The η_c-meson tPDF in lattice QCD:
Light-cone correlators from Euclidean setups

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Physique des 2 Infinis et des Origines

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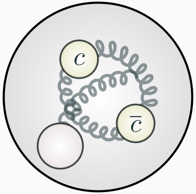
Introduction: η_c-meson

η_c meson	
Composition:	$c\bar{c}$
J^{PC}:	0^{-+}
Mass:	2983.9 ± 0.4 MeV
Width:	32.0 ± 0.7 MeV



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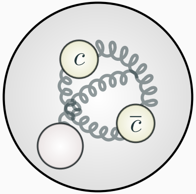


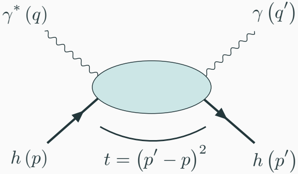
η_c-hadron structure

- How does it emerge from the bounding of a pair $c\bar{c}$?
- Comparison with lighter 0^{-} mesons: Assess quark-mass effect on hadron structure.

Introduction: t-dependent Parton Distribution Functions (tPDFs)

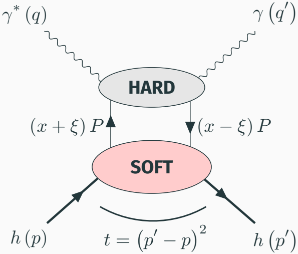
Hadron structure

How do quarks and gluons combine to make hadrons up?



Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

Factorization
[Phys.Rev.D59(1999)074009]



$$\mathcal{H}\left(\xi, t, Q^2\right) = \sum_{p=q,g} \int_{-1}^{1} \frac{dx}{\xi} \mathcal{K}^p \left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s\left(\mu_F^2\right)\right) \boldsymbol{H_{p/h}\left(x, \xi, t, \mu_F^2\right)}$$

Generalized Parton distributions: **Off-forward parton distribution functions**

Introduction: t-dependent Parton Distribution Functions (tPDFs)

Off-forward parton distribution functions: Non-local, light-like separated, quark or gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light-front. [Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}(x,\xi,t,\mu^2) = \frac{1}{2}\int \frac{dy^-}{2\pi} e^{ix\frac{p+p'}{2}\cdot z} \left\langle h\left(p'\right)\right| \overline{\psi}_q\left(-z/2\right)\gamma^+\widehat{\mathcal{W}}\left[-z/2, z/2\right]\psi_q\left(z/2\right)\left|h\left(p\right)\right\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

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t-dependent Parton Distribution Functions

$$q_h(x,t,\mu^2) = H_{q/h}(x,\xi=0,t,\mu^2) \quad [p\cdot z = p'\cdot z \equiv p^+]$$

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Properties:

- Universality *i.e.* hadron-specific objects.
- Contain parton distribution functions and electromagnetic form factors.
- Non-perturbative description of hadron structure: (3D) Tomography.

Project: To compute tPDFs of the η_c-meson

Lattice QCD (I)

(Continuum) Quantum field theory

$$\langle\Omega|\,\mathcal{O}\,|\Omega\rangle \propto \int \mathcal{D}\left[A_\mu,\overline{\psi},\psi\right](x)\,\mathcal{O}\left[A_\mu,\overline{\psi},\psi\right](x)\,e^{iS\left[A_\mu,\overline{\psi},\psi\right](x)}$$

Extremely hard to assess beyond perturbation theory

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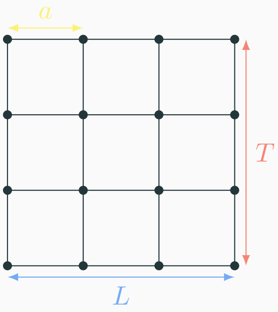
(Lattice) Quantum field theory

- Analytic continuation: $t \to -it_E \Rightarrow e^{iS} \to e^{-S_E}$
- Spacetime discretization:
  - a (lattice spacing): UV cut-off.
  - $L^3 \times T$ (finite box): Finite number of degrees of freedom.

Amenable for numerical evaluation of the path integral:
Non-perturbative calculations!



