QuantFunc

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Burjasot Campus quantfunc2024@gmail.com

QUANTFUNC VALENCIA 2024

Distribution and fragmentation functions of light and heavy mesons

in collaboration with Fernando Serna, Roberto Correa da Silveira and Gastão Krein

Bruno El-Bennich

Departamento de Física
Universidade Federal de São Paulo

UNIFESP
UNIVERSIDADE FEDERAL DE SÃO PAULO
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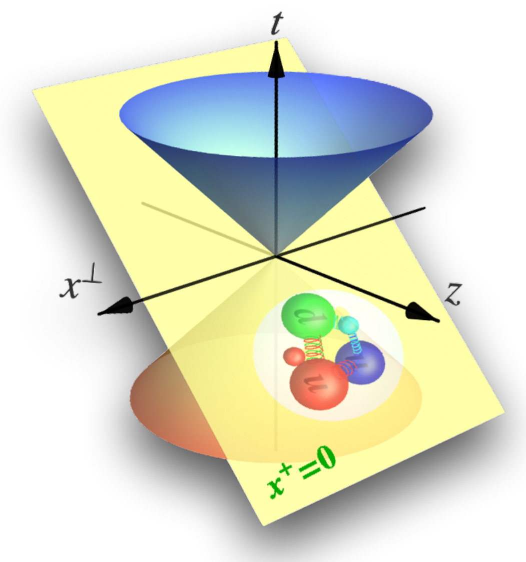
Before coming to the meat of the talk, namely Light Front Wave Functions, let's motivate them with LCDAs ...



Light-Cone Distribution Amplitudes

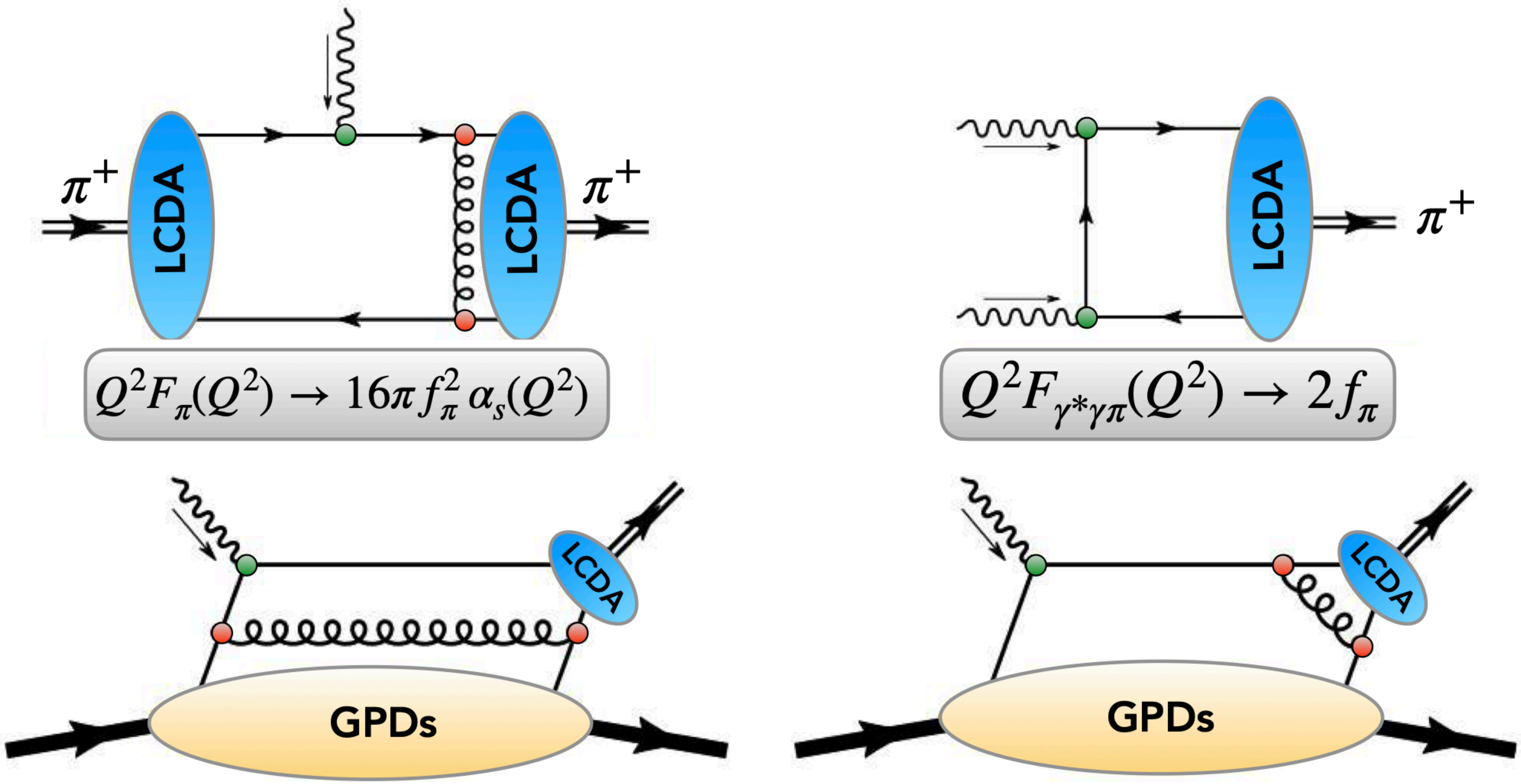
- Hadron light-cone distribution amplitudes (LCDAs) have been introduced four decades ago in the context of the QCD description of hard exclusive reactions.

- The LCDAs are scale-dependent nonperturbative functions that can be interpreted as quantum-mechanical amplitudes.

- $\phi_M(x, \mu)$ is a probability amplitude that describes the momentum distribution of a quark and anti-quark in the bound-state's valence Fock state.

- x is the light-front momentum fraction: $\frac{k^+}{P^+}$ and μ is the renormalization scale.

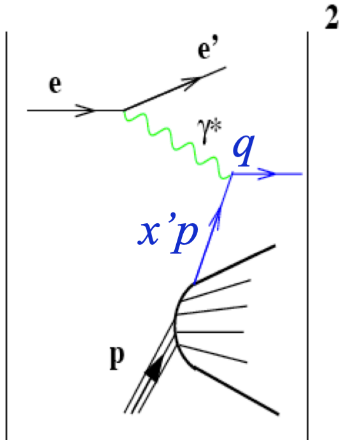
Hard exclusive scattering processes



DIS & Parton Distribution Functions

- Assumes fast moving hadron appears as a jet of partons moving in same direction and sharing its total momentum.
- DIS cross section is an incoherent sum of elastic scattering cross sections off individual partons.
- Parton model should work perfectly for $Q^2 \rightarrow \infty$ where coupling constant vanishes.

$$d\sigma \equiv \sum_i \int dx_i q_2^2$$



$$\mathrm{d}\sigma = \sum_i \mathrm{d}\hat{\sigma}(\hat{s}, \hat{t}, \hat{u})\ f_i(x)\mathrm{d}x$$

Define parton momentum distribution: $f_i(x) \equiv \dfrac{dP_i}{dx}$ where $\sum_i \int_0^1 dx f_i(x) = 1$

This defines the probability to find a parton with light-front momentum fraction $x = \dfrac{k^+}{P^+}$ of the hadron. PDF related to structure functions:

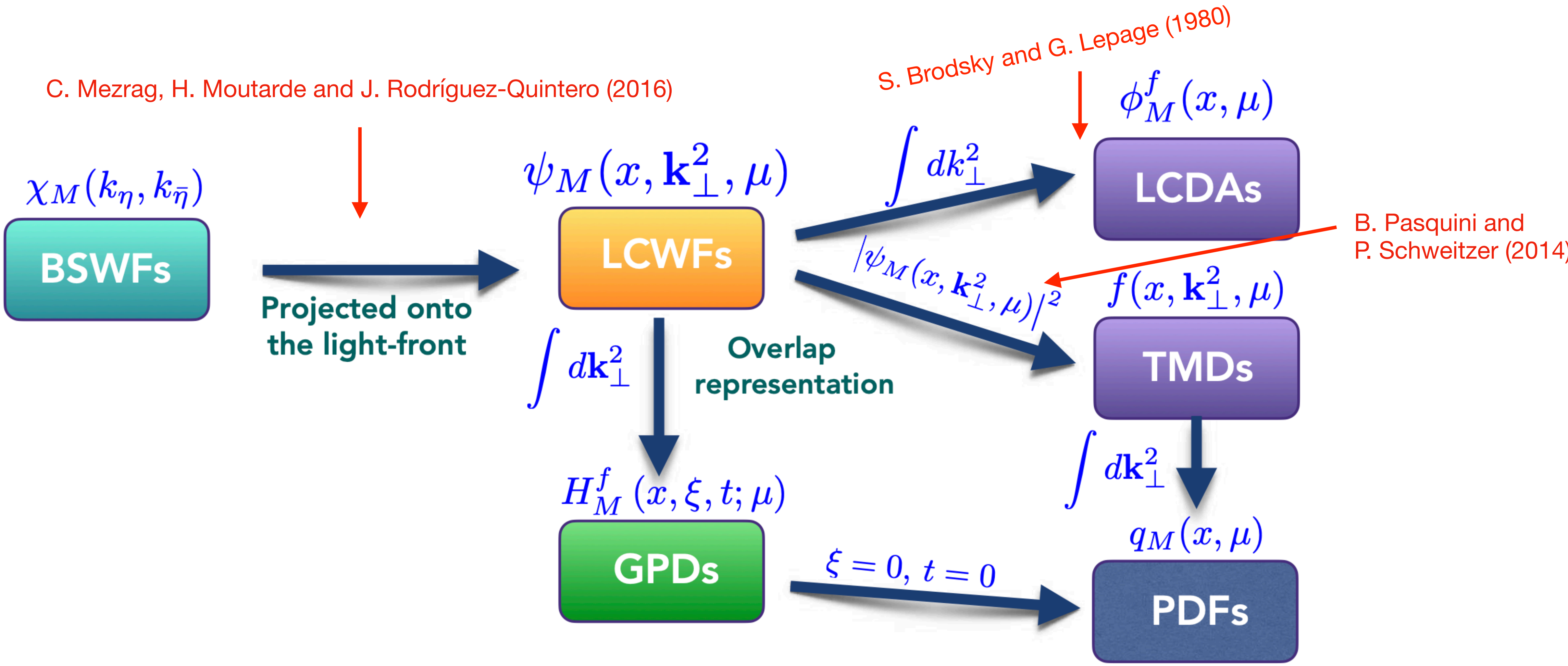
$$F_2(x) = \sum_i q_i^2 x f_i(x)$$

$$F_1(x) = \frac{1}{2x} F_2(x) \quad \text{Callan-Gross relation}$$

What if we could obtain all kind of distribution functions from one unified calculation ?

Light-Front Wave Functions

With a particular projection of the Bethe-Salpeter wave functions we arrive the light-front wave functions, a more general object to describe probability amplitudes.



Light-Front Wave Functions

- M. Burkardt, X.-D. Ji, and F. Yuan (2002) showed that for pseudoscalar mesons there are two independent light front wave functions for the leading Fock state, with $l_z = 0$ and $l_z = 1$.
- Minimal $\bar{q}q$ Fock-state configuration is given by: $|M\rangle = |M\rangle_{l_z=0} + |M\rangle_{|l_z|=1}$

$$|M\rangle_{l_z=0} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_0\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) - b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

$$|M\rangle_{|l_z|=1} = i \int \frac{d^2\boldsymbol{k}_T}{2(2\pi)^3} \frac{dx}{\sqrt{x\bar{x}}} \psi_1\left(x, \boldsymbol{k}_T^2\right) \frac{\delta_{ij}}{\sqrt{3}} \frac{1}{\sqrt{2}} \left[k_T^- b^\dagger_{f\uparrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\uparrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right) + k_T^+ b^\dagger_{f\downarrow i}\left(x, \boldsymbol{k}_T\right) d^\dagger_{h\downarrow j}\left(\bar{x}, \overline{\boldsymbol{k}}_T\right)\right] |0\rangle$$

Light-Front Wave Functions

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The LFWFs are obtained from the Bethe-Salpeter wave function via the light front projections:

$$\psi_0\left(x, \mathbf{k}_\perp^2\right) = \sqrt{3}i \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[\gamma^+\gamma_5\chi(k,P)\right]$$

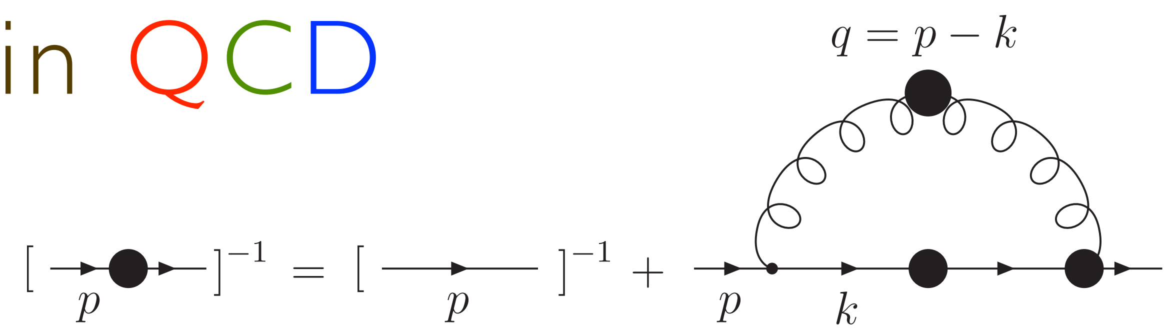
$$\psi_1\left(x, \mathbf{k}_\perp^2\right) = -\frac{\sqrt{3}i}{\mathbf{k}_\perp^2} \int \frac{dk^+ dk^-}{\pi} \delta\left(xP^+ - k^+\right) \mathrm{Tr}_D\left[i\sigma_{+i}k_T^i\gamma_5\chi(k,P)\right]$$

C. Mezrag, H. Moutarde, and J. Rodriguez-Quintero, Few Body Syst. 57 (2016)
C. Shi and I. C. Cloët, Phys. Rev. Lett. 122, 082301 (2019)

$$\left(\Box_x + m^2\right) G(x,y) = -\delta(x-y)$$

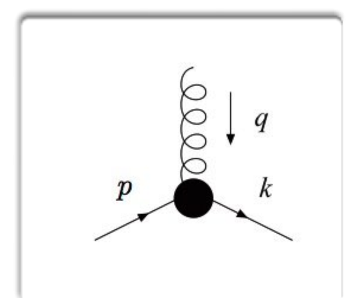
Green functions in functional approaches to QCD

