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A. Bou, C. Ramos, J. Schnedier, S. Carignano LT arXiv:2409.xxxx

S. Carignano (BSC) arXiv:2405.14706

S. Carignano (BSC), C. Ramos (UB) arXiv:2307.11649 PRR (2024)

On generalized temporal entropy, its experimental measure and its use to characterize quantum dynamics

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Entanglement

$$|\psi_A\rangle \otimes |\psi_B\rangle$$

$$|\psi_{AB}\rangle = \sum_i c_i \left(|\psi_A^i\rangle \otimes |\psi_B^i\rangle \right)$$

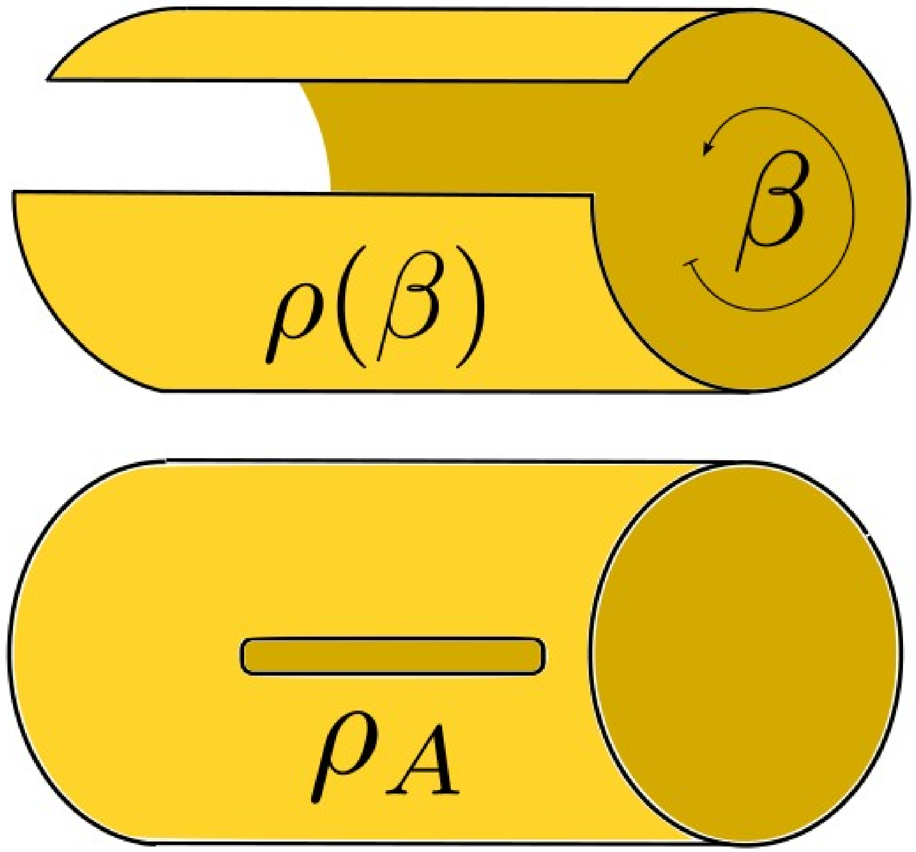
$$\rho_A = \mathrm{tr}_B \left(|\psi_{AB}\rangle\langle\psi_{AB}| \right)$$

$$\mathrm{S}_n = \frac{1}{1-n} \log(\mathrm{tr}\left(\rho_A^n\right))$$

Entanglement measures
correlation among different constituents

Entanglement in quantum many-body/ QFT

$$\rho_\beta = \frac{1}{Z} e^{-\beta H},$$



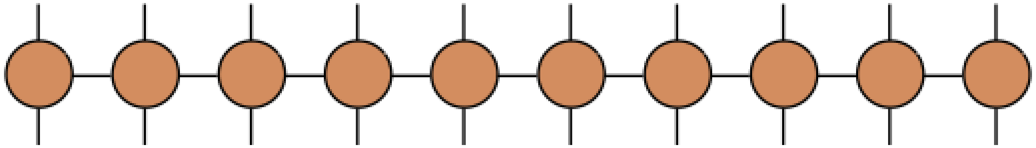
$$\rho_A = \mathrm{tr}_B \rho(\beta)$$

$$H = \sum_i h_i$$

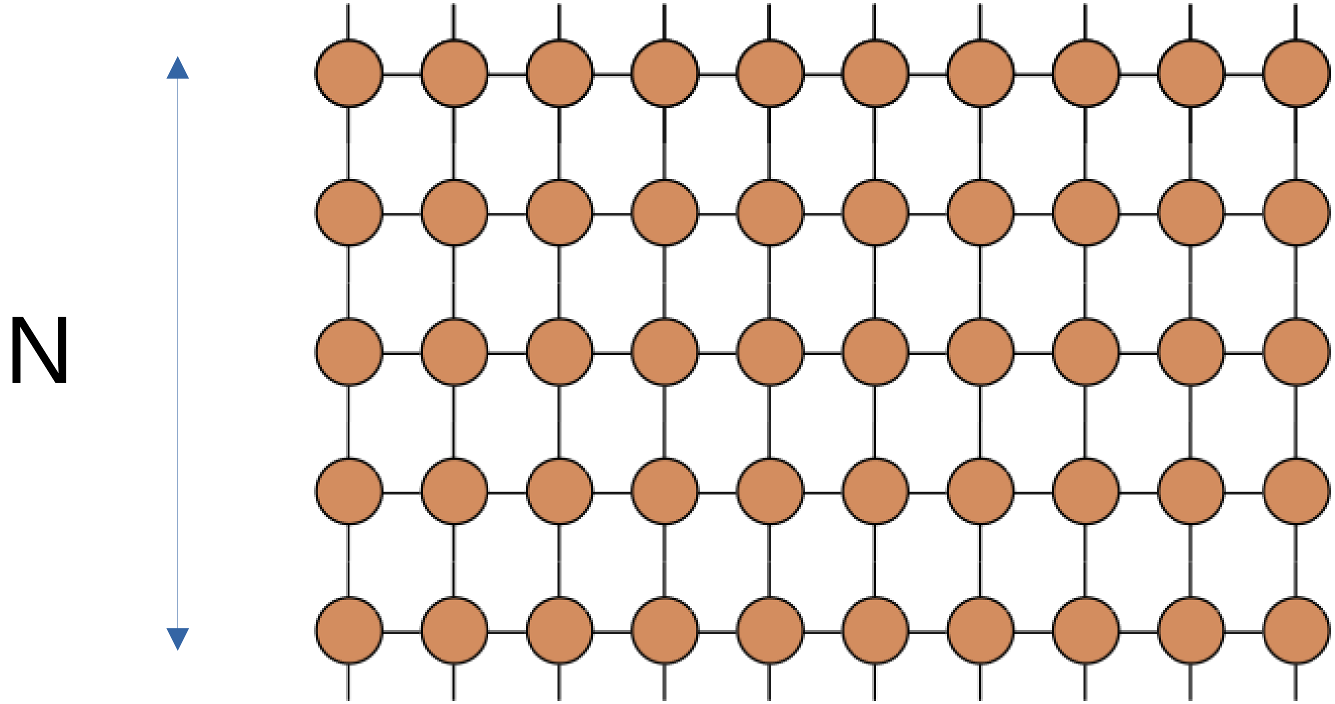
$$U(\beta) = \exp\left(-\beta H\right)$$

