Many-body magic in strongly correlated systems

QuantFunc2024,

Valencia, 4.9.24

Marcello Dalmonte

ICTP, Trieste

Joint work with T. Chanda, M. Frau, **P. Tarabunga, E. Tirrito**, and collaborations with M. C. Bañuls, M. Collura, **P. Falcao**, R. Fazio, G. Fux, A. Hamma, G. Lami, L. Leone, S. Oliviero, and J. Zakrzewski.

References: PRXQ 4, 040317 (2023) and 2409.01789
See also: PRA 109, L040401 (2024), PRL 133, 010601 (2024), PRB 110, 045101 (2024).

Question we want to address is:

Assuming that at some point a *correctly* working quantum computer exists, what would be the corresponding resources needed to simulate/represent a lattice gauge theory?

Outline

Motivation: quantum computers and their struggle to represent states - state complexity

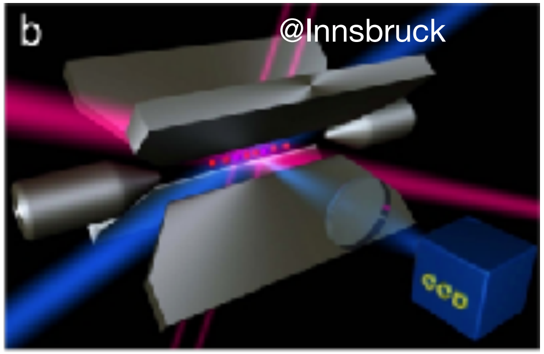
Magic: what is that?!?! Why does it characterize complexity?

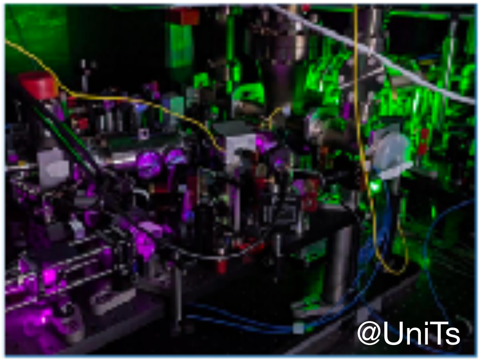
How to measure magic in numerical experiments

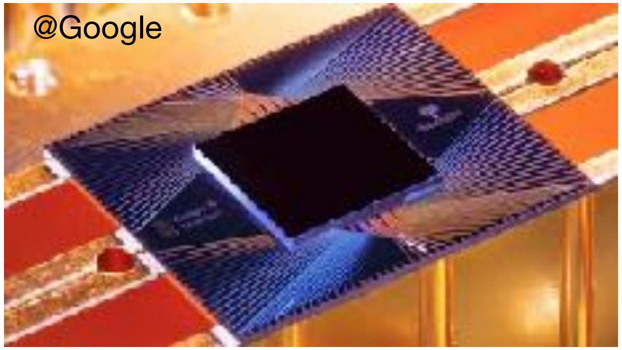
First many-body results for lattice gauge theory

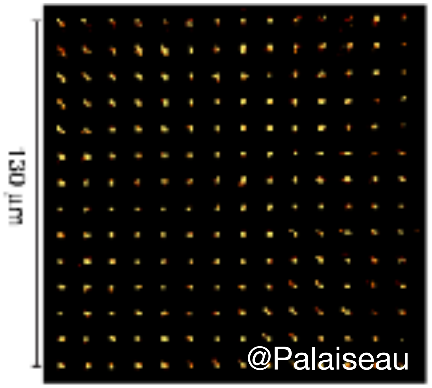
Quantum computing for physics problems

Recent experiments are harnessing quantum matter at the single quantum level, a fundamental step for quantum computing









Quantum computing and simulation

Plenty of Room at the Bottom

Richard P. Feynman
December 1959

What I want to talk about is the problem of manipulating and controlling things on a small scale.

In the year 2000, when they look back at this age, they will wonder why it was not until the year 1960 that anybody began seriously to move in this direction.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

Simulating Physics with Computers

Richard P. Feynman

a thing which is usually described by local differential equations. But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics—which is what I really want to talk about, but I'll come to that later. So what kind of simulation do I mean?

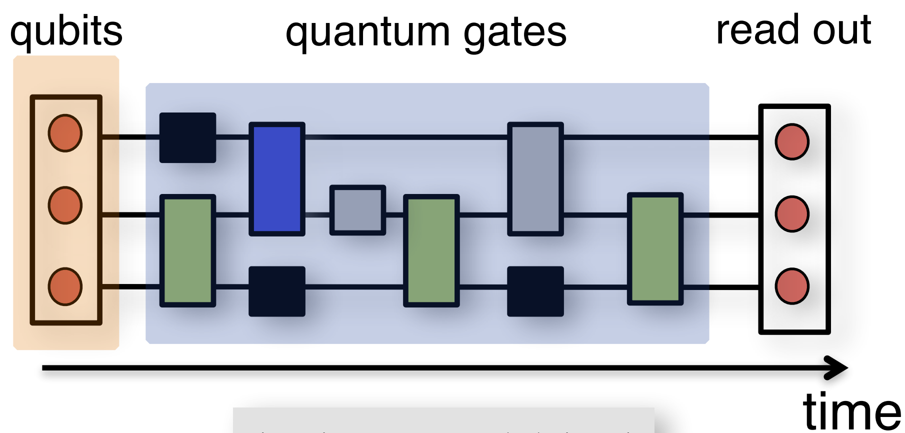
Example: digital quantum simulation

$|\Psi\rangle$ $\quad$ $U(t)|\Psi\rangle$

$|\Psi(t)\rangle$

‘Memory’
Qubits = spins

Deutsch, 80’s;
S. Lloyd (1996)



$$|\Psi\rangle \to U(t)|\Psi\rangle$$

‘Processor’

