The high temperature QCD static potential

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Introduction

The static potential gives the **dominant** contribution between heavy quark and antiquark at low E

- At T=0 It is attractive and Coulomb like at small r (and linearly rising at large r)

- At high T, it is a Yukawa-like potential, with screening mass ~ T

J/Psi suppression at high T Matsui and Satz, '86, as bound states melt

The potential develops an imaginary part
Laine, Philipsen, Romatschke, Tassler, '07

(**damping** instead of **melting** of bound states)

Goal

Push the initial high T computations of the static potential

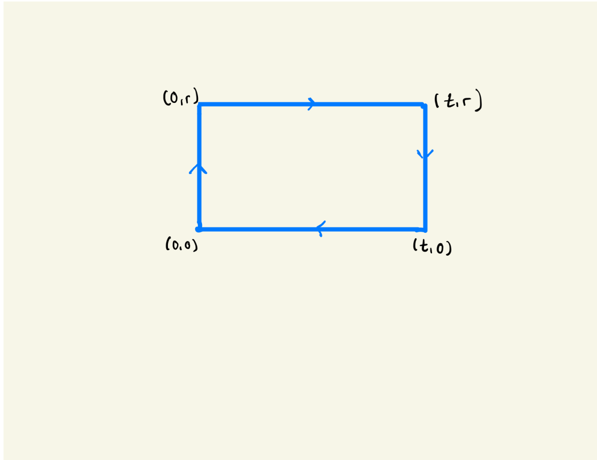
(computations were carried out only to leading order)

Provide physical motivated forms of the potential V, information that might be of great help for lattice

CALCULATIONAL METHOD

Potential obtained from the QCD Wilson loop

$$V(\mathbf{r}) = \lim_{t\to\infty} \frac{i}{t} \ln[W(t,\mathbf{r})]$$



$$W(t,\mathbf{r}) = \frac{1}{N_c} \left\langle \exp\left(ig\mathcal{P} \int dx^\mu A_\mu(x) \right) \right\rangle$$

(similar results if we apply standard diagrams in p space)

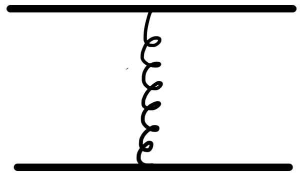
- Heavy quarks with M >> any other scale, unthermalized
- Thermal plasma, at high T - we use the real time formalism of TFT
- Computation carried out in Coulomb gauge; use of dimensional regularization

Leading order potential

Obtained from the time ordered longitudinal gluon propagator, when p << T

In momentum space

$$C_F = (N_c^2 - 1)/(2N_c)$$



$$V_{1\mathrm{lo}}(p) = g^2 C_F\, G(0,p)$$

$$G(0,p) = -\frac{1}{m_D^2 + p^2} + \frac{i\pi T m_D^2}{p\,(m_D^2 + p^2)^{\,2}}$$

$$m_D^2 = \frac{g^2 T^2 (N_c + N_f/2)}{3}$$

Debye mass squared

$$V_{1\mathrm{lo}}(r) = -\frac{g^2 C_F}{4\pi \hat{r}} \left(m_D e^{-\hat{r}} - 2iT\, I_2(\hat{r})\right)$$

