Shear and Bulk Viscosity for a pure glue theory Using an effective matrix model

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Semi-QGP region

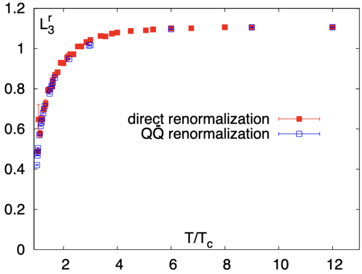

Figure 1: Gupta, Hubner and Kaczmarek, 0711.2251

- Range: T_d to $4T_d$ of the deconfined medium.
- $0.4 < \ell_1 < 1$ for $SU(3)$ gauge theory, without quarks. $\left(\ell_1 = \text{Tr}L(\mathbf{x})\right)$

(our) Matrix Model

We take the background field as,

$$A_{cl,\mu} = \frac{\mathbf{Q}}{g}\delta_{0,\mu} = \frac{2\pi T}{g}\mathbf{q}\,\delta_{0,\mu} \tag{1}$$

Where the q^a is related to the thermal Wilson line

$$L(\mathbf{x}) = \exp\left(\frac{2\pi i}{N_c}\mathbf{q}\right) \tag{2}$$

where $(\mathbf{q})_{ab} = q^a\delta_{ab}$ - is a diagonal matrix, with $N_c - 1$ independent element, satisfying

$$\sum_{a=1}^{N_c} q^a = 0$$

Hence the total field,

$$A_\mu = A_{cl,\mu} + B_\mu \tag{3}$$

(our) Matrix Model

Inverse Gluon propagator for the fluctuation $B_\mu \rightarrow$

$$(\Delta^{cl}_{\mu\nu})^{-1} = -(D^{cl})^2\delta_{\mu\nu} + D^{cl}_\mu D^{cl}_\nu \left(1 - \frac{1}{\xi}\right) + 2ig[G_{\mu\nu}, \cdot] \tag{4}$$

Here, $D^{cl}_\mu = \partial_\mu - igA^{cl}_\mu$, and $D^{cl}_\mu t^{ab} \equiv -iP^{ab}_\mu t^{ab}$ where

$$P^{ab}_\mu = (p_0 + 2\pi(q^a - q^b), \mathbf{p}) = (p^{ab}_0, \mathbf{p}) \tag{5}$$

Hence the Gluon propagator becomes,

$$\left\langle B^{ab}_\mu(P) B^{cd}_\nu(-P) \right\rangle = \left(\delta_{\mu\nu} - (1-\xi)\frac{P^{ab}_\mu P^{ab}_\nu}{(P^{ab})^2}\right) \frac{1}{(P^{ab})^2} \mathcal{P}^{ab,cd} \tag{6}$$

and $\mathcal{P}^{ab,cd} = \delta^{ac}\delta^{bd} - \frac{1}{N_c}\delta^{ab}\delta^{cd}$

(our) Matrix Model

- To leading order, the potential for holonomy, q,

$$V_{pert} \sim \text{Tr}\log\Big((\Delta^{cl}_{\mu\nu})^{-1}\Big) \sim T^4 B_4(q) \sim T^4 q^2 (1-q)^2 \qquad (7)$$

with $q = mod(q, 1)$.
This includes, $q^2 \sim \text{tr}A_0^2, q^4 \sim \text{tr}A_0^4$ and one problematic q^3 term, implying a first order phase transition from $q = 0$ to $q \neq 0$ in the deconfined region.

- Lattice study shows no phase transition in the deconfined phase, above T_d.

Evidence of (our) Matrix Model

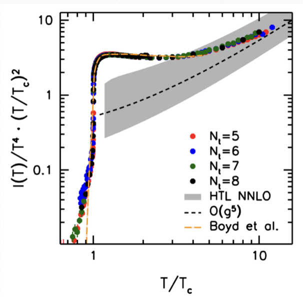

Figure 2: Borsanyi, Endrodi, Fodor, Katz, Szabo, 1204.6184

- Perturbative study can only give logarithmic correction in pressure to, $p \sim T^4$
- But Lattice suggests, near the transition region, viz., $\sim 1.2T_d$ to $4.0T_d$, leading correction to pressure in pure gauge theory $\sim T^2$. Not Exact, however, Approximate!