$$\delta\beta = \beta/N \ll 1 \quad U(\beta) \simeq U(\delta\beta)^N$$

$U(\delta\beta)$

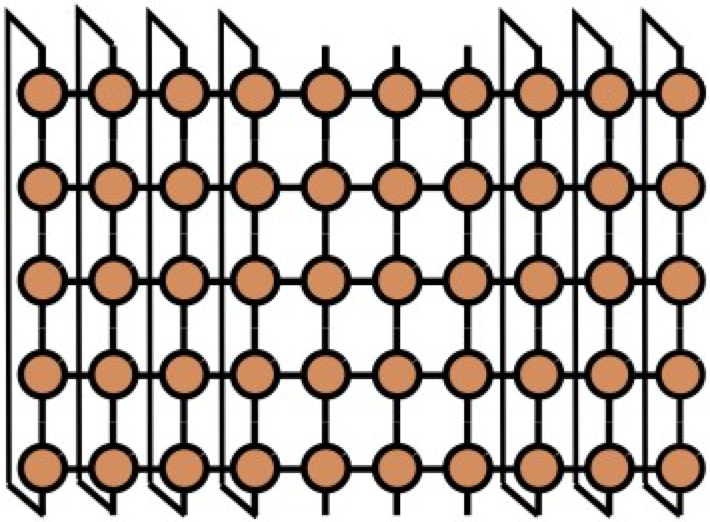


$$\rho_\beta = \frac{1}{Z} e^{-\beta H},$$



$$U(\beta) \simeq U(\delta\beta)^N$$

$$\rho_A = \mathrm{tr}_B \rho(\beta)$$



A

Temporal Entanglement

Can we measure the correlation among the same constituents at different times?

- Computational approaches → quantum complexity (see Bañuls et al, Abanin et al, Chan et al,…) see also our

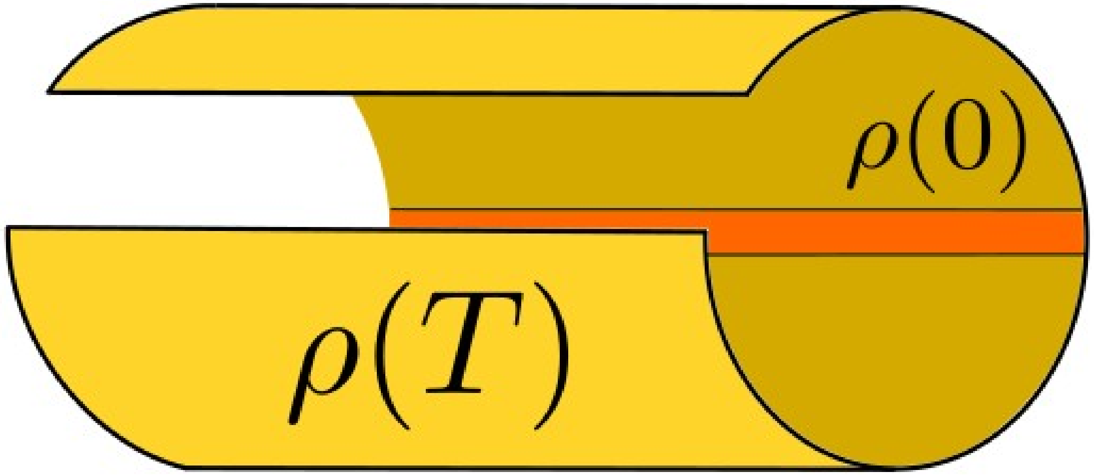
  arXiv:2307.11649

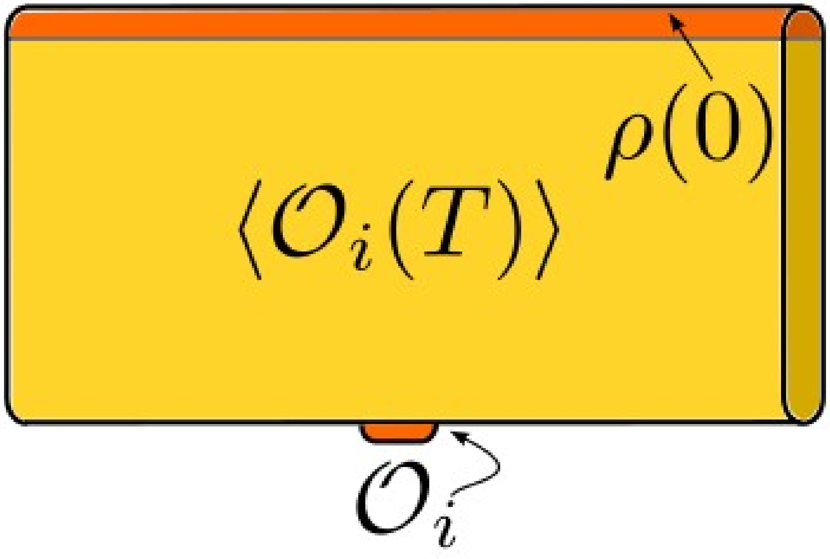
- Holographic field theories, dS/CFT (Takayangi et al, Heller et al...) for CFT see also our

  arXiv:2405.14706 (S. Carignano talk Thur)