$$\hat{r} = r m_D$$

Note: when screening is important $p \sim m_D$

$$\mathrm{Im}(V_{1\mathrm{lo}}) \gg \mathrm{Re}(V_{1\mathrm{lo}})$$

And then narrow resonances cannot exist

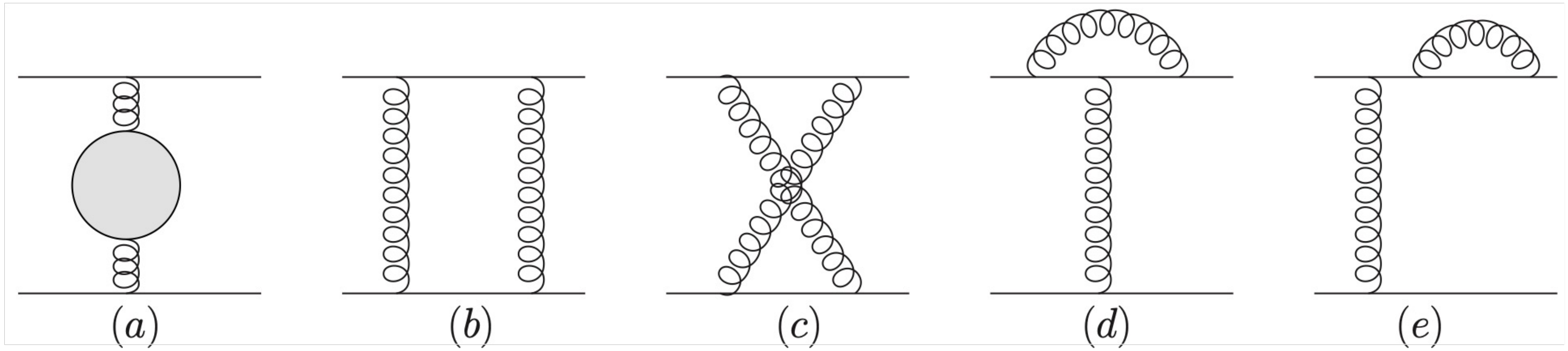
$\mathrm{Im}(V_{1\mathrm{lo}}) \sim \mathrm{Re}(V_{1\mathrm{lo}})$ for $p_d \sim (m_D^2 T)^{1/3} \sim g^{2/3} T$

Narrow resonances exist if the typical momentum is semi-hard

$$m_D \ll p \ll T$$

Potential beyond LO

Diagrams needed in the computation



We use the Hard Thermal Loop (HTL) effective field theory in the computation

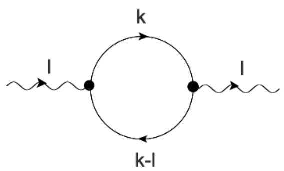
(Braaten and Pisarski, ’92)

When the momentum is soft ~ gT, resumed (and non-local) propagators and vertices have to be used

But (fortunately!) for semi-hard momentum enormous simplifications arise

HTL effective theory in QED and corrections

HTL physics corresponds to integrate out the scale T



$\Pi_{\rm HTL}(l) \sim g^2T^2$ $\qquad \frac{\Pi_{\rm HTL}(l)}{l^2} \sim 1$ for soft momentum

$$\mathcal{L}^{(1)}_{\rm HTL} = \frac{e^2}{2}\int \frac{d^3q}{(2\pi)^3}\left\{\frac{2n_F(q)}{q}\left(F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^2}F^\rho{}_\beta\right) - \frac{2(n_F(q)+n_B(q))}{q}\left(\bar\psi\frac{v\cdot\gamma}{(iv\cdot D)}\psi\right)\right\}$$

In QED , corrections to the HTL physics arise from

- Power corrections: p/T corrections to HTL

Stetina, CM, Soto, Carignano. '18
Carignano, Carrington, Soto , '19

- Two-loop corrections ~

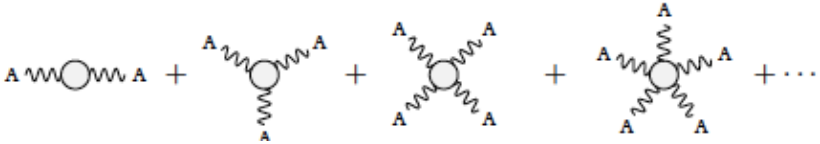
$$\mathcal{L}^{(3)\gamma}_{\rm HTL} = \frac{e^2\nu^{3-d}}{4}\int \frac{d^dq}{(2\pi)^d}\frac{1-2n_F(q)}{q^3}\left\{F_{\rho\alpha}\frac{v^\alpha v^\beta}{(v\cdot\partial)^4}\partial^4 F_\beta{}^\rho\right\}$$

$$\mathcal{L}^{pert} = -\frac{e^4T^2}{16\pi^2}\int\frac{d\Omega_v}{4\pi}F_{\rho\mu}\frac{v^\mu v^\nu}{(v\cdot\partial)^2}\left(\frac{1}{2}+\frac{\partial_0}{v\cdot\partial}\right)F_\nu{}^\rho$$

HTL theory in QCD and corrections

Not only the propagators are modified, also effective (non-local) propagators are needed

$$S^{+}_{\mathrm{HTL}} = -\frac{m_D^2}{4}\int_{x,v} \mathrm{Tr}\left(F_{\alpha\mu}\,\frac{v^\alpha v^\beta}{(v\cdot D)^2}\,F_\beta{}^\mu\right)$$

