No planar degeneracy for the Landau gauge quark-gluon-vertex

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Motivation

Why studying the **quark-gluon vertex** (QGV) to a high precision?

- It is the most important correlation function *connecting QCD's matter to its YM sector*:
  - the only primitively divergent "mixed" *n*-point function;
  - highly non-trivial role in Dynamical Chiral Symmetry Breaking (DχSB);
  - influences strongly the analytic structure of the quark propagator (RA, Detmold, Fischer, Maris) and the transition to the conformal window (Hopfer, Fischer, RA) (relation to confinement?).
- QGV investigated since QCD was formulated five decades ago:
  - construct from **symmetry considerations, renormalisibiity,** etc. (Ball & Chiu, Curtis & Pennington, Bashir et al., and many others).
  - **consistency requirement** for the quark propagator DSE (e.g. von Smekal, Amundsen &RA) or Stingl's alternating chain of poles and zeros -> confinement
  - **direct calculation:** gauge-fixed large-volume lattice calculations or functional methods, see, e.g., refs in Aguilar et al, 2408.15370.

Motivation

Anticipate the **time-like side** including analytic structure:

EW theory: FMS mechanism relates poles of gauge-invariant physical states with the ones of gauge-dependent unphysical states.

QCD: Confinement implies that physical *S*-matrix elements knows only about the singularities related to intermediate hadronic resonances; the singularities of elementary correlation functions cancel.

NB: (i) cf. IR saturation of gluon e.om. (Mader, Schaden, Zwanziger, RA)
(ii) relates to dispersion theory methods, quark-hadron duality, Chesire Cat Principle, etc.

In particular, hypothesis of alternating chain of poles and zeros with cancelations in the hierarchy of *n*-point functions imply:

Poles (zeros) of quark propagator $\Longleftrightarrow$ Zeros (poles) of the QGV
Zeros (poles) of QGV $\Longleftrightarrow$ Poles (zeros) of four-point functions / scattering kernels.

Motivation

To test this aspect of confinement:

- Analytic structure (incl. poles, branch points and zeros) of propagators, three-point and four-point functions.
  (NB: Three-point functions possess at least Landau singularities, cf. Huber, Kern & RA)
- Mechanism relating the analytic structure of different functions.

In this talk:
(i) Full kinematical dependence of the QGV in Euclidean domain
(ii) A surprising observation on planar (non-)degeneracy of QGV
(iii) Low-lying singularities of DχSB QGV form factors via extrapolation

A note on the calculations

Two calculations, both in quenched approximation:

(I) Self-consistently determined YM input and
coupled quark propagator and QGV 1PI DSE with non-Abelian
[plus Abelian] diagram

(II) YM input from M.Q. Huber [2003.13703]
(3-gluon vertex with single variable S_0 and correction factor)
coupled quark propagator and **QGV 3PI DSE** with non-Abelian
[plus Abelian] diagram

Disclaimer: High-precision data not yet available,
computations still running on Uni Graz' HPC cluster ...

QGV: Kinematics and decomposition

Fully dressed QGV in the Landau gauge including its full kinematics

$$\Gamma_\mu^{A\bar\psi\psi,a,ij}(k;-p,q) = igt^{a,ij}\sum_{i=1}^{8}\Gamma^{(i),A\bar\psi\psi}(k^2,\bar p^2,k\cdot\bar p)R_\mu^{(i)}(k;\bar p)$$

Gluon momentum $k = p - q$, averaged quark momenta $\bar p = (p+q)/2$, and the angle θ in between them, resp., $w = \cos\theta$.

Eight transverse tensor structures:

- Tree-level structure γ_μ^T which is χS.
- Four χS and four χSB transverse tensor structures.
  (The four partly longitudinal ones are not needed in the Landau gauge.)
- The numbering of the tensors follows A. Windisch, PhD thesis, 2014.