- QM foundation, the role of time (review of Spekkens)

$$\rho(T) = U^\dagger(T)\rho_0 U(T)$$

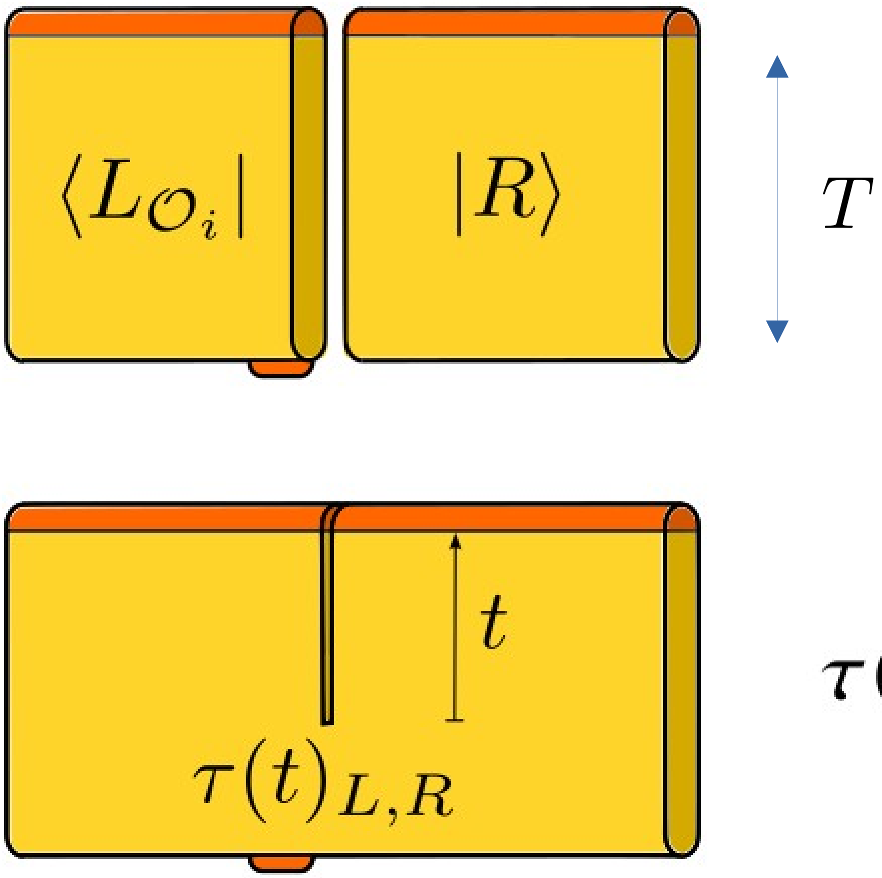




$$U(T) = \exp\left(-iHT\right)$$

$$\langle \mathcal{O}_i \rangle = \mathrm{tr}(\rho(T)\mathcal{O}_i)$$

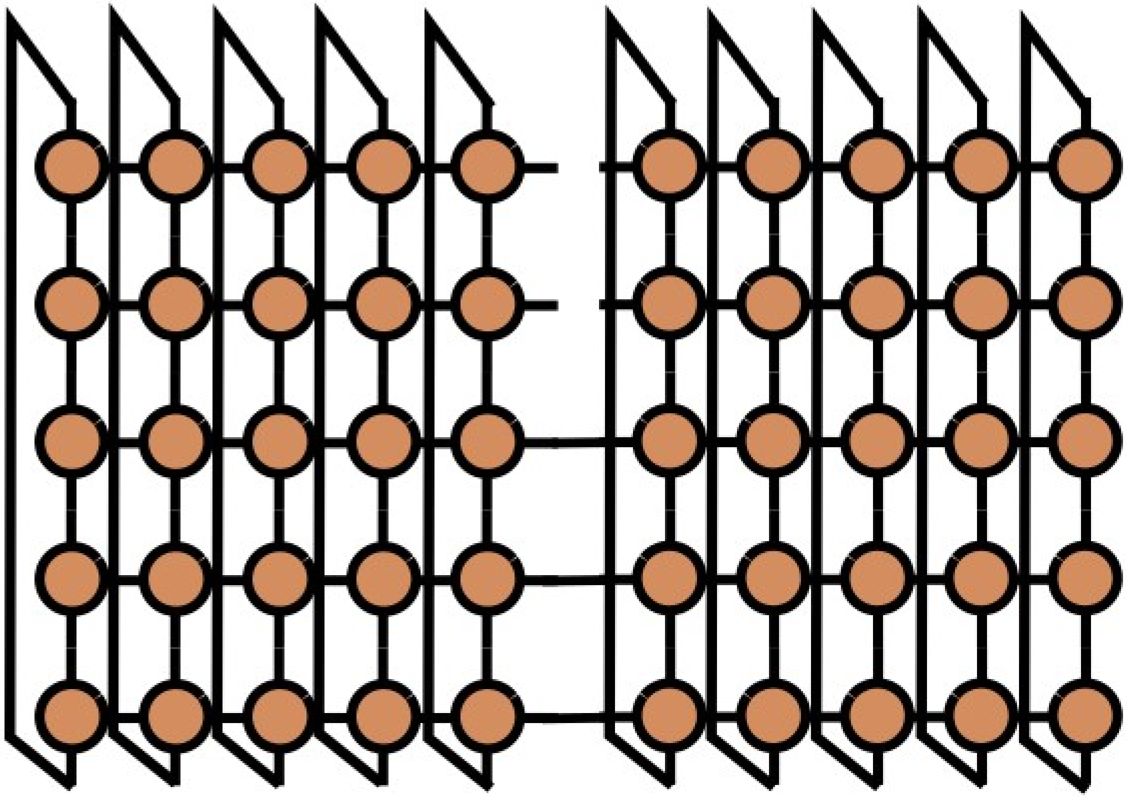
$$\langle O_i(T)\rangle \equiv \left\langle L_{r\,O_i}\middle| R_r\right\rangle$$



