

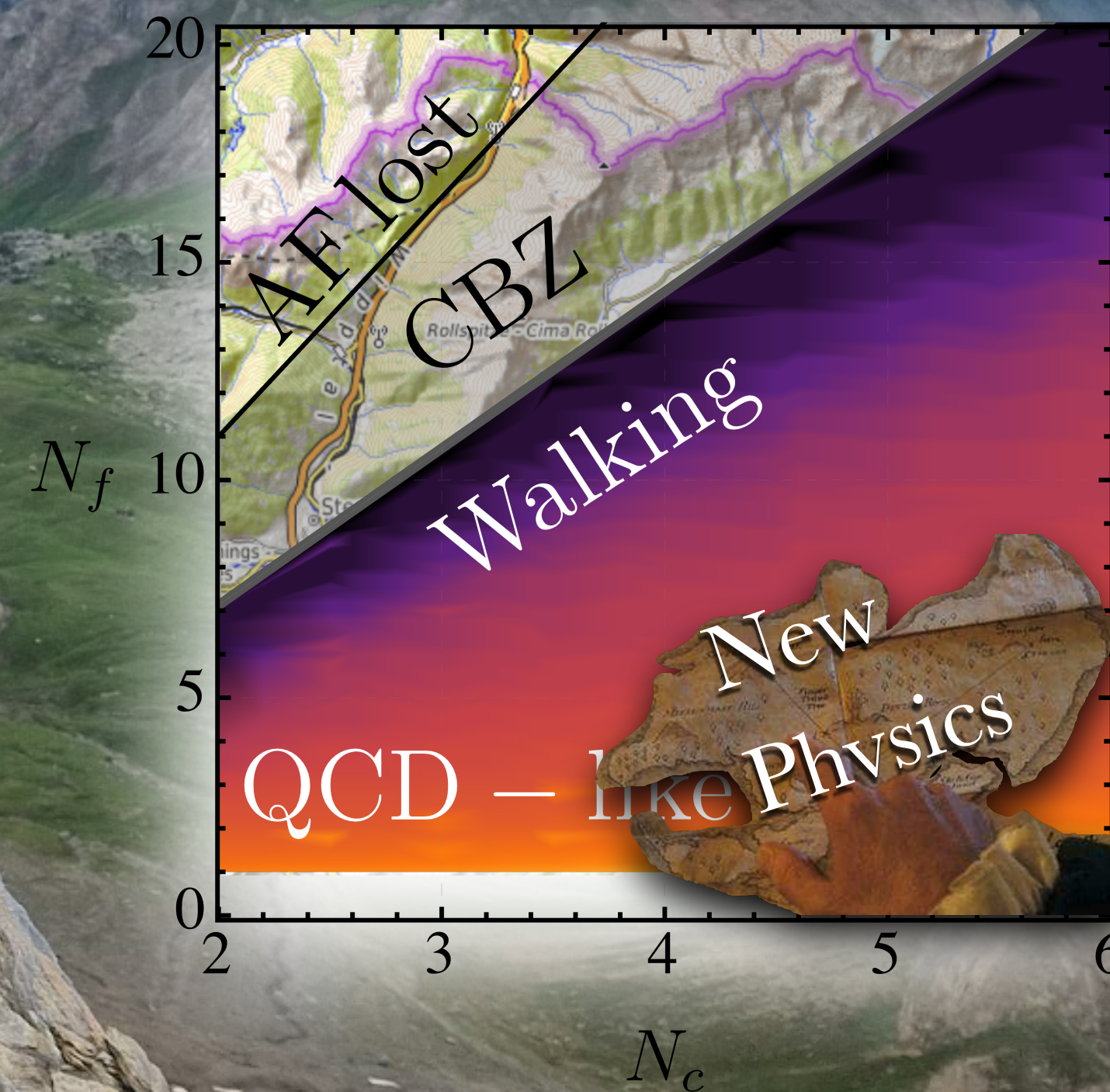
Cartography of gauge-chiral dynamics

1. Confinement

2. $D\chi$ SB

3. Conformality

Work in collaboration with
Florian Goertz and Jan M. Pawłowski



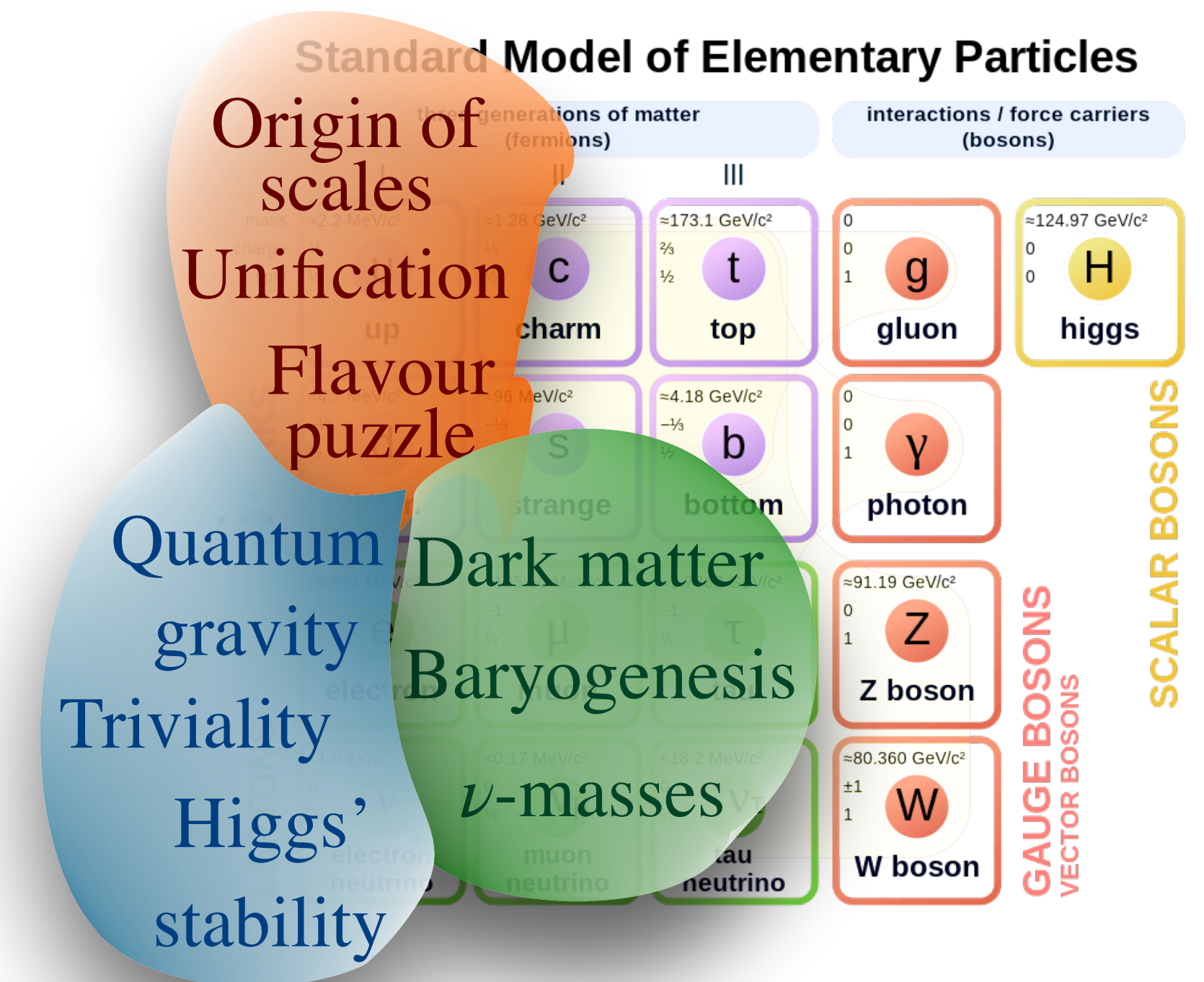
Álvaro Pastor Gutiérrez

Universitat de València 2.09.2024

Gauge-chiral theories

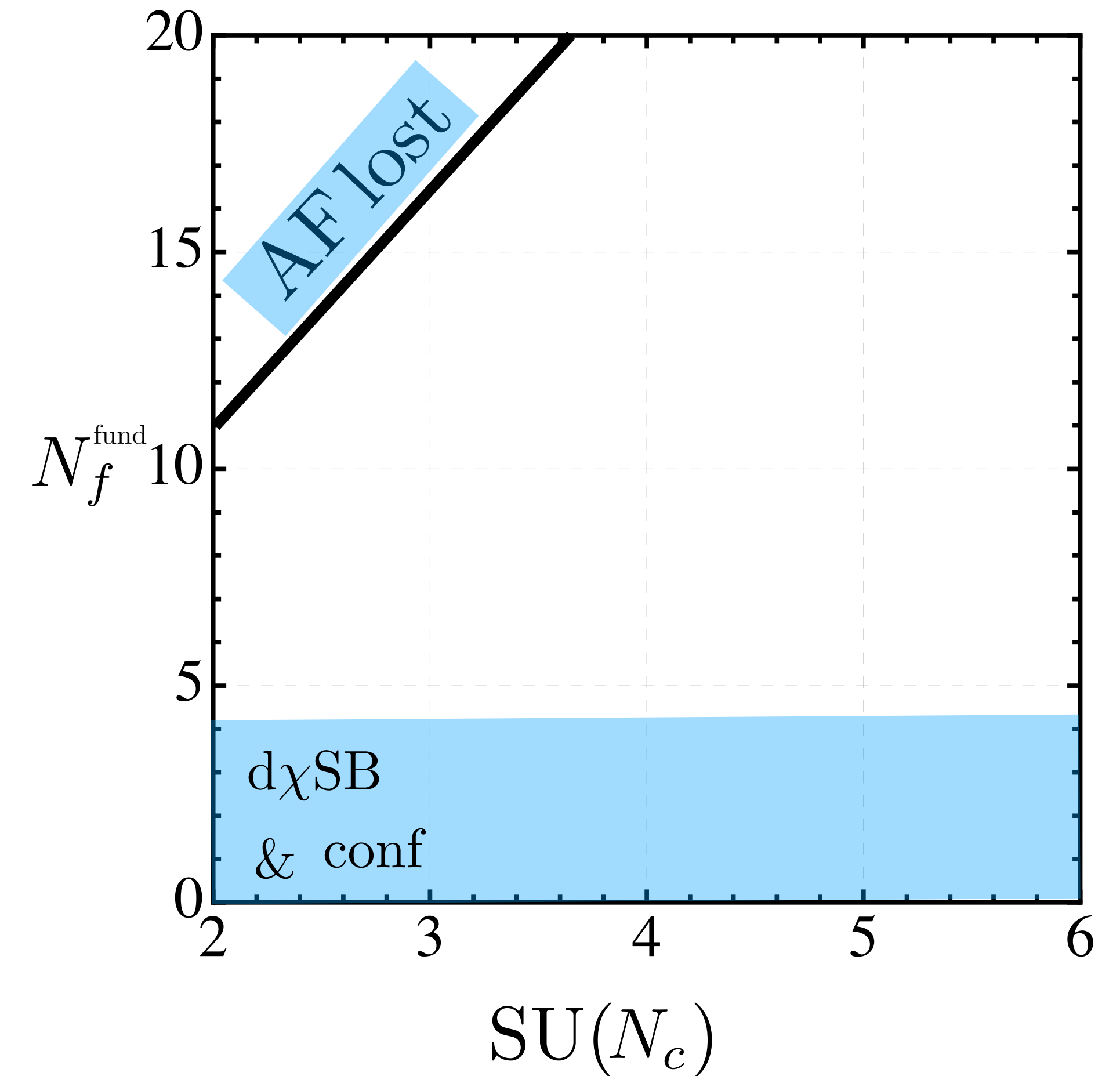
$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi} \not{D} \psi$$

- ♦ Importance in the **natural world**
- ♦ Non-trivial **dynamics** and **features**:
 - Confinement
 - Dynamical chiral symmetry breaking (dχSB)
 - Walking dynamics
 - Conformality
- ♦ Can be the **solution** to our **problems** and **puzzles**:
 - Dark strong sectors
 - Composite Higgs and Technicolour
 - Cosmological phase transitions and gravitational wave signatures



What we know so far from first-principles?