Different than in precious calculations we do NOT normalise any momenta! All form factors / dressing functions are therefore non-vanishing and finite at $k^2 = \bar p^2 = 0$.

QGV: Kinematics and decomposition

Charge conjugation property of the vertex $\Longrightarrow$:

$$\Gamma^{(i),A\bar{\psi}\psi}(k^2,\bar{p}^2,w) \stackrel{!}{=} \Gamma^{(i),A\bar{\psi}\psi}(k^2,\bar{p}^2,-w)$$

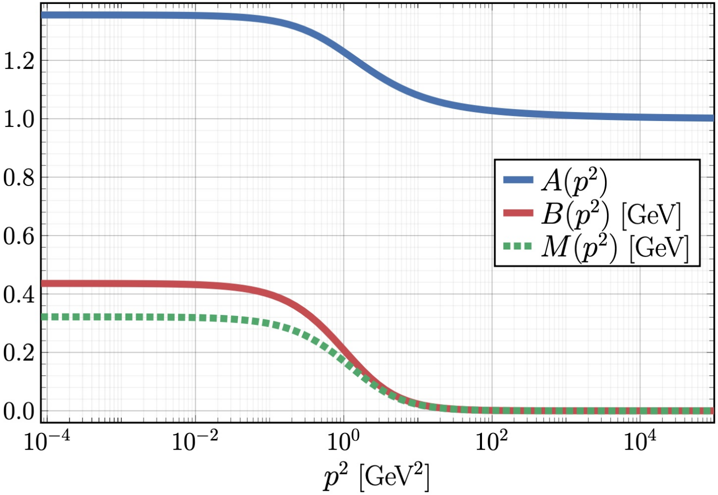
Implies that the Γ are **even functions of $w = \cos\theta$.**

Transverse structures
(numbering, χS[B], Lorentz structure & energy dimension):

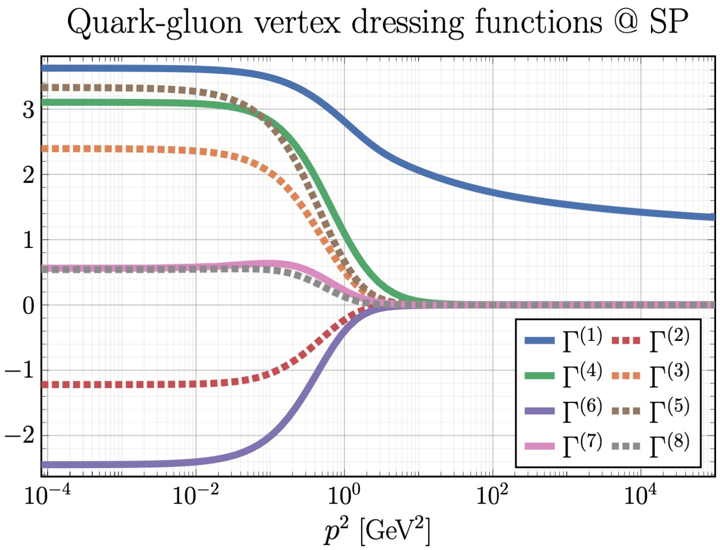
		χS	S	dim			χS	S	dim	
$\Longrightarrow$	1	y	V	0		2	n	T	-3	
	4	y	DV	-2		3	n	T	-1	$\Longleftarrow$
	6	y	V	-2		5	n	S	-1	$\Longleftarrow$
	7	y	V	-4		8	n	T	-3	

Numerical results (Computation II)