Basic operations as stroboscopic evolution (Trotter)

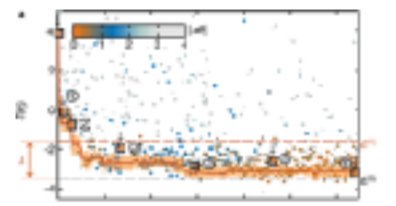
$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots^{-iH\Delta t_1/\hbar}$$

$$e^{-iH\Delta t/\hbar} \simeq e^{-iH_1\Delta t/\hbar}\, e^{-iH_2\Delta t/\hbar}\, e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}$$

First examples in the context of gauge theories

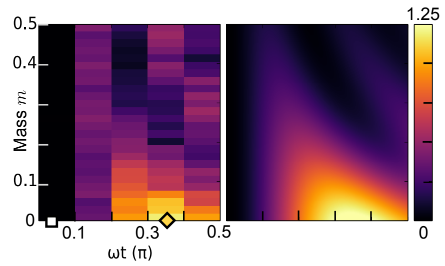
So far, mostly 1D, U(1) lattice gauge theory (LGT)

Hybrid



Kokail et al., Nature 2019

Digital



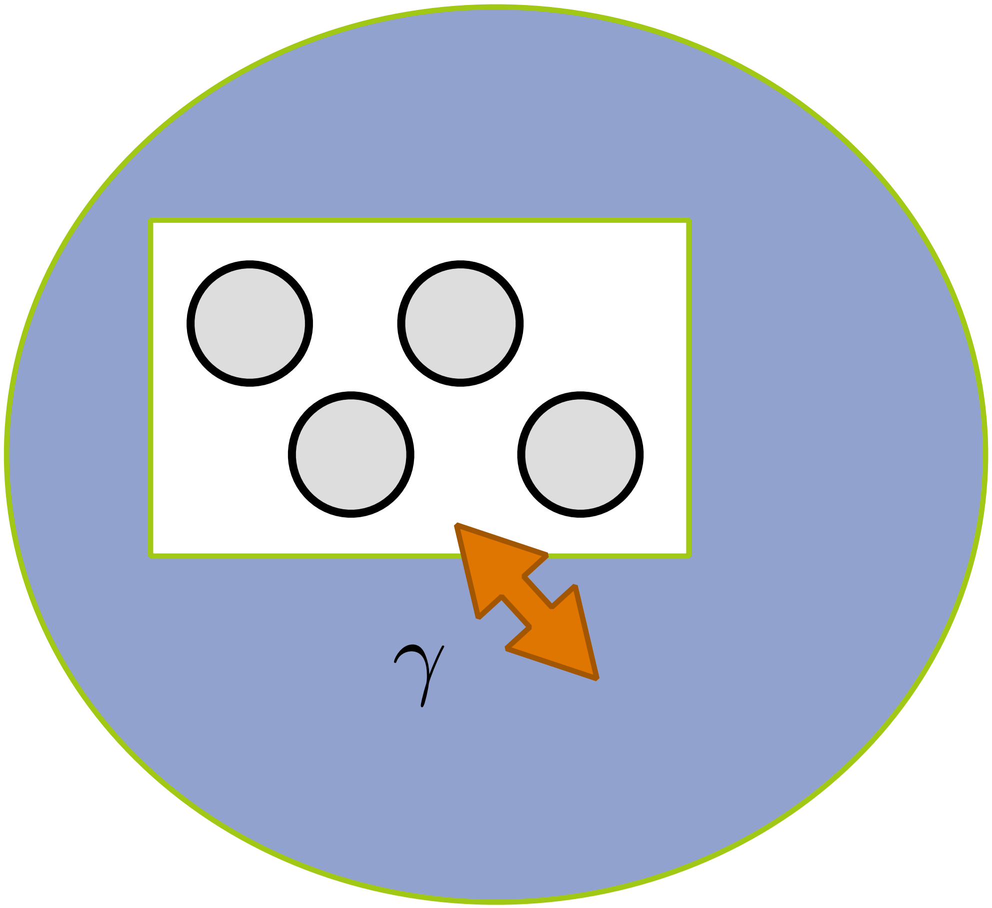
Martinez et al., Nature 2016

Analog

Bernier et al., Nature 2017

+ more experiments (Pan, Oberthaler, IBM,)

Challenge: what are the fundamental limits in studying gauge theories with quantum computers?



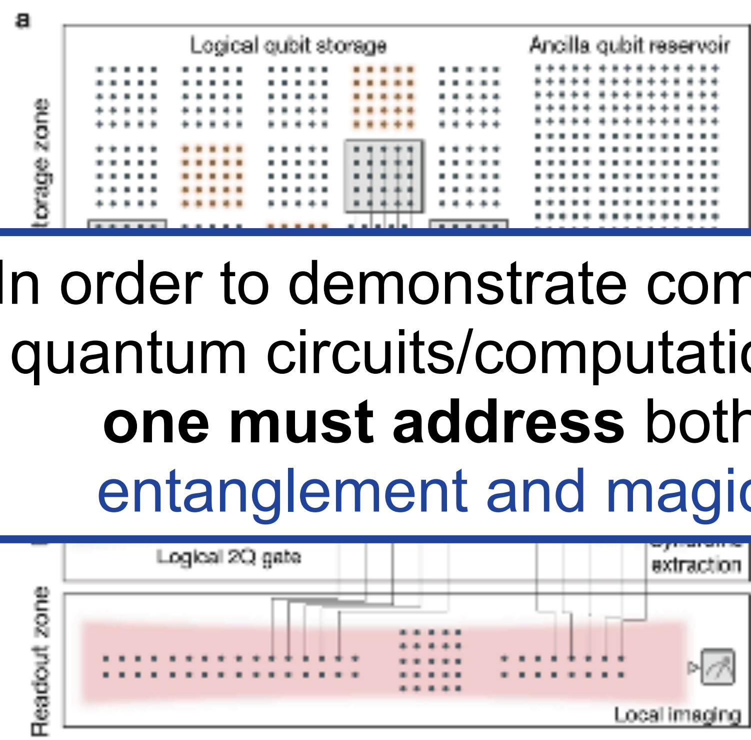
Key challenge of quantum simulation and computing: **reliable storing of information**