L, R are temporal states, states of constituents over time

$$\tau(t)_{O_i} = \frac{\mathrm{tr}_{T-t}\, |R\rangle\langle L_{O_i}|}{\langle L_{O_i}|R\rangle}\,.$$

$$\tau(t)_{O_i} = \frac{\mathrm{tr}_{T-t} \left|R\right\rangle\!\left\langle L_{O_i}\right|}{\left\langle L_{O_i} \middle| R \right\rangle} .$$

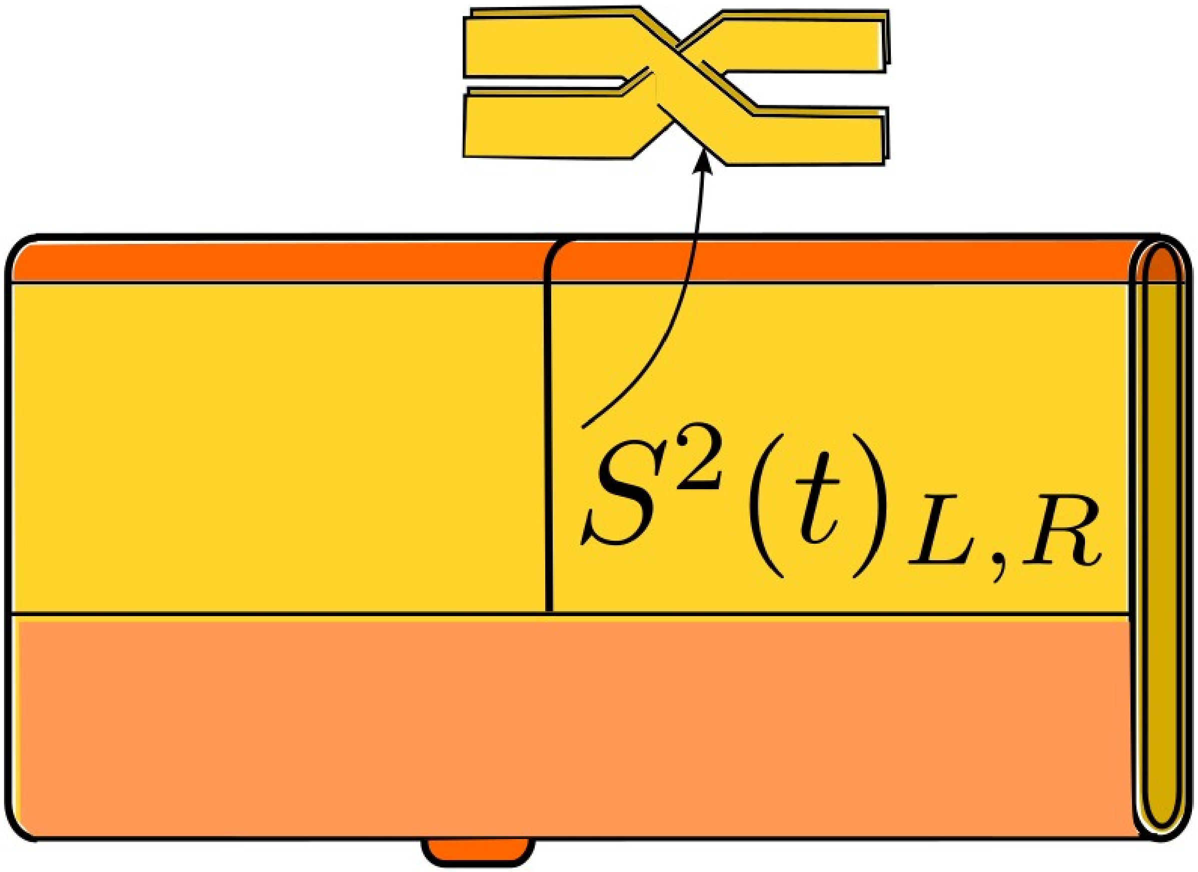


Reduced transition matrices,
Not a legitimate quantum state
Not positive definite
Not even Hermitian

$$\tau(t)_{O_i} = \frac{\mathrm{tr}_{T-t} \left|R\right\rangle\left\langle L_{O_i}\right|}{\left\langle L_{O_i}\middle|R\right\rangle} \, .$$