Quark-Gap Equation in QCD



The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4 q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu S(q)\, \Gamma^a_\nu(q,p)
\end{aligned}
$$



with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$D_{\mu\nu}$: dressed-gluon propagator

$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

Z_2 : quark wave function renormalization constant

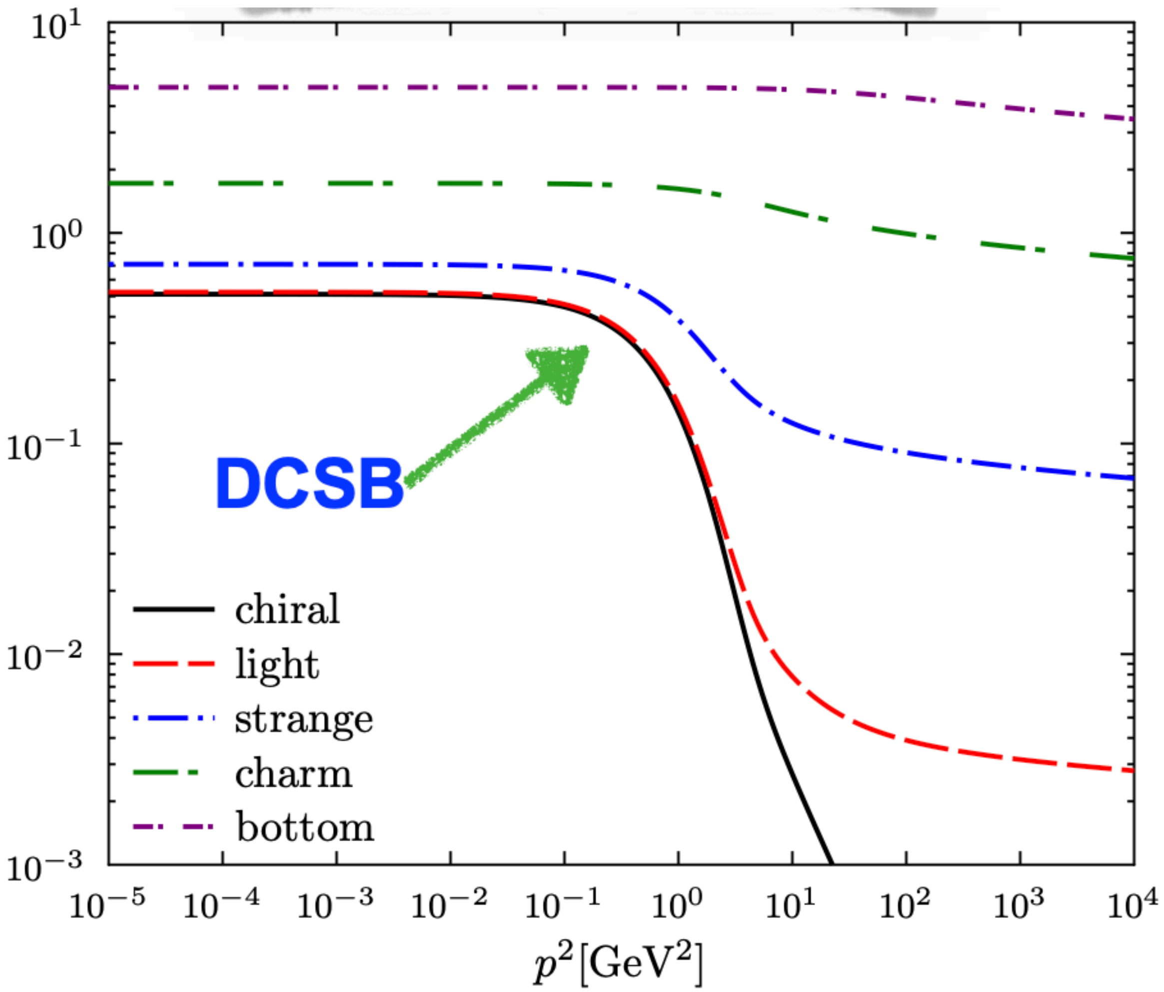
Z_1 : quark-gluon vertex renormalization constant

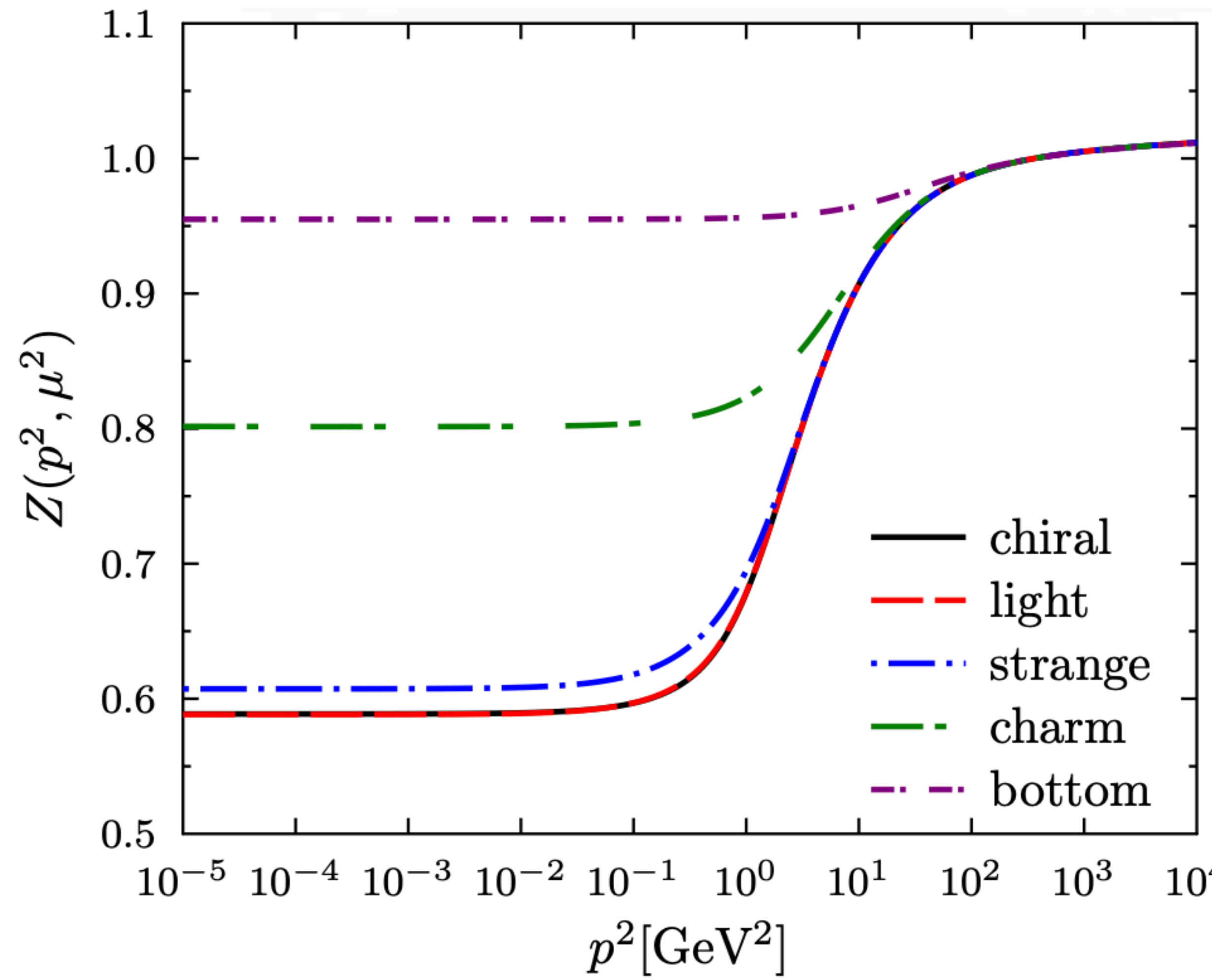
Each satisfies it's own DSE !

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

DSE solutions — flavor dependence





$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

L. Albino, A. Bashir, B.E., L.X. Gutiérrez Guerrero, E. Rojas PRD 100 (2019)

L. Albino, A. Bashir, B.E., E. Rojas, F. E. Serna, R.C. Silveira, JHEP11 (2021)

J. R. Lessa, F. E. Serna, B.E., A. Bashir, O. Oliveira, PRD 107 (2023)

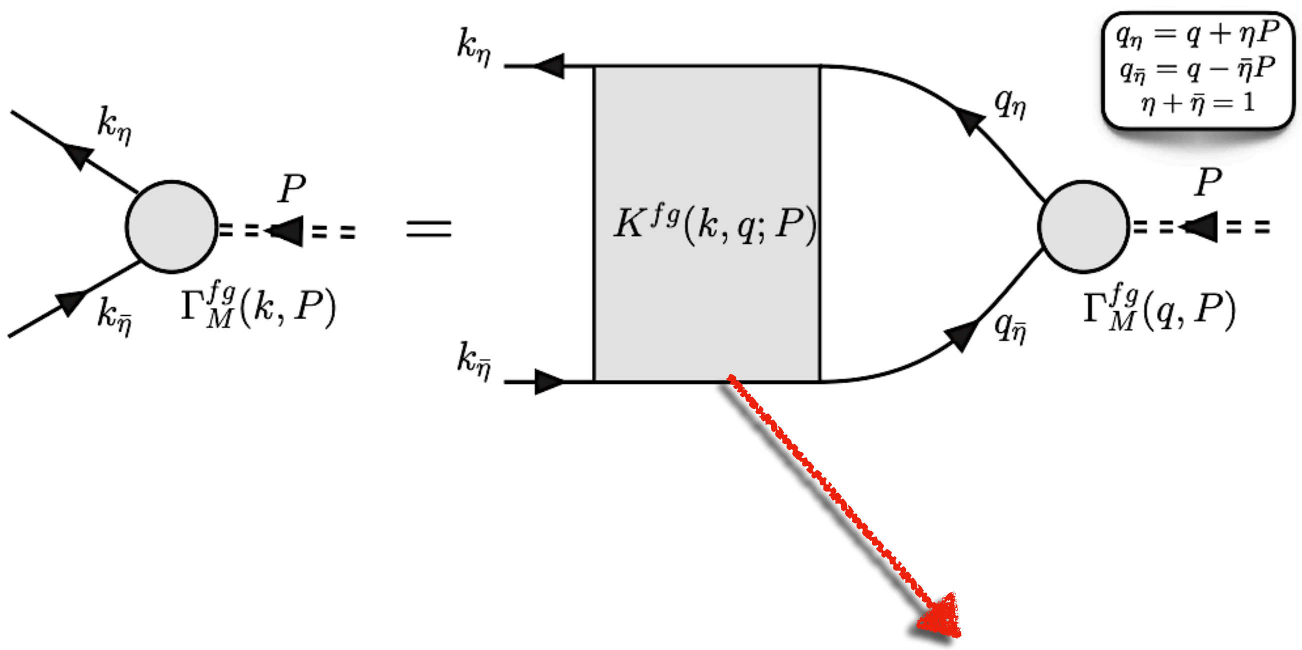
Bethe-Salpeter Equation for QCD Bound States

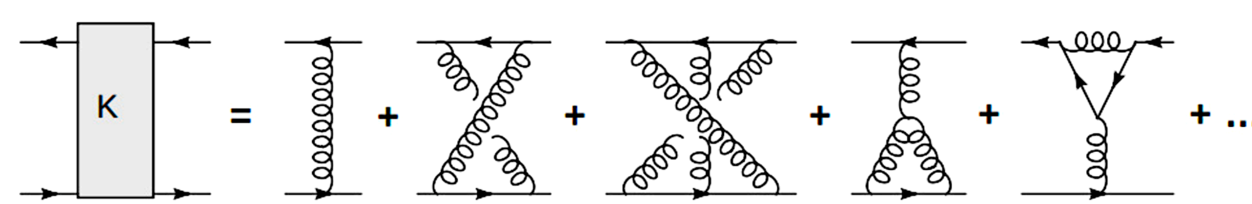
$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)





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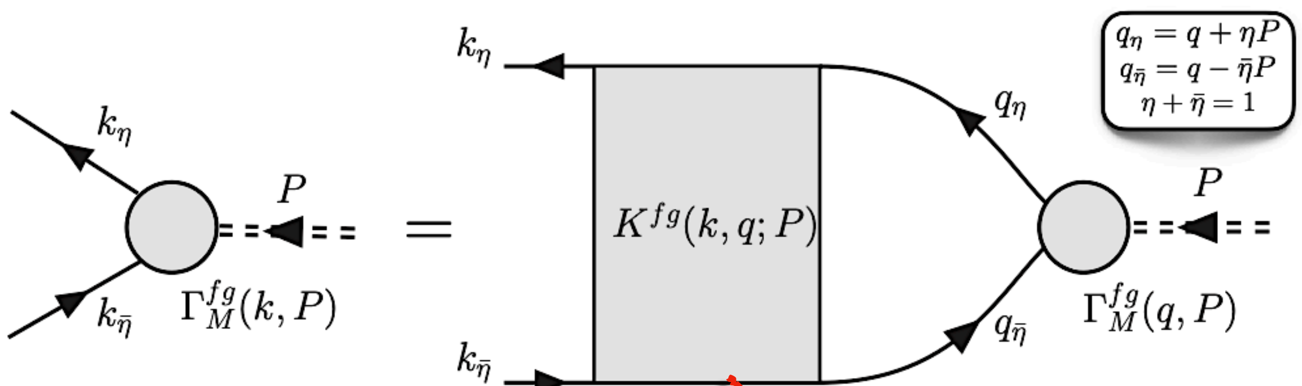
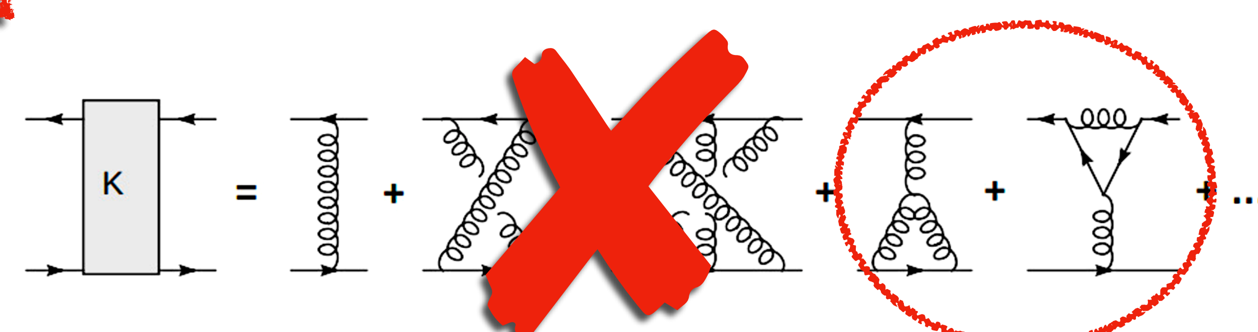