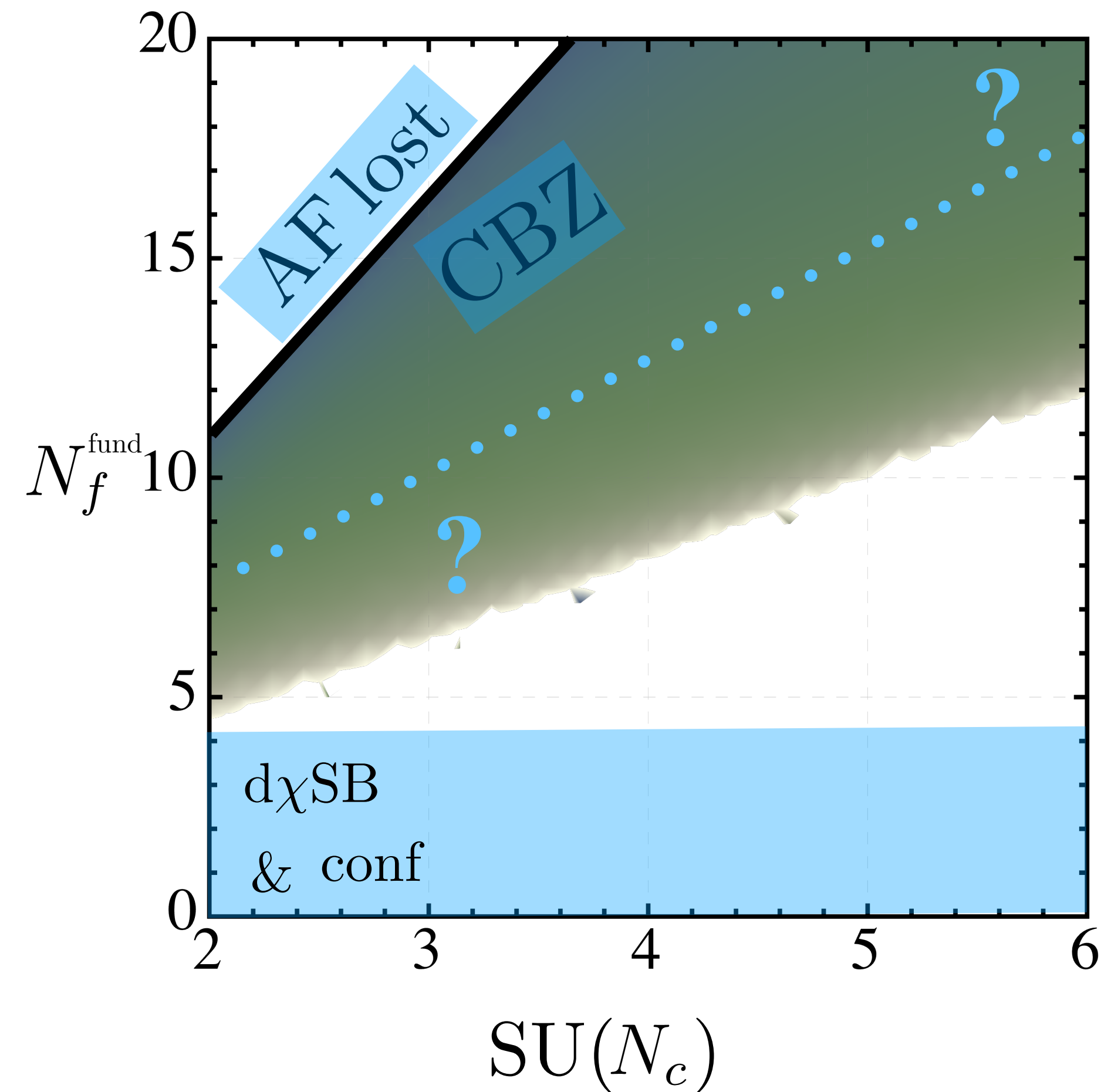
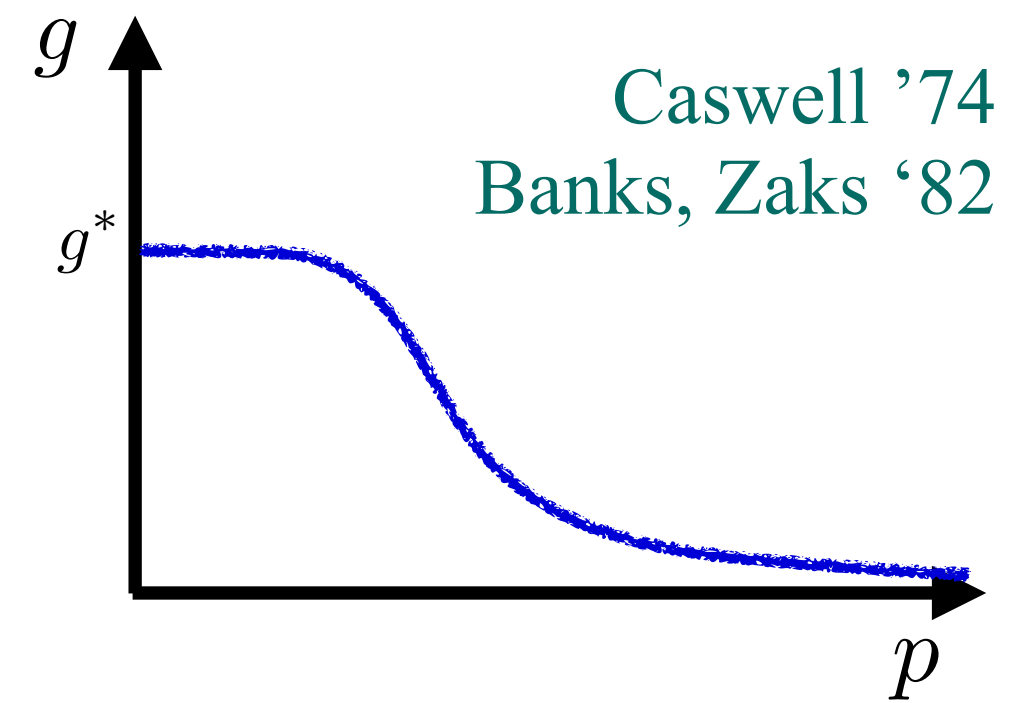
- ◆ Where **asymptotic freedom** is lost
- ◆ **Few flavours**: QCD-like theories



What we know so far from first-principles?

- ◆ Where **asymptotic freedom** is lost
- ◆ **Few flavours**: QCD-like theories
- ◆ **CBZ IR fixed-point** (perturbative)

$$\beta_g = -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3} C_A - \frac{4}{3} T_F N_f \right) - \frac{g^5}{(4\pi)^4} \left(\frac{34}{3} C_A - 4 C_F T_F N_f - \frac{20}{3} C_A T_F N_f \right) + \dots$$