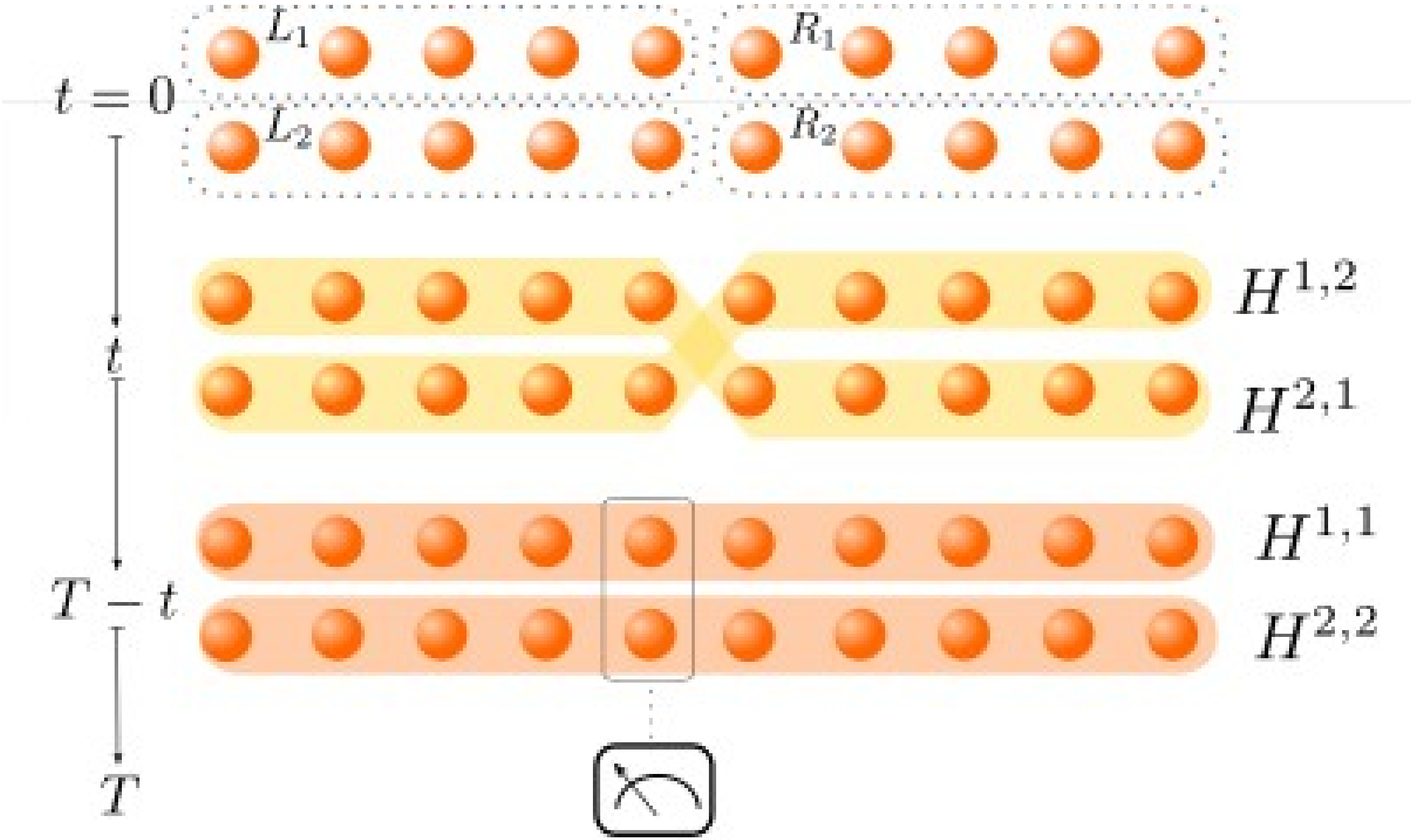
Generalized entropies as the expectation value of Hermitian operators

$$S_\alpha = -\frac{1}{1-\alpha}\,\mathrm{tr}\left(\tau(t)_{O_i}\right)^\alpha$$



The double quenche

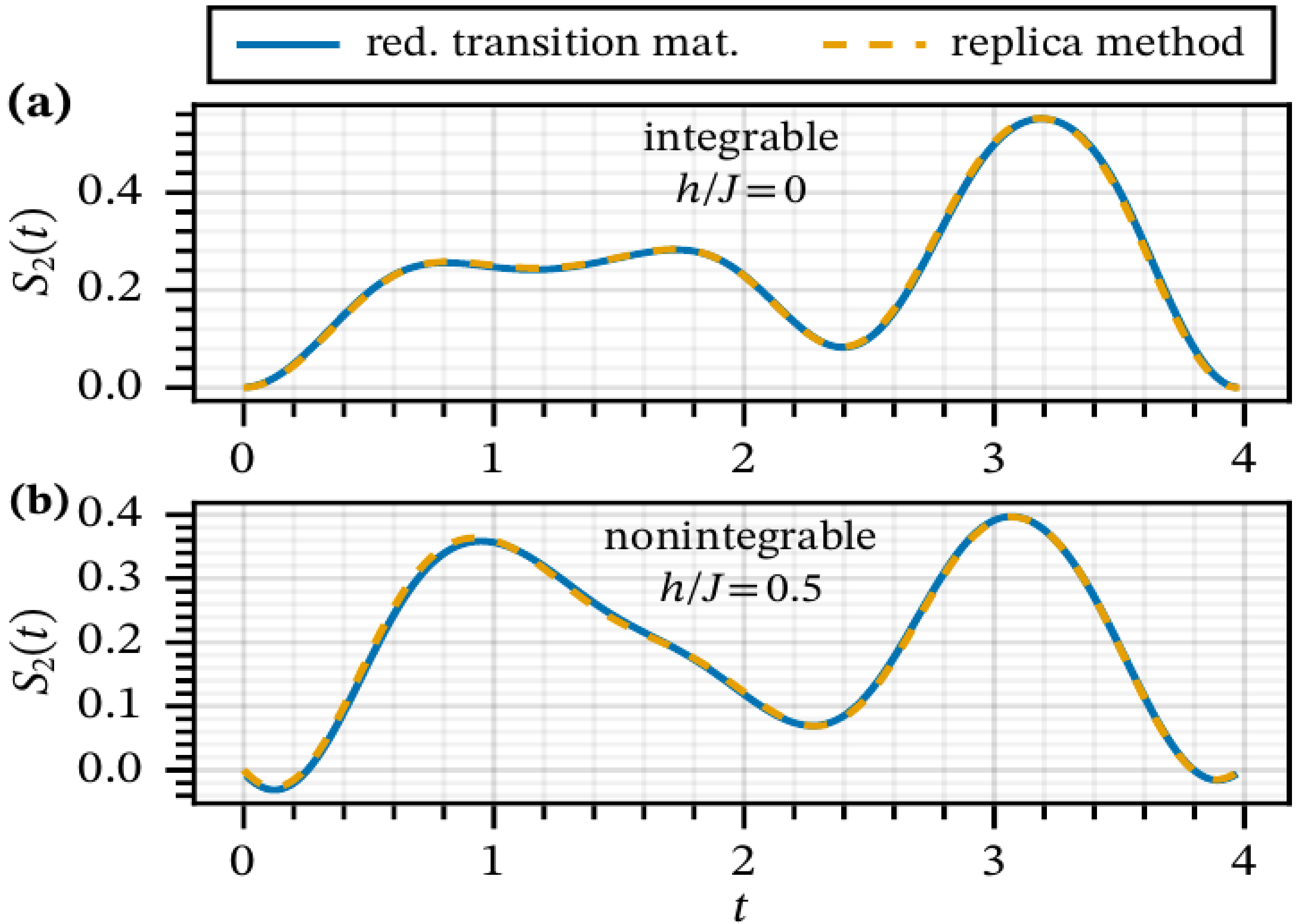
$$\mathrm{tr}\Big(\tau(t)^2_{O_i}\Big) \equiv \frac{\mathrm{tr}\left(O_i^1 \otimes O_i^2 \rho^{1,2}(T)\right)}{\langle O(T)\rangle^2}$$



