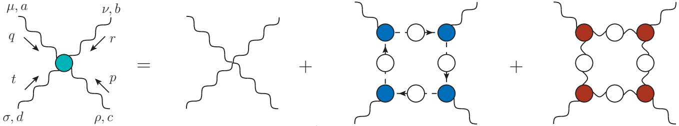

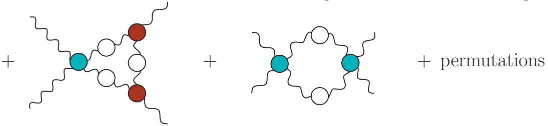

- Focus on

$$D_{4\mathrm{g}}(q,r,p,t) := \frac{\Gamma^{abcd}_{0\,\mu\nu\rho\sigma} P^{\mu'\mu}(q) P^{\nu'\nu}(r) P^{\rho'\rho}(p) P^{\sigma'\sigma}(t) \mathbb{I}\Gamma^{abcd}_{\mu'\nu'\rho'\sigma'}}{\Gamma^{abcd}_{0\,\mu\nu\rho\sigma} P^{\mu'\mu}(q) P^{\nu'\nu}(r) P^{\rho'\rho}(p) P^{\sigma'\sigma}(t) \Gamma^{abcd}_{0\,\mu\nu\rho\sigma}} \quad \begin{array}{l} \nearrow D_{4\mathrm{g}}(x,p) \\ \searrow D_{4\mathrm{g}}(q,r,p,0) \end{array}$$

where $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$.

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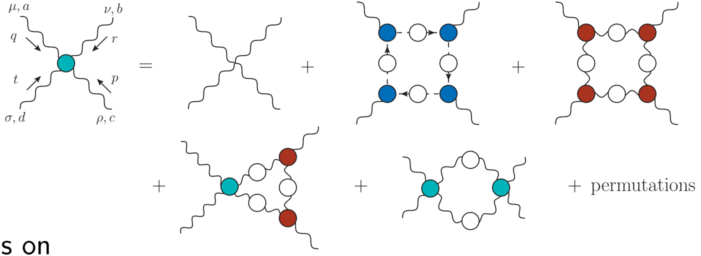

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where $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$.

- Problem: how to deal with the Four-gluon vertex dependence?
- Goal: Capture leading nonperturbative features

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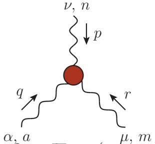
Planar degeneracy of the three-gluon vertex

- Key insight from the *three-gluon vertex*: **Planar Degeneracy**



$g f^{amn} \mathbb{\Gamma}_{\alpha\mu\nu}(q, r, p)$

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To a high degree of accuracy, we find

$$\overline{\mathrm{I}\Gamma}^{\alpha\mu\nu}(q,r,p) \approx L_{sg}(s^2)\overline{\Gamma}_0^{\alpha\mu\nu}(q,r,p)\,,$$

where $s^2 = (q^2+r^2+p^2)/2$ and

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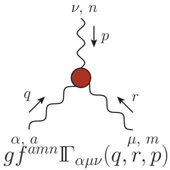

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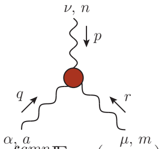

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- And in the continuum[2]

[1] F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou, and J. Rodríguez-Quintero, Phys. Lett. B 838, 137737 (2023).