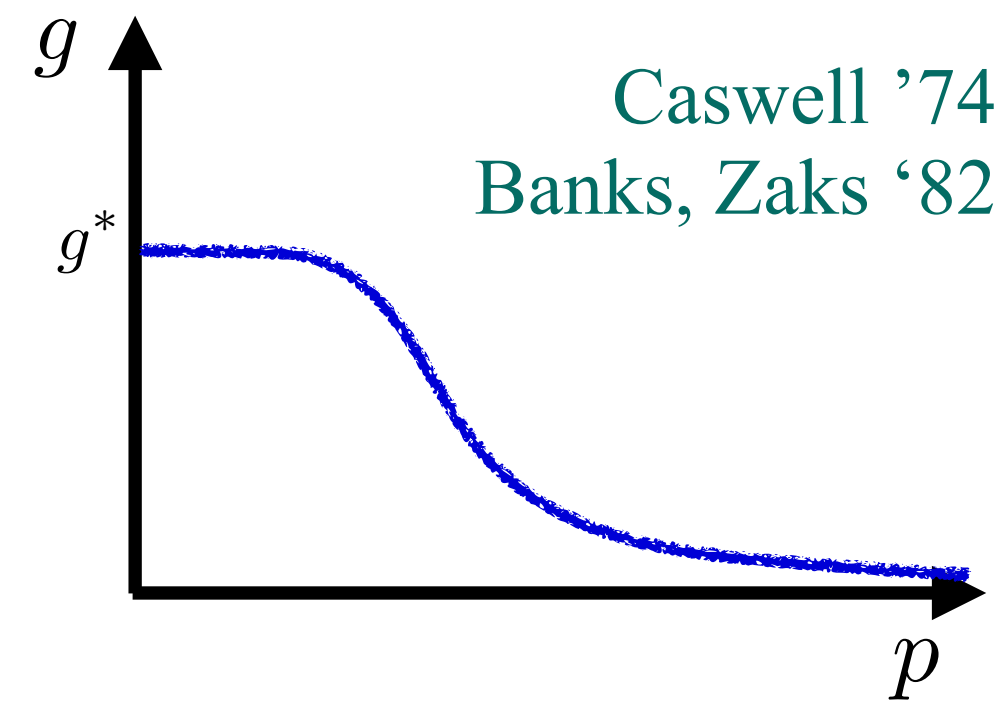
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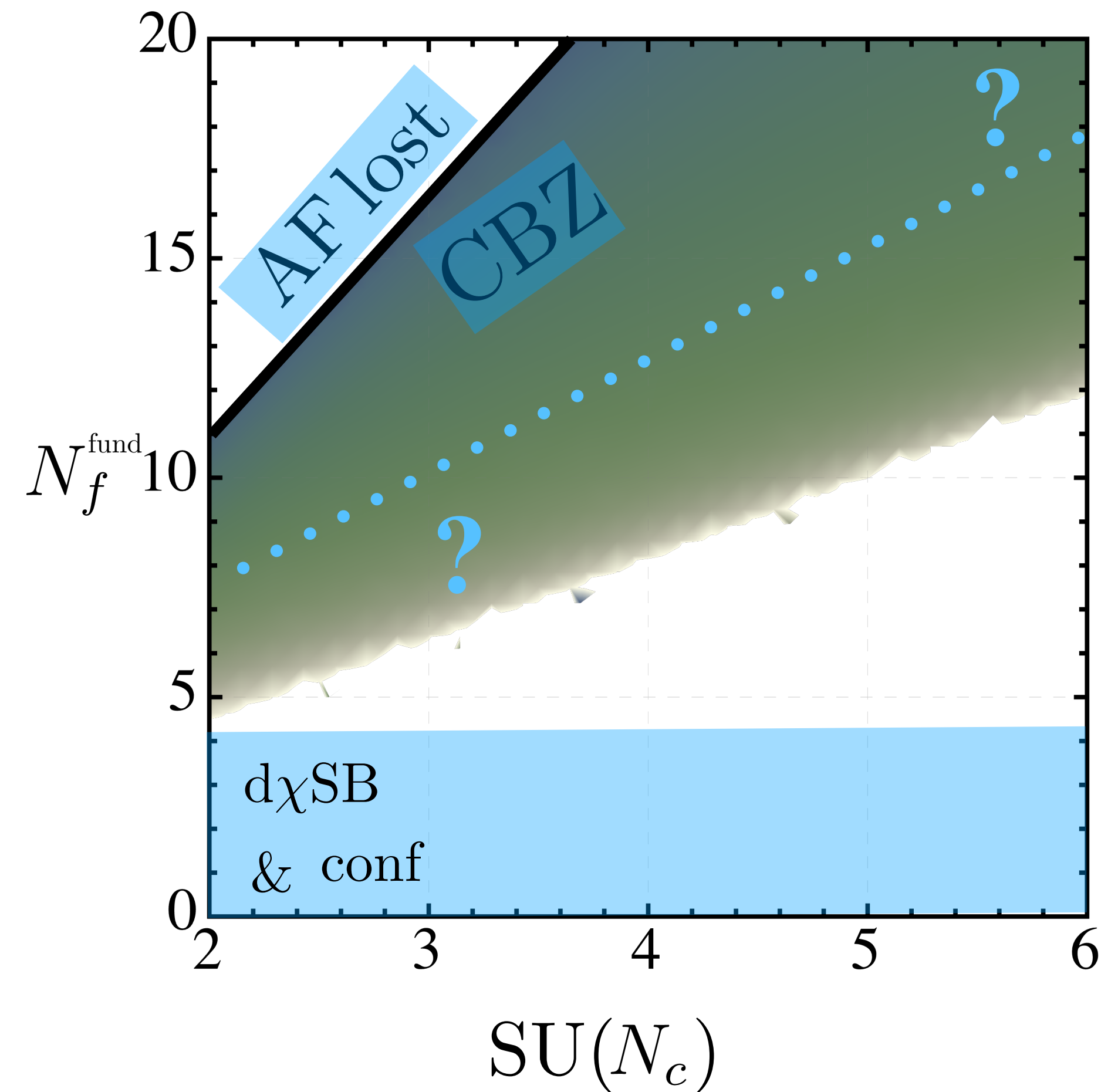
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◆ Tailor-made task for the **functional Renormalisation Group**: $\Gamma_k[\phi]$

- Quantitative [Ihssen, Pawłowski, Sattler, Wink\[2408.08413\]](#)
- Versatile: easily study large range of parameter and theory space
- Chiral limit with no difficulties



Functional Renormalisation Group

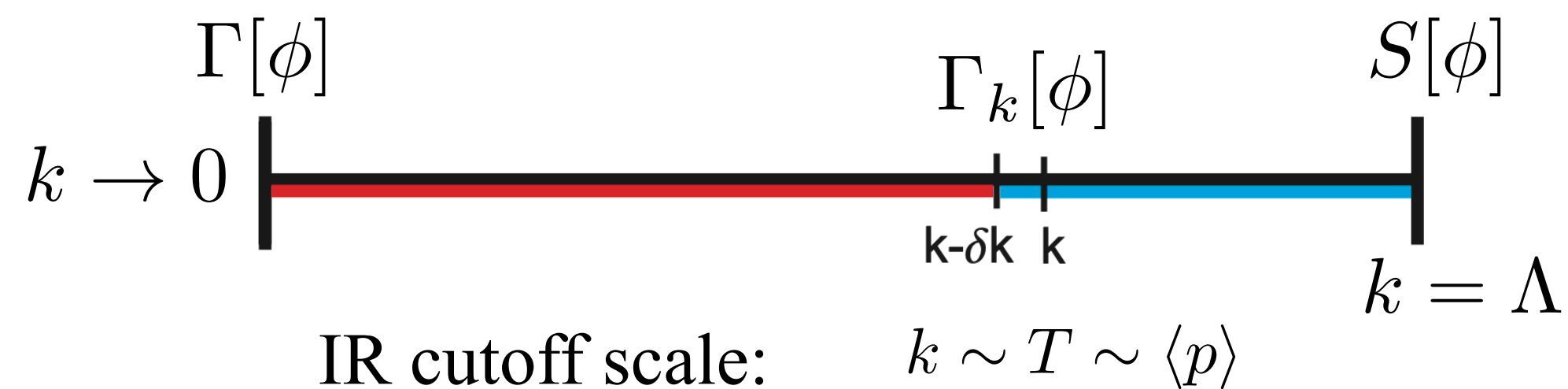
♦ Effective **average** action: $\Gamma_k[\phi]$

- Average action of fields over a k^{-d} volume
- Kadanoff's block-spinning idea in the continuum limit

$$\Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x) \phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$

Wetterich '89



♦ Pedagogical introduction to fRG: Gies [0611146]

♦ Review of achievements: Dupuis, Canet, Eichhorn,
Metzner, Pawłowski, Tissier, Wschebor [2006.04853]

Functional Renormalisation Group

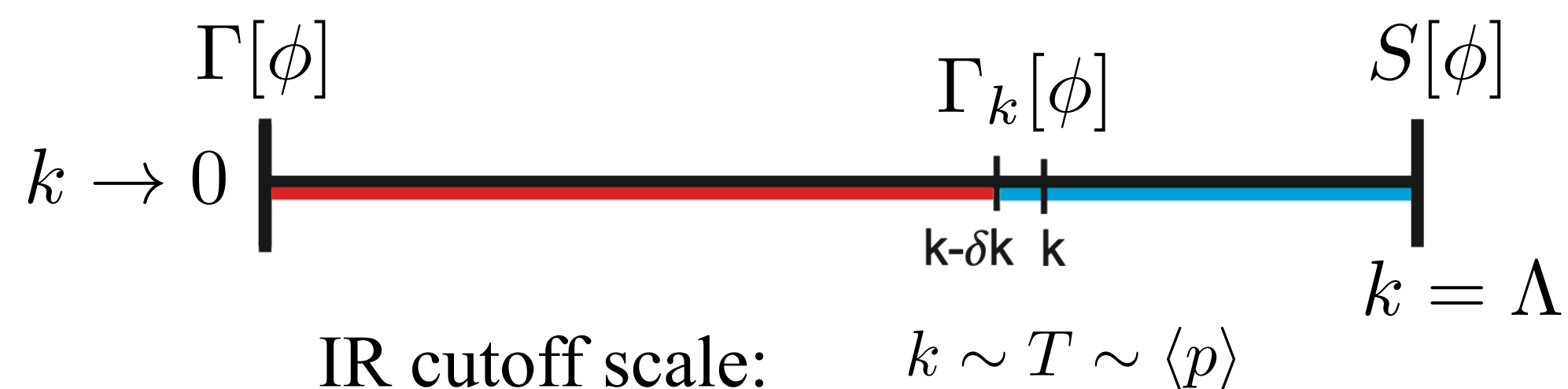
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Wetterich '89



Flow equation:

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right] = \frac{1}{2} \text{ (loop diagram) }$$

$$\partial_t \equiv k \partial_k$$

Wetterich '93

- | | |
|------------------------------|--------------------------------|
| ♦ One loop exact | ♦ Systematic expansion schemes |
| ♦ Non-perturbative | ♦ UV-IR finite |
| ♦ Mass-dependent | ♦ Diagrammatic |
| ♦ Analytic regulators | ♦ Real-time formulation |
| ♦ Versatile | |

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Functional Renormalisation Group

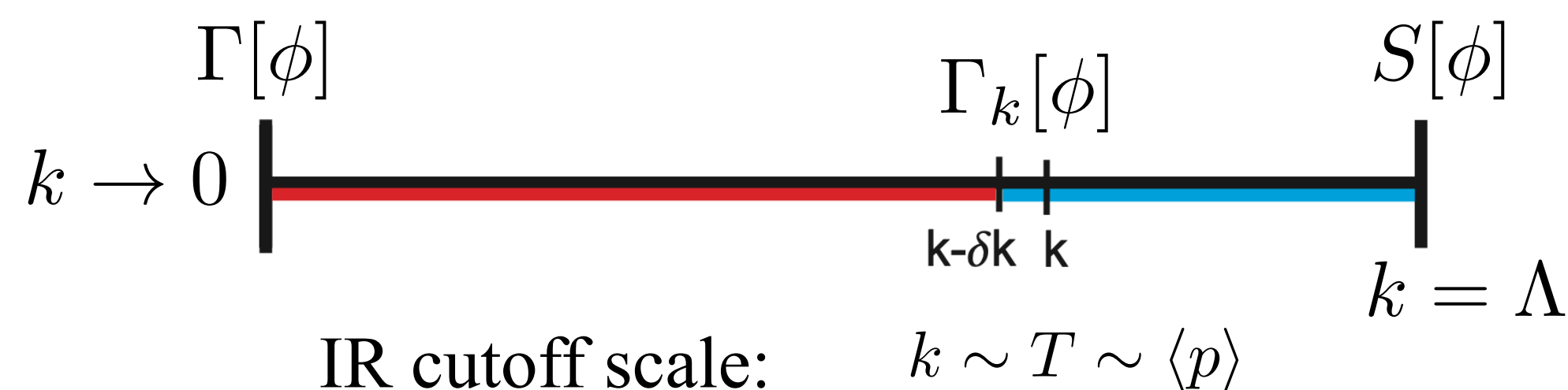
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- ♦ **Mass-dependent**
- ♦ **Analytic regulators**
- ♦ **Versatile**
- ♦ Systematic expansion schemes
- ♦ UV-IR finite
- ♦ Diagrammatic
- ♦ Real-time formulation