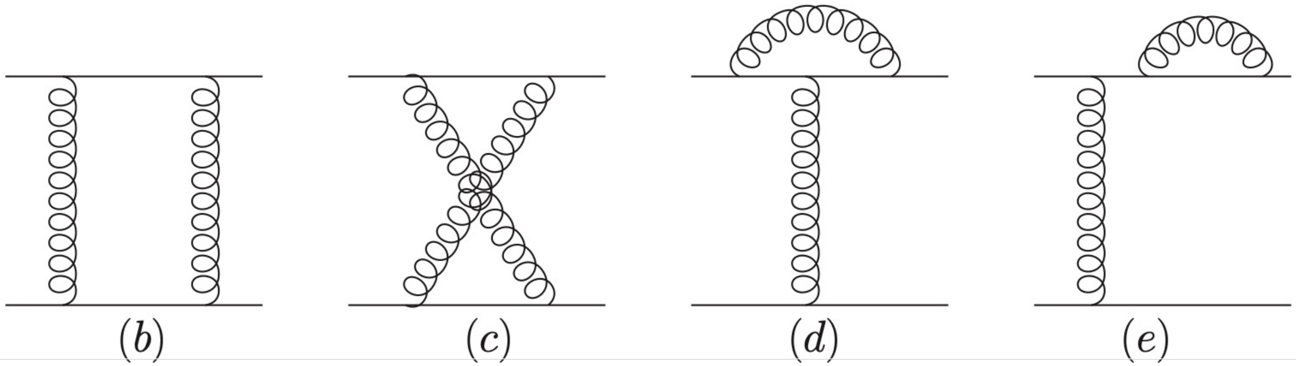


Corrections to HTL physics

- Diagrams with soft loop momentum (because gluon contribution is then Bose-enhanced), which are hard to compute as they need HTL are more important
- Power corrections p/T
- Two-loop diagrams

For semi-hard momentum the leading order term can be easily computed by expanding in (m/p)^2 (then one can use free classical propagators!)

and two-loops are subleading



In all the above diagrams p is semi-hard, k the internal momentum

These diagrams are dominated by $k \sim m_D$ (use of the HTL effective theory !)

But these diagrams can be computed in an expansion in k/p and m_D/p

Fully expanded potential in momentum space

$$V_{2,\exp}(p) = -\frac{g^4 C_F N_c T}{16\pi m_D p^2}\left(1 - \frac{3\pi^2}{16} + \frac{4\pi m_D}{p} + \frac{m_D^2}{p^2}\left(\frac{5\pi^2}{24} - \frac{4}{3}\right)\right)$$
$$-i\frac{g^4 C_F T^2}{16 p^4}\left(N_c\left(\frac{56}{3\pi} - \left(1 - \frac{3\pi^2}{16}\right)\frac{m_D}{p}\right) - \frac{4}{\pi}\left(N_c - \frac{N_f}{2}\right)\frac{p}{T}\right)$$

For $p = g^a T$ $\qquad \frac{1}{3} < a < \frac{2}{3}$

this holds up to corrections to the real part of order g^2

And to the imaginary part of order g^{3a} $\qquad (g^{3a}, g^{2-a})$

Keeping the momentum semi-hard: in coordinate space the potential is expected to be valid only for

$$r m_D \ll 1 \ll rT$$

Damped approximation:
keep in the gluon propagators unexpanded factors $1/(p^2 + m_D^2)$

