Infrared dynamics of the quark-gluon vertex in general kinematics

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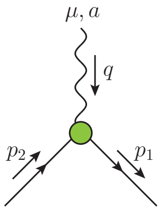
In collaboration with: A. C. Aguilar, M. N. Ferreira, G. T. Linhares, and J. Papavassiliou

Based on: A. C. Aguilar, M. N. Ferreira, G. T. Linhares, B. M. O. and J. Papavassiliou, arXiv:2408.15370 [hep-ph]

Importance of the study the quark-gluon vertex

$$\mathbb{\Gamma}^a_\mu(q, p_2, -p_1) =$$



- The quark-gluon vertex is a key ingredient of QCD
- A crucial role in understanding significant aspects of the theory
  - Chiral symmetry breaking[1]
  - Quark mass generation[2]
  - Hadronic bound state description[3]

- Motivation of this work:
  - Development on the understanding of the three-gluon vertex[4]
  - Recent lattice data available[5]

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[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961)

[2] C. S. Fischer and R. Alkofer, Phys. Rev. D67, 094020 (2003)

[3] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Prog. Part. Nucl. Phys. 91, 1 (2016)

[4] A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, J. Rodríguez-Quintero, and S. Zafeiropoulos, Eur. Phys. J. C80, 154 (2020)

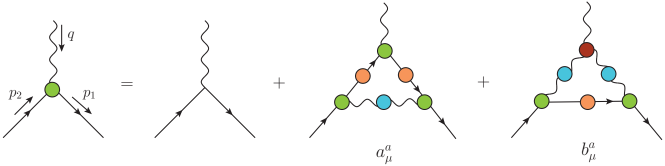
[5] A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, and A. Sternbeck, Phys. Rev. D 103,114515 (2021)

The quark-gluon vertex

- Determined the form factors of the transversely projected quark-gluon vertex in general kinematics, in the Landau gauge, with two degenerate light dynamical quarks ($N_f = 2$):

$$\overline{\Gamma}_\mu(q, p_2, -p_1) := P_{\mu\nu}(q)\Gamma^\nu(q, p_2, -p_1)\,, \quad P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

- Use the SDE - 3PI effective action, at the three-loop level[1 2]



  - ◇ Charge conjugation symmetry preserved

$$C\overline{\Gamma}_\mu(q, p_2, -p_1)C^{-1} = -\overline{\Gamma}^T_\mu(q, -p_1, p_2)$$

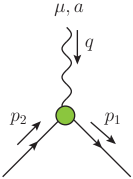
[1] R. Alkofer, C. S. Fischer, F. J. Llanes-Estrada, and K. Schwenzer, Annals Phys. 324, 106 (2009)
[2] R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D93, 034026 (2016)

- Decomposing the vertex in a basis in terms of the **form factors**:

$$\overline{\Gamma}_\mu(q,p_2,-p_1) = \sum_{i=1}^{8} \lambda_i(q,p_2,-p_1) P_{\mu\nu}(q)\tau_i^\nu(p_2,-p_1)$$

- with the tensorial elements of the basis given by:[1 2 3]

$$\begin{aligned}
\tau_1^\nu &= \gamma^\nu & \tau_2^\nu &= (p_1+p_2)^\nu \\
\tau_3^\nu &= (\not{p}_1+\not{p}_2)\gamma^\nu & \tau_4^\nu &= (\not{p}_1-\not{p}_2)\gamma^\nu \\
\tau_5^\nu &= (\not{p}_1-\not{p}_2)(p_1+p_2)^\nu & \tau_6^\nu &= (\not{p}_1+\not{p}_2)(p_1+p_2)^\nu \\
\tau_7^\nu &= -\frac{1}{2}[\not{p}_1,\not{p}_2]\gamma^\nu & \tau_8^\nu &= -\frac{1}{2}[\not{p}_1,\not{p}_2](p_1+p_2)^\nu
\end{aligned}$$