Treated with kernel beyond RL introducing flavor dependence

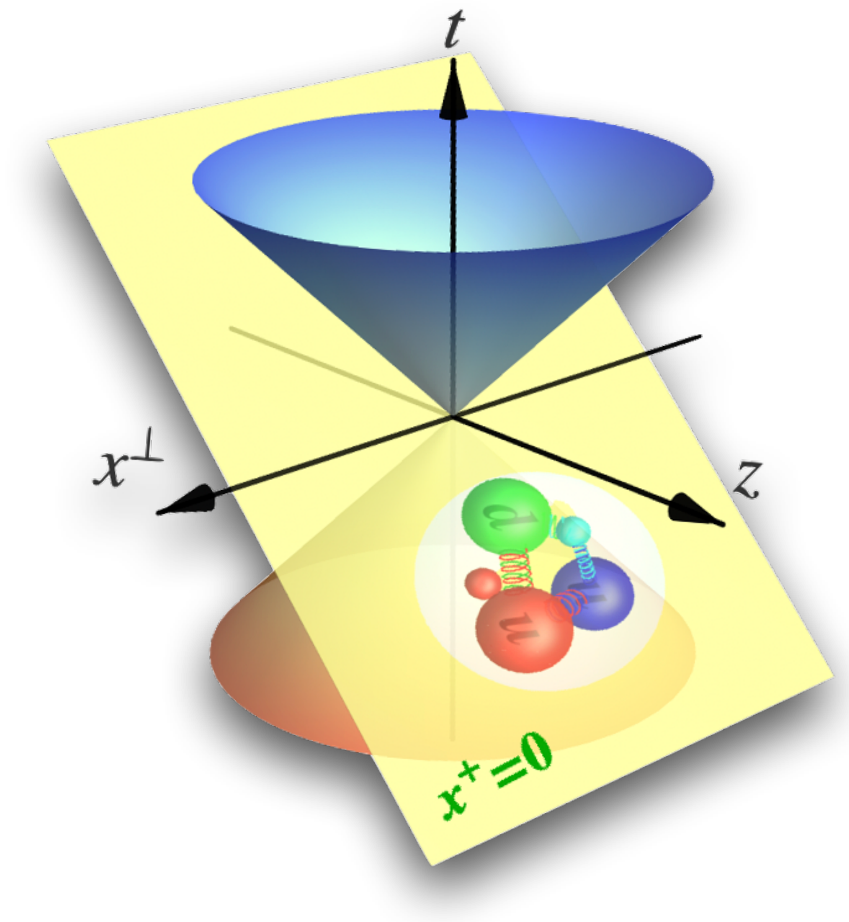


Bethe-Salpeter Amplitudes

- The general form of $\Gamma_M(k;P)$ is given by

$$\Gamma(k;P) = \sum_{i=1}^{N} \mathcal{T}^i(k,P)\mathcal{F}_i(k^2, z_k, P^2), \quad z_k = k.P/|k||P|,$$

- where $\mathcal{T}^i$(k,P) are Dirac's covariants;
- $\mathcal{F}_i(k^2, z_k, P^2)$ are Lorentz invariant amplitudes;
- N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$



Light-Front Wave Functions

Light-Front Wave Functions

- The **LCWFs** are obtained from the **BSWF** via the light front projections:

$$\psi_0\left(x,\mathbf{k}_\perp^2\right)=\sqrt{3}i\int\frac{dk^+dk^-}{\pi}\delta\left(xP^+-k^+\right)\mathrm{Tr}_D\left[\gamma^+\gamma_5\chi(k,P)\right]$$

$$\psi_1\left(x,\mathbf{k}_\perp^2\right)=-\frac{\sqrt{3}i}{\mathbf{k}_\perp^2}\int\frac{dk^+dk^-}{\pi}\delta\left(xP^+-k^+\right)\mathrm{Tr}_D\left[i\sigma_{+i}k_T^i\gamma_5\chi(k,P)\right]$$

- With the **LCWF** one can readily derive two distributions:

  - The leading-twist **TMD** [B. Pasquini and P. Schweitzer (2014)]

$$f_M\left(x,\mathbf{k}_\perp^2,\mu\right)=\frac{1}{(2\pi)^3}\left|\psi_M^0\left(x,\mathbf{k}_\perp^2,\mu\right)+\mathbf{k}_\perp^2\psi_M^1\left(x,\mathbf{k}_\perp^2,\mu\right)\right|^2$$

  - The **PDF**

$$q_M(x,\mu)=\int d\mathbf{k}_\perp^2 f_M(x,\mathbf{k}_\perp^2,\mu)$$

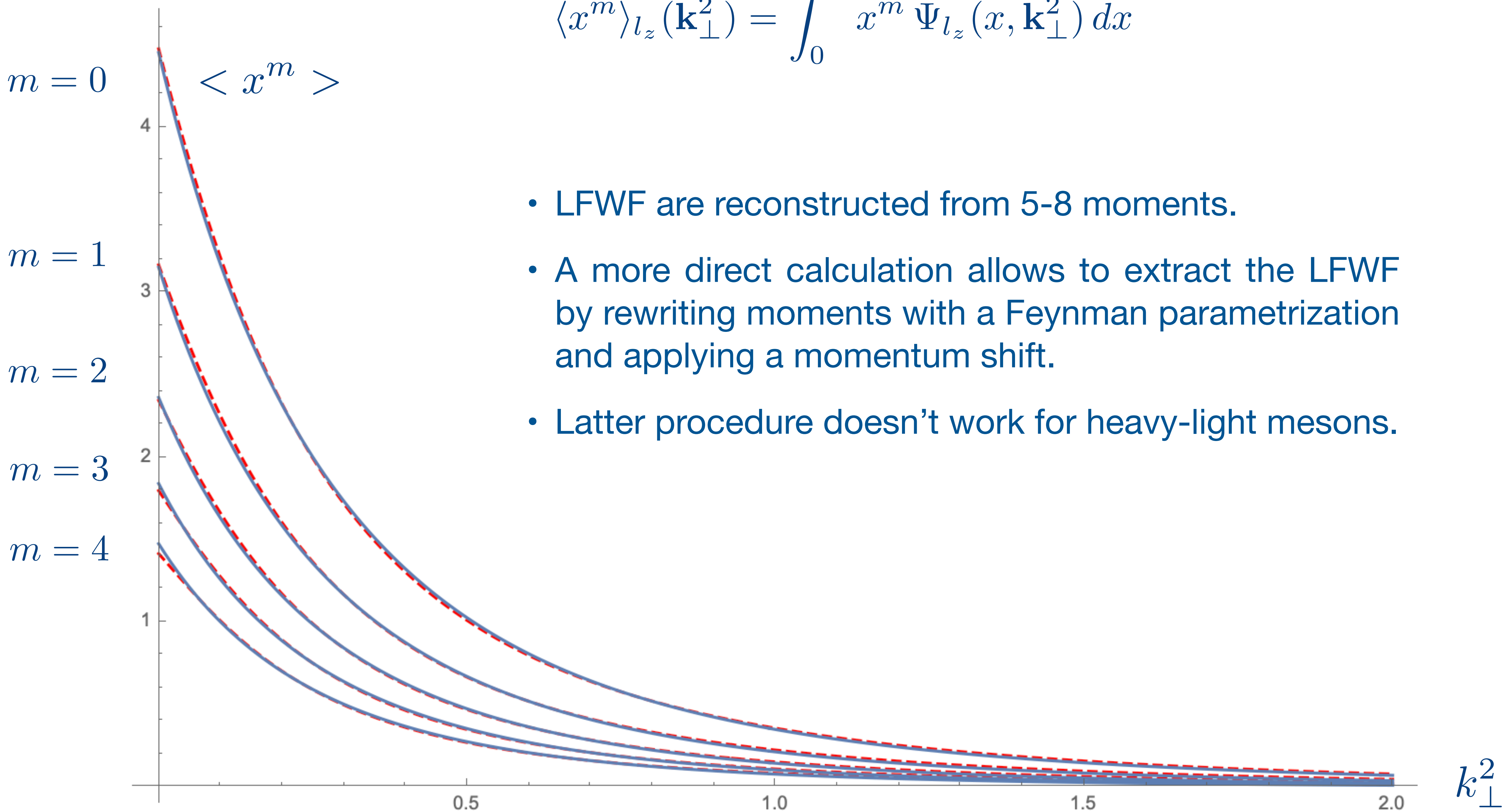
Normalization:

$$\int_0^1 dx\, q_M\left(x,\mu\right)=1.$$

Light-Front Wave Functions

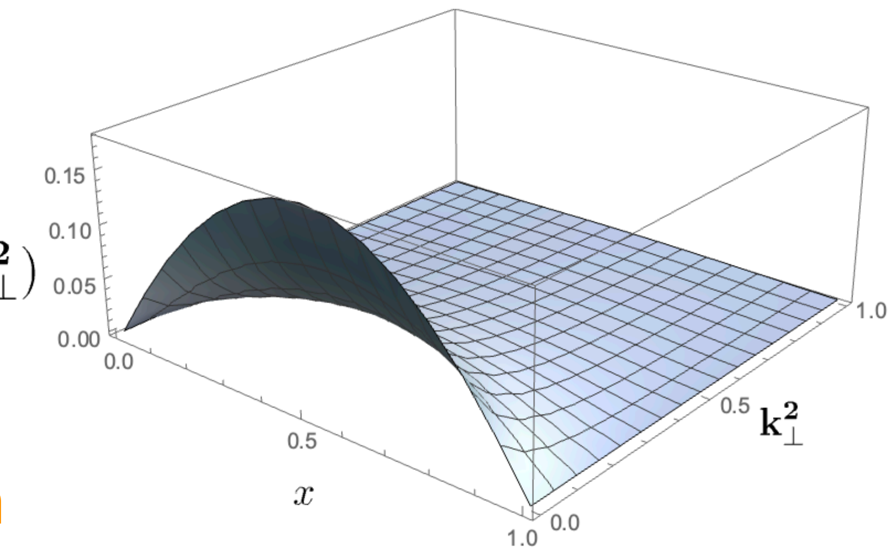
We calculate transverse momentum dependent moments:

$$\langle x^m \rangle_{l_z}(\mathbf{k}_\perp^2) = \int_0^1 x^m \, \Psi_{l_z}(x, \mathbf{k}_\perp^2) \, dx$$

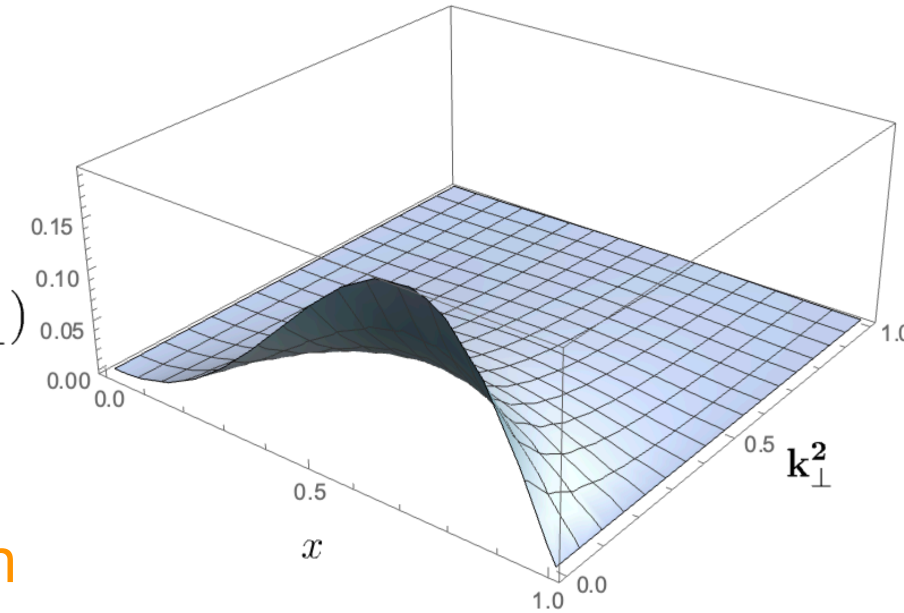


- LFWF are reconstructed from 5-8 moments.
- A more direct calculation allows to extract the LFWF by rewriting moments with a Feynman parametrization and applying a momentum shift.
- Latter procedure doesn't work for heavy-light mesons.