Qubit inevitably decays (quantum optics: Wigner-Weisskopf theory)

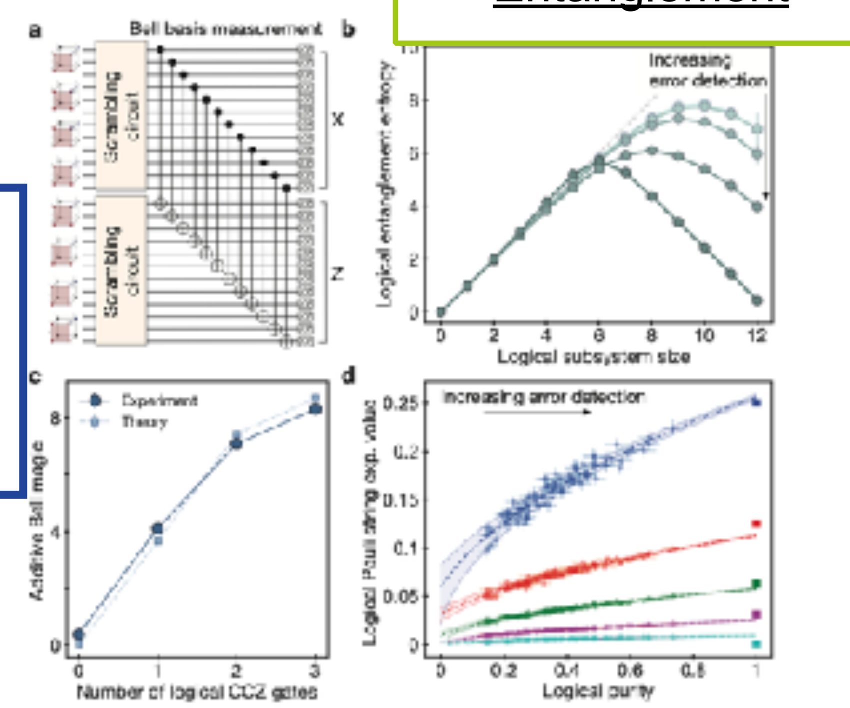
Quantum resources - quantum simulators beyond classical computability

Rydberg atom arrays

Lukin's group, Nature 2023



In order to demonstrate complex quantum circuits/computations, **one must address** both entanglement and magic



Entanglement

Magic

Platform: see pioneering works by Browayes, Lahaye, Saffman, Grangier
Alkaline-earth: Endres, Kaufman, Thompson

Why magic

- Computing: understanding how hard simulating systems is: bottleneck is often T-gates, so magic is a lower bound

- State certification: estimating magic provides rigorous errors on efficient certification tasks (provided magic is small enough)

- Compilation: understanding the structure of magic can further relax bounds on T-gate resources

Outline

Motivation: quantum computers and their struggle to represent states - state complexity

Magic: what is that?!?! Why does it characterize complexity?

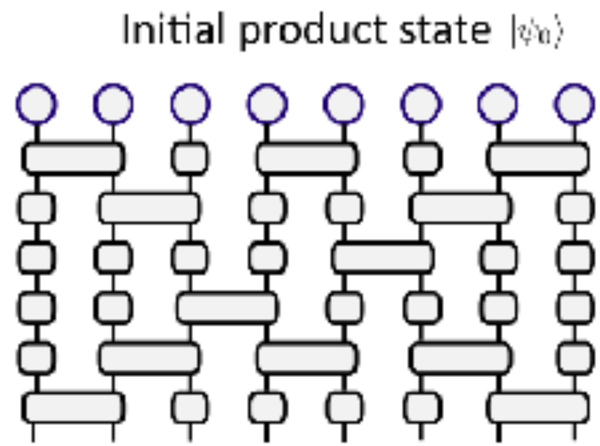
How to measure magic in numerical experiments

First many-body results for lattice gauge theory

Magic / non-stabilizerness

Sloppy versions

Given a target state $|\Psi\rangle$



$|\Psi\rangle$

Its magic is the (minimal) amount of non-Clifford resources required to realize it in a circuit starting from: $|\psi_0\rangle = |000...\rangle$

Essence: quantifies incapability of writing down a state as result of (arbitrary many) Clifford operations

Bravyi & Kitaev, Phys. Rev. A, 71, 022316 (2005).
Gottesman's phd thesis, 1997; Phys. Rev. A 57, 127 (1998).

A bit more details: non stabilizerness and relation to computational complexity

Clifford group (e.g., spin-1/2)

$$\mathcal{C}_N = \{\text{Hadamard}, \text{CNOT}, P(\pi/4)\}$$

Physics: maps Pauli strings into Pauli strings

$$\mathcal{P}_N = \left\{ e^{\frac{i\theta\pi}{2}} \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N} | \theta, j_k = 0, 1, 2, 3 \right\}$$

Gottesman-Knill theorem: states produced by Clifford gates can be very entangled but they can be *simulated efficiently with a classical computer.*

A bit more details: non stabilizerness and relation to computational complexity

Clifford group (e.g., spin-1/2)

$$\mathcal{C}_N = \{\text{Hadamard}, \text{CNOT}, P(\pi/4)\}$$

Needs T-gates to realize universal quantum computation

$$|T\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + e^{i\pi/4}|1\rangle\Big)$$

Universal quantum computing =
Clifford operations + T-state injection

Bravyi & Kitaev, *Physical Review A*, *71*, 022316 (2005)

<u>Resource</u>: T-states

A new viewpoint on complexity

Big picture questions:

Is there a relation between entanglement and magic?