$q = p_1 - p_2$

- Separated in two subsets: chirally symmetric (cs) or chiral symmetry breaking (csb) - tensors with an odd (even) number of γ matrices:

$$\tau_{cs} = \{\tau_1^\nu, \tau_5^\nu, \tau_6^\nu, \tau_7^\nu\} \qquad \tau_{csb} = \{\tau_2^\nu, \tau_3^\nu, \tau_4^\nu, \tau_8^\nu\}$$

[1] M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D91, 054035 (2015)

[2] F. Gao, J. Papavassiliou, and J. M. Pawlowski, Phys. Rev. D 103, 094013 (2021)

[3] A. K. Cyrol, M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D97, 054006 (2018)

Charge conjugation symmetry

- The full vertex should respect the property:

$$C\overline{\Gamma}_\mu(q, p_2, -p_1)C^{-1} = -\overline{\Gamma}^T_\mu(q, -p_1, p_2)$$

- This will create constrains on the form factors:

Symmetric under the exchange $p_1 \to -p_2$:

$$\lambda_i(q, p_2, -p_1) = \lambda_i(q, -p_1, p_2)\,, \quad i = 1, 4, 6, 7, 8$$

Anti-symmetric:

$$\lambda_3(q, p_2, -p_1) = -\lambda_3(q, -p_1, p_2)$$

Mixture:

$$\lambda_2(q, p_2, -p_1) + 2\lambda_3(q, p_2, -p_1) = \lambda_2(q, -p_1, p_2)\,,$$
$$\lambda_5(q, p_2, -p_1) - \lambda_7(q, p_2, -p_1) = -\lambda_5(q, -p_1, p_2)\,.$$

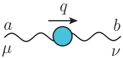
Simplification:

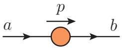
- $\overline{\Gamma}_{\mu}(q,p_2,-p_1) \to \lambda_1(q,p_2,-p_1)P_{\mu\nu}(q)\gamma^{\nu}$
- Equations decouple - one integral equation for λ_1, one integration depending on λ_1 for the others

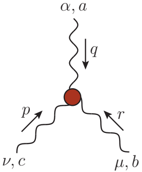
- Solve the equation in Euclidean space: $\lambda_i(q,p_2,-p_1) \to \lambda_i(p_1^2,p_2^2,\theta)$

$$\lambda_i(p_1^2,p_2^2,\theta) = Z_1\delta_{i1} + \underbrace{\int_{\mathrm{E}}\mathcal{K}_{i\mathbb{A}}\lambda_1^3}_{\text{Abelian}} + \underbrace{\int_{\mathrm{E}}\mathcal{K}_{i\mathbb{B}}\lambda_1^2}_{\text{Non-Abelian}}$$

- Quark and gluon propagator, and the three-gluon vertex - **external inputs**

$\Delta^{ab}_{\mu\nu}(q) =$ 

$S^{ab}(p) =$ 

$\Gamma^{abc}_{\alpha\mu\nu}(q,r,p) =$ 

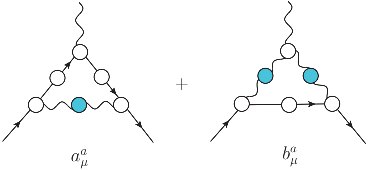
External inputs

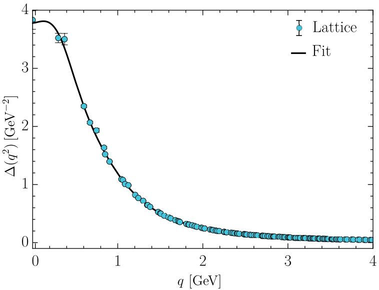
- **Gluon propagator**[1,2]:

$$\Delta_{\mu\nu}(q^2) = \Delta(q^2)P_{\mu\nu}(q)$$

- Gluon dressing function:

$$\mathcal{Z}(q^2) = q^2\Delta(q^2)$$



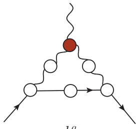


[1] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti, and J. Rodriguez-Quintero, Phys. Rev. D86, 074512 (2012)