[2] G. Eichmann, R. Williams, R. Alkofer, and M. Vujinovic, Phys. Rev. D89, 105014 (2014)

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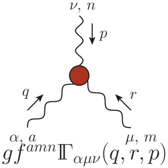

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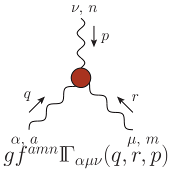

- Solid lattice evidence[1]
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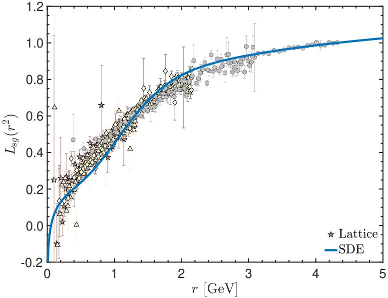
- And in the continuum[2]
- *As a **working hypothesis** it leads to considerable technical simplification in the SDE*[3]
- Leads to excellent SDE/Lattice agreement



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Methodology

- Employing **Planar Degeneracy** for the **Four-gluon vertex**

$$\overline{\mathbb{\Gamma}}^{abcd}_{\mu\nu\rho\sigma}(q,r,p,t) \approx D^*(\bar{s}^2)\overline{\Gamma}^{abcd}_{0\,\mu\nu\rho\sigma}(q,r,p,t)\,,$$

where now

$$\bar{s}^2 = \frac{1}{2}(q^2 + r^2 + p^2 + t^2)\,,$$

generalizes the three-gluon vertex s^2

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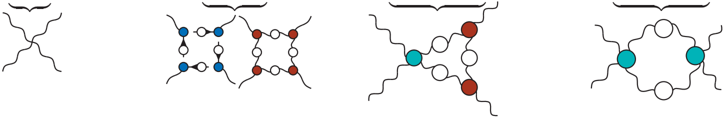
$$D_{4g}(q,r,p,t) = \underbrace{Z_4} \quad + \quad \left[\ \underbrace{\int_k K_1} \quad + \quad \underbrace{\int_k D^* K_2} \quad + \quad \underbrace{\int_k D^{*2} K_3}\ \right]$$



→ **Collinear** : $D_{4g}(x,p)$

→ **Soft** : $D_{4g}(q,r,p,0)$

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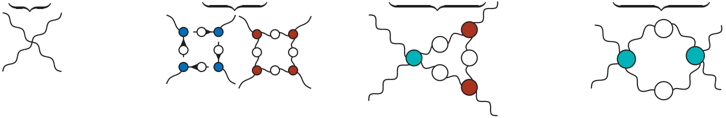

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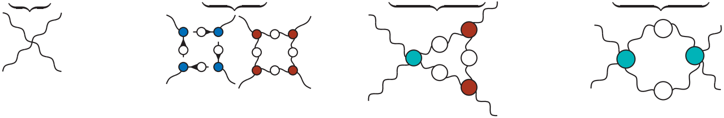


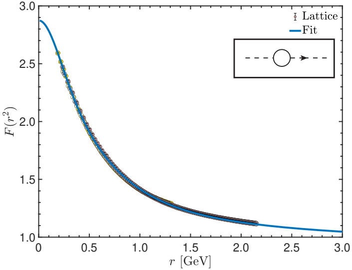
$\longrightarrow$ **Collinear** : $D_{4g}(x,p)$

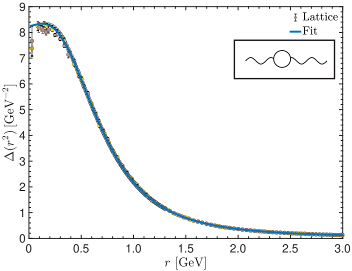
$\longrightarrow$ **Soft** : $D_{4g}(q,r,p,0)$

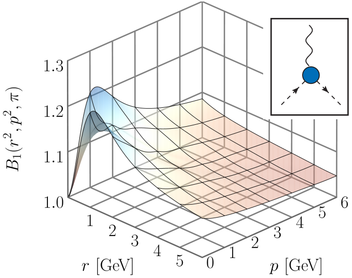
- **Planar Degeneracy** $\rightarrow$ to be explicitly tested!
- Solution method: Iterative procedure defined by employing

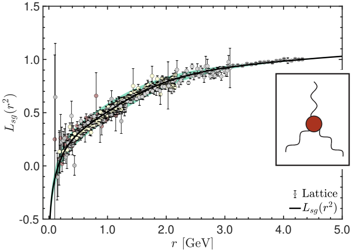
$$D_{4g}(q,r,p,t) \approx D^*(\bar{s}^2)$$