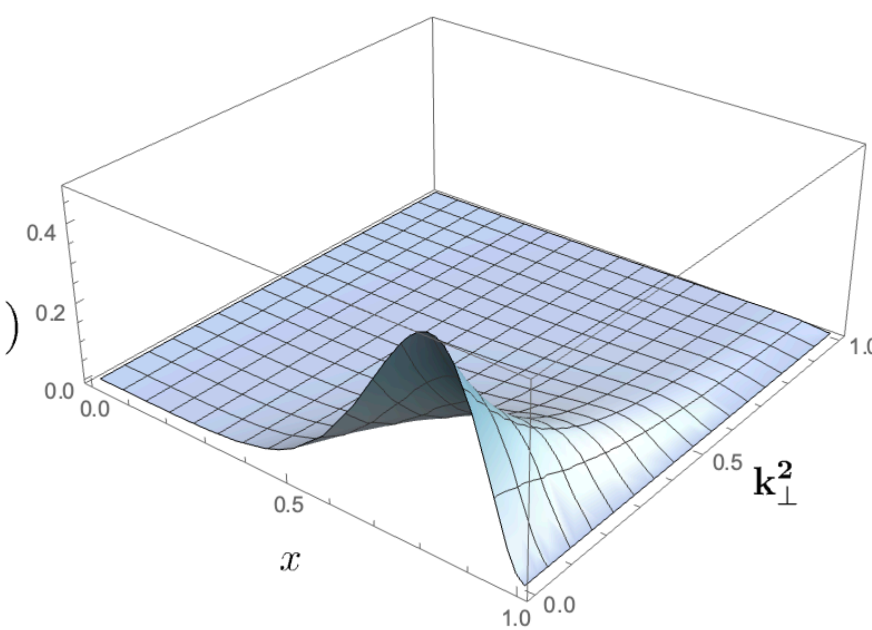
Transverse Momentum Distributions



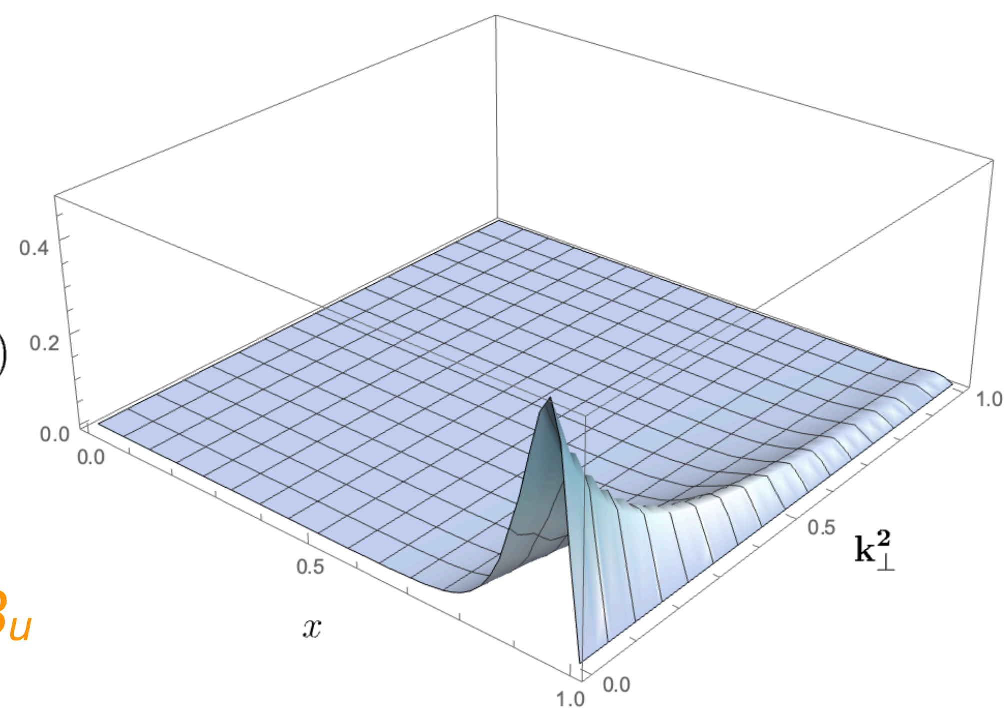
Pion



Kaon

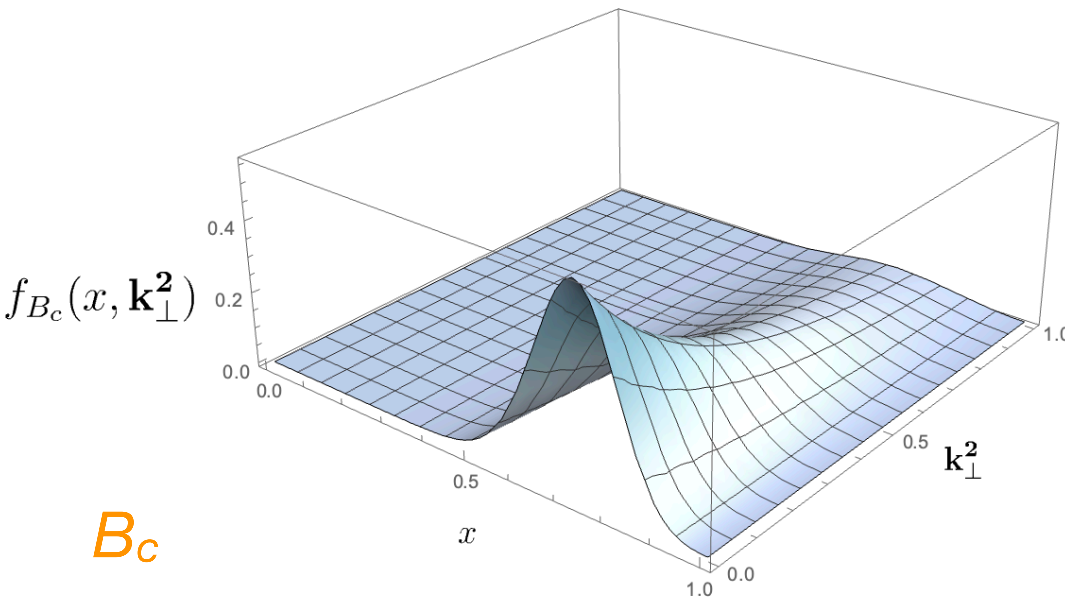


D_u

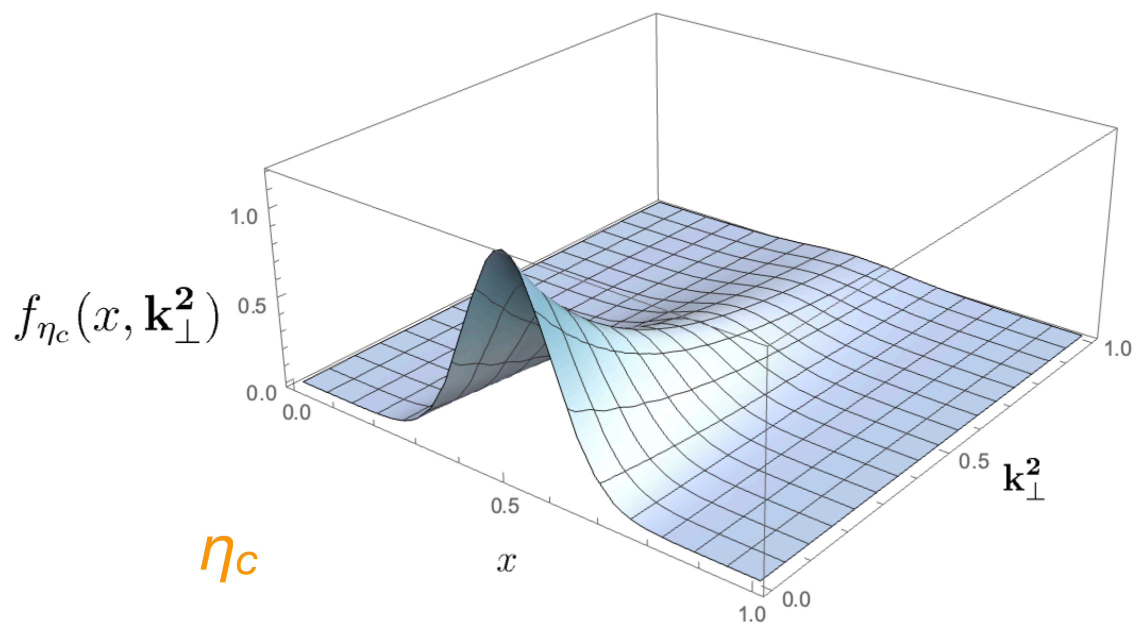


B_u

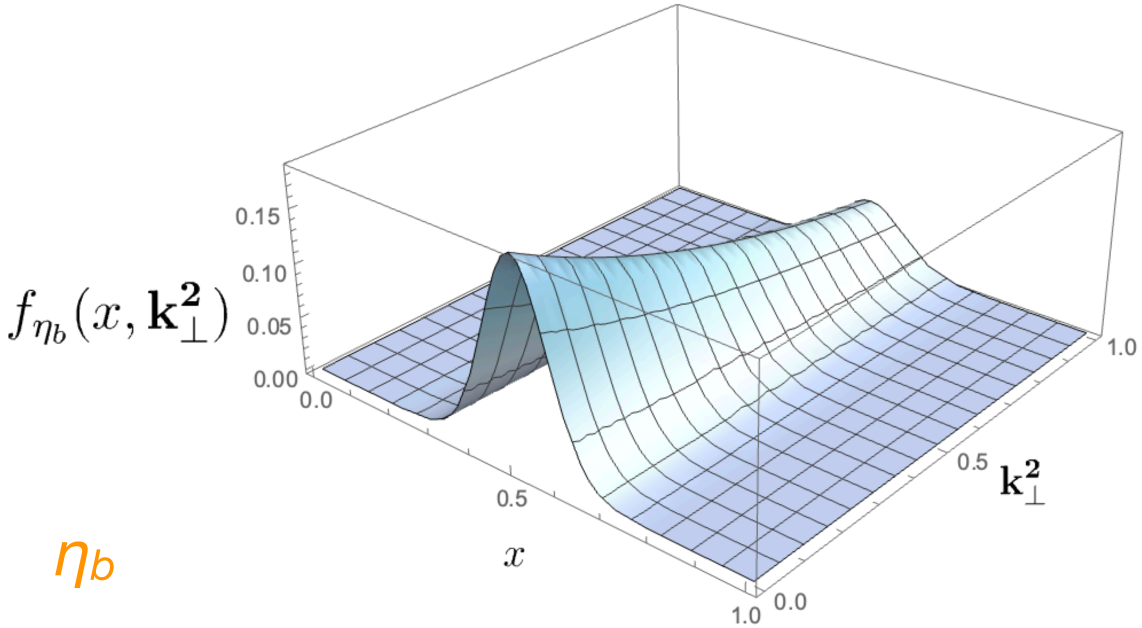
Transverse Momentum Distributions



B_c

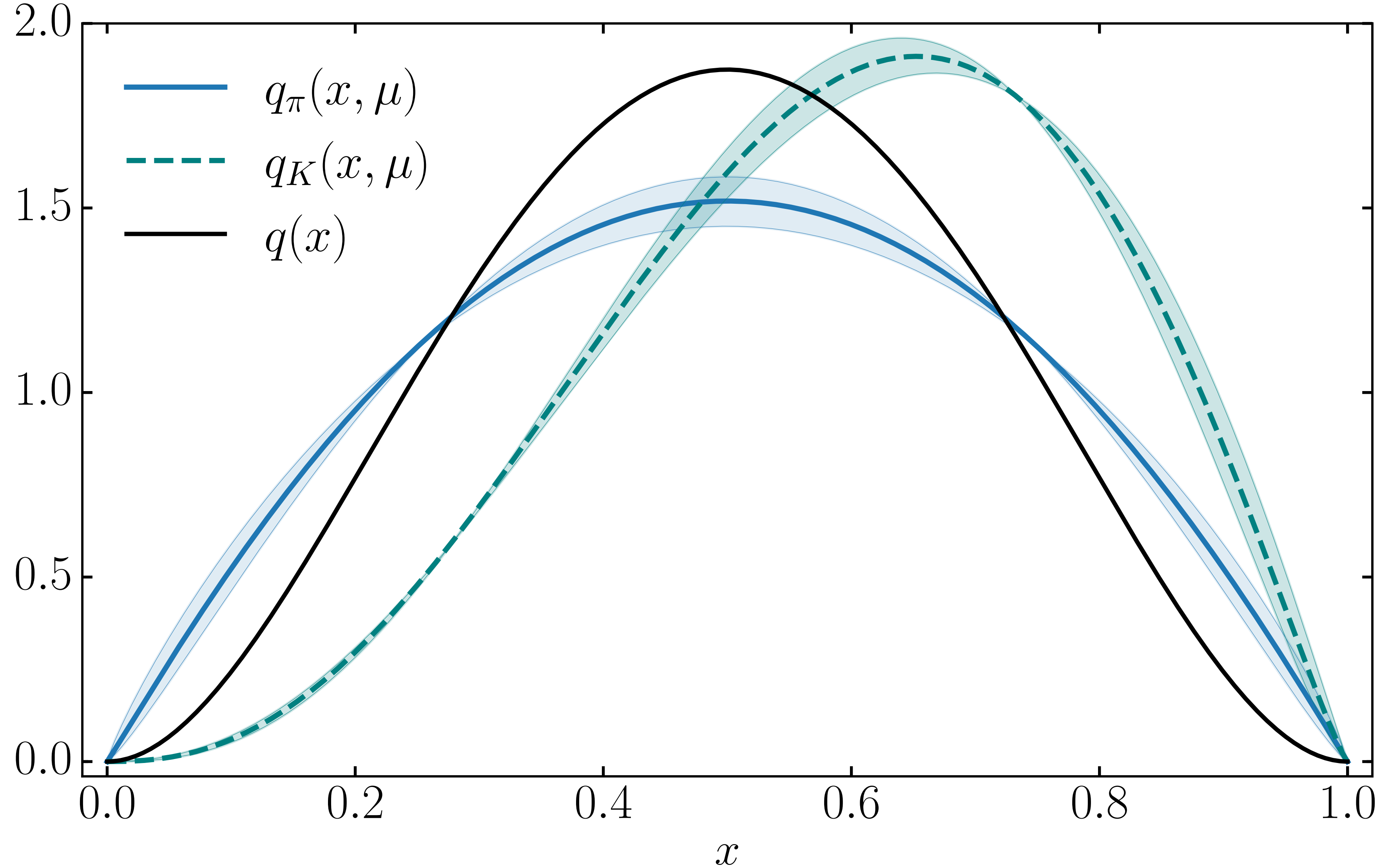


η_c



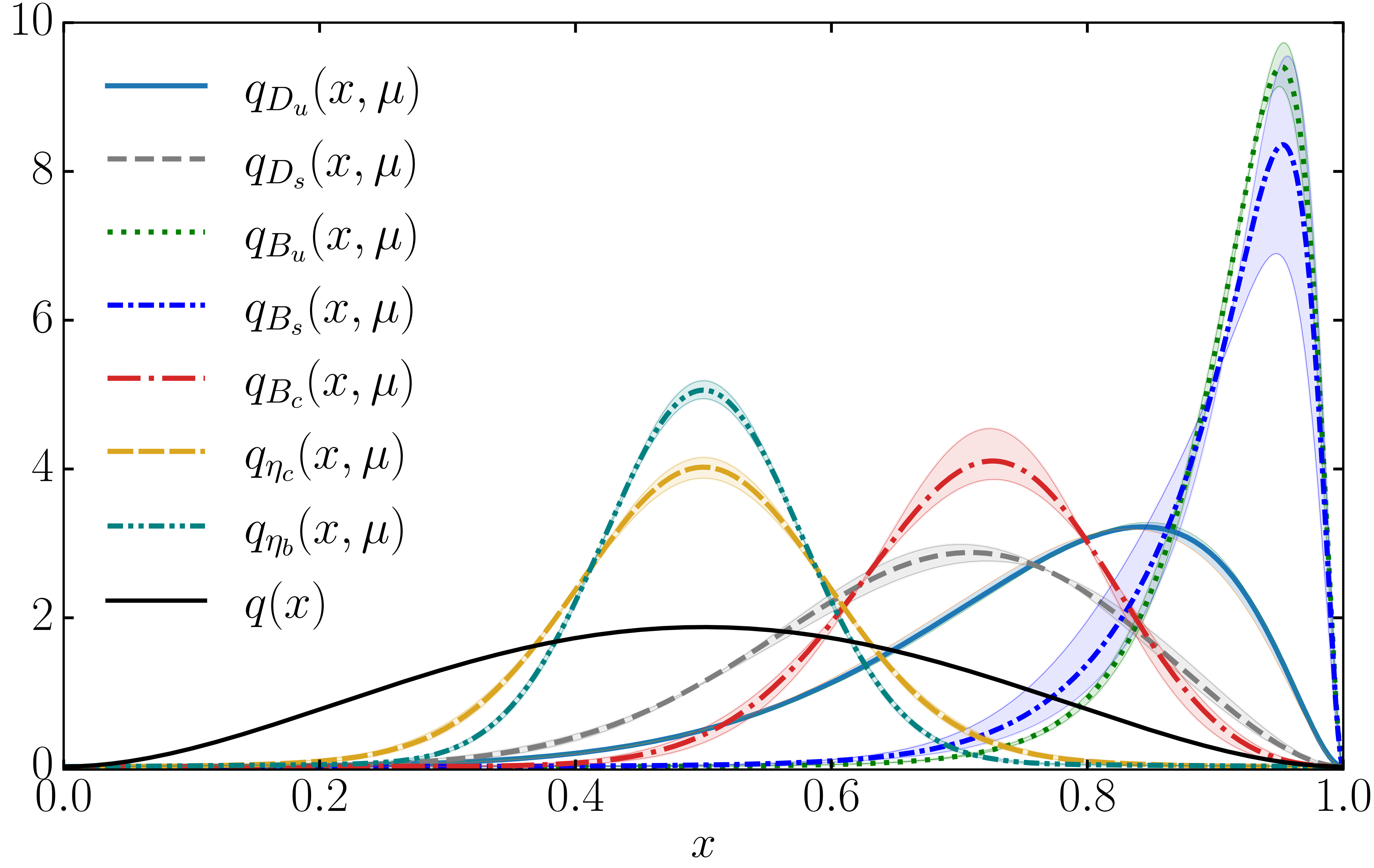
η_b

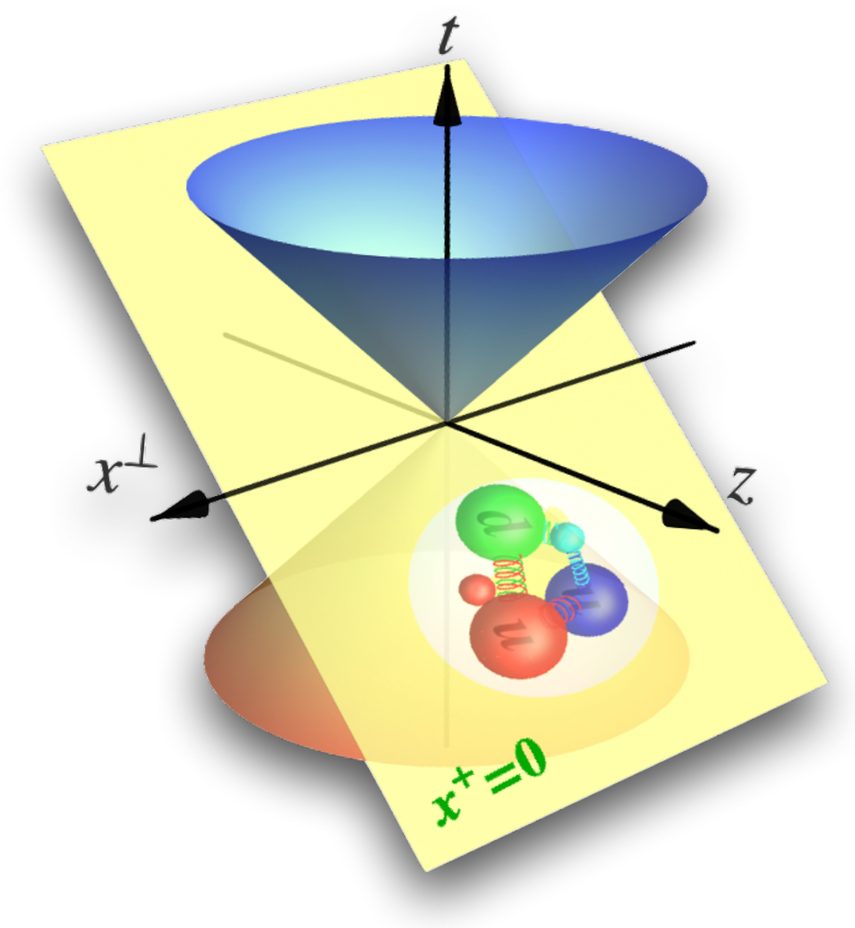
Parton Distribution Functions



The PDFs can be parametrized by: $q(x;\mu_0) = 30[x(1-x)]^2 \left[1 + \sum_{j=1}^{j_m} a_j C_j^{5/2}(2x-1)\right]$

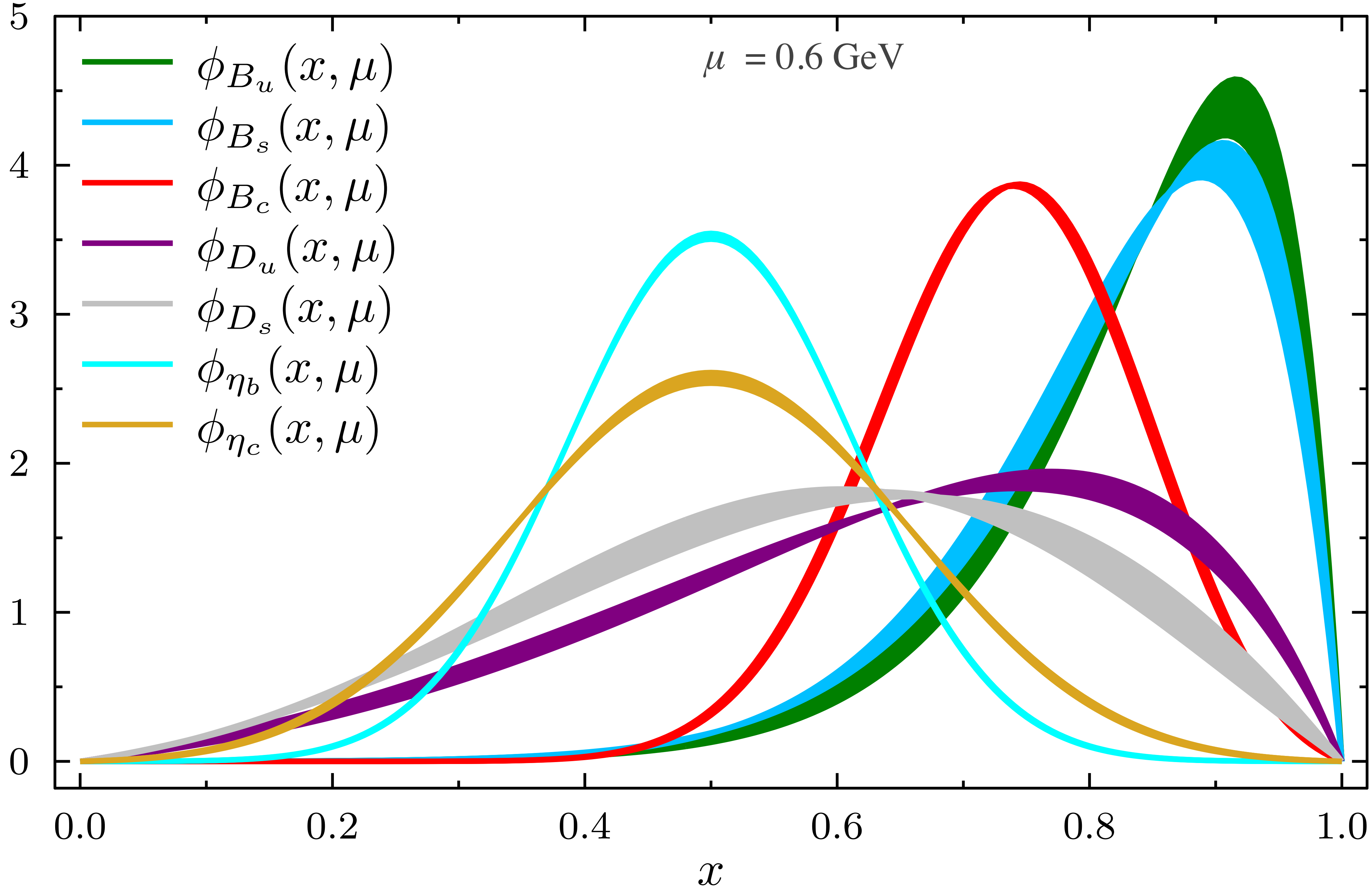
Parton Distribution Functions





Meson Distribution Amplitudes on the Light Front

Light-Cone Distribution Amplitudes



F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Vector Meson LCDA

- Vector mesons are described by two distribution amplitudes $\phi_V^{\parallel}(x;\mu)$ and $\phi_V^{\perp}(x;\mu)$