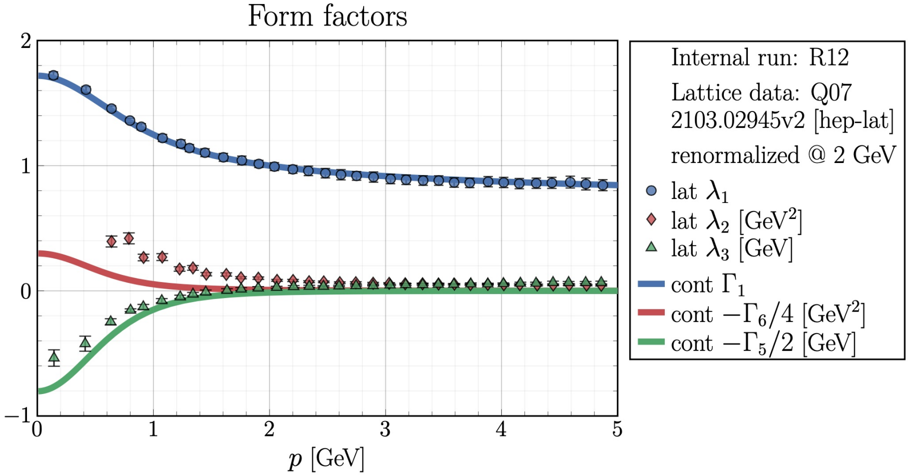
Quark propagator in chiral limit:



Numerical results (Computation II)

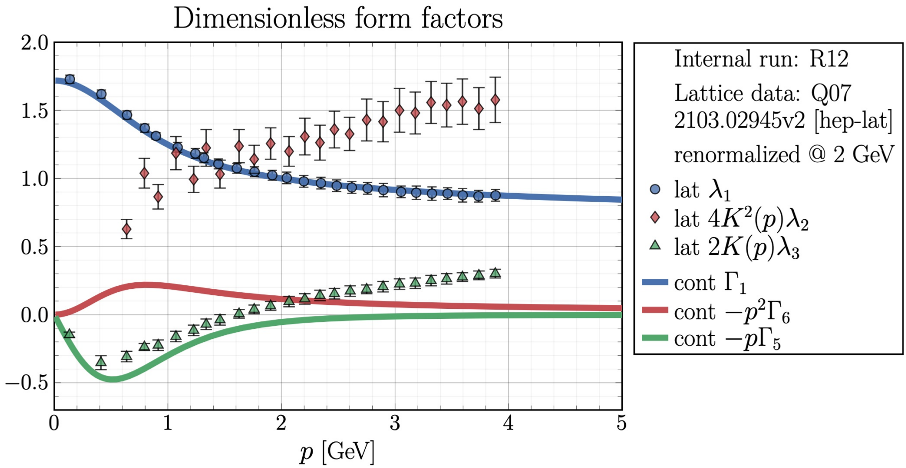


Comparison to lattice at vanishing gluon momentum:

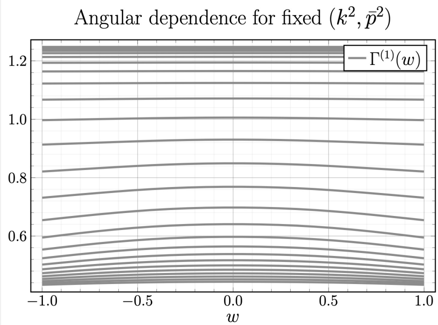


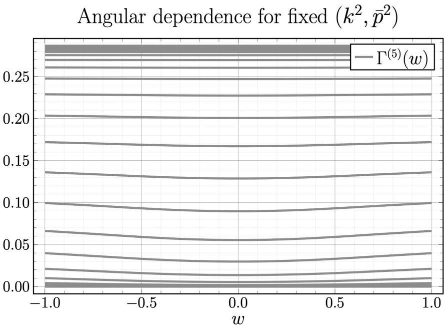
Numerical results (Computation II)

Comparison to lattice for *dimensionless* functions:



Angular dependence:



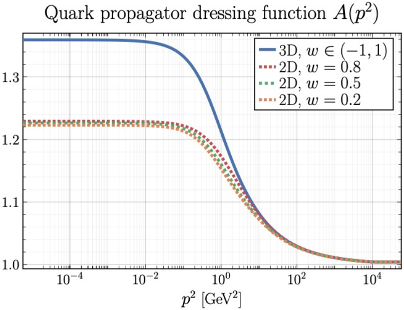


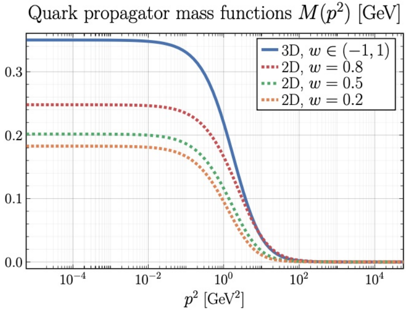
$$\propto a(k, \bar{p}) \left(1 + b(k, \bar{p})\, w^2\right) + \text{“}tiny\text{”}$$

$-0.5 \leq b(k, \bar{p}) \leq 0.5$ and for most momenta $|b| < 0.1$.

Numerical results

Dependence of the vertex dressings on $w = \cos(\angle(k,\bar{p}))$ seems relatively small!
And solving for $\Gamma^{A\bar{\psi}\psi}(k^2,\bar{p}^2)$ with fixed w would simplify the numerical calculations significantly...
⚠️But: Explicit calculations show notable differences in the quark propagator ⚠️





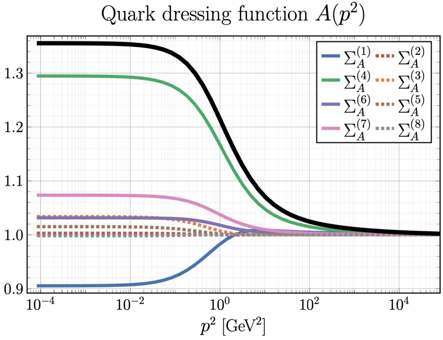
cf. alsoTang, Gao, Liu, Phys. Rev. D **100** (2019); 1902.01679 [hep-ph]

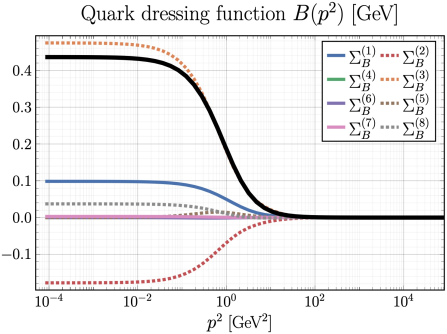
Important:

No planar degeneracy
despite seemingly small angular dependence!

Numerical results

χS part of QGV contributes to χS quark propagator dressing $A(p^2)$,
χSB part of QGV drives DχSB!



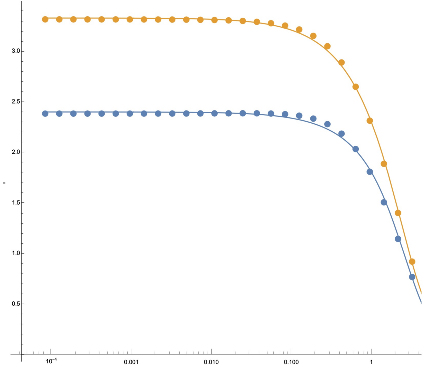


Even tree-level structure contribute less than 1/4!!!
RAINBOW-LADDER TRUNCATION IS UNJUSTIFIED!

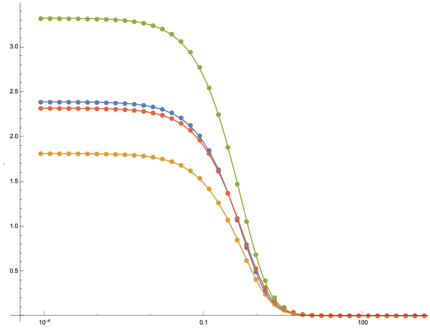
Numerical results

Fits of Γ^5 and Γ^3 at $w = 0$:

for $\bar{p}^2 = 0$ vs. k^2:



for $k^2 = 0$ and $k^2 \approx 1$ GeV^2 vs. $\bar{p}^2$:



Numerical results

Different dependence on gluon and quark momentum!
(Support region: $k \approx 1.2$ GeV, $\bar{p} \approx 0.4$ GeV)

NB: χS form factors decrease even slower with gluon's k^2.