Numerical inputs





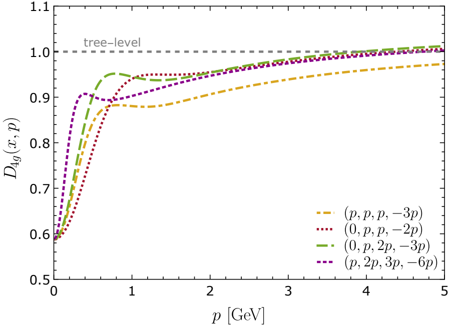




I. Bogolubsky, E. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Phys. Lett. B676, 69 (2009)
A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).

Results: Collinear configurations

- Firstly focus on four configurations



- IR suppression

A. C. Aguilar, M. N. Ferreira, J. Papavassiliou, L. R. Santos, Eur. Phys. J. C 84, 676 (2024)

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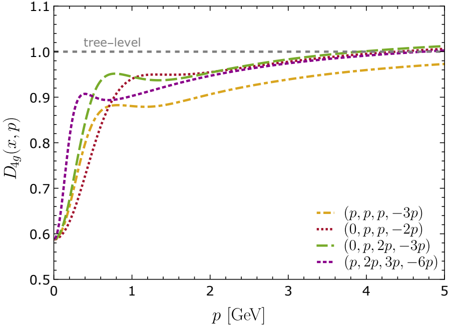

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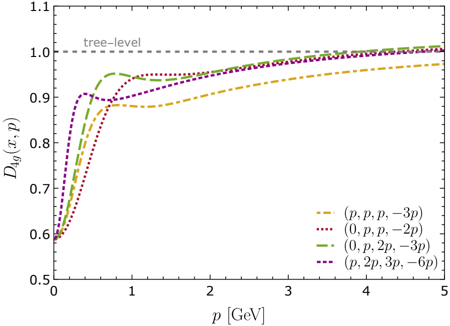

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- Configuration dependent transformation from p to $\bar{s}$

$$\bar{s}^2 = \frac{p^2}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2) \longrightarrow p^2 = 2\bar{s}^2(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{-1}.$$

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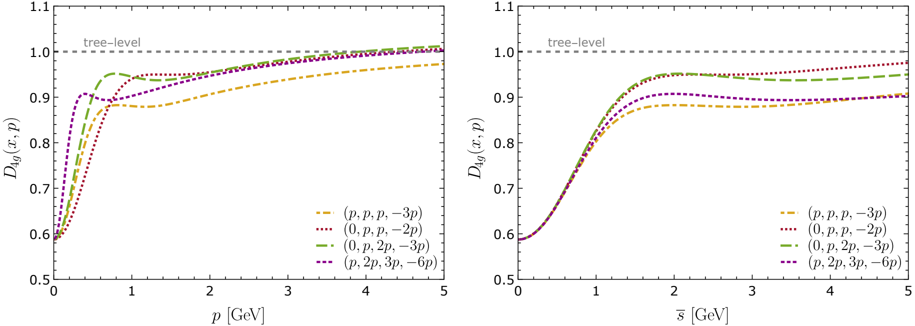

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- Evidence for **Planar Degeneracy!**

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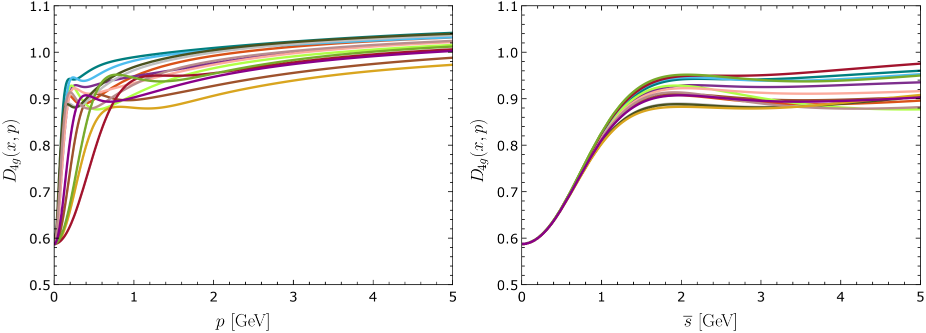
Results: Collinear configurations