- The distributions are obtained via the projections :

$$(n\cdot P) f_V \phi_V^{\parallel}(x;\mu) = \frac{m_V N_c \mathcal{Z}_2}{\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P)\gamma\cdot n\, n_\nu \chi_{V\nu}^{fg}(k;P),$$

$$f_V^{\perp} \phi_V^{\perp}(x;\mu) = -\frac{N_c \mathcal{Z}_T}{2\sqrt{2}} \mathrm{Tr}_D \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x\, n\cdot P) n_\mu \sigma_{\mu\rho} \mathcal{O}_{\rho\nu}^{\perp} \chi_{V\nu}^{fg}(k;P),$$

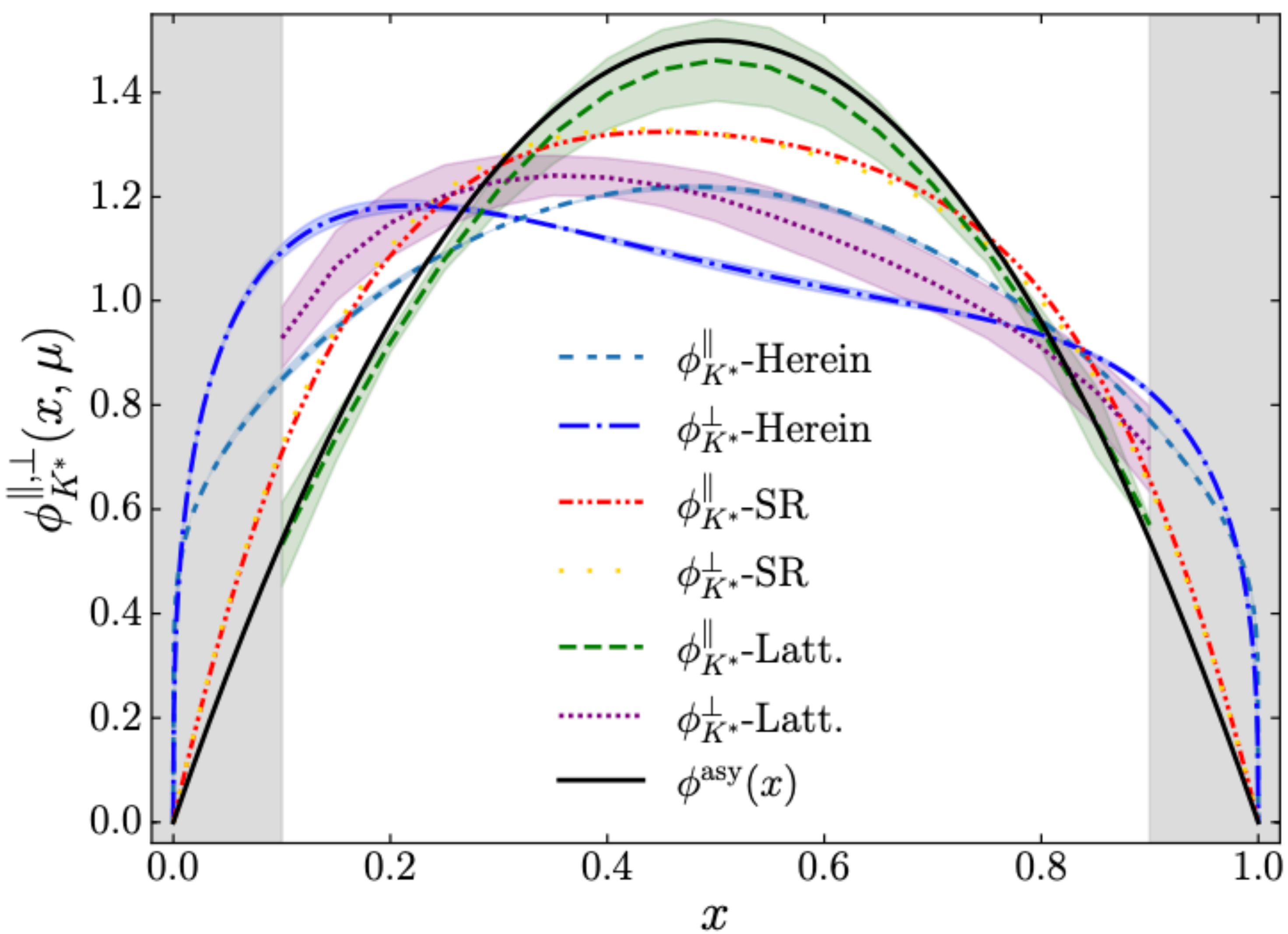
with

$$\chi_{V\nu}^{fg}(k;P) = S_f(k_\eta)\Gamma_{V\nu}^{fg}(k;P)S_g(k_{\bar\eta}), \quad \text{and} \quad \mathcal{O}_{\rho\nu}^{\perp} = \delta_{\rho\nu} + n_\rho \bar{n}_\nu + \bar{n}_\rho n_\nu,$$

$n^2 = 0$; $P^2 = -m_V^2$ and $n\cdot P = -m_V$; $\bar{n}$ is a conjugate light-like four-vector, $\bar{n}^2 = 0$, $n\cdot\bar{n} = -1$, $\bar{n}\cdot P = -m_V/2$