$\Gamma^5 \propto \Gamma^3$ **up to numerical precision (a few %, to be improved)**
for the whole kinematical range!

Off-shell version for the on-shell Gordon identity?!?

Numerical results

For fixed $\bar{p}^2$ and w fits of the form

$$\frac{a}{1 + bk^2 + (ck^2)^{5/2}}$$

are very precise! Likewise for fixed k^2 and w fits of the form

$$\frac{a}{1 + rk^2 + (s\bar{p}^2)^{5/2}}$$

are very precise!

Hereby $b, c, r, s = \mathcal{O}(1)$ GeV^2.

Indicate poles close to the origin on the time-like axis!

Location at appr. $-M^2(0) \dots$

An observation . . .

Suppose the difference between the decoupling solutions (with a finite gluon curvature mass scale) and the scaling (power-law) solutions are a non-perturbative gauge-fixing issue (hypothesis by Axel Maas):

As YM and matter sector are independently gauge invariant the matter sector Green functions obtained from different types of solutions should be Landau-Khalatnikov transforms of each other!?!

We observe even (PRELIMINARY!):

Taking from the YM sector consistent input for decoupling and the scaling solutions, respectively, and include the full kinematical dependence for the QGV, the results for the quark propagator and the QGV even coincide for the different solutions!

(I.e., within the numerical precision
which is as already stated up to now a few % !)

Conclusions and Outlook

Results for the Landau gauge QGV:

- Different behaviour of χS and χSB form factors.
- **χSB QGV form factors: self-consistent drivers of DχSB.**
- Scalar and tensor couplings are proportional to each other (off-shell version of Gordon identity?) and are besides the form factor for the tree-level structure the important ones.
- Kinematical variables:
  Gluon, averaged quark momenta, angle $(k, \bar{p}, w = \cos\theta)$
  $\Longrightarrow$ weak angular dependence $\propto a(k,\bar{p})\left(1 + b(k,\bar{p})w^2\right) + \text{``}tiny\text{''}$
  $-0.5 \geq b(k,\bar{p}) \geq 0.5$ and for most momenta $|b| < 0.1$.
- Significant effect of angular dependence on, e.g., $M(p^2)$:
  NO planar degeneracy!

Conclusions and Outlook

- Different dependence on gluon and quark momentum!
  (Support region: $k \approx 1.2$ GeV, $\bar{p} \approx 0\ .4$ GeV)
- Highly accurate fits available!
- **Different analytic structure for χS and χSB form factors.**
- Conjecture:
  χSB form factors possess **pole close to origin** ($\bar{p}^2 \approx -M^2(0)$),
  and **branch points** according to Landau singularities.

Conclusions and Outlook

Outlook

- Apply QGV for an analysis of the decays of exotics
  (with F. Llanes-Estrada, A. Salas-Bernadez, E. Swanson, G. Wieland)
  cf. Alex' talk on the role of the QGV in $^3P_0$ mechanism for low-momentum meson decays.
- Calculate QGV for time-like momenta via contour deformation.
- Beyond the quenched approximation:
  - N_f-dependence in chiral limit / small quark masses
  - Physical case $N_f = 2 + 1$
  - Dependence on current mass m for heavy quarks
    (cf. results in G. Wieland's master thesis:
    different mass scales for the onset of suppression)
  - Phase transition to conformal window
  - Other gauge groups (BSM physics or Dark Matter candidates)
- Test hypothesis of canceling poles and zeros.