- Adding more cases: total 14



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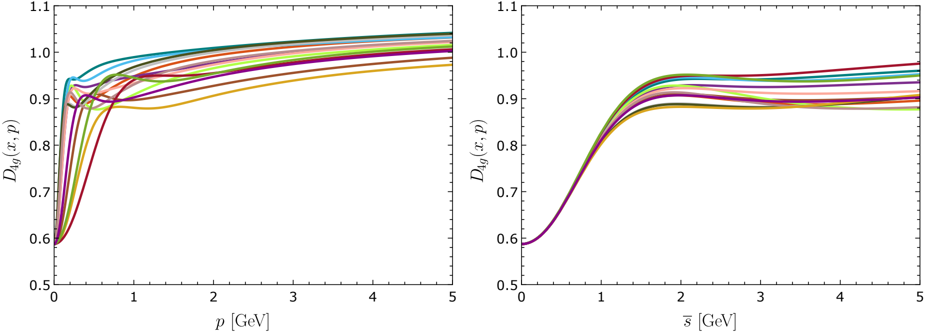

- Excellent agreement for $\bar{s} < 1$ GeV
- Mild dependence on the x_i for larger $\bar{s}$
- Planar degeneracy as an *infrared property*

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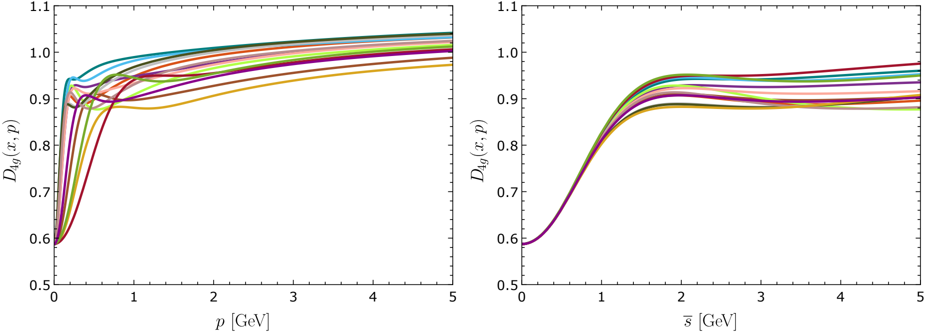

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Relative deviation with respect to $D^*(\bar{s}^2)$

$$\delta(\bar{s}^2) \leq 1.7\% \text{ for } \bar{s} \leq 1\text{GeV}$$

$$\delta(\bar{s}^2) \leq 4.4\% \text{ for } \bar{s} \leq 2\text{GeV}$$

Results: Collinear configurations

- Direct comparison

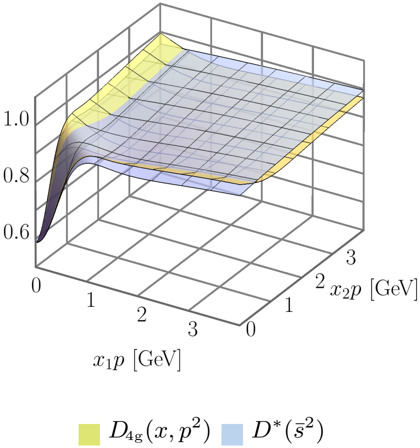
$$D_{4g}(x, p^2) \ v.s. \ D^*(\bar{s}^2)$$

through the mapping

$$\bar{s}^2 = \frac{p^2}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2)$$