Lattice QCD (II)

In Lattice field theory, the expectation value of an observable, $\langle \mathcal{O} \rangle$, is obtained as:

$$\langle \mathcal{O} \rangle \propto \int \mathcal{D}\left[U, \overline{\psi}, \psi\right] \mathcal{O}\left[\overline{\psi}, \psi, U\right] e^{-S_E\left[U, \overline{\psi}, \psi\right]} \simeq \frac{1}{N} \sum_i^N p\left(U_i\right) \langle \mathcal{O} \rangle_F \left[U_i\right],$$

with

- $p\left(U_i\right)$: Boltzmann probability distribution - Obtained, numerically, through Monte Carlo sampling of the Euclidean path integral.
- $\langle \mathcal{O} \rangle_F$: Fermionic expectation value - Evaluated, exactly, through Wick theorem.

Computed expectation values are connected to (Euclidean) correlation functions
[Comm.Math.Phys.:42(1975)281, Comm.Math.Phys.:54(1977)283]

How do we compute tPDFs (light-cone correlator) on an Euclidean setup?

Pseudo-distributions in a nutshell (I)

Definition: Ioffe-time tPDF ($\nu \equiv -p \cdot z$)
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h(\nu, t, \mu^2) \equiv \int_{-1}^{1} dx e^{i\nu x} q_h(x, t, \mu^2) = \frac{1}{2p^+} \left\langle h\left(p'\right) \left|\overline{\psi}_q\left(-z/2\right) \gamma^+ \widehat{\mathcal{W}}\left[-z/2; z/2\right] \psi_q\left(z/2\right)\right| h\left(p\right)\right\rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

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1. Consider a **generic matrix element** with $z^\mu \in \mathbb{R}^{3,1}$ (or even $z^\mu \in \mathbb{R}^4$)
[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$\begin{aligned} M_q^\mu\left(p, p', z\right) &= \left\langle h\left(p'\right)\right| \overline{\psi}_q\left(-z/2\right) \gamma^\mu \widehat{\mathcal{W}}\left[-z/2; z/2\right] \psi_q\left(z/2\right) \left|h\left(p\right)\right\rangle \\ &= (p+p')^\mu \mathcal{F}(\nu, t, z^2) - (p'-p)^\mu \mathcal{G}(\nu, t, z^2) + z^\mu \mathcal{Z}(\nu, t, z^2), \quad \nu \equiv -p \cdot z \end{aligned}$$

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2. Light-front projection, *i.e.* $z^\mu \propto (1, 0, 0, -1)$ and $\mu = +$

$$M_q^+(p, p', z)\Big|_{\substack{z^+ \to 0 \\ \boldsymbol{z}_\perp \to \boldsymbol{0}_\perp}} = \langle h(p') | \overline{\psi}_q(-z/2) \gamma^+ \widehat{\mathcal{W}}[-z/2; z/2] \psi_q(z/2) | h(p) \rangle = 2p^+ \mathcal{F}(\nu, t, z^2 \to 0)$$

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3. Connection with Ioffe-time distributions

$$q_h(\nu, t, \mu^2) = \mathcal{F}(\nu, t, z^2 \to 0)$$

Pseudo-distributions in a nutshell (II)

$$\textbf{Pseudo-tPDF: } \widetilde{q}_h(\nu, t, z^2) = \mathcal{F}(\nu, t, z^2)\Big|_{\mathrm{R}}, \quad \lim_{z^2 \to 0} \widetilde{q}_h(\nu, t, z^2) \to q_h(\nu, t, \mu^2)$$

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Euclidean setup:

- $p^\mu = (E, \boldsymbol{p}_\perp, p_3)$ and $p'^\mu = (E, -\boldsymbol{p}_\perp, p_3)$
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1. Compute

$$M^0\left(p, p', z\right) = \langle h\left(p'\right)|\overline{\psi}_q\left(-z/2\right)\gamma^0\widehat{\mathcal{W}}\left[-z/2; z/2\right]\psi_q\left(z/2\right)|h\left(p\right)\rangle = 2E\mathcal{F}(\nu, t, z^2)$$

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2. Renormalize through a RGI ratio
[Phys.Lett.B:767(2017)314]

$$\mathcal{F}(\nu,t,z^2)\Big|_{\mathrm{R}} \equiv \frac{M^0(p,p',z)}{M^0(0,0,z)}\frac{M^0(0,0,0)}{M^0(p,p',0)} = \frac{\mathcal{F}(\nu,t,z^2)}{\mathcal{F}(0,0,z^2)}\frac{\mathcal{F}(0,0,0)}{\mathcal{F}(0,t,0)} = \widetilde{q}_h(\nu,t,z^2) + \text{h.t.}$$