[2] D. Binosi, C. D. Roberts, and J. Rodriguez-Quintero, Phys. Rev. D 95, 114009 (2017)

- **Three-gluon vertex**:
  - Appears transversally projected:

$$\overline{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) = P_{\alpha}^{\alpha'}(q)P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(p){\rm I\Gamma}_{\alpha'\mu'\nu'}(q,r,p)$$



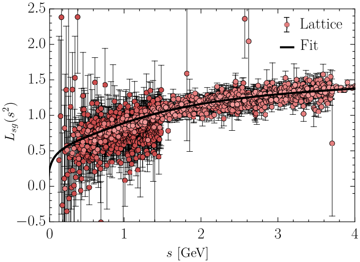
  - Planar degeneracy:

$$\overline{\rm I\Gamma}^{\mu\alpha\beta}(q,r,p) = L_{sg}(s^2)\overline{\Gamma}_0^{\mu\alpha\beta}(q,r,p)$$

    - The classical form factor is dominant and can be expressed in terms of one Bose-symmetric variable:[1 2 3]

$$s^2 = \frac{1}{2}(q^2 + r^2 + p^2)$$

    - Lattice data for $N_f = 2 + 1$[4]



[1] G. Eichmann, R. Williams, R. Alkofer, and M. Vujinovic, Phys. Rev. D89, 105014 (2014)

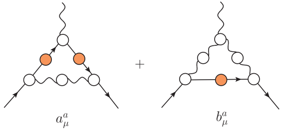
[2] R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D93, 034026 (2016)

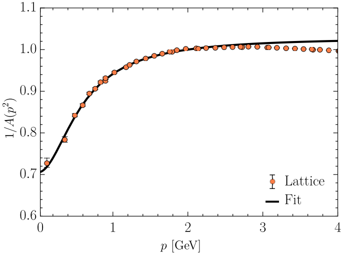
[3] F. Pinto-Gómez, F. De Soto, M. N. Ferreira, J. Papavassiliou, and J. Rodríguez-Quintero, Phys. Lett. B 838, 137737 (2023)

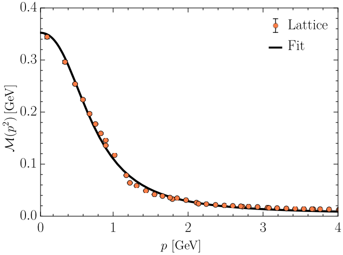
[4] A. C. Aguilar, F. De Soto, M. N. Ferreira, J. Papavassiliou, J. Rodríguez-Quintero, and S. Zafeiropoulos, Eur. Phys. J. C80, 154 (2020)

External inputs

- **Quark-propagator**: $S^{-1}(p) = A(p^2)\not{p} - B(p^2)$
  - Quark mass: $\mathcal{M}(p^2) = B(p^2)/A(p^2)$
  - Setup "L08" from lattice[1 2]
    - $m_q = 6.2$ MeV, and $m_\pi = 280$ MeV







[1] A. Kızılersü, O. Oliveira, P. J. Silva, J. I. Skullerud, and A. Sternbeck, Phys. Rev. D 103, 114515 (2021)
[2] O. Oliveira, P. J. Silva, J.-I. Skullerud, and A. Sternbeck, Phys. Rev. D 99, 094506 (2019)

Solution of the equation

$$\lambda_i(p_1^2, p_2^2, \theta) = Z_1\delta_{i1} + \int_{\mathrm{E}} \mathcal{K}_{i\mathbb{A}}\lambda_1^3 + \int_{\mathrm{E}} \mathcal{K}_{i\mathbb{B}}\lambda_1^2$$

- Renormalization of the equation:
  - Renormalization scheme - $\widetilde{\text{MOM}}$[1]:

$$\Delta_R^{-1}(\mu^2) = \mu^2\,,\;\; A_R(\mu^2) = 1\,,\;\; \lambda_{1,R}^{sg}(\mu^2) = 1$$

  - Z_1 is obtained through the soft-gluon limit:

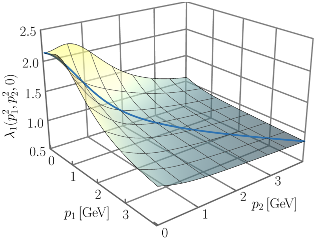
$$Z_1 = 1 - \lim_{q\to 0}\left[\int_{\mathrm{E}} \mathcal{K}_{1\mathbb{A}}\lambda_1^3 + \int_{\mathrm{E}} \mathcal{K}_{1\mathbb{B}}\lambda_1^2\right]_{p^2=\mu^2}$$

  - Renormalization point: $\mu = 2\,\text{GeV}$, $\quad \alpha_s(\mu) \equiv g^2/4\pi = 0.55$
- We solve the equation for the classical form factor, λ_1, iteratively
- With the result for λ_1 we solve the other seven equations as one integral

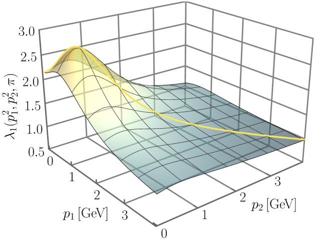
[1] J. Skullerud and A. Kizilersü, J. High Energy Phys. 09, 013 (2002)

Results

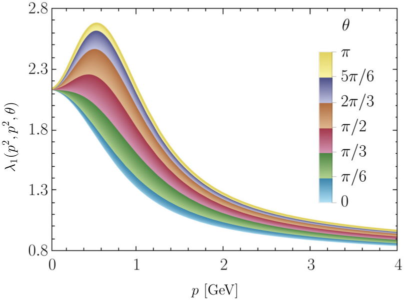
- 3D for $\theta = 0$ and soft-gluon config.:



- 3D for $\theta = \pi$ and asymmetric config.:



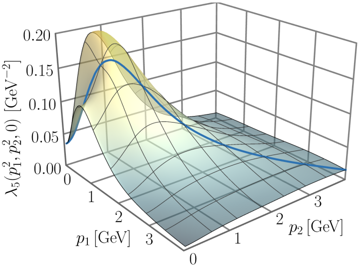
- **Strong** angular dependence:

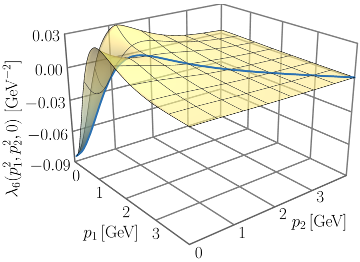


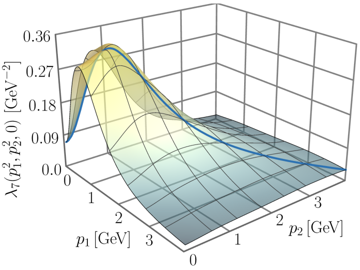
  - 27% difference between the peaks
  - Precludes planar degeneracy (see Alkofer's presentation this morning)
  - Non-Abelian contribution dominates

Results - non classical form factors

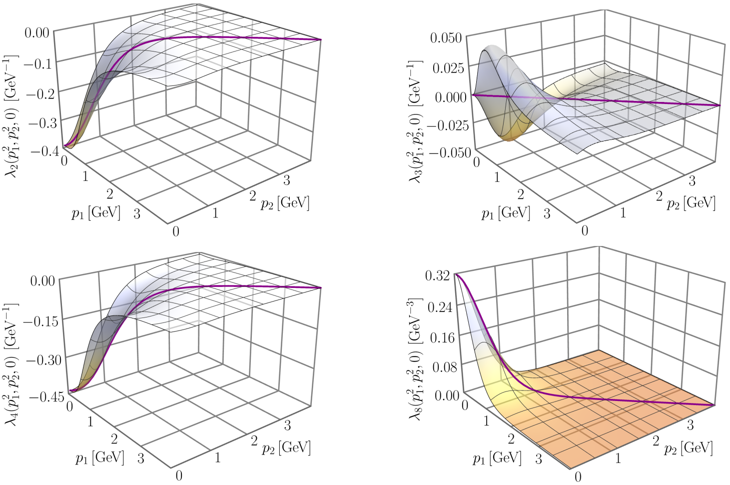
- With the results for λ_1 we compute the other form-factors as one integral
- Soft-gluon limit are highlighted in the diagonal
- The charge conjugation relations are all numerically satisfied
- Infrared finite and perturbative behaviour in the UV