which is more realistic for momenta getting close to the soft scale

In the damped approximation

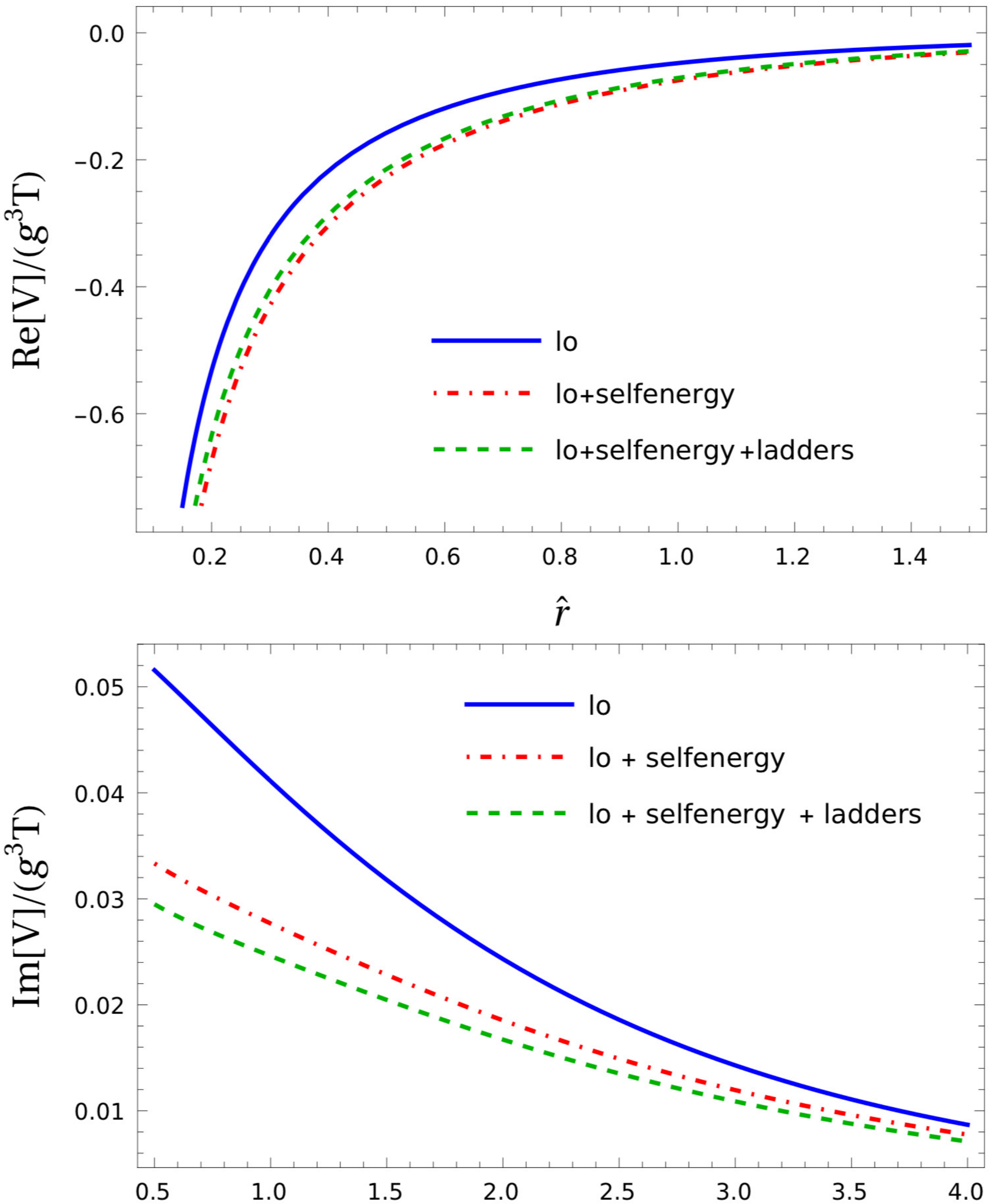
$$V_{1\mathrm{lo}}(r) = -\frac{g^2 C_F}{4\pi\hat{r}}\left(m_D e^{-\hat{r}} - 2iT\, I_2(\hat{r})\right) \qquad\qquad \hat{r} = r m_D$$

$$\mathrm{Re}[V_2] = \frac{g^4 N_c C_F T}{64\pi^2\hat{r}}\left\{8\left(I_2(\hat{r}) - I_1(\hat{r})\right) + \frac{e^{-\hat{r}}}{16}\left(3\pi^2 - 16 + \frac{\hat{r}}{6}\left(16 - \pi^2\right)\right)\right\}$$

$$i\mathrm{Im}[V_2] = -i\frac{g^3 C_F\, T}{16\pi^2\hat{m}_D}\left\{\frac{3\pi^2 - 16}{32\,\hat{r}} I_2(\hat{r}) + \frac{7}{3}\, N_c e^{-\hat{r}} - \frac{2g\hat{m}_D}{\pi\hat{r}}\left(N_c - \frac{N_f}{2}\right)\left(I_1(\hat{r}) - I_2(\hat{r})\right)\right\}$$

$$m_D = gT\hat{m}_D$$

$$I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \sin\left(\hat{p}\hat{r}\right)\left(\hat{p}^2 + 1\right)^{-j}$$