♦ Physics from correlation functions:

$$\Gamma_{k, \phi_{i_1} \dots \phi_{i_n}}^{(n)}(p_1, \dots, p_n) = \frac{\delta}{\delta \phi_{i_1}(p_1)} \dots \frac{\delta}{\delta \phi_{i_n}(p_n)} \Gamma_k[\phi]$$

Colour confinement: gluon mass gap

- ♦ **Absence of coloured asymptotic states**
- ♦ **Massive spectrum** of bound states (glueballs)
- ♦ Existence of a gluon **mass gap**:
 - Wilson area law
 - Kugo-Ojima conditions
 - Confinement-deconfinement phase transition

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- ♦ **fRG bootstrap approach** to confinement:

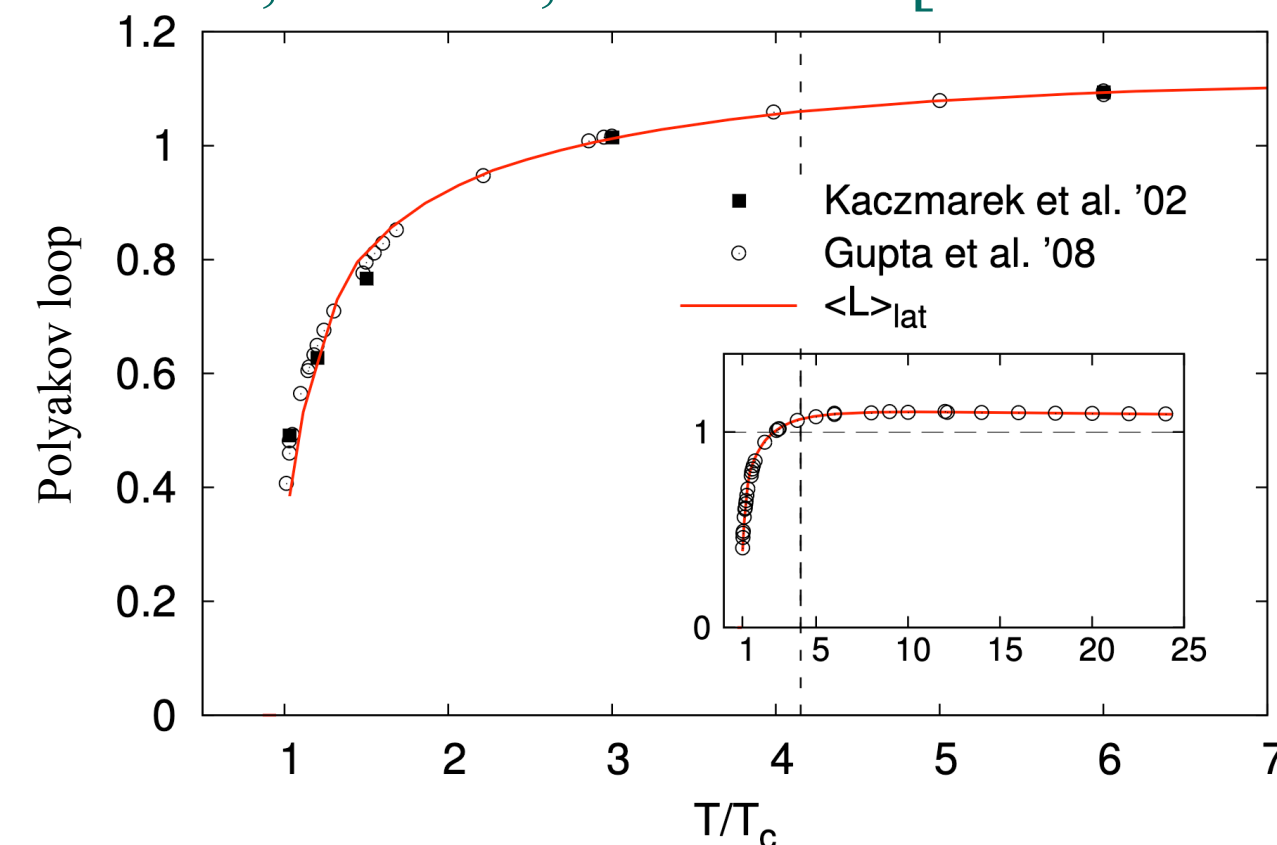
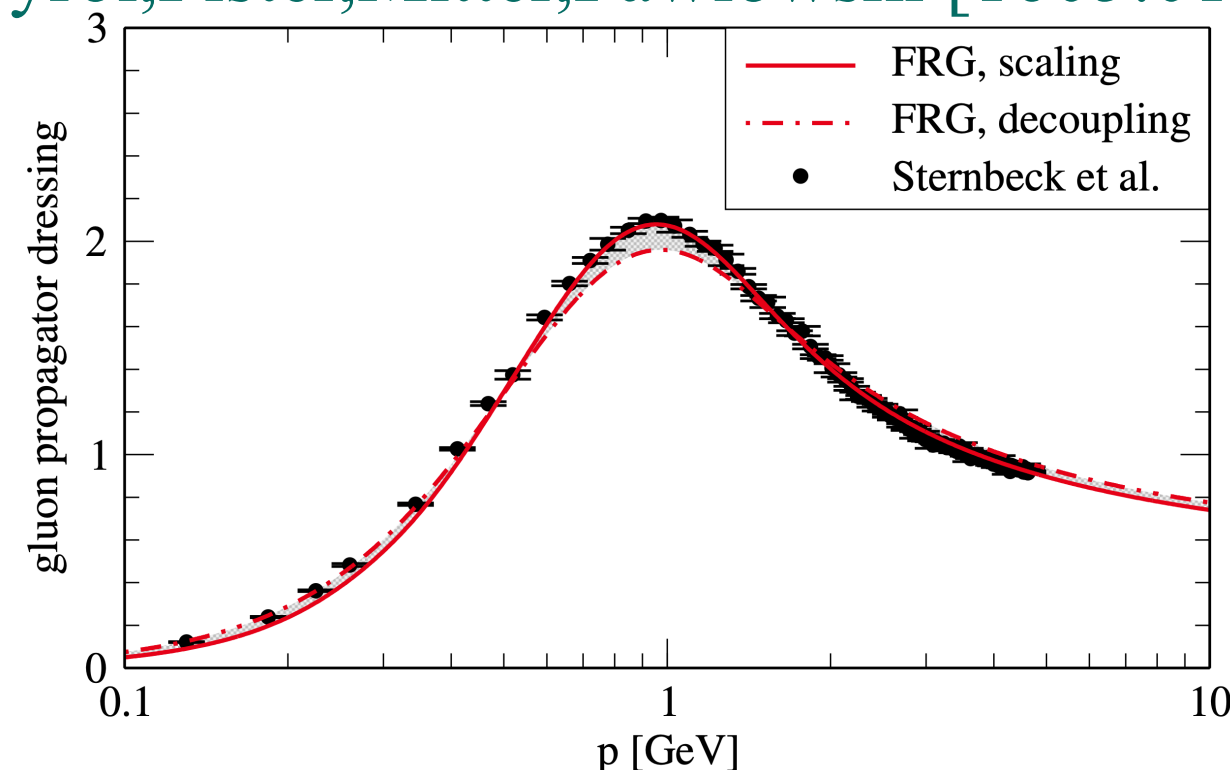
- **Mass gap** generated by **quantum fluctuations**

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p) (p^2 + m_{A,k}^2) = \hat{Z}_{A,k}(p) p^2$$

- **Uniquely defined** confinement condition

$$\lim_{p \rightarrow 0} Z_c(p^2) \propto (p^2)^\kappa \quad \lim_{p \rightarrow 0} Z_A(p^2) \propto (p^2)^{-2\kappa}$$

Cyrol,Fister,Mitter,Pawlowski [1605.01856] Herbst,Luecker,Pawlowski[1510.03830]



- ♦ New: “*easy*” confinement

- **Cutoff dependences suffice**
- Semi-analytical
- Facilitate study beyond QCD-limit

Confinement in correlation functions

♦ Flows computed: $\{g_{A\bar{\psi}\psi}, g_{A\bar{c}c}, g_{A^3}, g_{A^4}, \bar{m}_A, Z_A, Z_c\}$

♦ **Exchange couplings:**

$$\alpha_{\psi\bar{\psi}A} = \frac{\left[\Gamma_k^{(\psi\bar{\psi}A)}\right]_{\mathcal{T}=1}^2}{4\pi Z_A Z_\psi^2} = \frac{g_{\psi\bar{\psi}A}^2}{4\pi (1 + \bar{m}_A^2)}$$

$$\alpha_{\psi\bar{\psi}A} = \text{[Feynman diagram: a box with a wavy line and two vertices]} \cdot p^2$$