Lattice QCD calculation

Numerical setup

- $N_f = 2$ ensembles (CLS) [Nucl.Phys.B:865(2012)397,PoSLATTICE2013:(2014)475]
  - Wilson gauge action.
  - $\mathcal{O}(a)$-improved Wilson fermions.
  - $\kappa_u = \kappa_d \equiv \kappa_k$.
  - No Symanzik improvement for M^μ.

id	β	a [fm]	L/a	am_π	m_π [MeV]	$m_\pi L$	κ_c	κ_l
A5	5.2	0.0755(9)(7)	32	0.1265(8)	331	4.0	0.12531	0.13594
E5	5.3	0.0658(7)(7)	32	0.1458(3)	437	4.7	0.12724	0.13625
F7			48	0.0885(3)	265	4.3	0.12713	0.13638
N6	5.5	0.0486(4)(5)	48	0.0838(2)	340	4.0	0.13026	0.13667

- One hadron-interpolator and four smearings (source and sink).

$$\eta_c^s(x) = \bar{\psi}_c^s(x)\,\gamma_5\psi_c^s(x) \quad , \quad J^{PC} = 0^{-+}$$

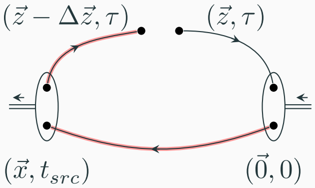
$$\psi_q^s(x) = (1 + 0.125\Delta_{\mathrm{APE}})^{N_s}\,\psi_q(x) \quad , \quad N_s \in \{0, 30, 50, 80\}$$

- Twisted boundary conditions and a symmetric frame.

Matrix elements from LQCD

Computation of hadron three-point functions

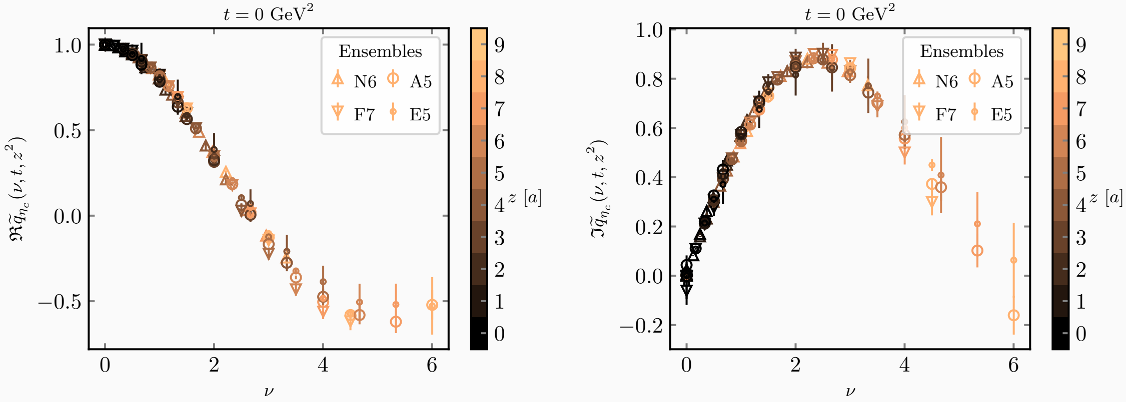
$$C_3^{(ss')}\left(\vec{p}, t_{src}\right) = \sum_{\vec{x},\vec{z}} e^{-i\vec{p}\cdot\vec{x}-i\vec{q}\cdot\vec{z}} \langle \eta_c^s(\vec{x}, t_{src}) \overline{\psi}_c\left(\vec{z}-\Delta\vec{z}, \tau\right) \gamma^0 \widehat{\mathcal{W}}[\vec{z}-\Delta\vec{z}, \tau; \vec{z}, \tau] \psi_c\left(\vec{z}, \tau\right) \overline{\eta}_c^s(\vec{0}, 0)\rangle$$