Re[V]/(g^3T)
lo
lo+selfenergy
lo+selfenergy+ladders
$\hat{r}$
Im[V]/(g^3T)
lo
lo + selfenergy
lo + selfenergy + ladders

$g = 1.8$

Comparison with lattice data

Our potential in p space describes the semi-hard scales
The damped approximation allows us to get closer to r ~ 1/m_D

For scales r << 1/m_D we miss the contribution of the soft modes p ~ m_D

e a universal form ~ a polynomial in r (up to logs), as one can expand in the Fo

$e^{i\vec{p}\cdot\vec{r}}$

We add contributions to the coordinate potential arising from the soft modes

$$\mathrm{Re}[V_{\mathrm{soft}}] = C + g^3 q_0 T$$

$$\mathrm{Im}[V_{\mathrm{soft}}] = g^3 i_0 T + g^5 i_2 r^2 T^3$$

C adjusts the origin of energies;
We obtain the coefficients by fitting to lattice

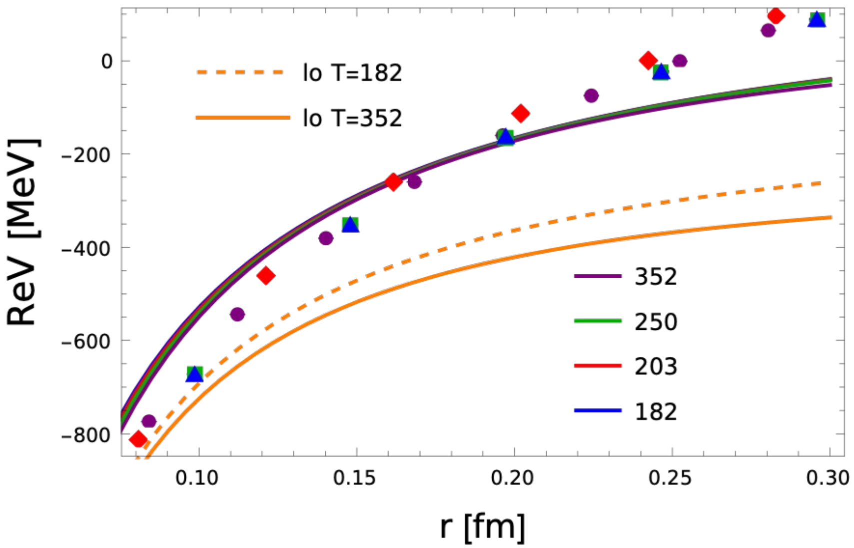
Potential [1]. Bzavov, Hoying, Kaczmarek, Larsen, Mukherjee, Petreczky, Rothkopf, Weber, 23

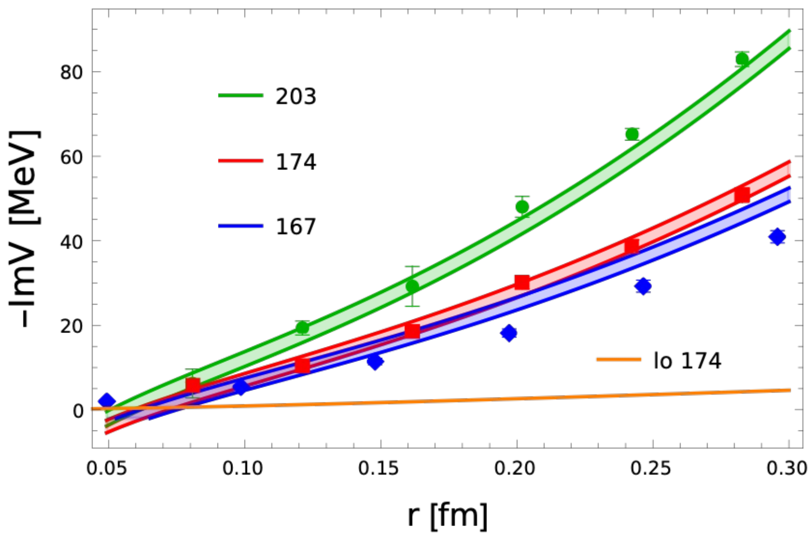
Mass spectrum and widths [2] Larsen, Meine, Mukherjee, Petreczky, 20