Light-Cone Distribution Amplitudes

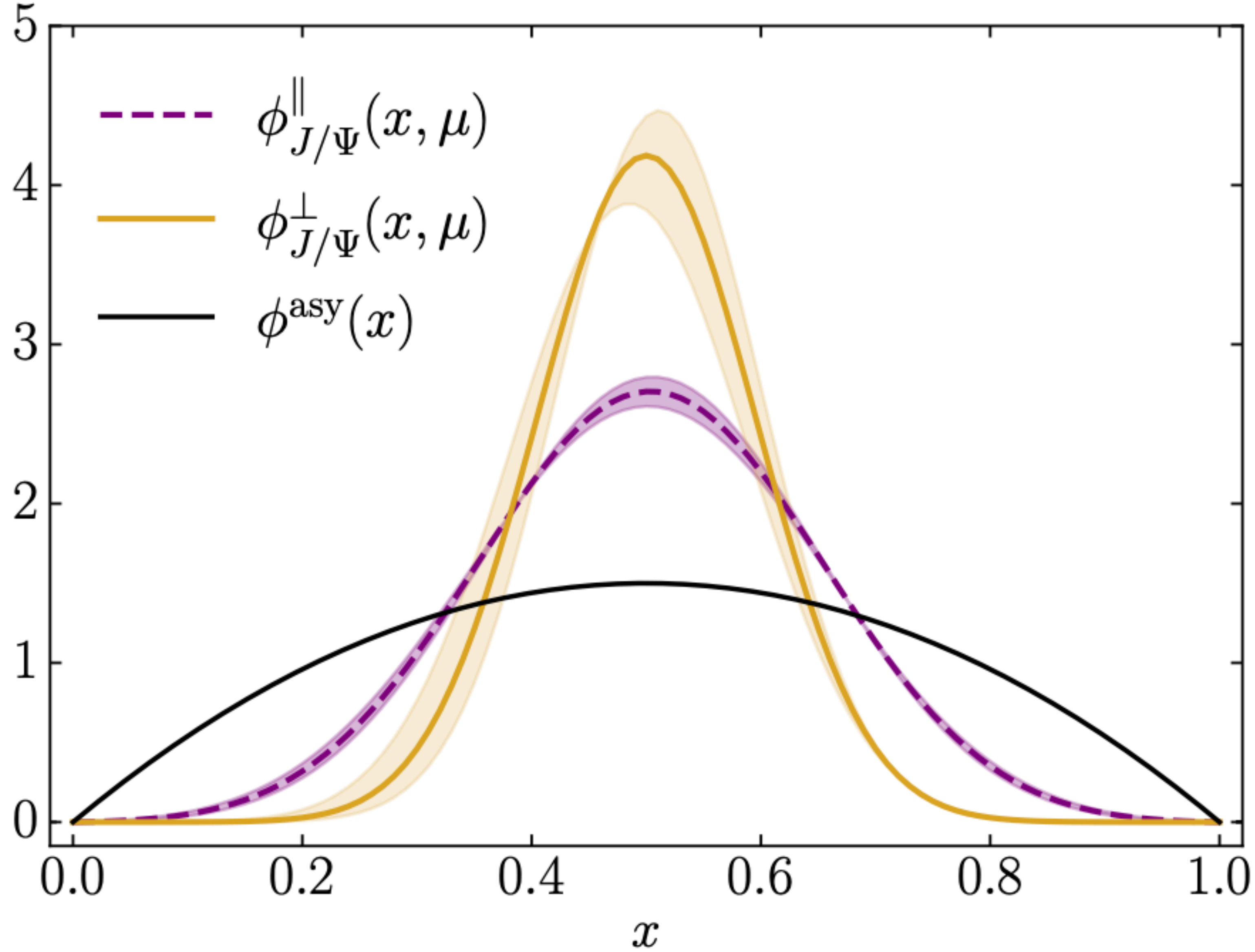
*K** meson longitudinal and transverse LCDA



F. Serna, R. Correa da Silveira, B.E., PRD Letters 106 (2022)

Light-Cone Distribution Amplitudes

J/ψ meson longitudinal and transverse LCDA



F. Serna, R. Correa da Silveira, B.E., PRD Letters 106 (2022)

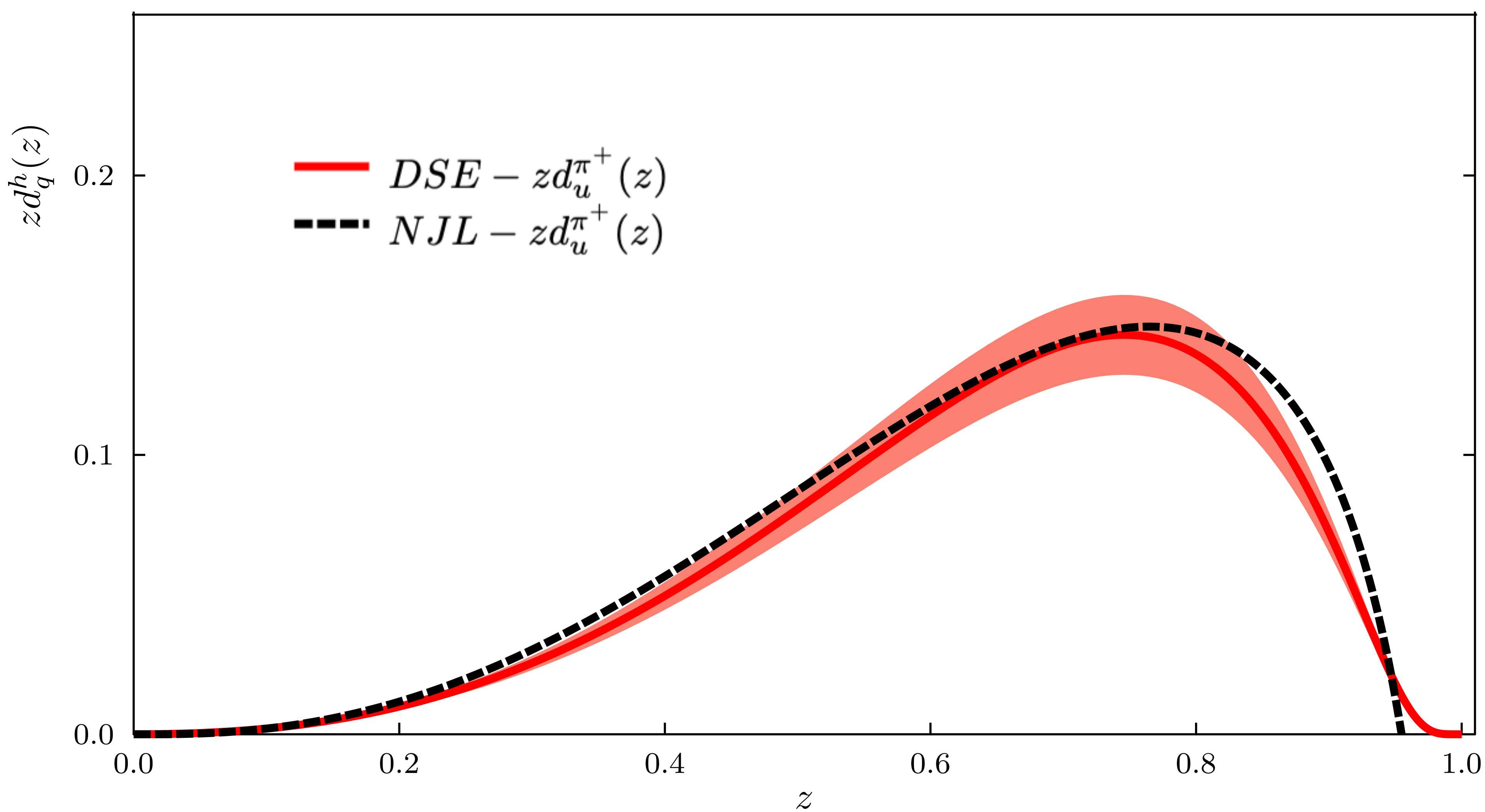
Quark Fragmentation Function

- Fragmentation functions describe the hadronization of highly energetic partons with nearly longitudinal momenta.
- The original parton model of fragmentation (Field & Feynman 1977) is nowadays understood as the process of very energetic quarks and gluons escaping the initial collision region and then fragmenting into jets of hadrons.
- The elementary fragmentation function of the process $q \to q\pi$ can be related to can be related to the parton distribution function for *unphysical* $x > 1$ using crossing and charge- conjugation symmetry:

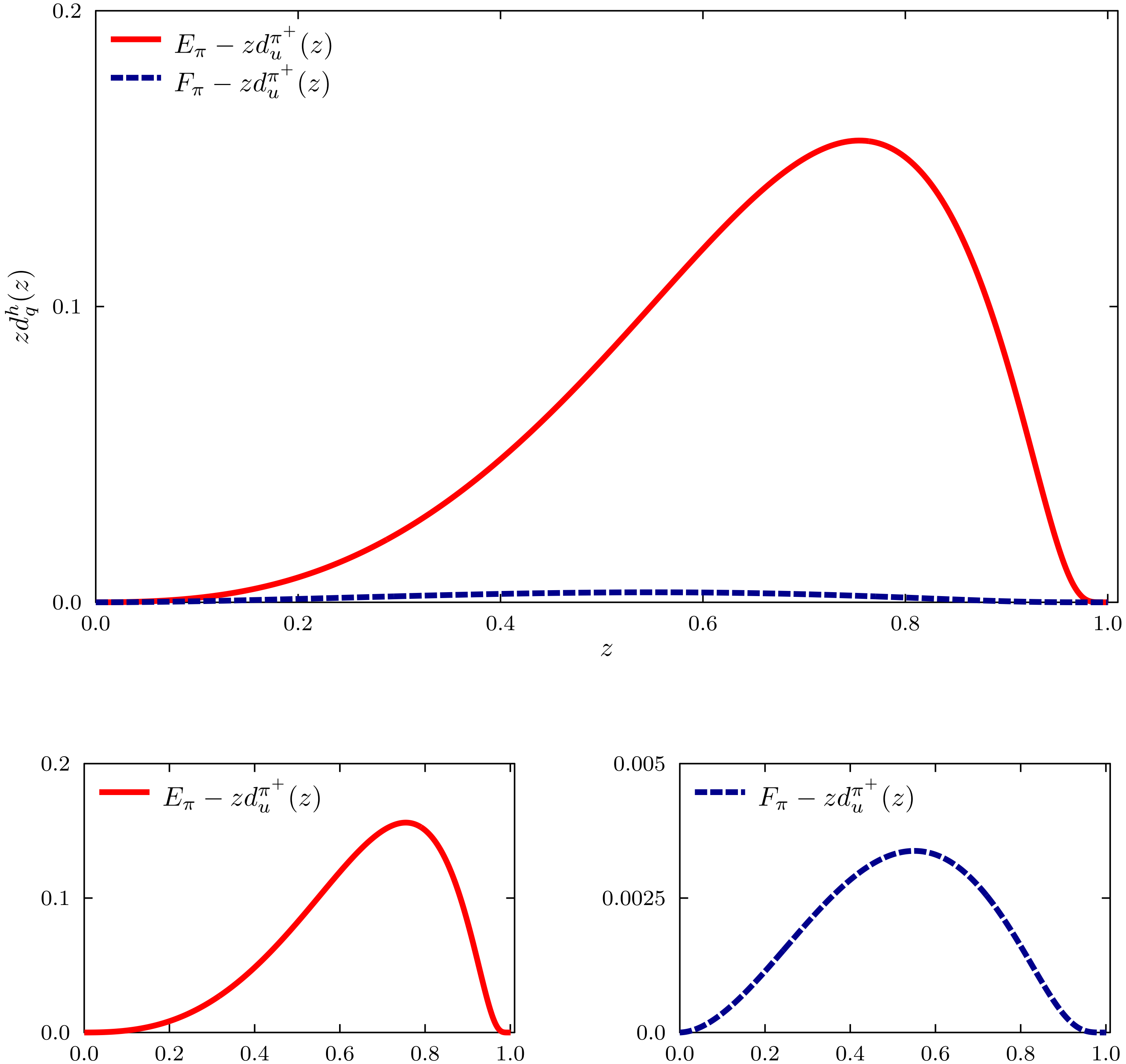
$$d_q^\pi(z) = -\frac{1}{6}\, f_q^\pi(x) \qquad \text{(Drell-Levy-Yan relation)}$$

- Fragmentation and parton distribution functions can also obtained from the discontinuity of the forward Compton amplitude via the optical theorem (cut-diagram).

Quark Fragmentation Function



Quark Fragmentation Function



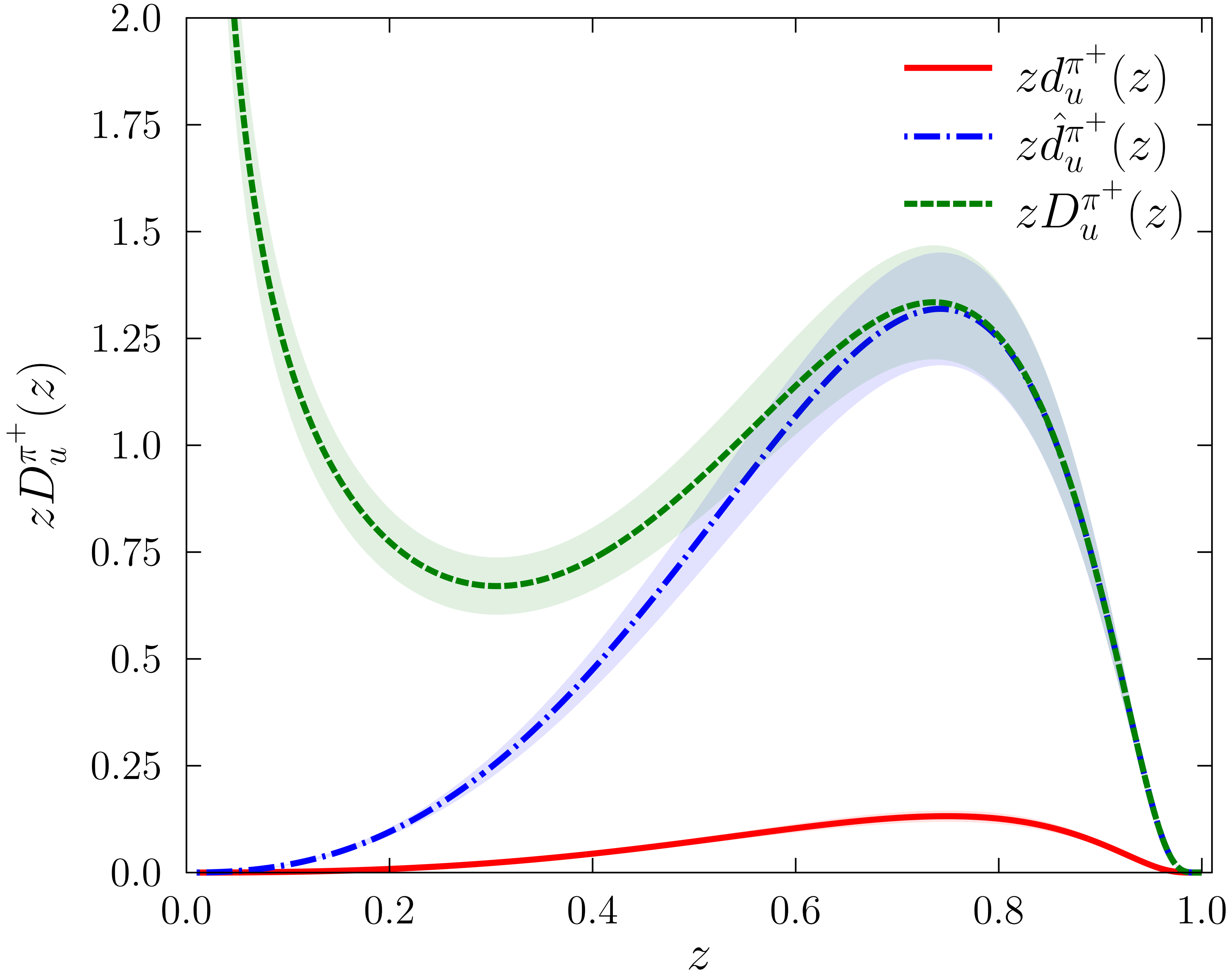
Quark-Jet Model (Field & Feynman)