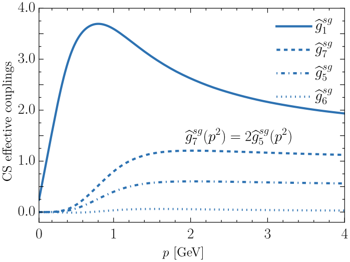


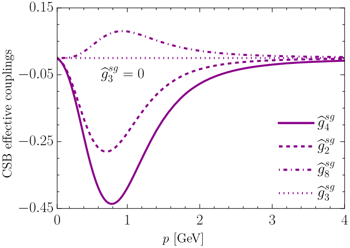
Results - non classical form factors



- λ_2 and λ_4 have the Abelian contribution very suppressed
- For the others, for $p \leq 1$ GeV, the Abelian contribution is significant, but still smaller in magnitude then the non-Abelian

Results - Effective coupling



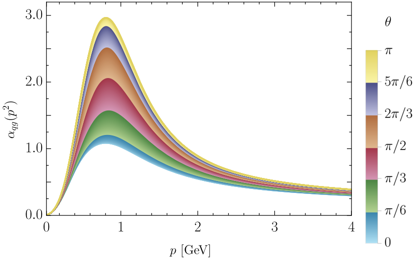


- Comparison through RGI combinations:[1,2]

$$\widehat{g}_i^{sg}(p^2) = g(\mu^2)\,[p^{n_i}\lambda_i^{sg}(p^2)]A^{-1}(p^2)\mathcal{Z}^{1/2}(p^2)$$

- Same hierarchy found in literature
- Effective charge: $\lambda_1^{sg}(p^2) \to \lambda_1(p^2, p^2, \theta)$

$$\alpha_{qg}(p^2,\theta) = \frac{\widehat{g}_1^2(p^2,\theta)}{4\pi}$$

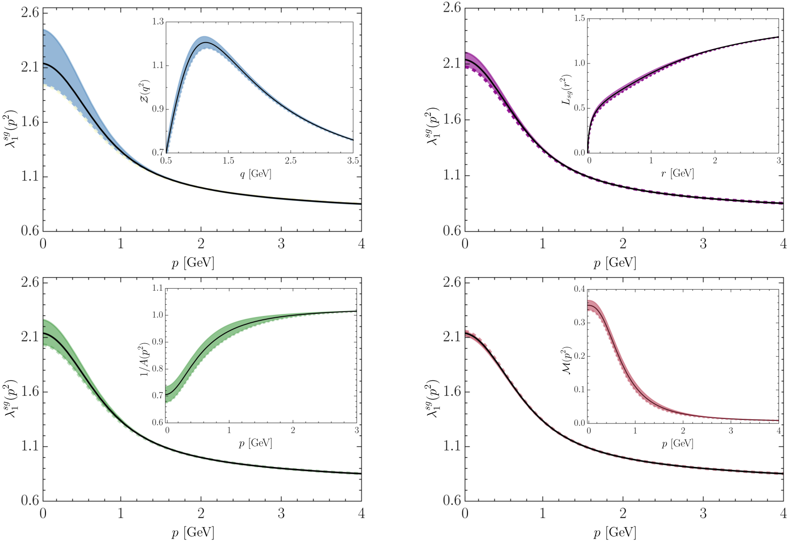


[1] M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D91, 054035 (2015)