$$\rho^{1,2}(T) \equiv W^{1,2}(T)\left(\rho_0^1 \otimes \rho_0^2\right) W^{1,2}(T)^\dagger$$

$$W^{1,2}(T) = \left(U^1(T-t) \otimes U^2(T-t)\right)\left(U^{1,2}(t) \otimes U^{2,1}(t)\right)$$

$$H = -J \sum_{i=1}^{N-1} \sigma_x^i \sigma_x^{i+1} + \sum_{i=1}^{N} \left(g\sigma_z^i + h\sigma_x^i \right) .$$



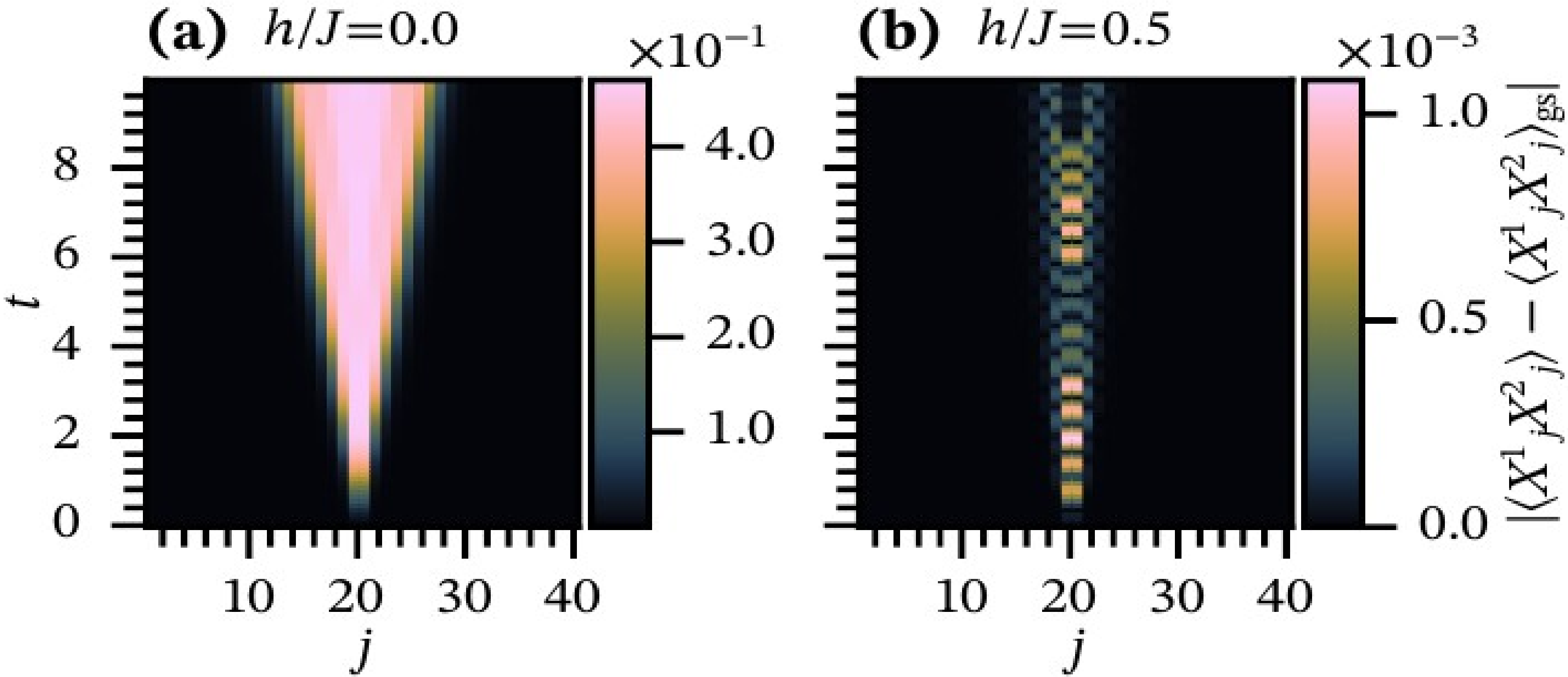
Application, witnessing the nature of the quantum dynamics

Generalized temporal entanglement of the ground state (equilibrium state)

$$\mathrm{tr}\left(\tau(t)^2_{O_i}\right) \equiv \frac{\mathrm{tr}\left(O_i^1 \otimes O_i^2 \rho^{1,2}(T)\right)}{\langle O(T)\rangle^2}$$