(our) Matrix Model

To incorporate these, we add to the Holonomous potential a non-perturbative term,

$$V_{non-pert} \sim CT^2 B_2(q) \sim -CT^2 q(1-q) \tag{8}$$

for small q. We want an auxilliary field that can dynamically generate $B_2(q)$ in the potential for q,so that there is no phase transition above T_d , where $q \neq 0$ always.

Two Dimensional Ghost - Teen (૭)

૭ (Teen) field propagator in momentum space,

$$\Delta_{૭}^{ab}(p_0^{ab}, \mathbf{p}) = \frac{1}{(p_0^{ab})^2 + p_{\|}^2 + p_{\perp}^2} \quad (9)$$

where $p_\perp \leq T_d$ and $T_d \ll gT \ll T$. Holonomic potential using Teen field →

$$V_{non-pert} = -\mathrm{Tr}\log\left((\Delta^{-1})_{૭}^{ab}(P^{ab})\right) \propto T_d^2 T^2 \sum_{a,b} |q^{ab}||(1-q^{ab})| \quad (10)$$

Teen Field (૭)

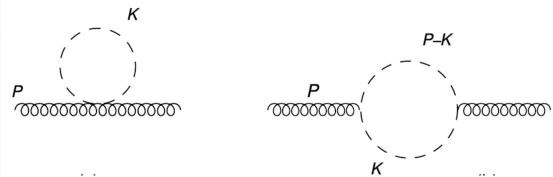

Figure 3: Teen contribution to gluon Self energy

Now the total self energy,

$$\Pi^{\mu\nu;ab,cd}_{total} = \Pi^{\mu\nu;ab,cd}_{gl} + \Pi^{\mu\nu;ab,cd}_{\text{૭}} = -(m^2)^{ab,cd}\delta\Pi^{\mu\nu}(P^{ab}) \qquad (11)$$

which satisfy the Ward identity.
NOTE: Due to non-zero holonomy, a $\sim g^2T^3/p$ term arises, but anyway vanishes because of equation of motion, which is essential for consistency.

Shear Viscosity

We use kinetic theory when $q^a \neq 0$ for gluons and teens. Then the Boltzmann equation,

$$\mathcal{S}_{\text{ব}}\, 2P^{\mu,a_1b_1}\partial_\mu f_{\text{ব},a_1b_1}(\mathbf{x},\mathbf{p},t) = -\mathcal{C}_{B,a_1b_1}[f_{\text{দ}}] \quad (12)$$

with ব $=$ gluon, teen, and $\mathcal{S}_{gl} = +1$, $\mathcal{S}_{teen} = -1$
For the LHS of the Boltzmann , we follow Chapman-Enskog method, where we expand the distribution function up to linear order in fluctuation.

Shear Viscosity

Stress Energy tensor in kinetic theory:

$$T^{\mu\nu}_{\rm gl}(\mathbf{r},t) = 2\int_{\rm g} d\Gamma_{ab}\; P^{\mu,ab}P^{\nu,ab}\; f_{{\rm gl},ab}(\mathbf{r},\mathbf{p},t) \tag{13}$$

where

$$f^0_{{\rm gl},ab} = f^0_{ab}(E_p) \equiv \frac{1}{{\rm e}^{(E_p - i(Q^a - Q^b))/T} - 1}$$

For the Teen field ($\circlearrowleft$), in equilibrium,

$$f^0_{\circlearrowleft,ab}(E_p) = \pm f^0_{{\rm g},ab}(E_p)$$

depending upon whether the teen field is being absorbed or emitted. Here,

$$\int_{\rm gl} d\Gamma_{ab} = \sum_s \sum_{a,b=1}^{N_c} \mathcal{P}^{ab,ba} \int \frac{d^3p}{(2\pi)^3 2E_p} \tag{14}$$

and

$$\int_{\circlearrowleft} d\Gamma_{ab} = \sum_s \sum_{a,b=1}^{N_c} \mathcal{P}^{ab,ba} \left(\int_0^{T_d} d^2p_\perp \int \frac{dp_\parallel d\Omega}{(2\pi)^3 2p_\parallel} = \frac{T_d^2}{2} \int \frac{dp_\parallel d\Omega}{(2\pi)^3 2p_\parallel} \right) \tag{15}$$