- Consider connected diagrams only: Sequential propagator technique.
- Project ground state ($\eta_c\left(1s\right)$) according to GEVP.
- Compute ratios to isolate matrix elements: [PoSLATTICE2005:(2006)360]

$$R\left(\tau\right) = \frac{C_3^{(\mathrm{P})}\left(\vec{p}, \vec{p}', t_{src}, \tau\right)}{\sqrt{C_2^{(\mathrm{P})}\left(\vec{p}', t_{src}\right) C_2^{(\mathrm{P})}\left(\vec{p}, t_{src}\right)}} \sqrt{\frac{C_2^{(\mathrm{P})}\left(\vec{p}, t_{src}-\tau\right) C_2^{(\mathrm{P})}\left(\vec{p}', \tau\right)}{C_2^{(\mathrm{P})}\left(\vec{p}', t_{src}-\tau\right) C_2^{(\mathrm{P})}\left(\vec{p}, \tau\right)}} \to \frac{M^0\left(p, p', z\right)}{4\sqrt{E\left(\vec{p}\right) E\left(\vec{p}'\right)}}$$

Pseudo-tPDF data



- Pseudo-distribution data approaching a universal line for all ensembles.
- Signal-to-noise ratio significantly good over the entire kinematic range.

Light-cone matching

Light-cone matching: $\lim_{z^2 \to 0} \widetilde{q}_h(\nu, t, z^2) \to q_h(\nu, t, \mu^2)$

From pseudo- to light-cone distributions (I)

1. Light-cone operator product expansion: $z^2 \to 0$
[Nucl.Phys.B:27(1971)541]

$$\widetilde{q}_h(\nu, t, z^2) \sim \sum_i \sum_{n=0}^{\infty} \mathcal{C}_n^{(i),\overline{\mathrm{MS}}}(z^2, \mu^2) \langle h(p') | \widetilde{\mathcal{O}}_{(i)}^{\{\mu\mu_1\cdots\mu_n\}} | h(p) \rangle^{\overline{\mathrm{MS}}} z_\mu \prod_{k=1}^{n} z_{\mu_k} + \mathrm{h.t.}$$

$$= \sum_i \sum_{n=0}^{\infty} \mathcal{C}_n^{(i),\overline{\mathrm{MS}}}(z^2, \mu^2)(-2)^n \widetilde{a}_n^{(i),\overline{\mathrm{MS}}}(t, z^2, \mu^2)\nu^n + \mathrm{h.t.}$$

With $\widetilde{a}_n^{(i),\overline{\mathrm{MS}}}(t, z^2, \mu^2) = (1 + z^2 \widetilde{f}_n^{(i)}(\nu, t, z^2, m^2))\widetilde{a}_n^{(i)}(t, \mu^2)$, $\widetilde{f}_n^{(i)}$ include targe-mass correct.

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2. Recast as convolution: Light-cone matching to $\overline{\mathrm{MS}}$ distribution

[Phys.Rev.D:98(2018)014019, Phys.Rev.D:98(2018)050004, Phys.Rev.D:97(2018)074508]

$$\widetilde{q}_h(\nu, t, z^2) = \int_0^1 dw \, \mathcal{C}(w, z^2\mu^2, \alpha_s) q_h(w\nu, t, \mu^2)$$

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Challenges of the light-cone matching

- Integral relation
- Acts on light-cone distributions

From pseudo- to light-cone distributions (II)

Solution: Ioffe-time distributions, onto and outside the light-front, are analytic functions in the Ioffe-time variable:

$$q_h(\nu, t, \mu^2) = \sum_{m=0}^{\infty} a_m(t, \mu^2)\nu^m, \qquad \widetilde{q}_h(\nu, t, z^2) = \sum_{m=0}^{\infty} \widetilde{a}_m(t, z^2)\nu^m.$$

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- Expansion coefficients identified in LC-OPE: Inclusion of target-mass corrections
- Connection with Mellin moments

$$\mathfrak{m}_m^{(i)}(t, \mu^2) = i^m m! a_m^{(i)}(t, \mu^2)$$

From pseudo- to light-cone distributions (III)

Strategy:

1. Fit pseudo-distribution data to

$$\widetilde{q}_h(\nu, t, z^2) = \sum_{m=0}^{N} \left(\widetilde{A}_m(t, z^2) + z^2 \widetilde{B}_m(t, z^2) \right) \nu^m$$