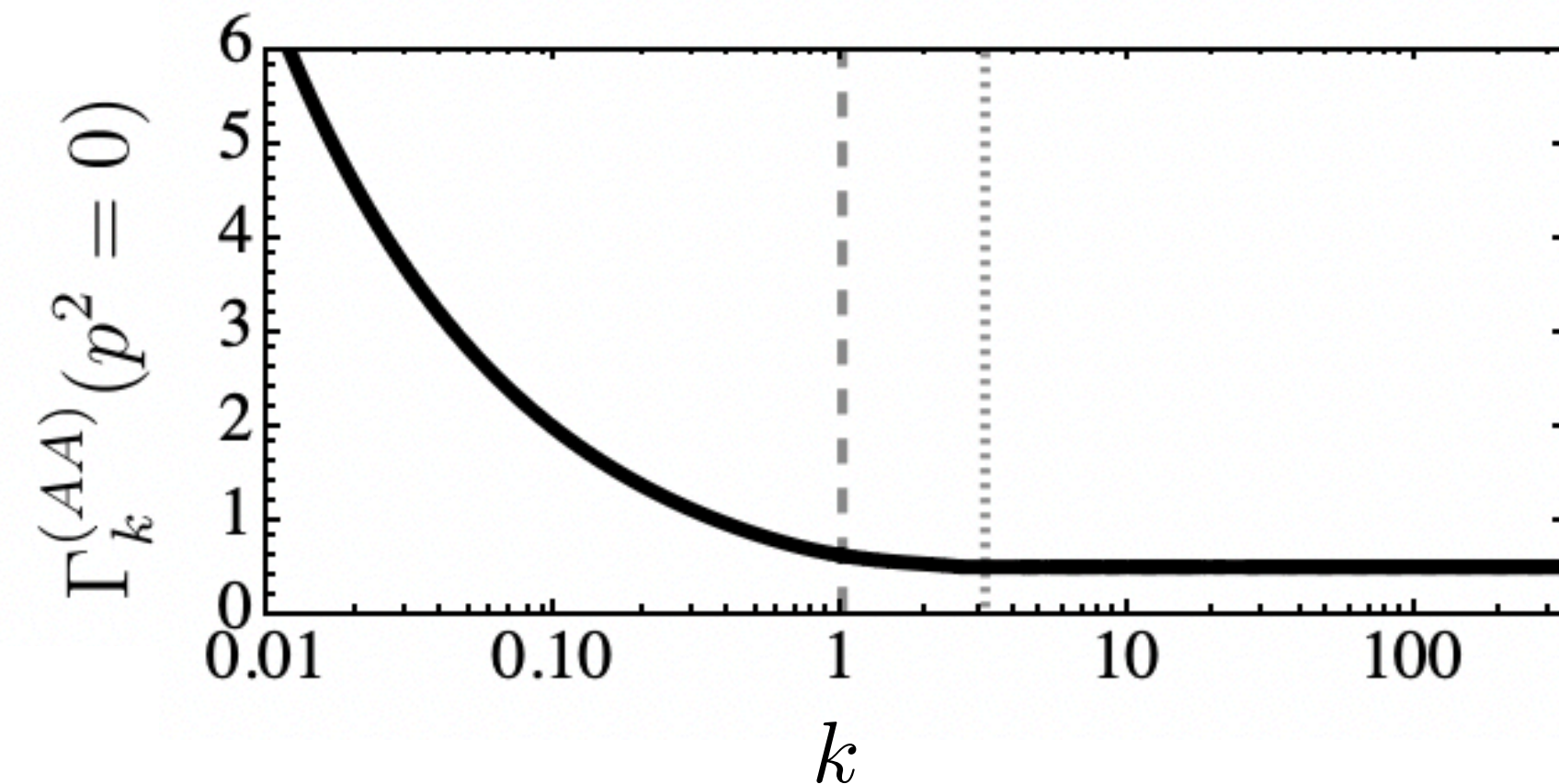
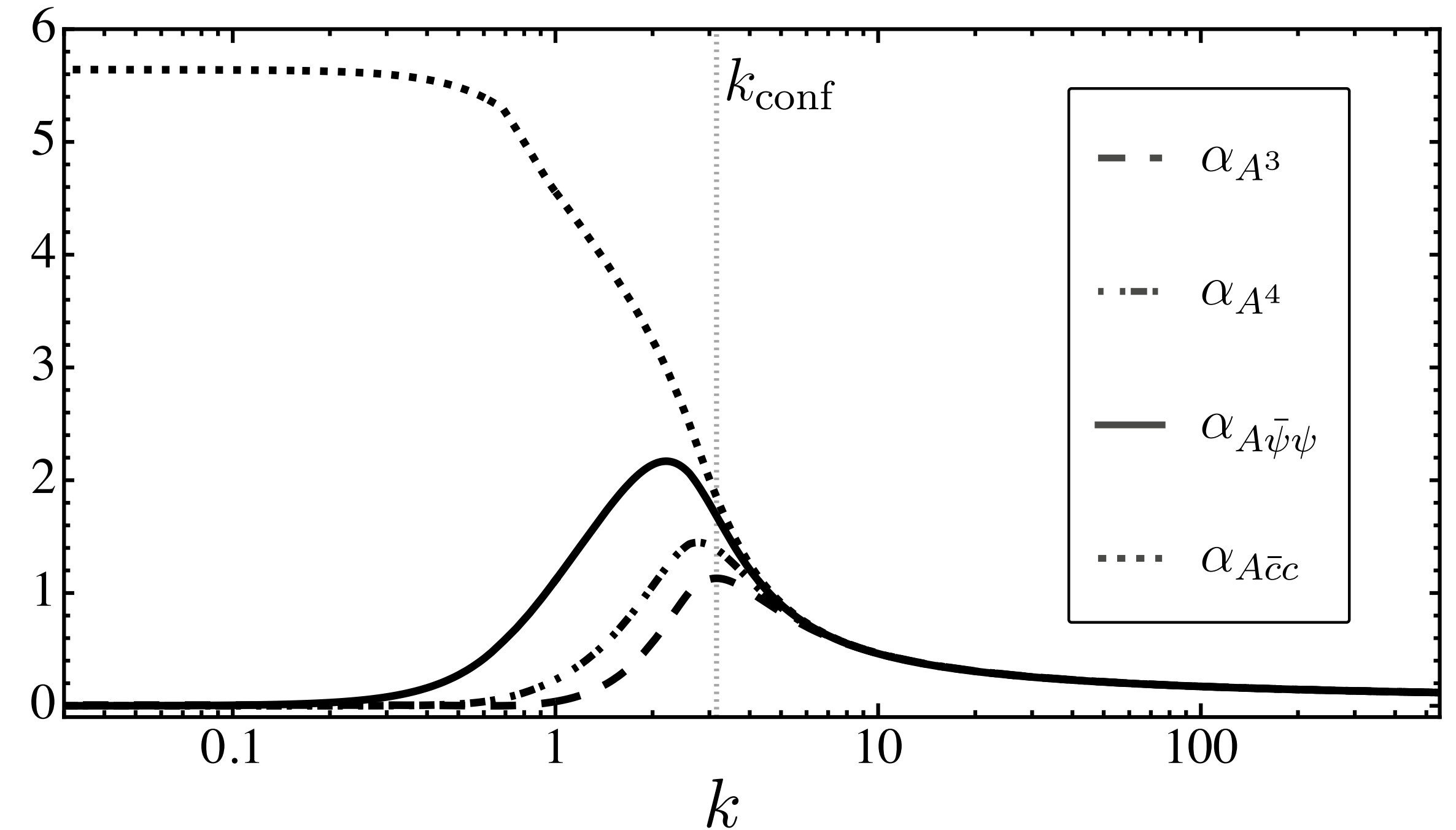
♦ **Decay of correlation functions below the mass gap scale**

$$k_{\text{conf}} \sim m_{A, \text{gap}} \sim T_{\text{conf}} \sim \Lambda_{\text{QCD}}$$

♦ The **interplay of gapped gauge and ghost contributions**

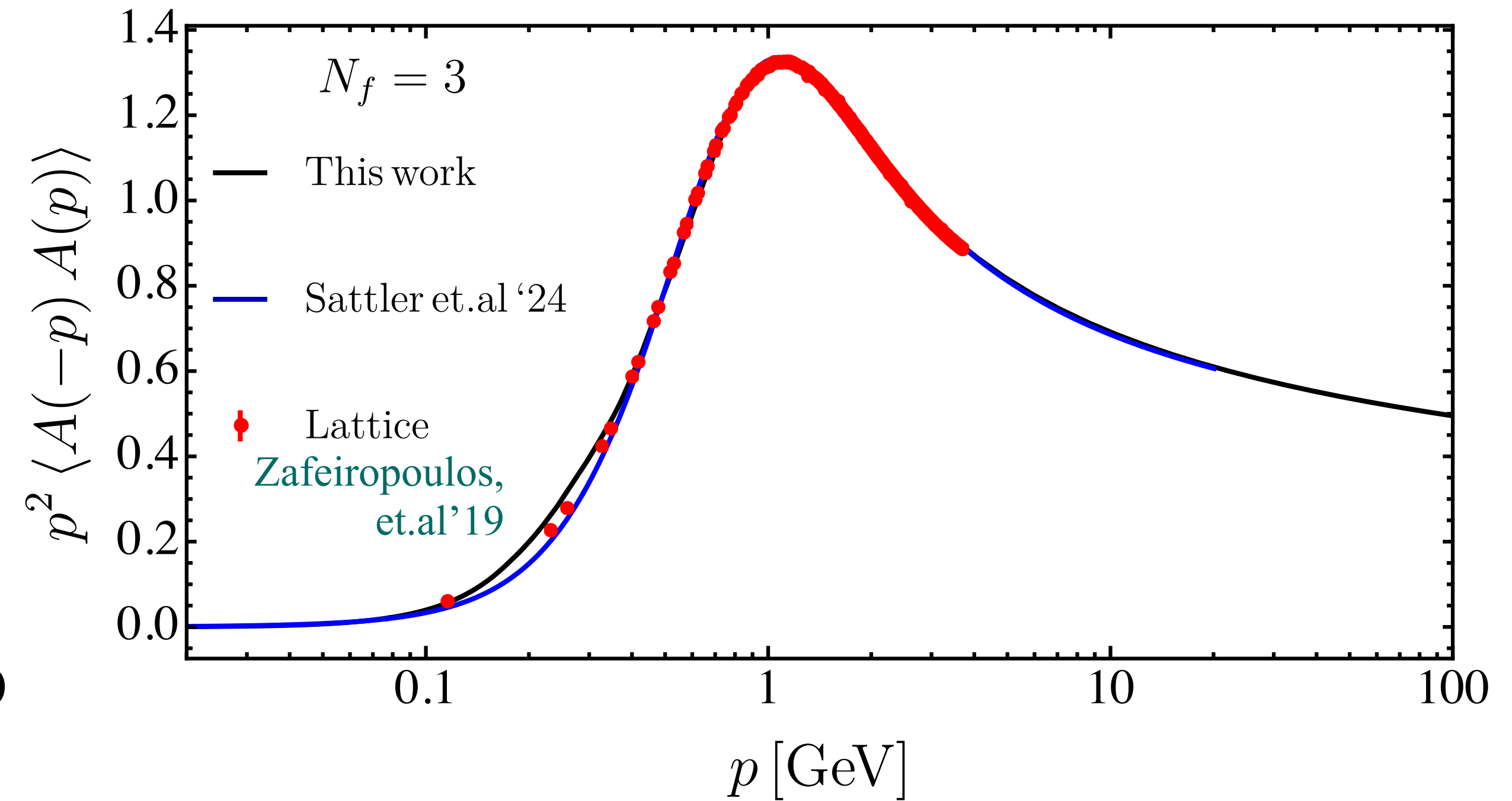
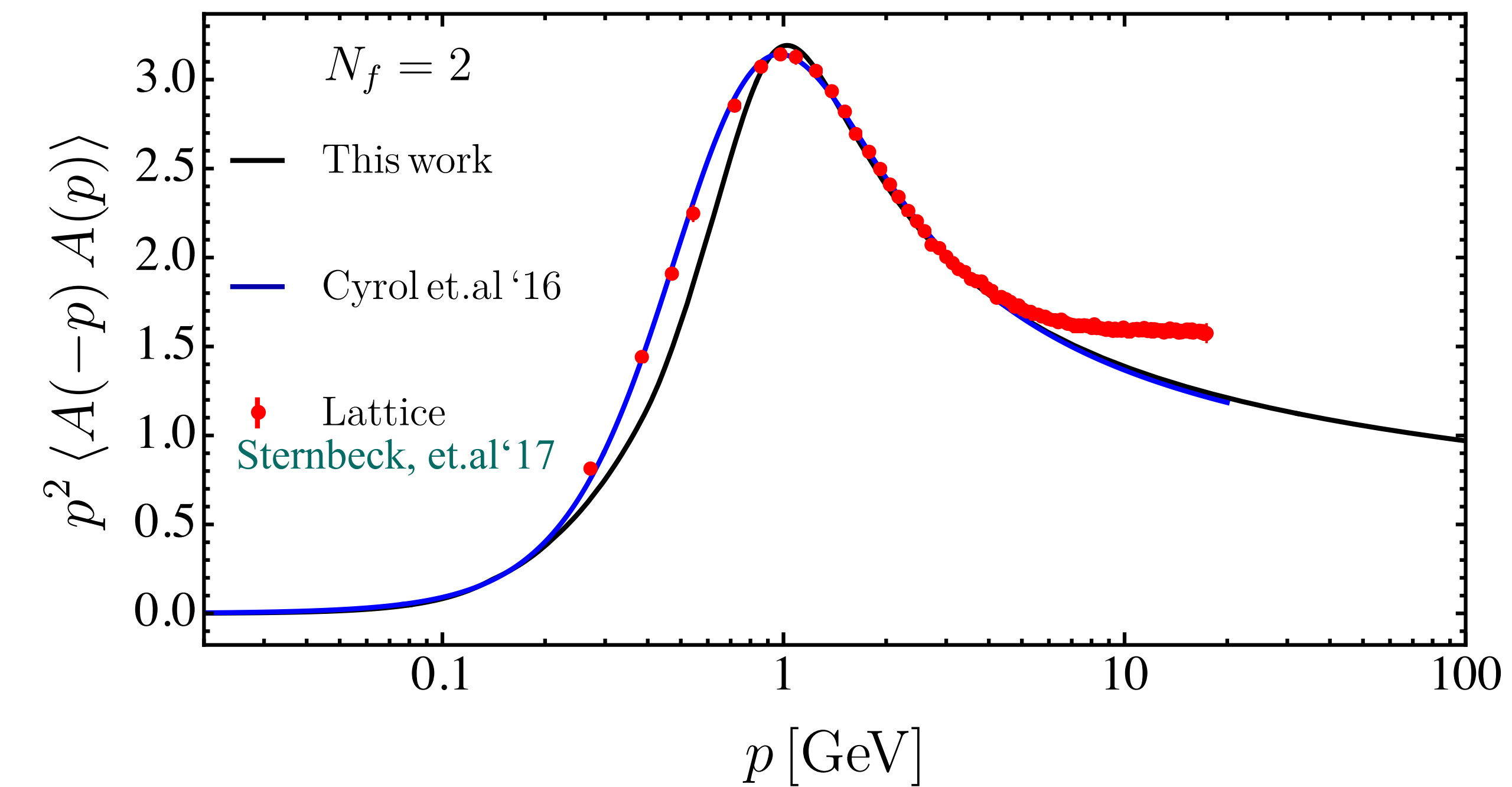
$$\partial_t \text{[wavy line]}^{-1} = \text{[Feynman diagram: wavy line with a loop]} - 2 \text{[Feynman diagram: wavy line with a loop and a cross]} - \frac{1}{2} \text{[Feynman diagram: wavy line with a loop and a cross]}$$