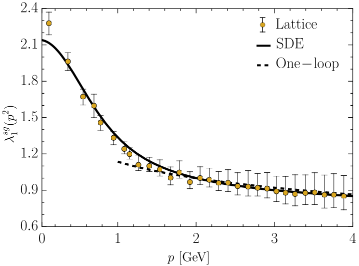
[2] F. Gao, J. Papavassiliou, and J. M. Pawlowski, Phys. Rev. D 103, 094013 (2021)

Results - Varying the inputs

- Varying the inputs $f = [\Delta, L_{sg}, A, \mathcal{M}]$ as: $f^{\pm}(q^2) = f(q^2) \pm \frac{\delta}{[1+(q^2/\kappa^2)^2]}$

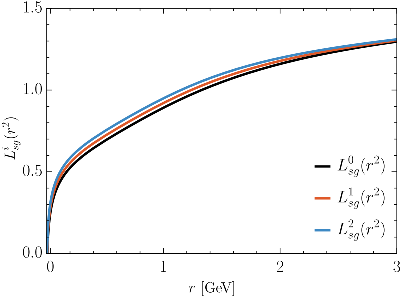


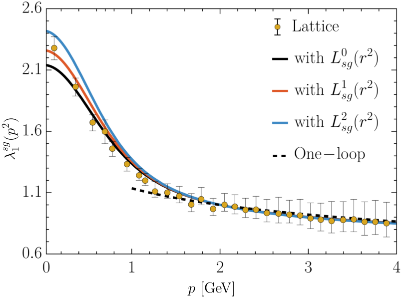
Results - Comparison with lattice



- The value of $\alpha_s = 0.55$ was adjusted to fit better the lattice data[1]
- Differ by 7% at the origin
- Small variations improves the coincidence

$$L^i_{sg}(r^2) = L_{sg}(r^2) + \frac{\epsilon_i}{1 + (r^2/\kappa^2)^2}$$





[1] A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, and A. Sternbeck, Phys. Rev. D 103,114515 (2021)

Conclusions

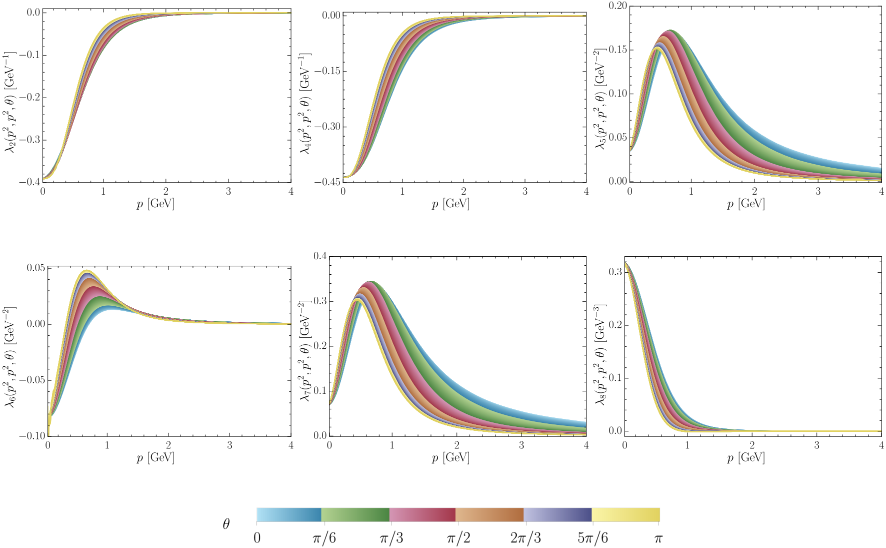
- In this work, we determined the form factors of the transversely projected quark-gluon vertex, employing reasonable approximations that simplified the numerical treatment of the equations, and preserved the vertex's charge conjugation symmetry
- Our findings reaffirmed several known properties of the vertex while also providing new insights and contributions
- We hope to use the results to address open issues in the near future

Thank you!