Are there states of matter that fundamentally require magic?

Does magic relate to physical phenomena?

2304.01175

PRXQ 4, 040317 (2023), 2312.02039, 2401.16498

See also works by Beri, Hamma, Haug, Kim, Lloyd, Piroli, Winter, Lukin's and Monroe's exp

Lack of separability / entanglement

Caveat: these are minimal conditions for complexity (in terms of probability distribution sampling)

First studies in LGTs: upper bounds

2D SU(2), state preparation

x	η	L	t/a_s	Δ	$\alpha_{\text{Trot.}}$	$\alpha_{\text{Newt.}}$	Schwinger bosons Qubits	Schwinger bosons T gates
1	4	100	1	0.01	90%	9%	2626	8.19713×10^{11}
1	4	100	1	0.001	90%	9%	2704	3.09951×10^{12}
1	4	100	10	0.01	90%	9%	2704	3.0993×10^{13}
1	4	100	10	0.001	90%	9%	2808	1.2146×10^{14}
1	4	1000	1	0.01	90%	9%	18904	3.12769×10^{13}
1	4	1000	1	0.001	90%	9%	19008	1.22564×10^{14}
1	4	1000	10	0.01	90%	9%	19008	1.22564×10^{15}
1	4	1000	10	0.001	90%	9%	19086	4.486[illegible]7 $\times 10^{15}$

Davoudi, Shaw, Stryker, Quantum 2024

Why do we know so little about magic and many-body?

Problem 1: useful measures of magic are rare as it requires minimization

Problem 2: for the few measures known, lack of scalable computational/analytical/experimental protocols!

Getting to the physics will require to address the two methodological problems above

Liu & Winter, *PRX Quantum*, *3*, 020333 (2022).

Challenge 1: how to measure?

Breakthrough: **Stabilizer Renyi entropies**

Leone, Oliviero & Hamma, PRL 128, 050402 (2022).

$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$

Renyi index

Physics: entropy of a distribution of Pauli strings

$$\mathcal{P}_N = \left\{ e^{\frac{i\theta\pi}{2}} \sigma_{j_1} \otimes \cdots \otimes \sigma_{j_N} \middle| \theta, j_k = 0, 1, 2, 3 \right\}$$

See Alioscia's talk on Tuesday

Stabilizer Renyi entropies

$$M_n(\rho) = \frac{1}{1-n} \log \sum_{P \in \mathcal{P}_N} \frac{|\mathrm{Tr}(\rho P)|^{2n}}{d^N}$$

- Formulated as expectation values of string operators ✓
- Satisfies properties of measures* ✓
- Still, requires to measure of measurements $\propto 4^N$ ✗

Leone, Oliviero & Hamma, PRL 128, 050402 (2022); *see also Haug & Piroli, Quantum 2023, for a discussion about allowed operations

Our tool: Pauli Markov chains

$$M^{(n)}\left(|\psi_N\rangle\right) = \frac{1}{1-n}\log\left\{\sum_{P\in P_N}\frac{Tr\left(\rho P\right)^{2n}}{2^N}\right\}$$

$$\Xi_P = \frac{\mathrm{Tr}\left(\rho P\right)^2}{d^N}$$

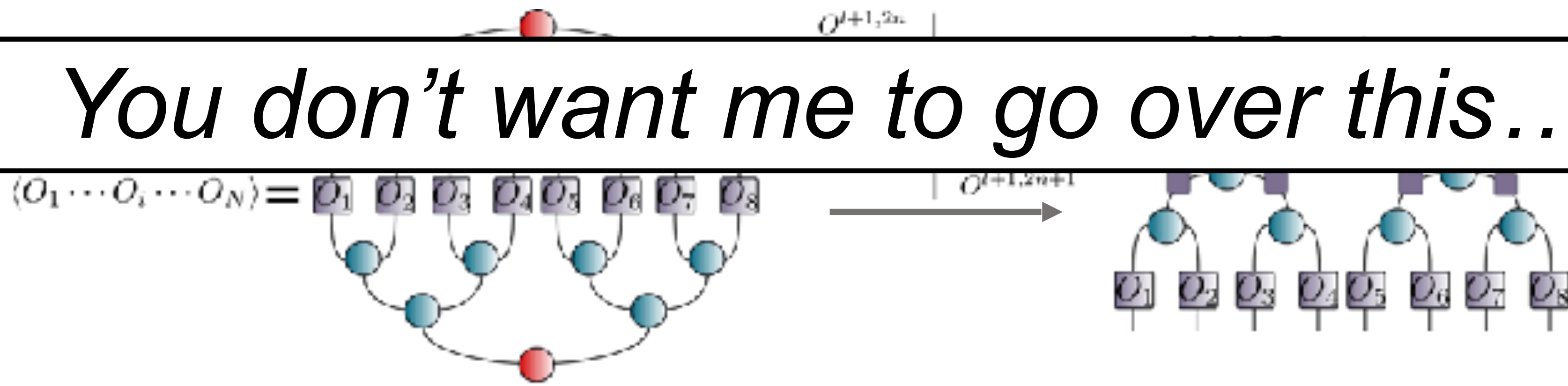
Sample not states, but the **distribution of Pauli strings**, with importance sampling!

Pauli-Markov chains

NB: straightforwardly applicable to experiments, albeit role of statistical errors unclear. Also applicable to variational wave functions.