Cyrol, Fister, Mitter, Pawłowski [1605.01856]



“Easy” confinement

Gluon propagator dressing:
$$p^2 \langle A(-p) A(p) \rangle = p^2 \left[\Gamma_k^{(AA)}(p^2 = 0) \right]_{k=c p}^{-1}$$



Dynamical χ SB

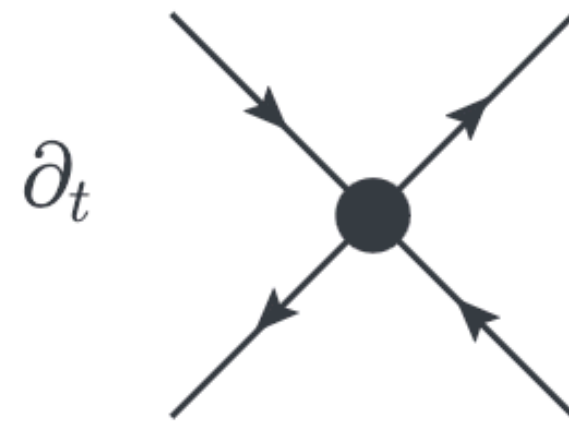
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Dynamical χ SB

$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi} \gamma_\mu D_\mu \psi + \lambda (\bar{\psi} \mathcal{T} \psi)^2 + \kappa (\bar{\psi} \mathcal{T} \psi)^3 + \dots$$