Backup slides

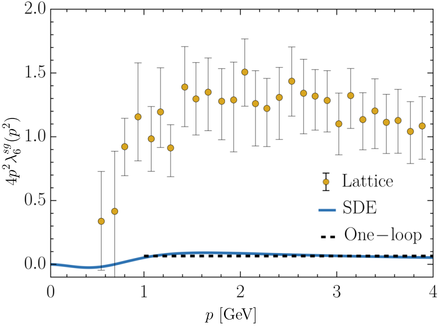
Results - non classical form factors

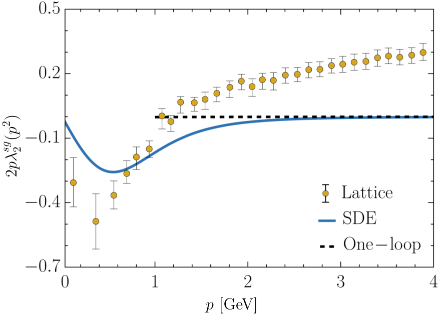
- Angular dependence is small for some form factors: λ_2, λ_4 and λ_8,
- and bigger for others: λ_5, λ_6 and λ_7



- In our notation the soft-gluon limit of the quark-gluon vertex is:

$$\overline{\rm I\Gamma}_\mu(0,p,-p) = \gamma_\mu \lambda_1^{sg}(p^2) + 2p_\mu \lambda_2^{sg}(p^2) + 4\not{p}\, p_\mu \lambda_6^{sg}(p^2)$$





Minkowski to Euclidean transformation

- We compare with literature that computes the vertex directly in Euclidean space imposing:[1]

$$\left[\overline{\Gamma}_\mu(q,p_2,-p_1)\gamma^\mu\right]_{\mathrm{E}} = \overline{\Gamma}^{\mathrm{E}}_\mu(q,p_2,-p_1)\gamma^\mu_{\mathrm{E}}$$

- In the transformation to Euclidean space we use:

$$\not{p} \to i\not{p}_{\mathrm{E}}\,, \qquad p^2 \to -p^2_{\mathrm{E}}\,, \qquad \lambda_i(q,p_2,-p_1) \to \lambda^{\mathrm{E}}_i(q^{\mathrm{E}},p^{\mathrm{E}}_2,-p^{\mathrm{E}}_1)$$

- An additional step may be required, depending on the conventions adopted for the Euclidean quark propagator: $p_{\mathrm{E}} \to -p_{\mathrm{E}}$[4]
    - ◇ In our notation: $S^{-1}_{0\mathrm{E}}(p_{\mathrm{E}}) = i\not{p}_{\mathrm{E}} - m_q$
    - ◇ In the references[2] [3]: $S^{-1}_{0\mathrm{E}}(p_{\mathrm{E}}) = i\not{p}_{\mathrm{E}} + m_q$.
- From this we obtain that the two following definitions are equivalent:

$$\overline{\Gamma}_\mu(0,p,-p) = \gamma_\mu\lambda^{sg}_1(p^2) + 2p_\mu\lambda^{sg}_2(p^2) + 4\not{p}\,p_\mu\lambda^{sg}_6(p^2)\,.$$

$$\overline{\Gamma}^{\mathrm{E}}_\mu(0,p_{\mathrm{E}},-p_{\mathrm{E}}) = \gamma^{\mathrm{E}}_\mu\lambda^{sg}_{1\mathrm{E}}(p^2_{\mathrm{E}}) - 2ip^{\mathrm{E}}_\mu\lambda^{sg}_{2\mathrm{E}}(p^2_{\mathrm{E}}) - 4\not{p}^{\mathrm{E}}p^{\mathrm{E}}_\mu\lambda^{sg}_{6\mathrm{E}}(p^2_{\mathrm{E}})\,.$$

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[1] J. Skullerud and A. Kizilersü, J. High Energy Phys. 09, 013 (2002)
[2] M. Mitter, J. M. Pawlowski, and N. Strodthoff, Phys. Rev. D91, 054035 (2015)
[3] A. Kızılersü, O. Oliveira, P. J. Silva, J.-I. Skullerud, and A. Sternbeck, Phys. Rev. D 103, 114515 (2021)
[4] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. 33, 477 (1994)