- View $D_{4g}(x, p^2)$ as a function of the momenta $x_1 p$ and $x_2 p$ (fix $x_3 = x_1 + x_2$)
  - Vary x_i continuously

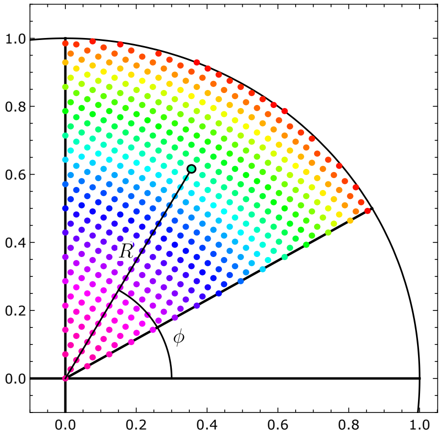
- $D^*(\bar{s}^2)$ accurately *reconstructs* $D_{4g}(x, p^2)$



More details: A. C. Aguilar, M. N. Ferreira, J. Papavassiliou, L. R. Santos, Eur. Phys. J. C 84, 676 (2024)

Soft kinematics

- More general configuration: Three-gluon vertex kinematics
- We employ $D_{4\mathrm{g}}(q, r, p, 0) \to D_{4\mathrm{g}}(s^2, R, \phi)$ where $s^2 = (q^2 + r^2 + p^2)/2$
- $R \in [0, 1]$ and $\phi \in [0, 2\pi]$
- By Bose symmetry[1] one sixth of the disk is independent, we compute $\phi \in [\pi/6, \pi/2]$
- Each choice of R and ϕ identifies a unique soft configuration

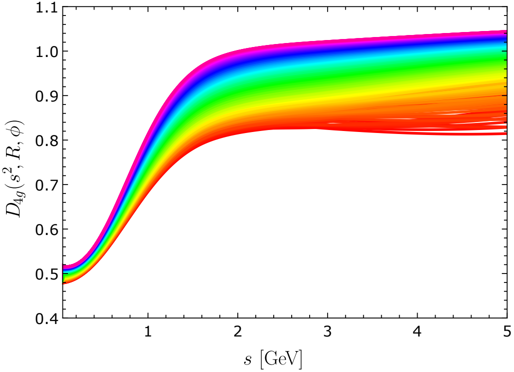


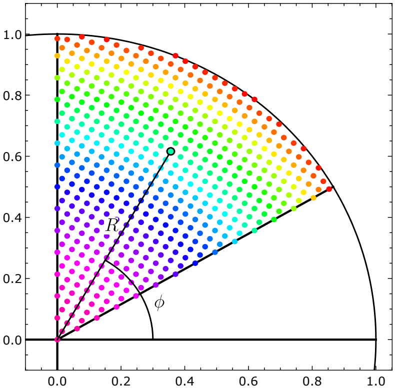
- Numerically probe $N = 491$ configurations

A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, F. Pinto-Gómez, J. Rodríguez-Quintero, L. R. Santos, arXiv:2408.06135 [hep-ph] (2024)

[1] F. Pinto-Gómez, F. De Soto, J. Rodríguez-Quintero, Phys. Rev. D 110 014005 (2024).

Results: Soft configurations





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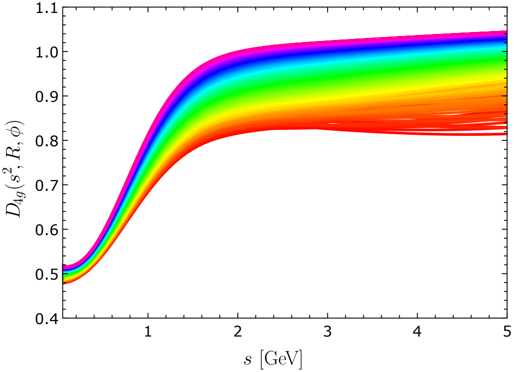