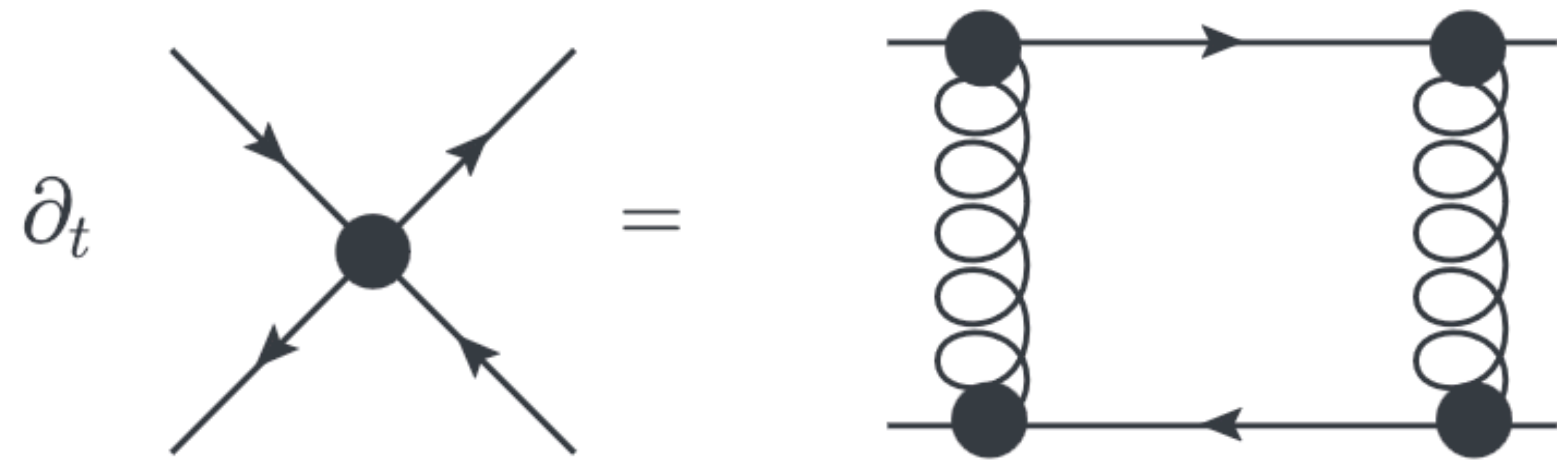
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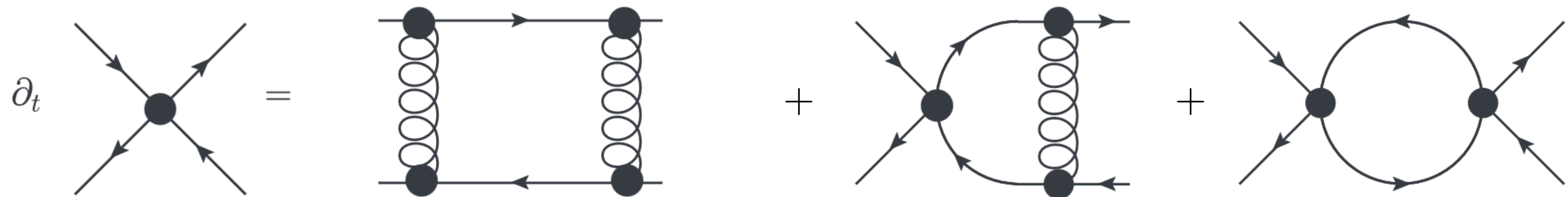
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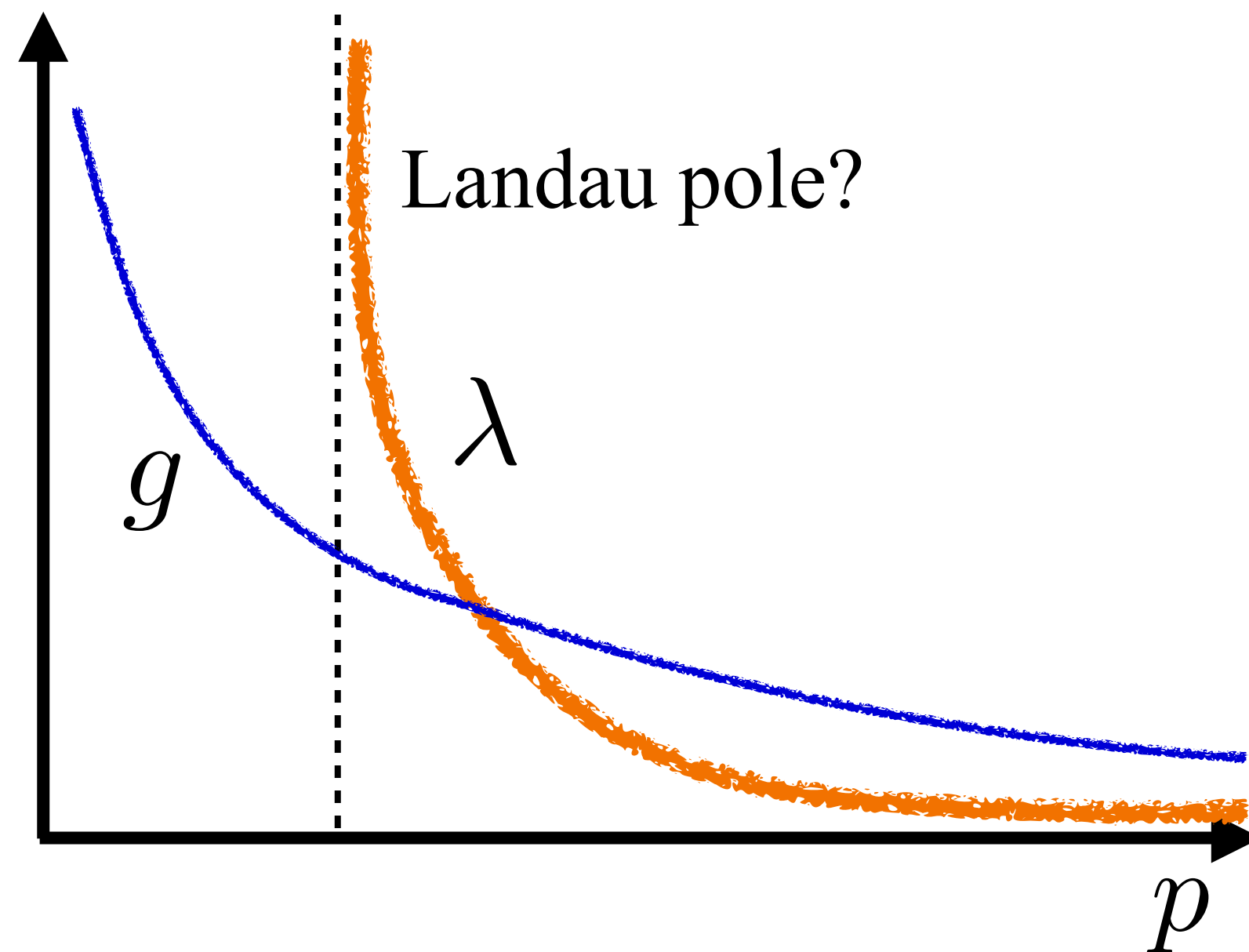
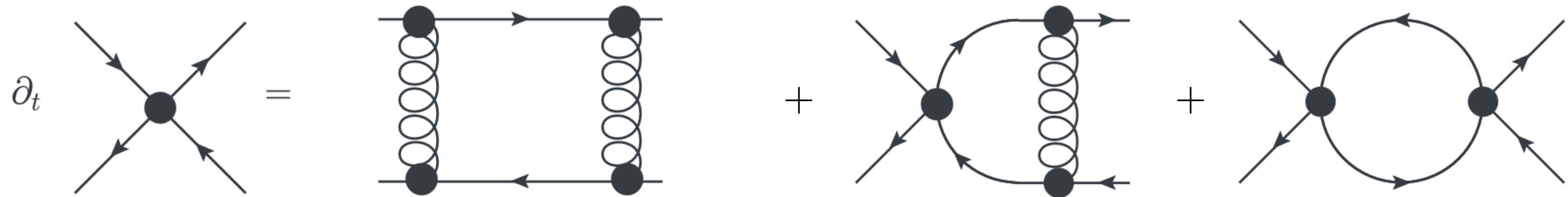
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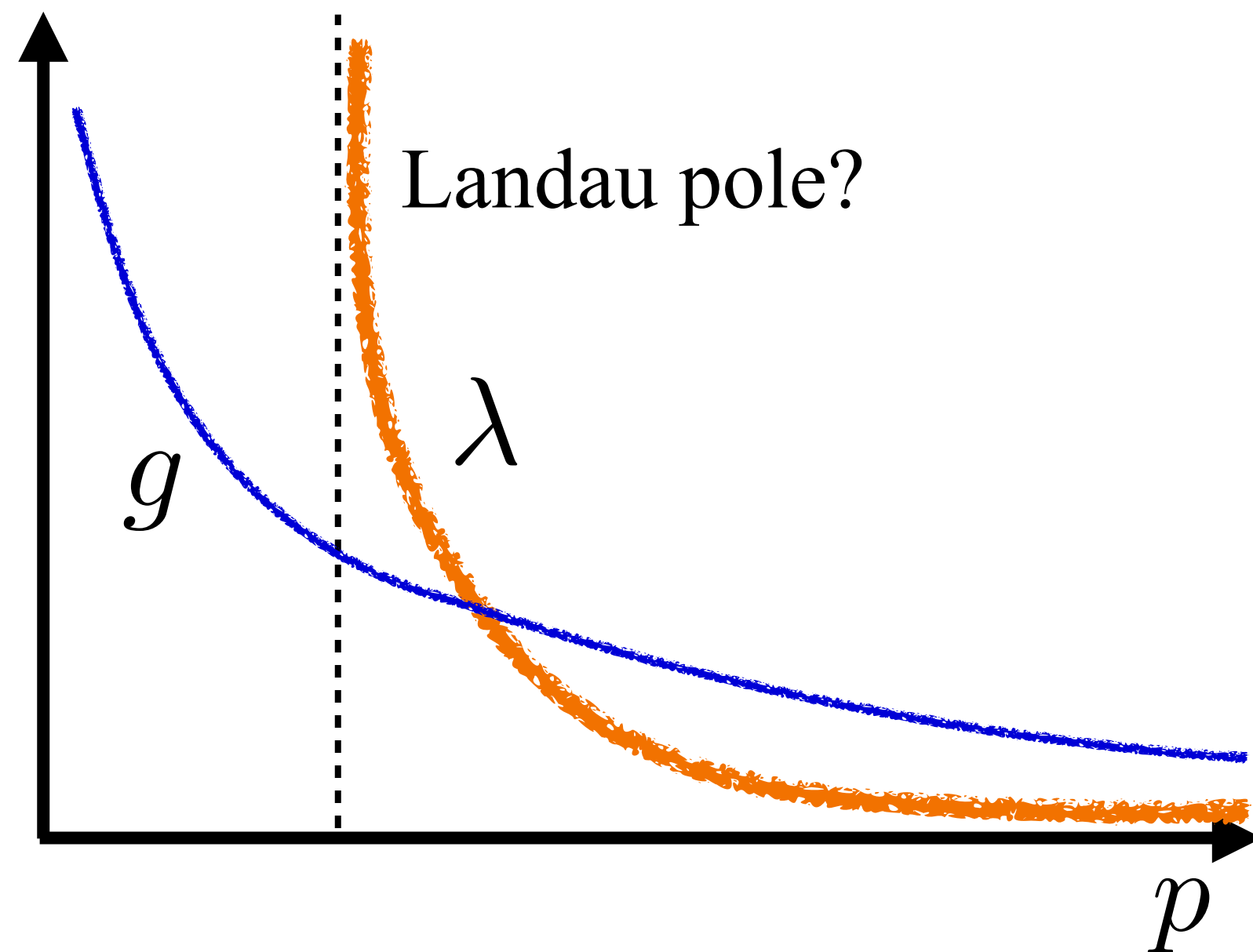
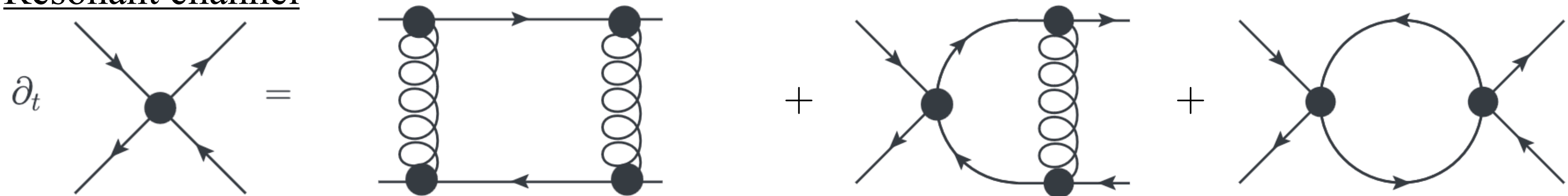
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Resonant channel

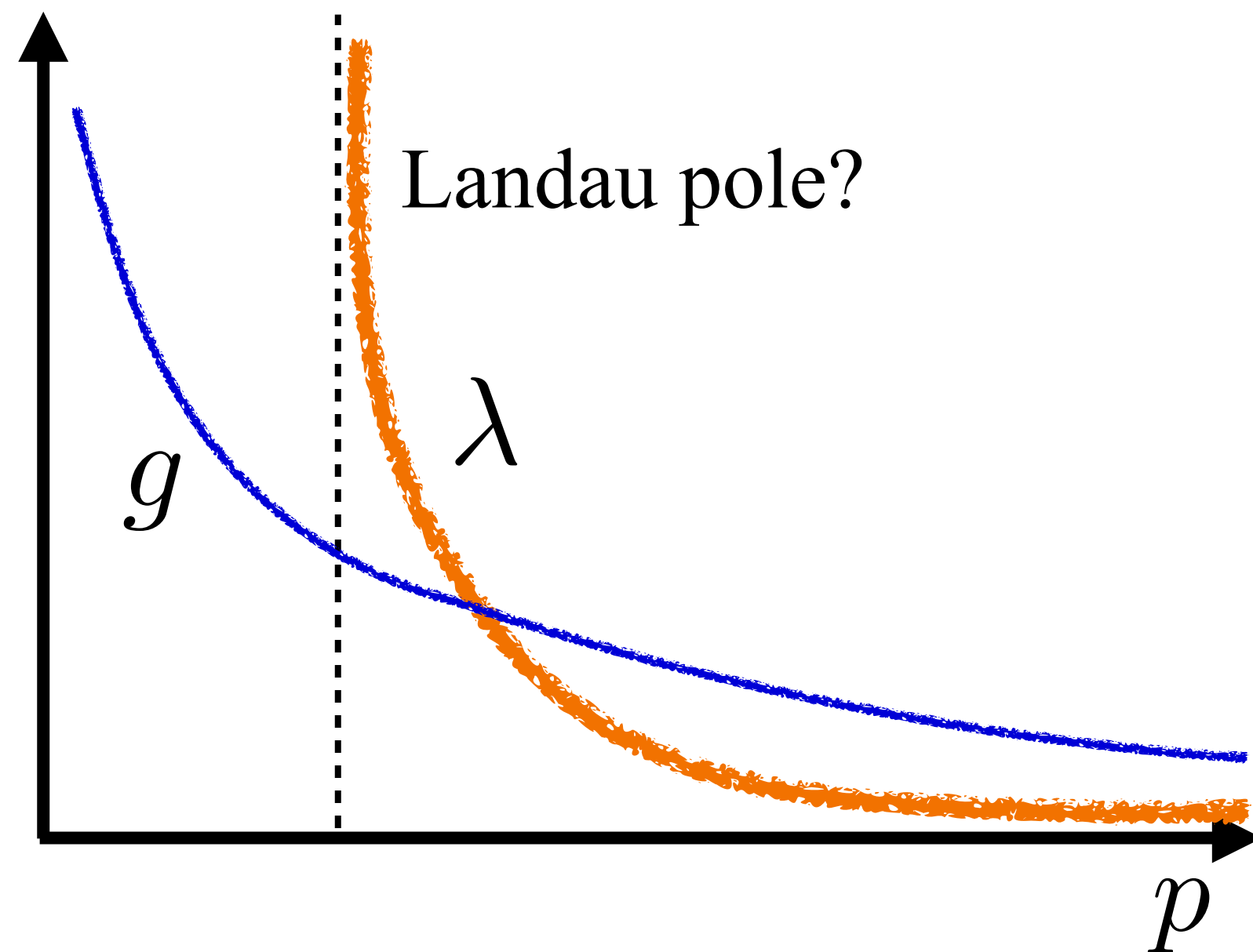
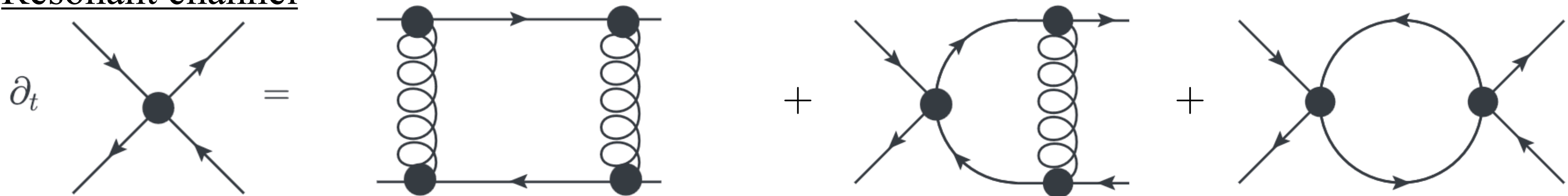


$$\langle (\bar{\psi} \psi)^2 \rangle \sim \langle \phi \phi \rangle^{-1} \longrightarrow \lambda \propto \frac{1}{p^2 + m_\phi^2}$$