- No reason to assume the quark fragments into a single pion.
- One must consider the possibility that the fragmenting quark produces a cascade of mesons (quark-jet model).
- The probability for finding a pion with light-front momentum fraction z in the jet is described by the full fragmentation function $D_q^\pi(z)$.
- $D_q^\pi(z)$ is obtained from a recursion relation that resums the entire series of such events (cascade):

$$D_q^\pi(z) = d_q^\pi(z) + \int_z^1 (dy/y) d_q^\pi(1 - z/y) D_q^\pi(y) \qquad \int_0^1 z D_q^\pi(z)\, dz = 1$$

- If the parton gives all its momentum to the pion, then there is none to contribute to the cascade: $D_q^\pi(z) \overset{z \approx 1}{\approx} d_q^\pi(z)$

Elementary fragmentation and jet functions



Conclusions & Progress

- Much progress has been made starting from QCD-based modeling to nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.

- This approach reproduces very well the charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.

- The three-dimensional momentum landscape of light and heavy mesons is obtained from different light-front projection of their Bethe-Salpeter wave function and don't involve the calculation of diagrams.

- For all LCDA, PDF and TMD we can readily provide *parametrized expressions*.

- Current progress: *improve beyond-leading corrections in BSE kernel for heavy-light mesons; computing GPDs and gravitational form factors; extension to nucleons.*

Back-up slides

Light-Front Distribution Amplitudes

QCD factorization in B decays involves matrix elements which are convolution integrals:

$$\langle \pi^+\pi^- | (\bar{u}b)_{\mathrm{V-A}}(\bar{d}u)_{\mathrm{V-A}} | \bar{B}_d \rangle \to \int_0^1 d\xi du dv \, \Phi_B(\xi)\, \Phi_\pi(u)\, \Phi_\pi(v)\, T(\xi, u, v; m_b)$$

The integrals are over a (hard) scattering kernel $\boldsymbol{T}$ (ξ, u, v, m) and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

$$\varphi_\pi(x;\tau) = \varphi_\pi^{\mathrm{asy}}(x)\left[1 + \sum_{j=2,4,\ldots}^{\infty} a_j^{3/2}(\tau)\, C_j^{(3/2)}(2x-1)\right]$$

$$\varphi_\pi^{\mathrm{asy}}(x) = 6x(1-x)$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, RQCD Collaboration, Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

Light-Front Distribution Amplitudes

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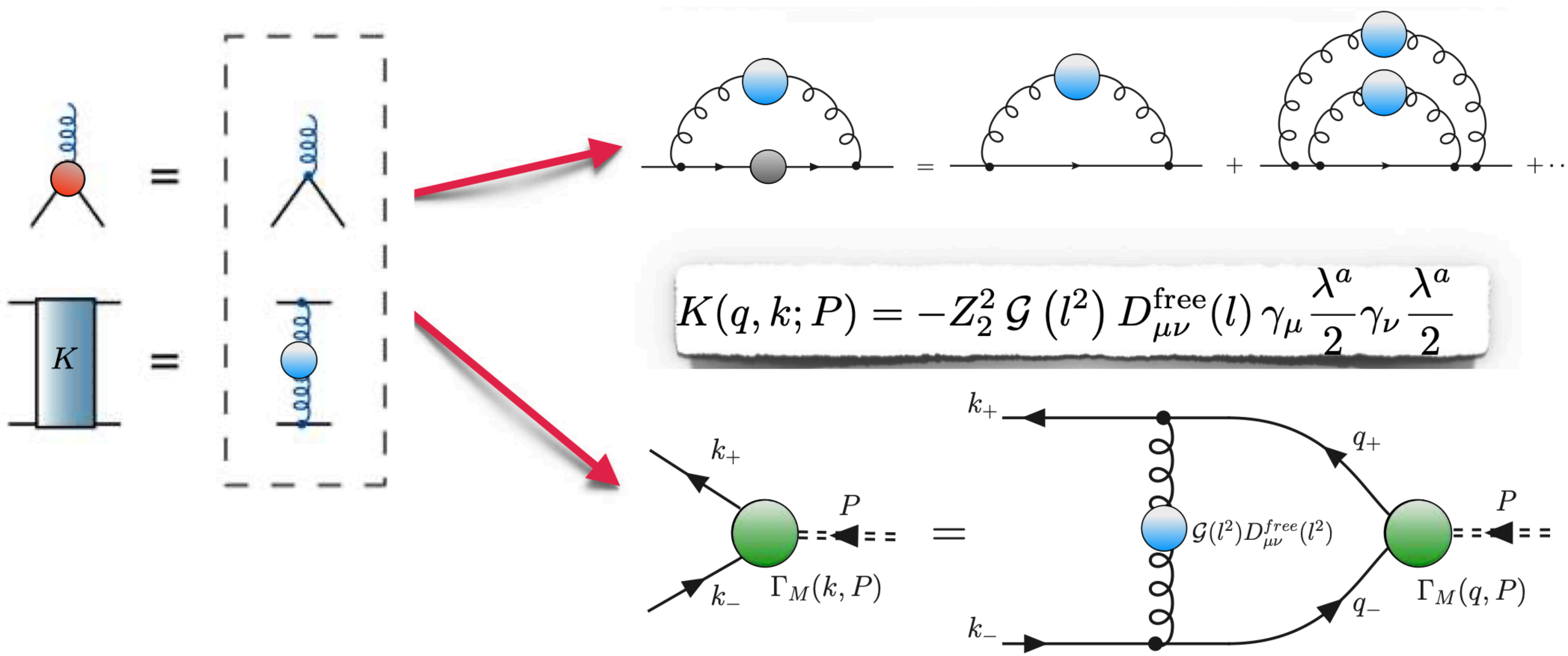
$$\langle\pi^+\pi^-|(\bar{u}b)_{\rm V-A}(\bar{d}u)_{\rm V-A}|\bar{B}_d\rangle \to \int_0^1 d\xi du dv\, \Phi_B(\xi)\, \Phi_\pi(u)\, \Phi_\pi(v)\, T(\xi,u,v;m_b)$$

The integrals are over a (hard) scattering kernel $\boldsymbol{T}$ (ξ,u,v,m) and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

$$\boxed{\phi_M(x) \neq \phi^{\rm asy}(x) = 6x(1-x)}$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, RQCD Collaboration, Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

Bethe-Salpeter equation for QCD bound states

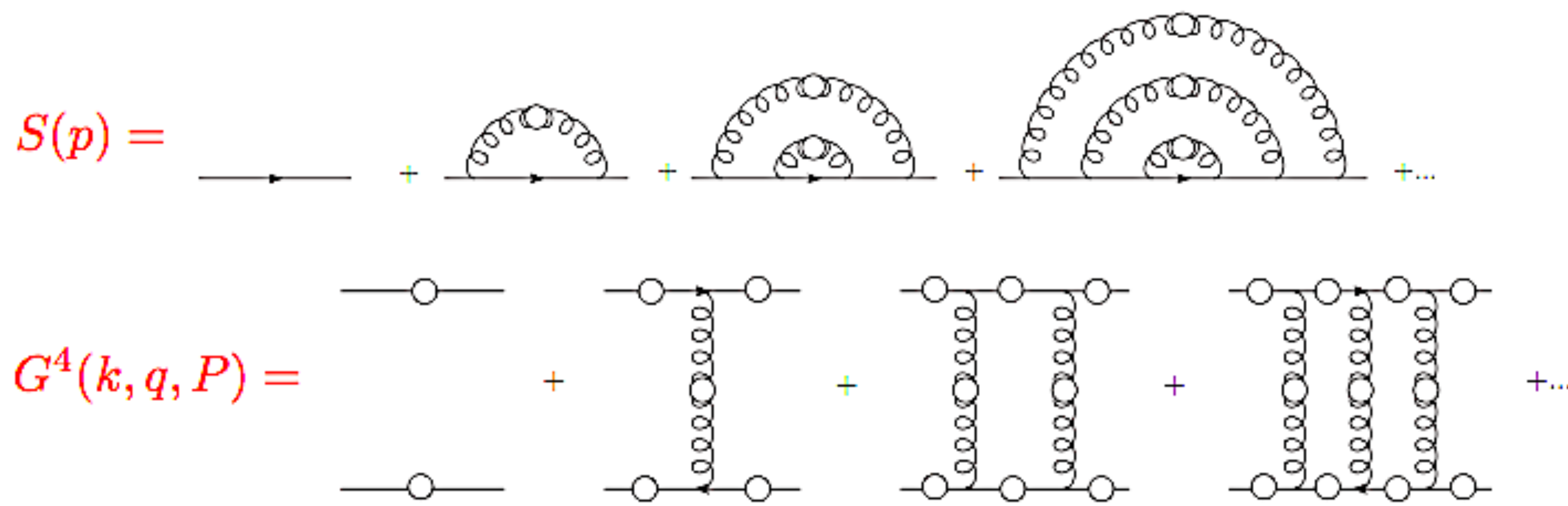


$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right) D_{\mu\nu}^{\text{free}}(l)\,\gamma_\mu \frac{\lambda^a}{2}\gamma_\nu \frac{\lambda^a}{2}$$

- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2$, $P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

Bethe-Salpeter equation for QCD bound states

Rainbow-ladder truncation (leading symmetry-preserving approximation)



Model gluon propagator, solve quark propagator and 4-point Green function.

- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2$, $P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

Pseudoscalar meson spectrum

	M_P	M_P^{exp}	ϵ_{M_P} [%]	f_P	$f_P^{\text{exp/lQCD}}$	ϵ_{f_P} [%]
$\pi(u\bar{d})$	0.140	0.138	1.45	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(u\bar{s})$	0.494	0.494	0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0
$D(c\bar{d})$	$1.867^{+0.008}_{-0.004}$	1.864	0.11	$0.144^{+0.001}_{-0.001}$	0.150 (0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03
$B(u\bar{b})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(s\bar{b})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.5
$B_c(c\bar{b})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	10.17

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Vector meson spectrum

	M_V	M_V^{exp}	ϵ_{M_V} [%]	f_V	$f_V^{\mathrm{exp/lQCD}}$	ϵ_{f_V} [%]
$\rho(u\bar{u})$	0.730	0.775	5.81	0.145	0.153(1)	5.23
$\phi(s\bar{s})$	1.070	1.019	5.20	0.187	0.168(1)	11.31
$K^*(u\bar{s})$	0.942	0.896	5.13	0.177	0.159(1)	11.32
$D^*(c\bar{d})$	2.021	2.009	0.60	0.165	0.158(6)	4.43
$D_s^*(c\bar{s})$	2.169	2.112	2.70	0.205	0.190(5)	7.90
$J/\psi(c\bar{c})$	3.124	3.097	0.87	0.277	0.294(5)	5.78
$\Upsilon(b\bar{b})$	9.411	9.460	0.52	0.594	0.505(4)	17.62