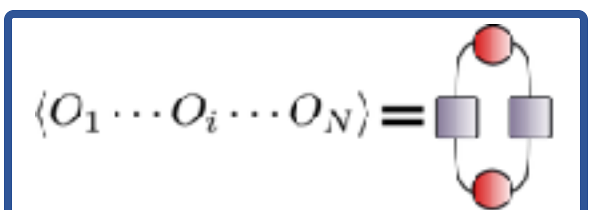
Computations with tree tensor networks

You don’t want me to go over this…



$$O(\log(N)\chi^4)$$

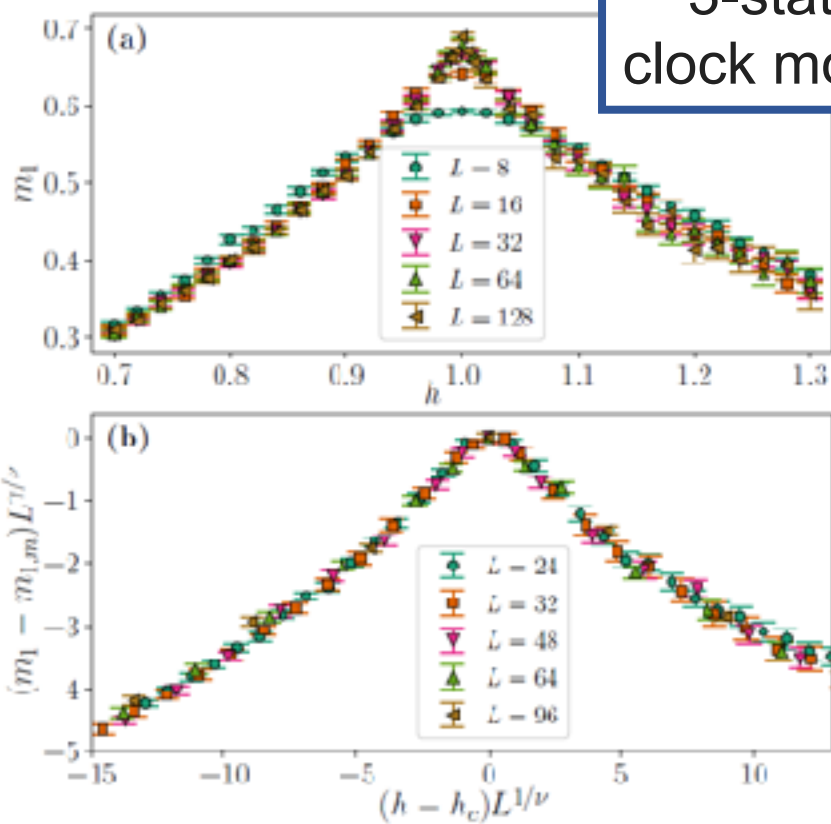
For local (e.g., 1- or 2-sites) updates, can be contracted very efficiently!!!



- arbitrary partitions
- PBC
- dimension plays very little role
- Stochastic
- Easily extended to MPS, PEPS etc.

Examples: magic at conformal critical points

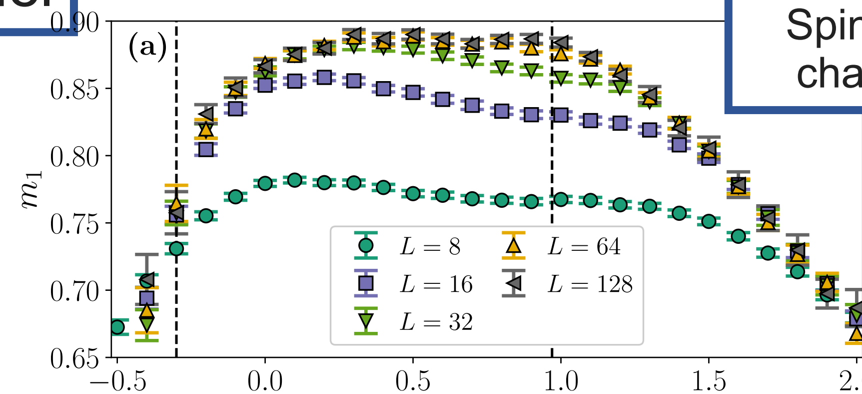
3-state clock model



- full state magic in some cases satisfied critical scaling

- but in some others, it fails…

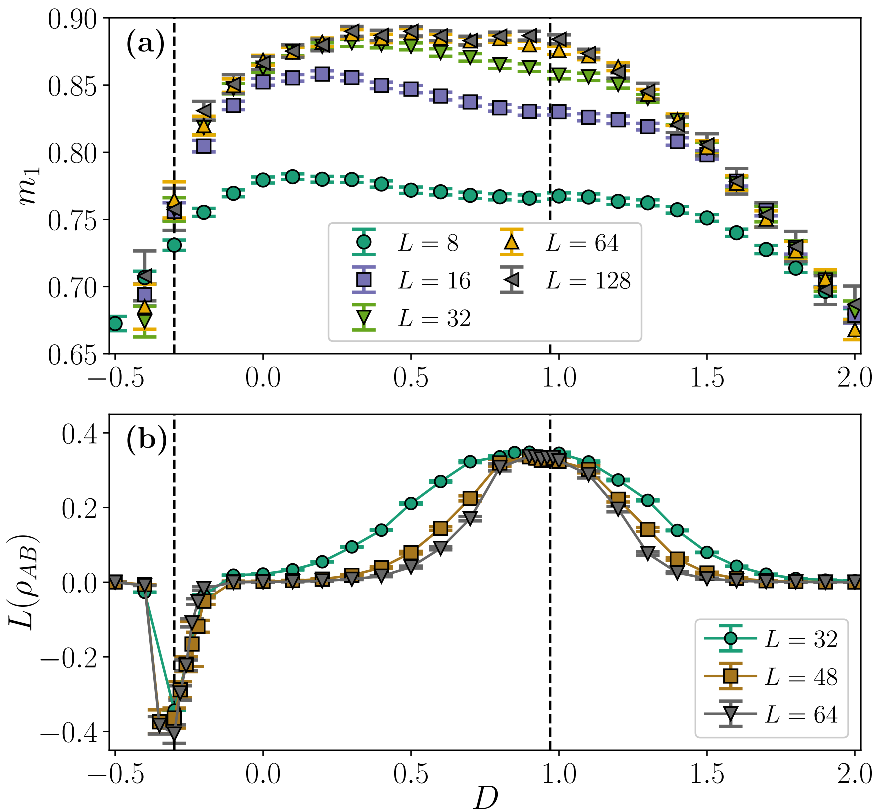
Spin-1 chain



For small partitions: White, Cao & Swingle; *Physical Review B*, *103*, 075145 (2021).
Ising: Haug and Piroli 2023