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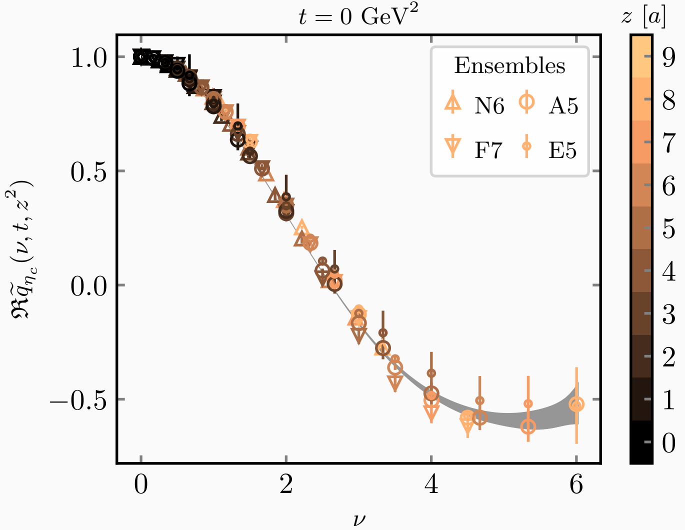
3. Reconstruct light-cone Ioffe-time $\overline{\text{MS}}$ distribution accordingly

$$q_h(\nu, t, \mu^2) = \sum_{m=0}^{N} a_m(t, \mu^2)\nu^m$$

Ioffe-time tPDFs: Non-singlet distribution

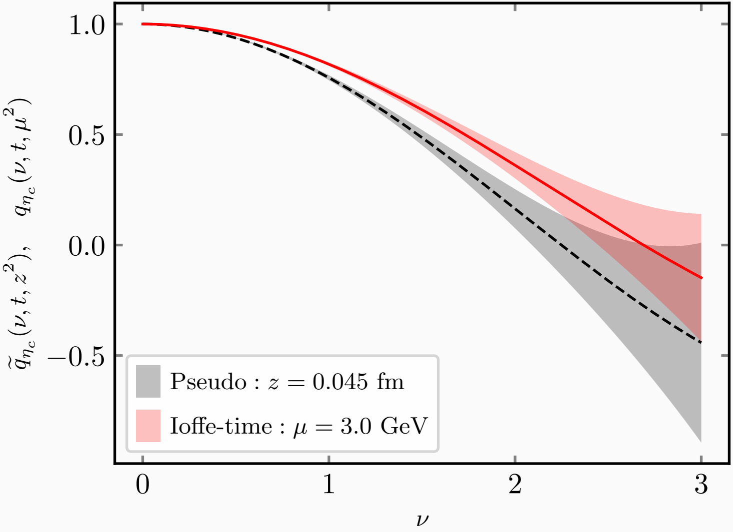


$\chi^2/\mathrm{dof} = 95.29/114$

$a_2(t, z^2)$	$-0.2552(87)$
$a_4(t, z^2)$	$-0.0124(25)$
$a_6(t, z^2)$	$-0.00022(20)$
$a_8(t, z^2)$	$-0.0000009(42)$

$z = 0.045$ fm

Ioffe-time tPDFs (II)



Conclusions and future steps

Summary

- Ongoing study of η_c-meson's structure through GPDs within lattice QCD.
- Study light-cone matching in Ioffe-time space:
  - Model bias reduction to truncation error.
  - Systematic extraction of "arbitrary" order Mellin moments.
  - Transparent treatment of target mass corrections.
  - Possible improvement in matching.

Future steps

- Extend analysis to the non-singlet sector.
- Extend kinematics and statistics: t-values and new ensembles.
- Refine analysis of lattice artifacts: e.g. finite volume or quark masses.
- Reconstruction of x-space distributions (?).

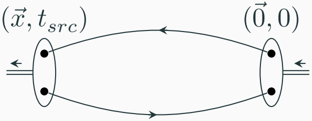
Thank you!