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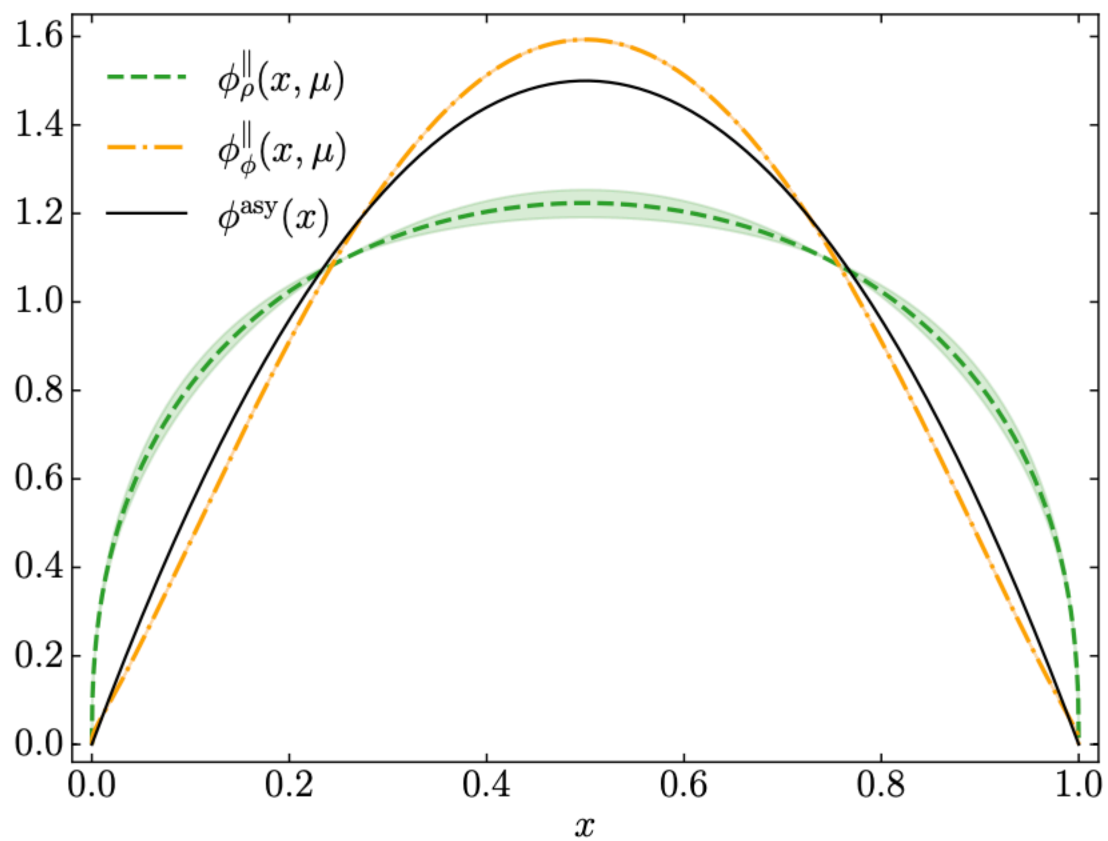
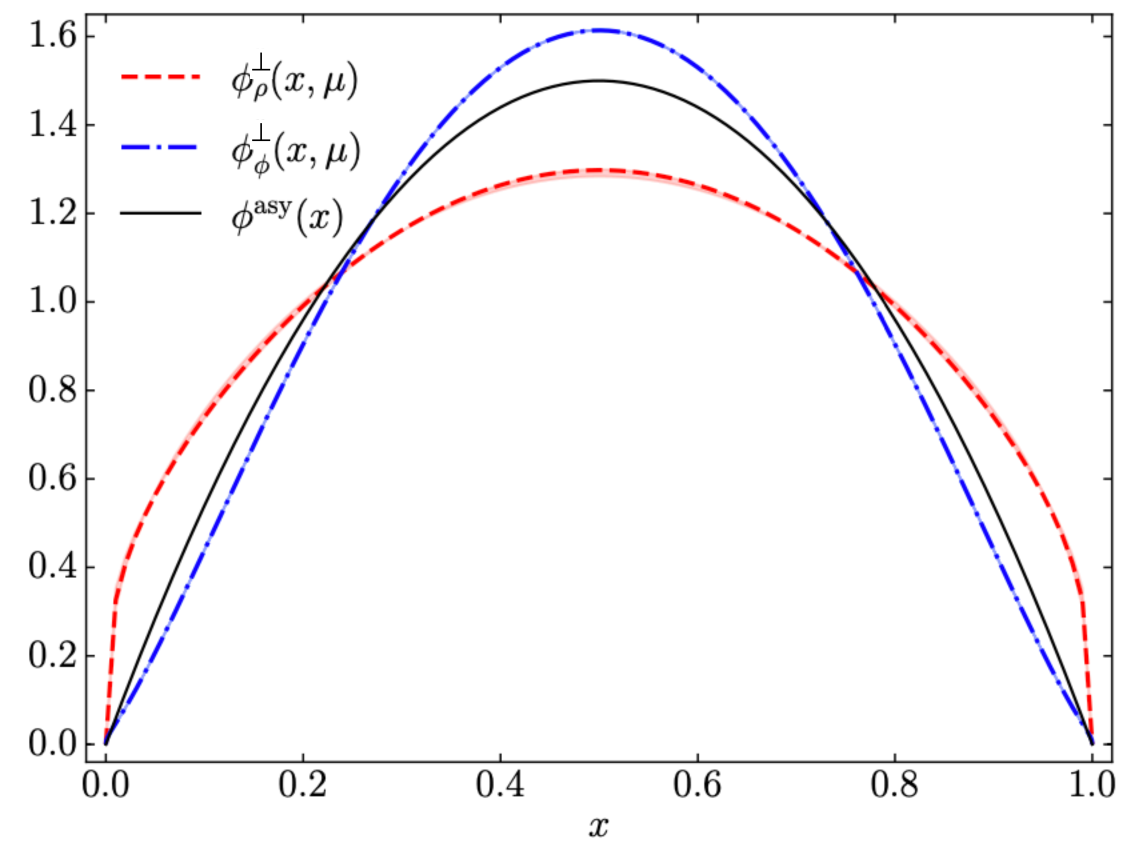
Vector Meson LCDA

ρ and **ϕ** mesons: longitudinal LCDA



ρ and **ϕ** mesons: transverse LCDA



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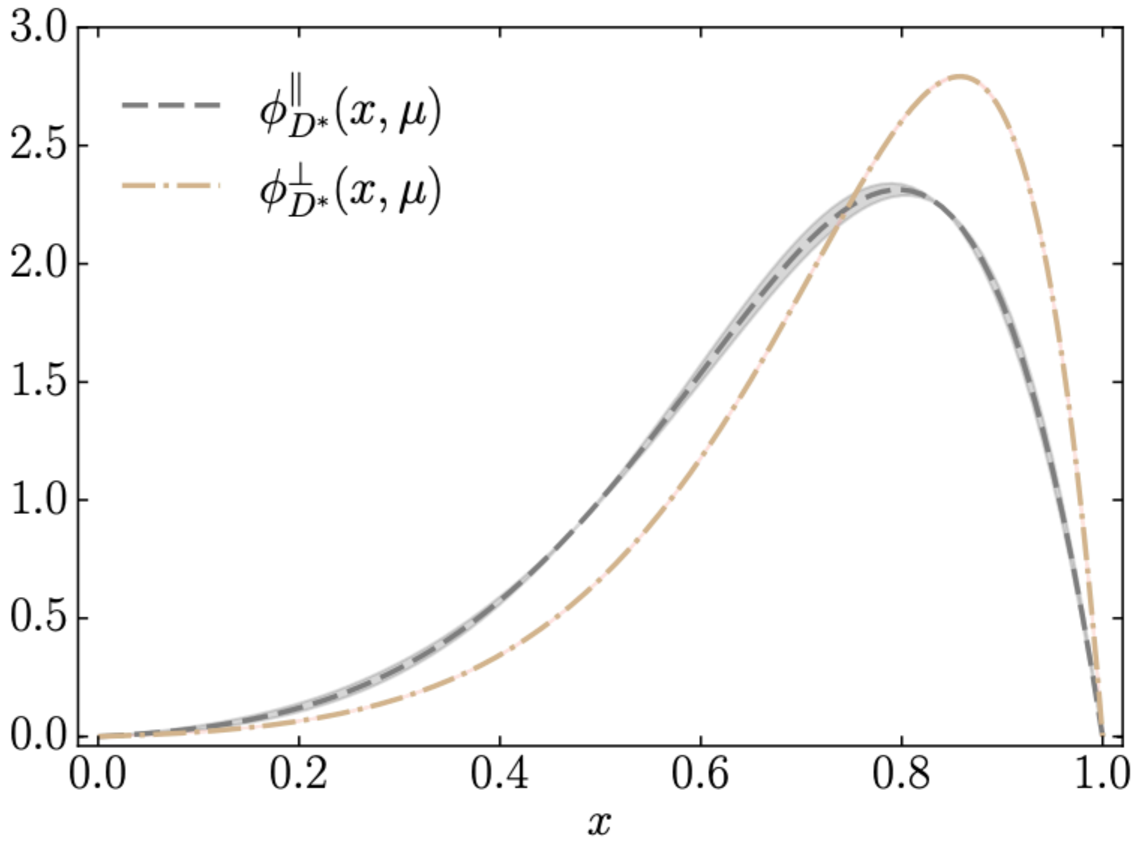
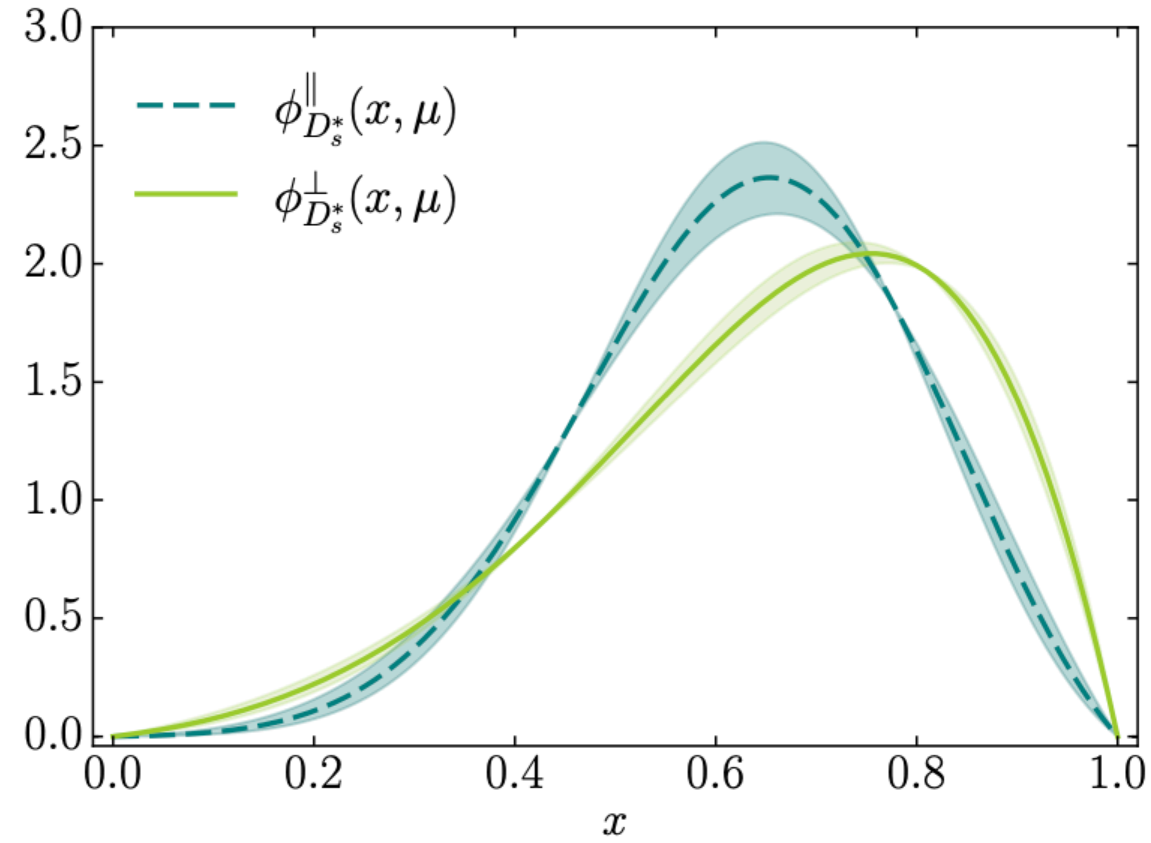
We find: $\phi_\phi^\parallel(x,\mu) \approx \phi_\phi^\perp(x,\mu)$

Light-Cone Distribution Amplitudes

D^* and D_s^* mesons longitudinal and transverse LCDA





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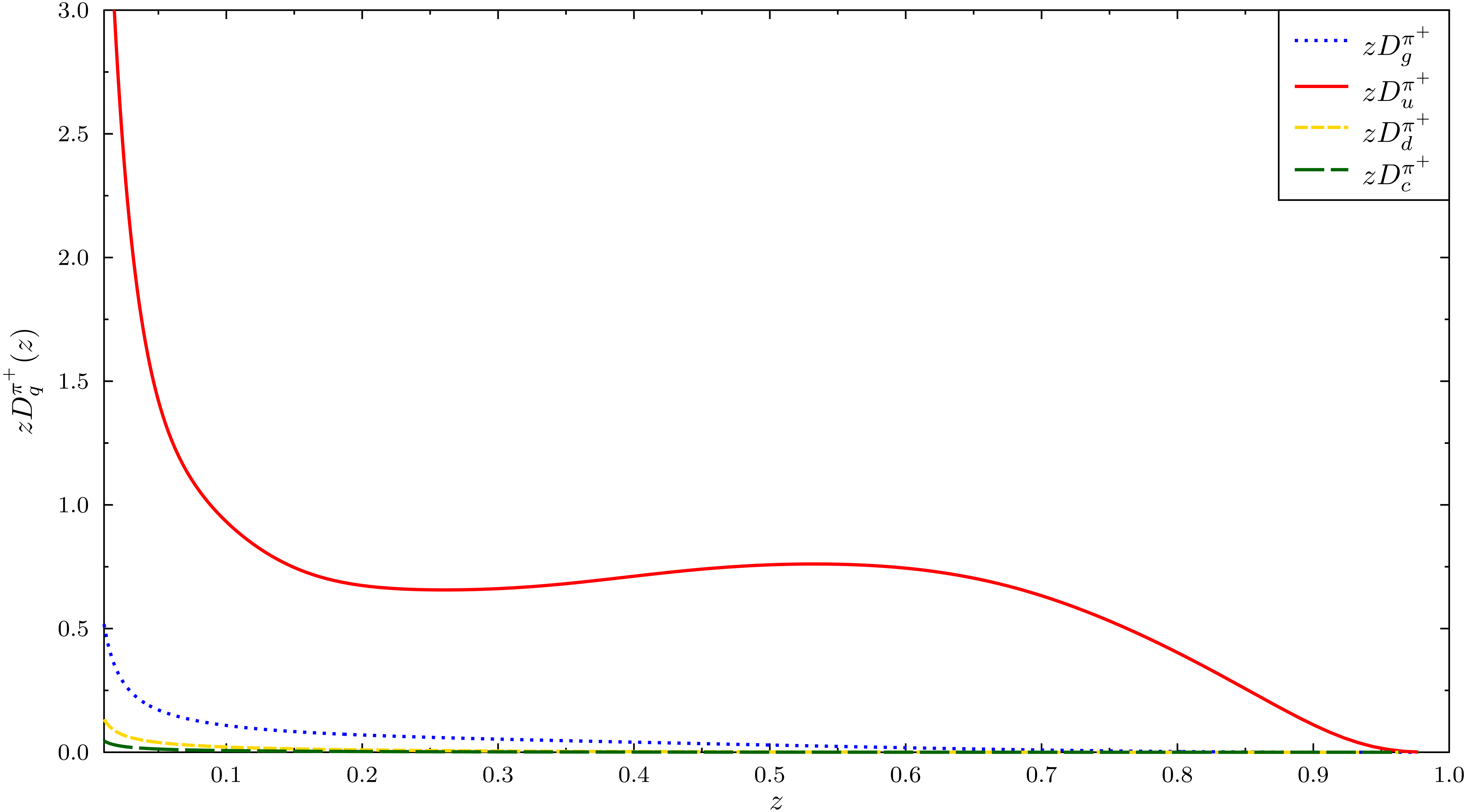
DGLAP Evolution to 2 GeV



PDF DGLAP evolution to 2 GeV