Long-range magic and conformal criticality

$$L(\rho_{AB}) = M_2(\rho_{AB}) - M_2(\rho_A) - M_2(\rho_B)$$



- cleans up “non-universal” contributions
- *always* displaying a peak at critical points

Summary: for CFTs, magic displays universal features that are only evident if disconnected partitions are considered

Outline

Motivation: quantum computers and their struggle to represent states - state complexity

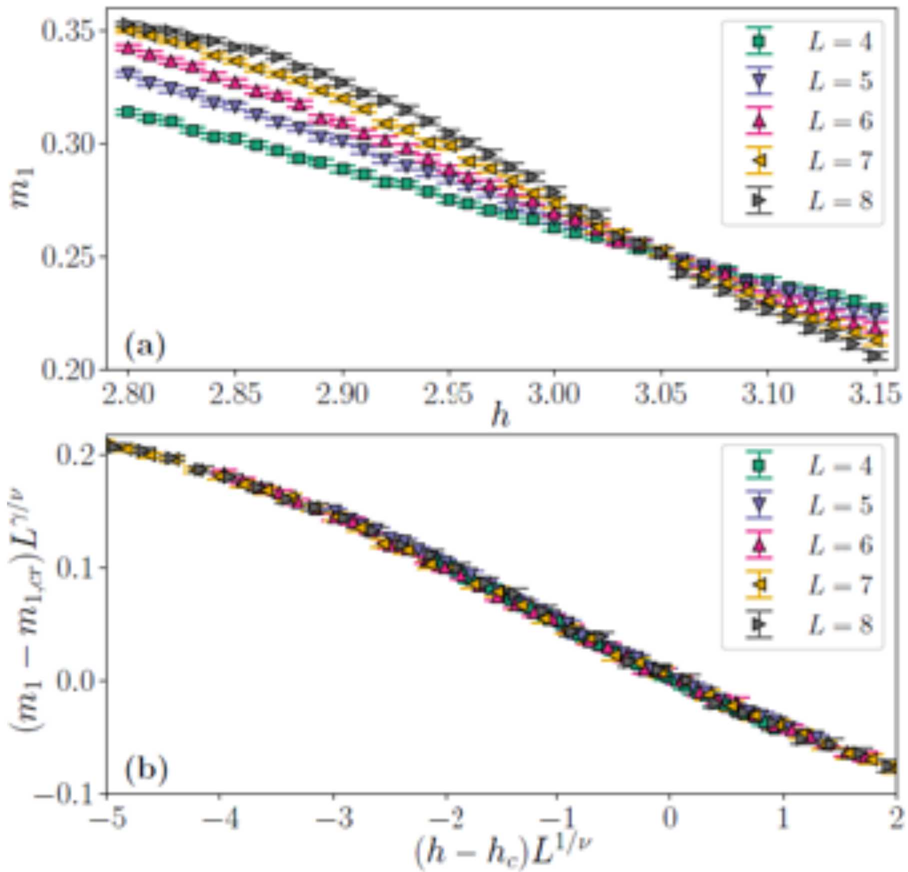
Magic: what is that?!?! Why does it characterize complexity?

How to measure magic in numerical experiments

First many-body results for lattice gauge theory

Lattice gauge theories: quenched

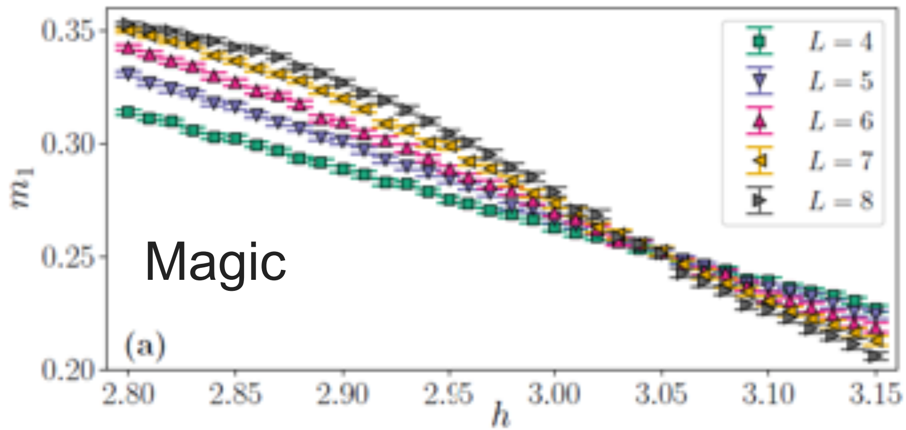
$$H_{Z_2-\text{Gauge}} = -h \sum_{\boxdot} \prod_{i \in \boxdot} \tau_i^x - \sum_i \tau_i^z$$

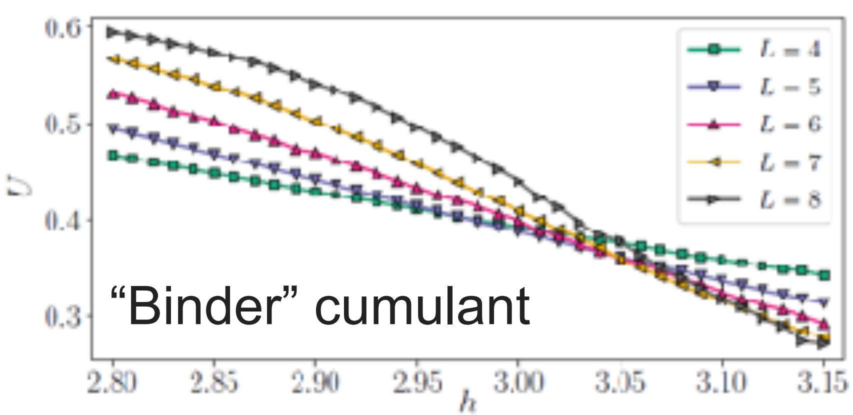


- very different from 1D: magic displays crossing at criticality

- very impressive collapse scaling!