Integrable

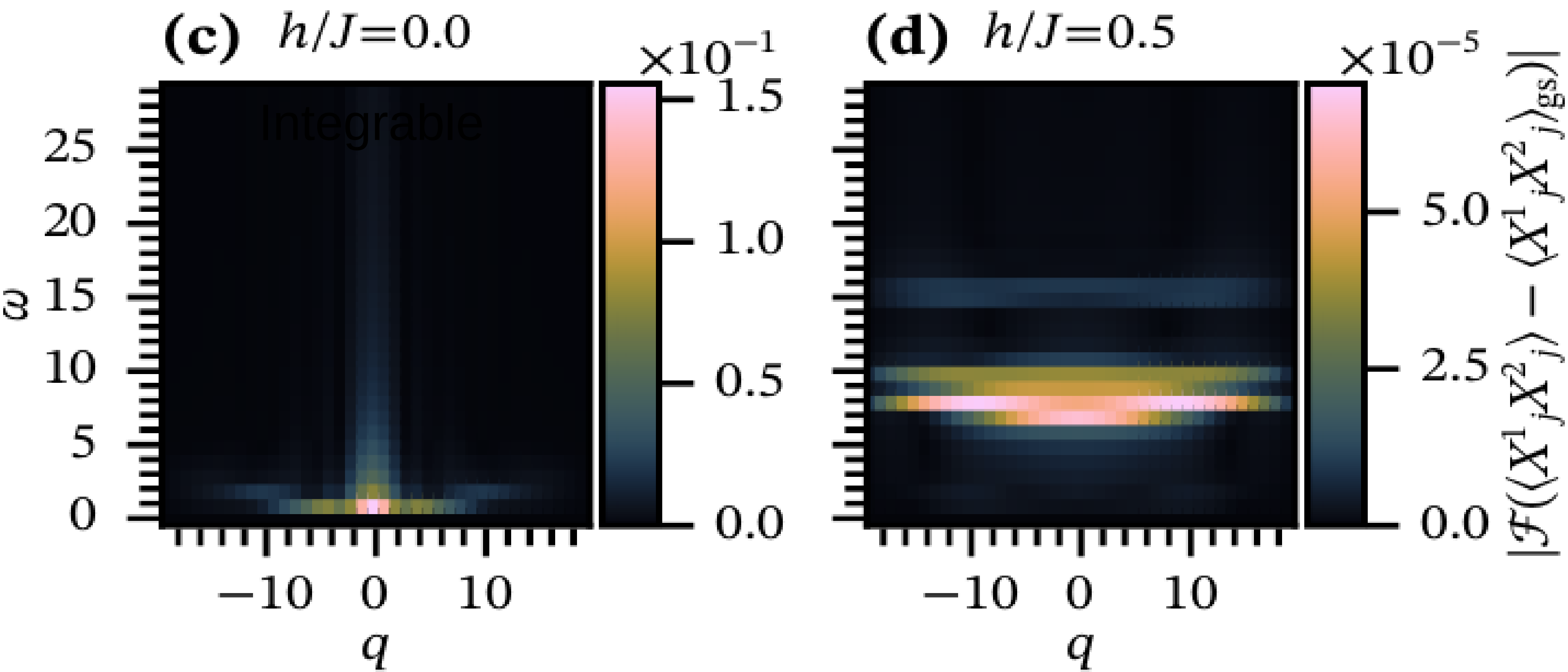
Non Integrable



$$\mathcal{F}_{\tau^2}(k,\omega) = \frac{2\pi}{LT}\delta t \sum_{j=1}^{N} e^{-ik(j-\frac{N}{2})} \sum_{n=0}^{t_N} e^{-i\omega t_n} \left(\mathrm{tr}\left(\tau(t_n)^2_{O_j}\right) - 1\right)$$