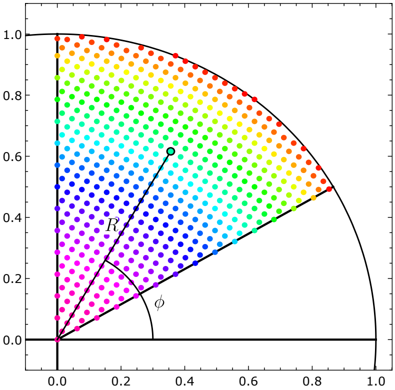

- Greater deviation of planar degeneracy compared to collinear
- Relative deviation: $\delta(\bar{s}^2) \leq 12\%$ for $\bar{s} \leq 1$GeV and $\delta(\bar{s}^2) \leq 15\%$ for $1 \leq \bar{s} \leq 2$GeV

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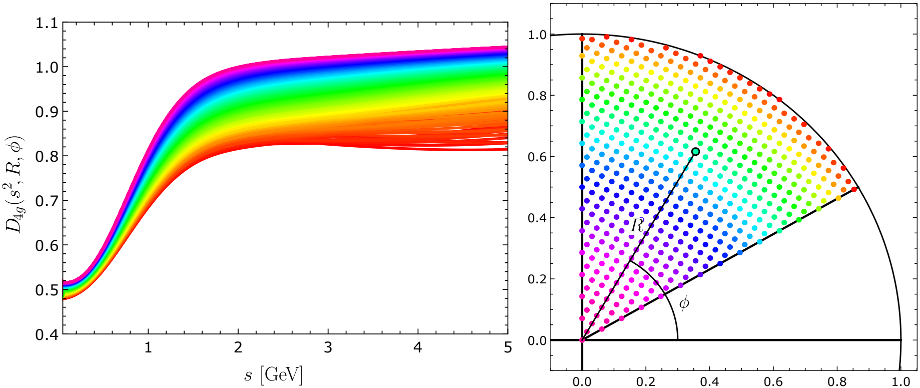

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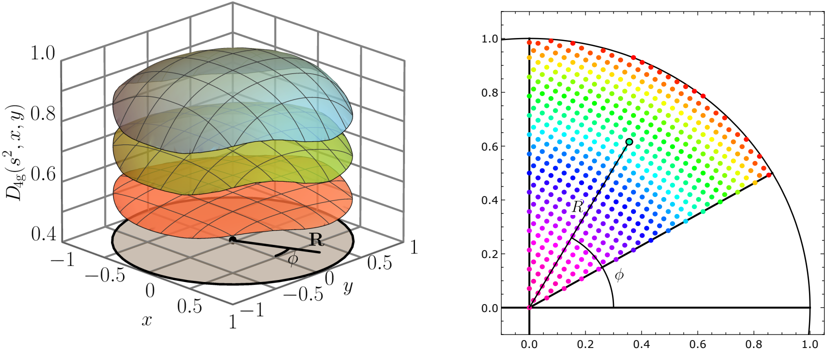
- Color gradient: clear indication of R dependence

Results: Soft configurations



■ $s = 0.5\,\text{GeV}$ ■ $s = 1\,\text{GeV}$ ■ $s = 2\,\text{GeV}$

- Deviation from Planar Degeneracy $\rightarrow$ **curvature** of the surfaces
- Minimal dependence on ϕ.

$$R = (x^2 + y^2)^{1/2}\,,$$
$$\phi = \arctan(y/x)\,.$$

G. Eichmann, C. S. Fischer, W. Heupel, Phys. Rev. D92 056006 (2015)
M. Q. Huber, Phys. Rev. D 101 114009 (2020).

Lattice comparison

- De Soto's talk $\rightarrow$ **Four-gluon vertex** soft kinematics on the lattice