Lattice gauge theories





- at finite “entanglement”, magic detects critical behavior *better* than the order parameter

Lattice gauge theories: U(1) + matter

Hamiltonian formulation of a generalized Schwinger model

Minimal coupling

Staggered mass

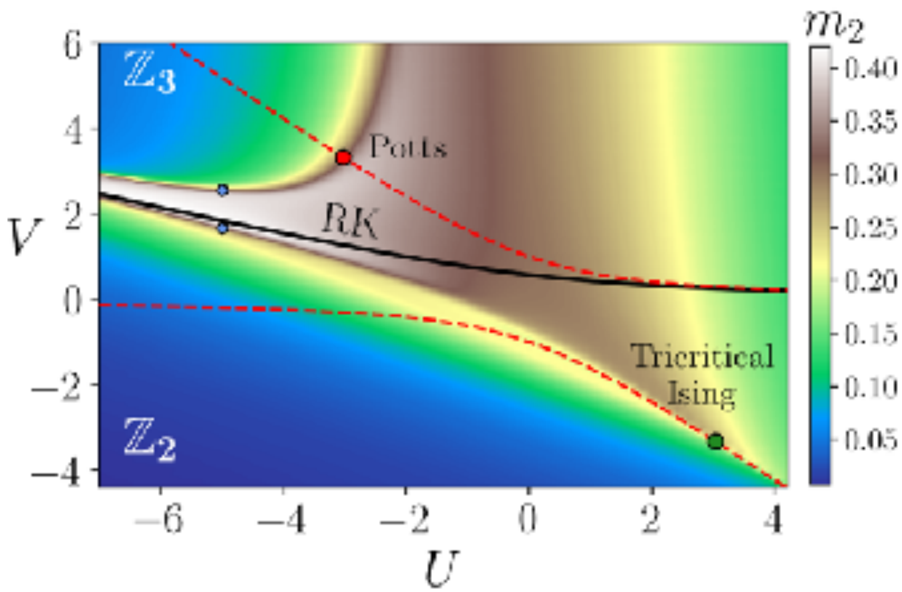
$$H = -w\sum_{j=1}^{L-1}(\Phi_j^\dagger S_{j,j+1}^+\Phi_{j+1} + \mathrm{h.c.}) - U\sum_{j=1}^{L}(-1)^j\Phi_j^\dagger\Phi_j$$
$$+V\sum_{j=1}^{L}\Phi_j^\dagger\Phi_j\Phi_{j+1}^\dagger\Phi_{j+1} + J\sum_{j=1}^{L-1}(S_{j,j+1}^z - \theta/\pi)^2.$$