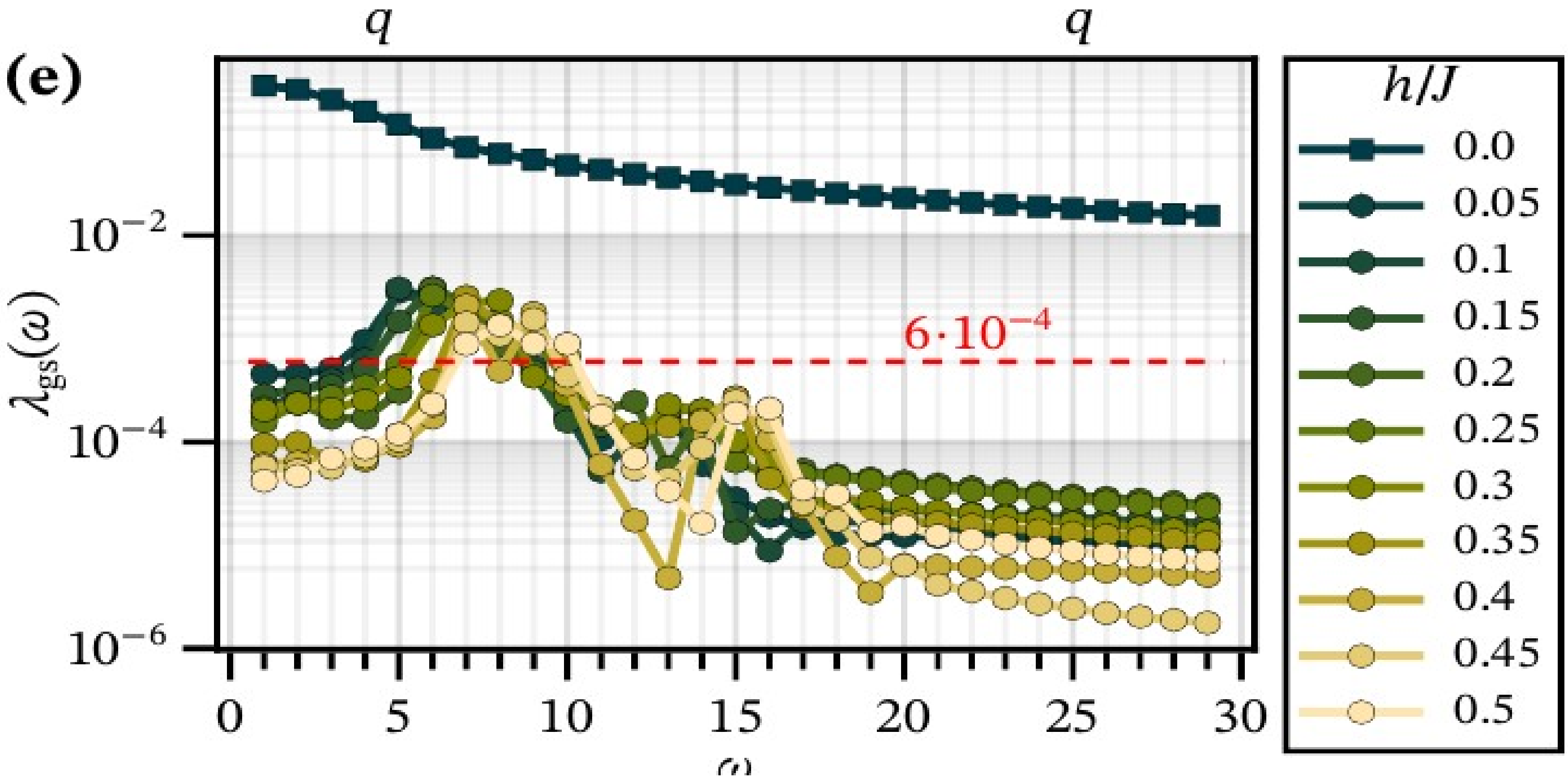
Integrable

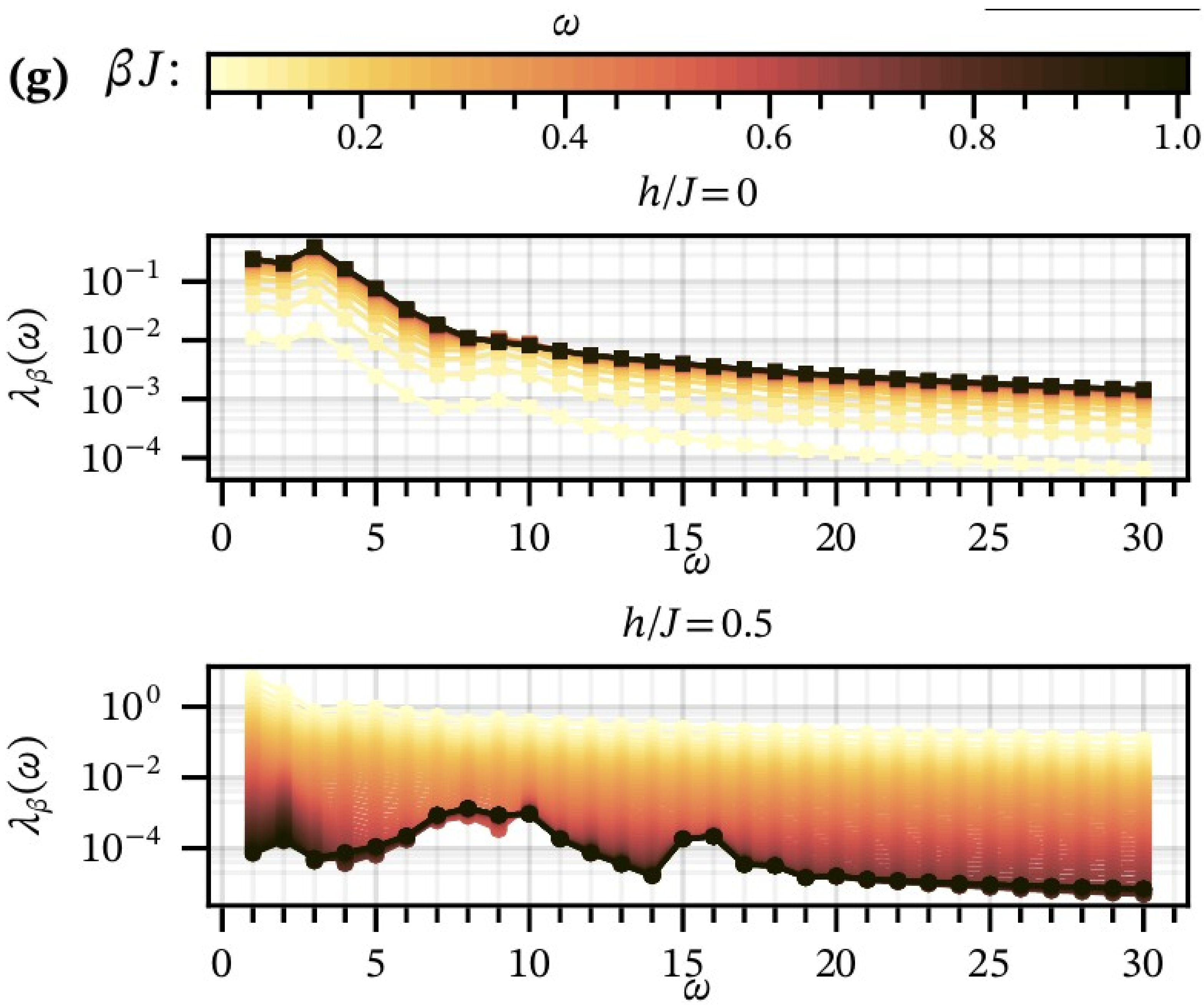
Non integrable



Soft mode → integrable dynamics

$$\lambda(\omega) = \frac{2\pi}{T}\delta t \sum_{n=0}^{t_N} e^{-i\omega t_n} \left(\mathrm{tr}\left(\tau(t_n)^2_{O_{N/2}}\right) - 1\right)$$




ω
(g) βJ:
0.2
0.4
0.6
0.8
1.0
h/J = 0
$\lambda_\beta(\omega)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30
ω
h/J = 0.5
$\lambda_\beta(\omega)$
10^{0}
10^{-2}
10^{-4}
0
5
10
15
20
25
30
ω

- Path integral formulation (tensor networks) allows for democratic treatment of space-time
- Generalized temporal entropies can be defined and mapped to expectation value of Hermitian operators after an appropriate quench
- When computed on the ground states and thermal states the operators couple to different modes,

  Massless for integrable dynamics

  Massive for non-integrable dynamics

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