Shear Viscosity- Collision term

For the RHS of the Boltzmann equation, the Collision term $\mathcal{C}_{\text{ব},a_1b_1}[f_{\text{দ}}]$ depend on the scattering matrix, at leading log order, such that,

$$\begin{aligned}\mathcal{C}_{\text{ব},a_1b_1}[f_{\text{দ}}] =& \frac{1}{2}\sum_{\text{দ},\text{ব}',\text{দ}'} \mathcal{S}_{\text{ব}}\mathcal{S}_{\text{দ}} \int_{\text{দ}} d\Gamma_{a_2b_2} \int_{\text{ব}'} d\Gamma_{a_3b_3} \int_{\text{দ}'} d\Gamma_{a_4b_4} (2\pi)^4 \\ & \delta^4(P_1+P_2-P_3-P_4)|\mathcal{M}_{\text{বদ}\to\text{ব}'\text{দ}'}|^2 \\ & \times \Big[f_{\text{ব},a_1b_1} f_{\text{দ},a_2b_2}(1+f_{B\text{ব}',a_3b_3})(1+f_{\text{দ}',a_4b_4}) \\ & - f_{\text{ব}',a_3b_3} f_{\text{দ}',a_4b_4}(1+f_{\text{ব},a_1b_1})(1+f_{\text{দ},a_2b_2}) \Big] \end{aligned} \tag{16}$$

Feynmann Diagram (Tree level)

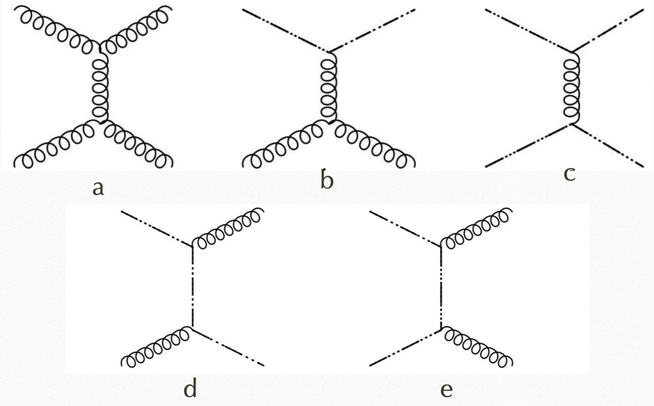

Figure 4: Feynmann diagrams contributing to the shear viscosity calculation at tree level. Among these,contribution from teen exchange diagrams vanishes at tree level for soft momentum exchange limit.

Viscosity (Ongoing)

We have obtained results for the Shear and Bulk Viscosity at leading log order for general ℓ. For simplicity, we present here:

- Shear viscosity at small Polyakov loop value:

$$\eta = \frac{t^3}{10(g^2)^2 \ln 1/(g^2 N_c)} \frac{128}{5 \times 45\, \zeta(4)} \frac{50t^2 + 3}{4t^2 + 1} \ell_1^2 \tag{17}$$

- Bulk viscosity at small Polyakov loop value:

$$\zeta = \frac{(1/3 - v_s^2)^2\, t^3}{g^4 \log(g^{-1})} 576 \frac{8(50t^4 + 43t^2 - 3)}{3(8t^4 - 7t^2 - 4)} \pi \ell_1^2 \tag{18}$$

Where, $t = T/T_d$.
Also Note that, We use two parameters w and κ, which enters through g and $\log(1/g)$ as $\log(\kappa/g)$ and $g^2 = \frac{24\pi^2}{\log(w 2\pi T/T_d)}$.
Now we need thermodynamics to fix the loops and get the η/s, ζ/s plots.

Thermodynamics!!