4-Fermi coupling

Electric field

Phase diagram

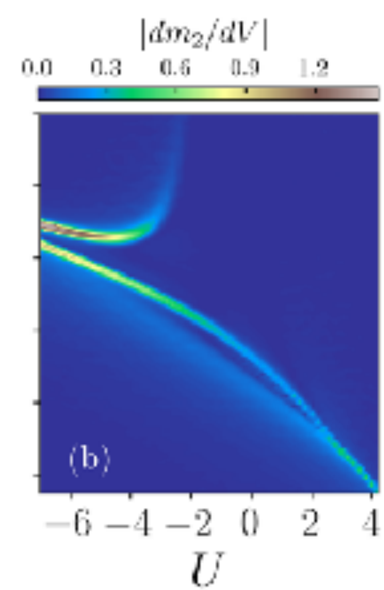
Magic



4-Fermi coupling

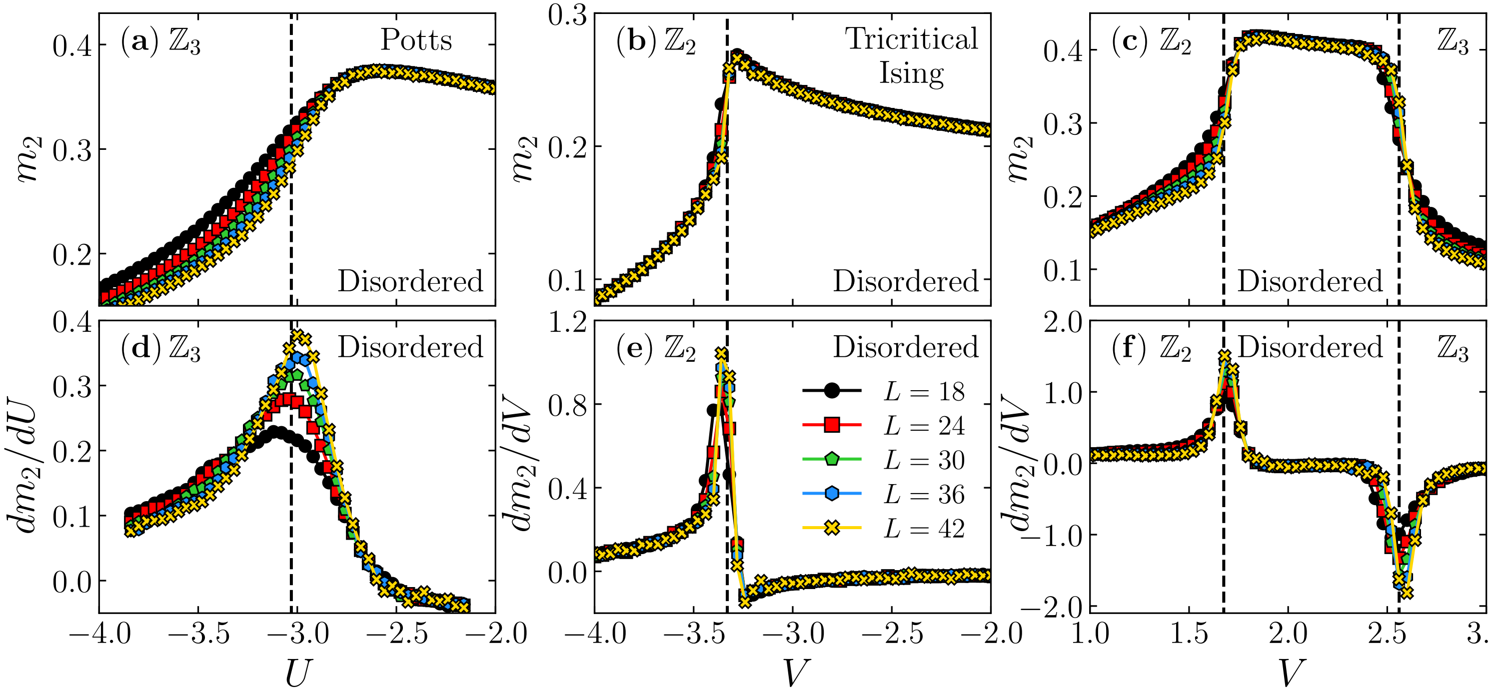
Mass

- Magic is extensive over the full phase diagram
- Magic does not peak at transition points



Dashed: integrable lines. Thin: exactly soluble
L=30. Error < 10^-3

Magic approaching the continuum limit

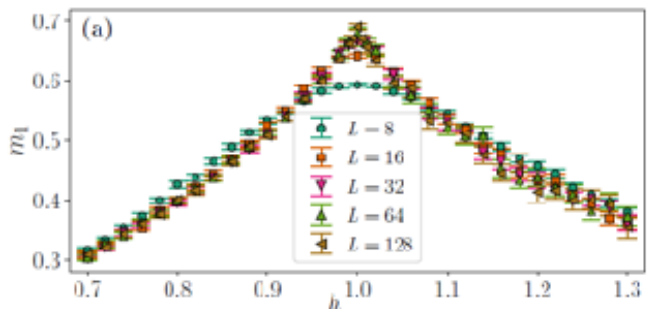


- Magic peaks away from criticality most of the time
- It s derivative is compatible with divergence, signalling its key role

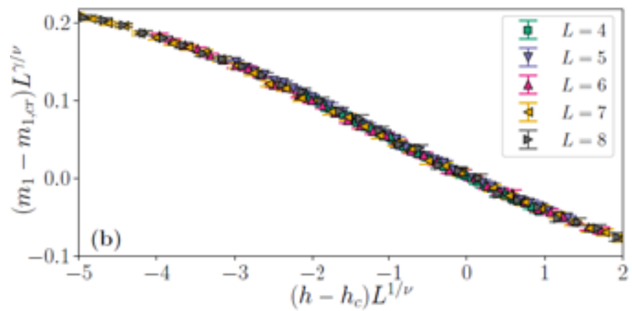
What have we learned?

Q: Does magic relate to physical phenomena?

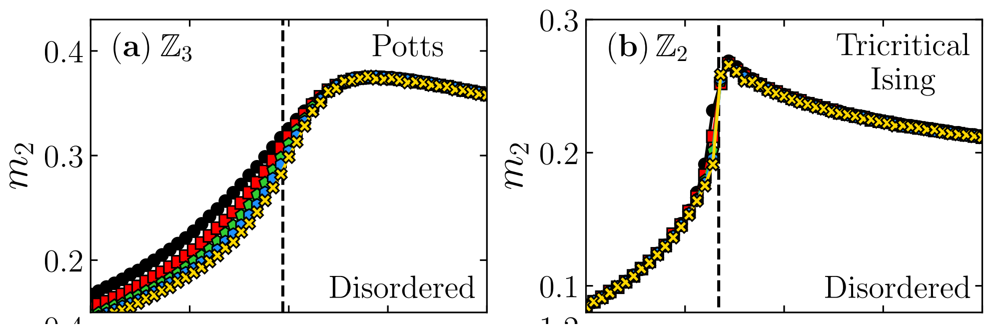
Conformal critical points



Lattice gauge theory



Q: Is magic gonna be a bottleneck for quantum computing and LGTs?



- Magic is always extensive
- Peaks is typically very off continuum limit

Open questions

We are barely scratching the surface of the interface magic/many-body systems

- Magic vs entanglement - are they related?

  2312.02039 (with Fux, Tirrito, Fazio); see also 2312.00132 (Beri's group) and 2403.19610

- Development of better computational methods, including Monte Carlo?

  PRL 133, 010601 (2024) (with Tarabunga, Tirrito, Bañuls); see also Collura's and Clarks's group

- (Other) relations to physical phenomena? Non-Abelian theories?

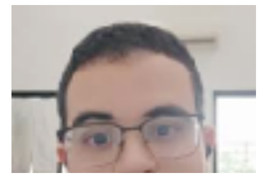
- Complete (?) understanding of where a quantum simulator is really beyond reach (compilation)

P. Tarabunga

M. Frau

T. Chanda

E. Tirrito



Thank you!

Kuba Zakrzewski   Pedro Falcao

M. Collura

MC. Bañuls

Based on PRXQ 4, 040317 (2023), PRL 133, 010601 (2024), and 2409.01789