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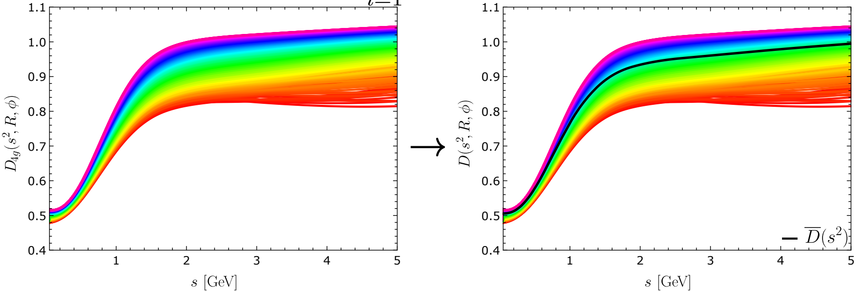
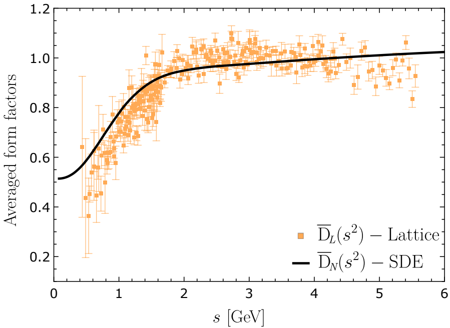
We define it as $\overline{D}(s^2) = \dfrac{1}{N}\displaystyle\sum_{i=1}^{N} D_{4g}(s^2, R_i, \phi_i)\,,$



- Matching renormalization schemes $\overline{D}(s^2) \rightarrow \overline{D}_N(s^2) := \overline{D}(s^2)/\overline{D}(\mu^2)$ where $\mu^2 = 18.64$ GeV^2

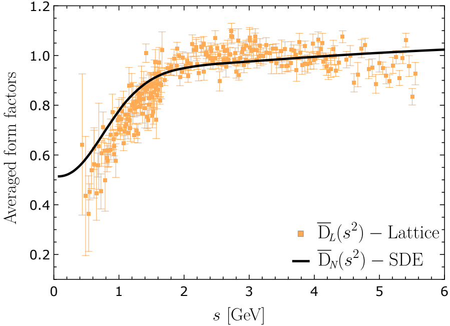
Lattice comparison

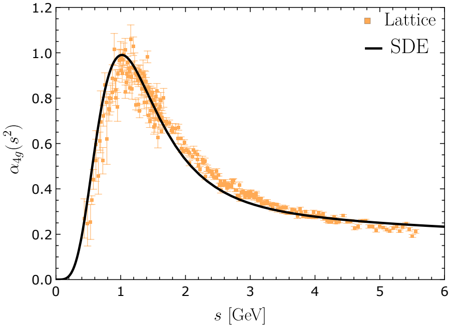


- Very nice agreement!
- Mean absolute percentage error $\sigma = 7\%$

$$\sigma = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\overline{D}_{\mathsf{N}}(s_i^2) - \overline{D}_{\mathsf{L}}(s_i^2)}{\overline{D}_{\mathsf{L}}(s_i^2)}\right| \times 100\% \,.$$

Lattice comparison





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- Definition of Effective Charge[1,2,3]

$$\alpha_{4g}(s^2) = \alpha_s(\mu^2)\overline{D}(s^2)\mathcal{Z}^2(s^2)\,,$$

with $\mathcal{Z}(p^2) = p^2\Delta(p^2)$

- $\sigma = 9\%$

[1] C. Kellermann, C. S. Fischer, Phys. Rev. D78 025015 (2008)
[2] A. K. Cyrol, M. Q. Huber, L. von Smekal, Eur. Phys. J. C75 102 (2015)
[3] M. Q. Huber, Phys. Rev. D 101 114009 (2020)

Conclusions

- **Planar Degeneracy** $\rightarrow$ Considerable technical simplifications
- **Planar Degeneracy** is only an *approximate* property
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Thank you!