At large N_c we minimize the potential

$$\mathcal{V}_{\text{eff}}^{q}(T) \sim d_2(T)B_4(q) - d_1(T)B_2(q) \tag{19}$$

Here $d_2(T) = (2\pi^2/3)\,T^4$, and $d_1(T) \sim T^2T_d^2$, but here we determine this, by considering $d(T)^2 = \frac{12d_2(T)}{d_1(T)}$. For the simplest ansatz, we use

$$d(T) = 2\pi T/T_d = 2\pi t, \qquad t = T/T_d$$

.

Apart from this simple ansatz, we also use two other ansatz which fits the $SU(5)$ loop data with our model pretty well, viz.,

$$\begin{aligned} d_A(T) &= 1.08\,t + 5.2032\,, \\ d_B(T) &= \frac{0.26}{t^3} + 1.105\,t + 4.9182\,,\ t = \frac{T}{T_d}\,. \end{aligned} \tag{20}$$

Loops!

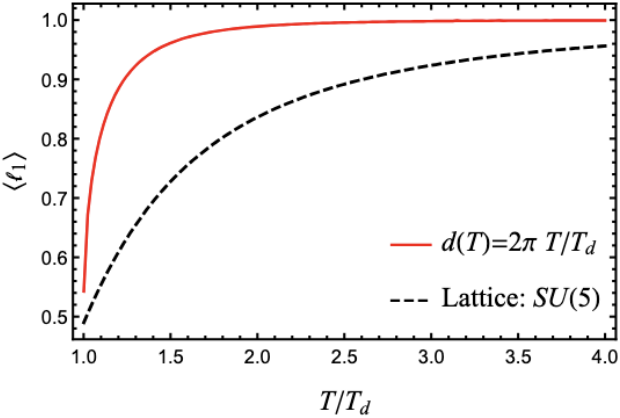

Figure 5: Comparison of loop of first order between our ansatz and Lattice data for $SU(5)$ pure gauge theory.

Loops!

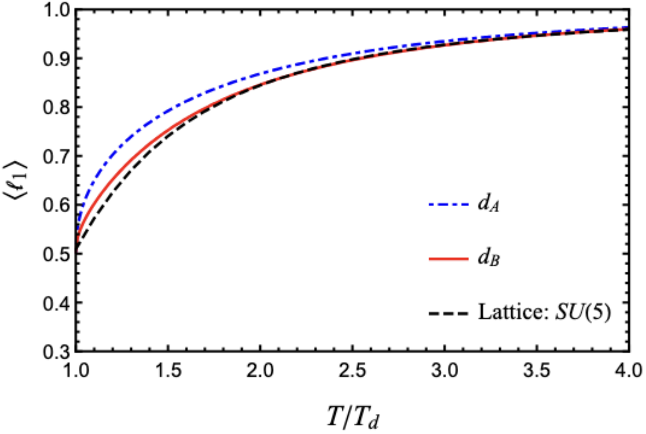

Figure 6: Loops using other two ansatz for fitting with lattice data.

Result (Shear Viscosity)

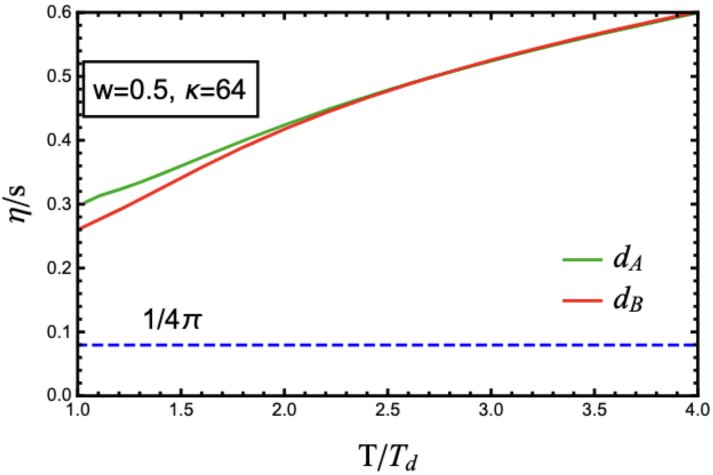

Figure 7: Shear viscosity to entropy ratio plot for two ansatz.