Back-up slides

Two-point functions (I)

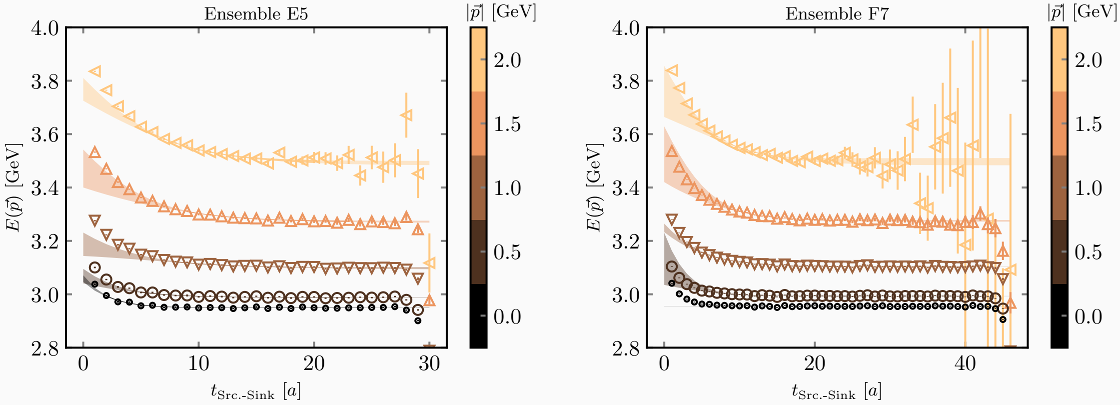
Computation of hadron propagators

$$C_2^{(ss')}(\vec{p}, t_{src}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \eta_c^s(\vec{x}, t_{src}) \overline{\eta}_c^{s'}(\vec{0}, 0) \rangle \propto \mathcal{N}(\vec{p})\, e^{-E(\vec{p})t_{src}}$$



- Consider connected diagrams only.
- Project ground-state ($\eta_c(1s)$) solving GEVP.
- Fit energies: Choose best fit range according to AIC.

Two-point functions: Effective masses

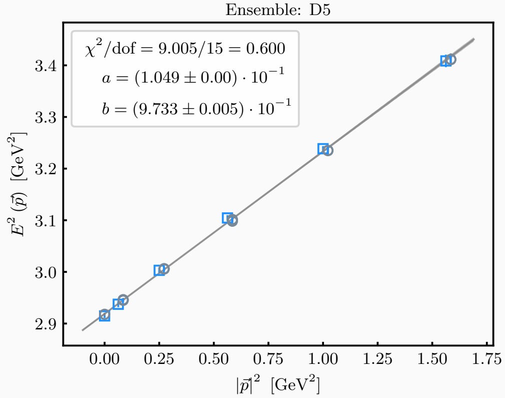


Energy spectrum compatible with expectation within finite-volume and cut-off effects.

Systematics: - Fit range: Model averaging (AIC) [Phys.Rev.D:103(2021)114502]

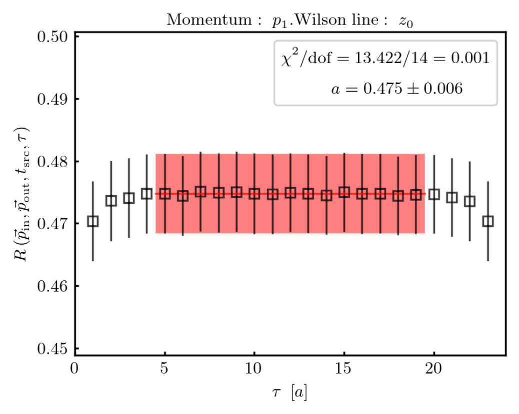
- Excited states: GEVP [Nucl.Phys.B:259(1985)58, JHEP:04(2009)094]

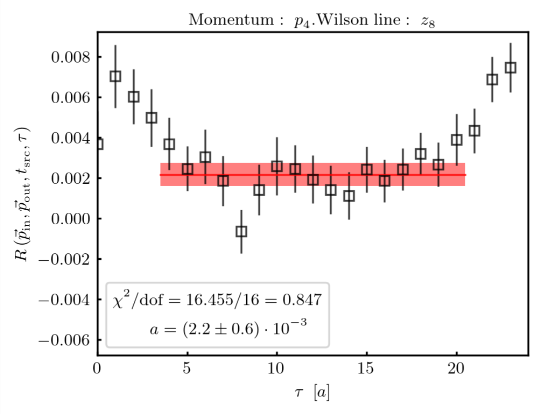
Two-point functions: Dispersion relations



Consistency check: Expected energy-momentum dispersion relations fulfilled.

Three-point functions: Ratio fits





Three-point functions