Use g =1.8 from fit T=0 lattice data;   bottom quark mass M = 4676 MeV

Find fitted constants to all available T (solid bands are uncertainties in the fitting with lattice [1])

$$(q_0, i_0, i_2) = (0.049, -0.021 \pm 0.002, 0.205 \pm 0.001) \qquad C = 219\,\mathrm{MeV}$$





The real part of V depends very little on T (like the lattice data)

The imaginary part of the V has big contribution from the soft region

We solve the Schrödinger equation using the real part of the potential and find the binding en

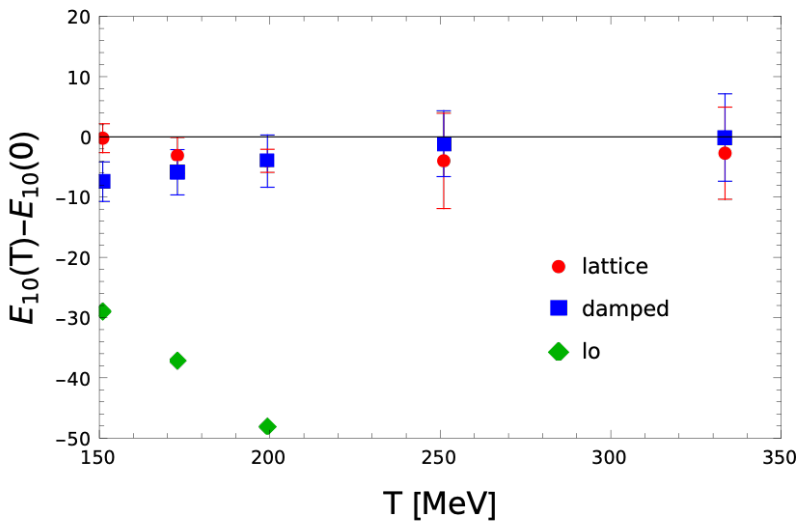
Decay rates are found as

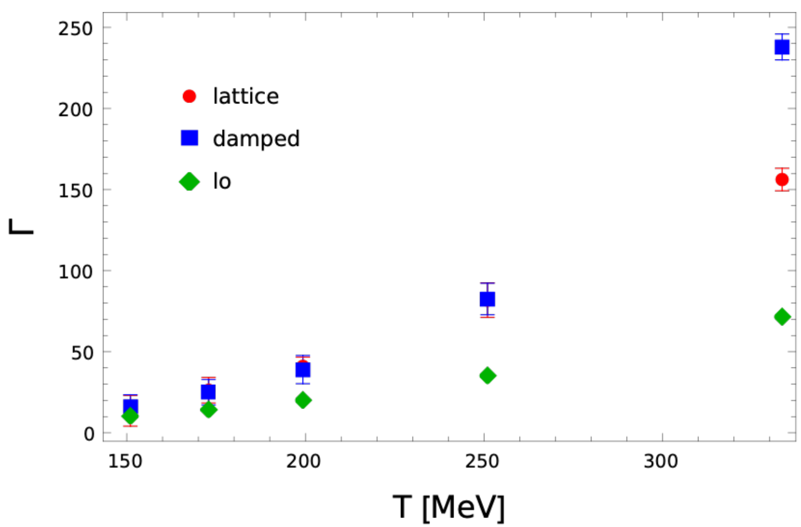
$$\Gamma = -2\langle \mathrm{Im} V \rangle$$

Fitting coefficients found by fitting with data in [2] (C plays no role …)

$$(q_0, i_0, i_2) = (0.078 \pm 0.004, -0.026 \pm 0.009, 0.053 \pm 0.002)$$

Similar values as with [1], except for that of i_2





Dissociation Temperature

It can be defined as the temperature at which the binding energies equal the decay width

At LO $T_d = 193\,\mathrm{MeV}$

BLO, with the fit to [1] $T_d = 151.8 \pm 1.2\,\mathrm{MeV}$

BLO, with the fit to [2] $T_d = 225 \pm 10\,\mathrm{MeV}$

(due to the different values in fitting coefficient; it suggests a problem with the data)

CONCLUSIONS

- We have computed corrections to the LO static potential in QCD in momentum space , valid in the regime $m_D \ll p \ll T$

- We have provided the static potential in configuration space, including pieces due to soft modes (which are universal)

- Reasonable description of lattice data