Result (Bulk Viscosity)

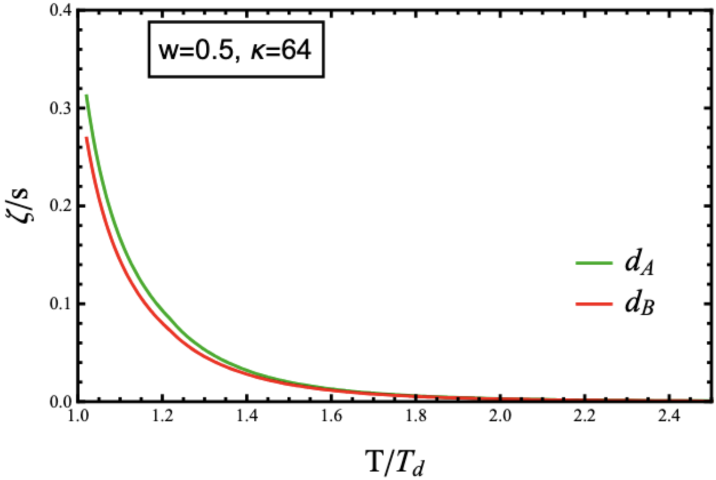

Figure 8: Bulk viscosity to entropy ratio for two ansatz

Result

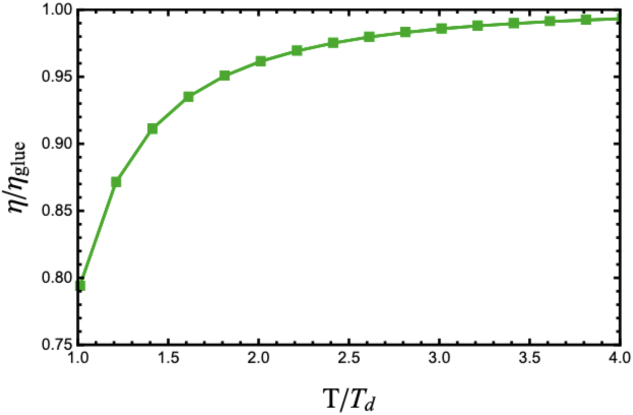

Figure 9: Comparison of Shear viscosity with and without Teen field.

Result

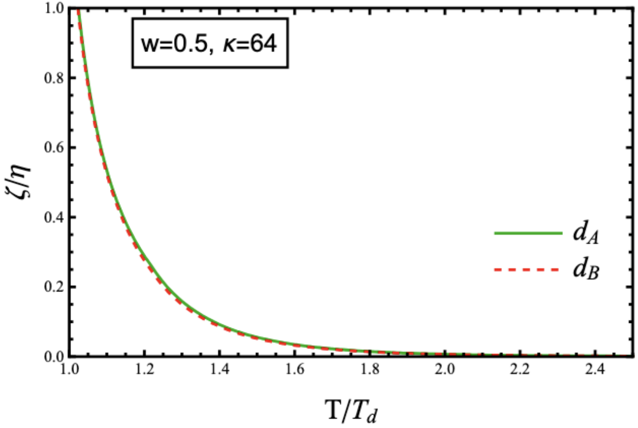

Figure 10: Bulk viscosity to shear viscosity ratio, with Teen.

Conclusion

This is a ongoing work. Complete conclusion/discussion awaits. Still few short comments:

• Teen field could explain the T^2 behaviour of pressure near transition region.

• Shear viscosity is suppressed near the transition region with the inclusion of Teen field , but still even at T_d, η/s is 4 times the AdS/CFT bound. With Quarks, ℓ_1 is smaller at T_χ, so η/s will be smaller. We want to compare our result with recent lattice data of η/s from Altenkort et al, 2211.08230 .

• Bulk viscosity depend on sound velocity as $(\frac{1}{3} - v_s^2)^2$.

Thank You for your attention!