Dynamical χ SB

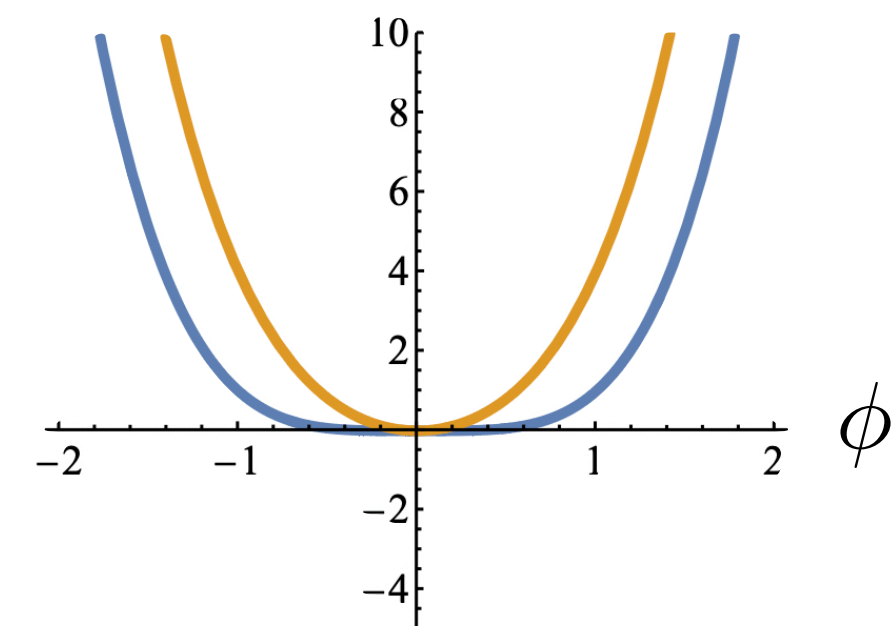
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Resonant channel



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$$\chi\text{SB} \longrightarrow (\lambda = \infty) \equiv (m_\phi = 0)$$

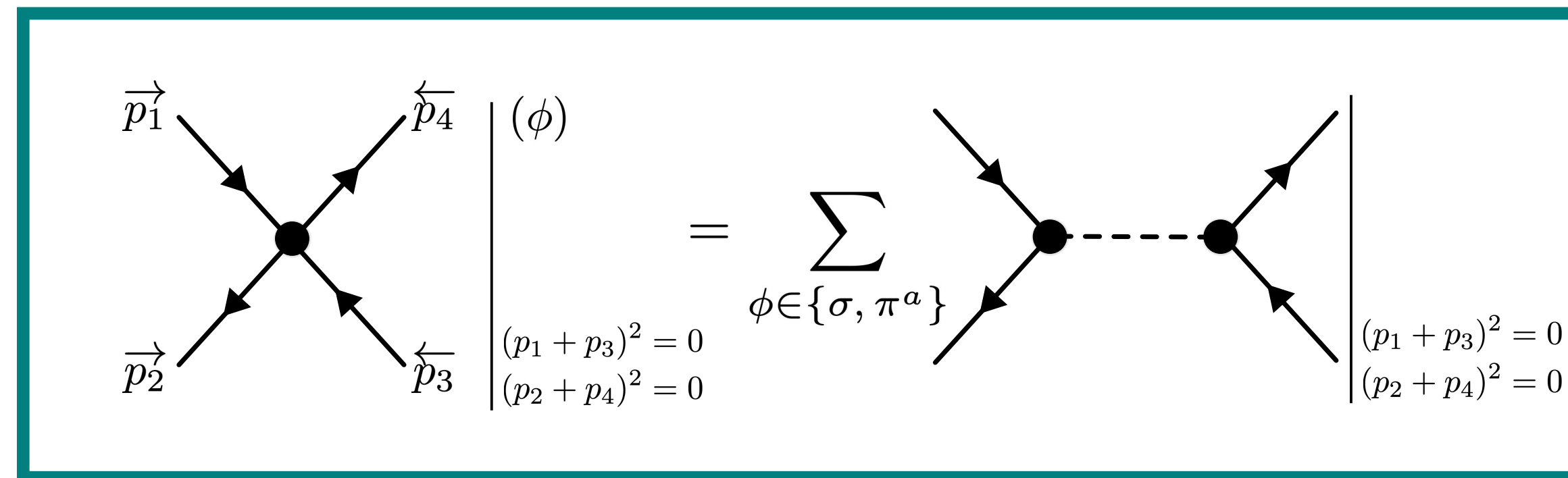


Higher dimensional operators carry information of bound states formation and $\mathbf{d}\chi\text{SB}$

Emergent composites

$$\Gamma = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (\partial_\mu \bar{c}^a) D_\mu^{ab} c^b + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 + \bar{\psi} [(\gamma_\mu D_\mu)] \psi - \lambda \left[(\bar{\psi} T_f^0 \psi)^2 + (\bar{\psi} i\gamma_5 T_f^a \psi)^2 \right]$$

Stratonovich'57 Hubbard'59 Gies, Wetterich '01

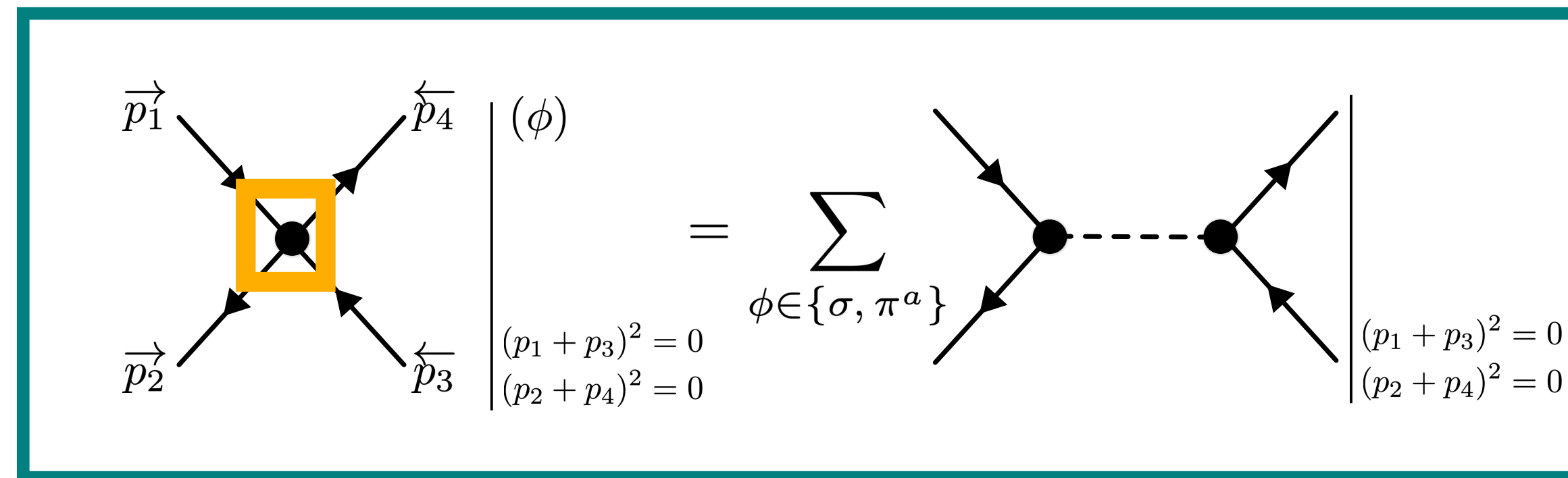


Pawlowski [hep-th/0512261] Fukushima, Pawlowski, Strodthoff [2103.01129]

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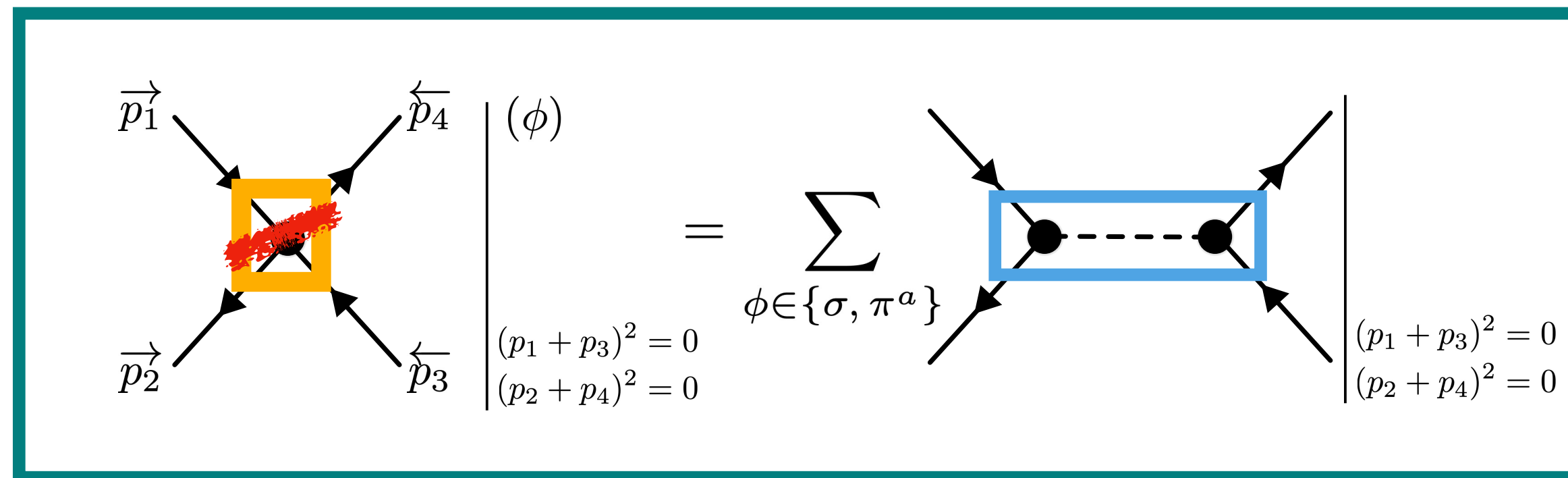
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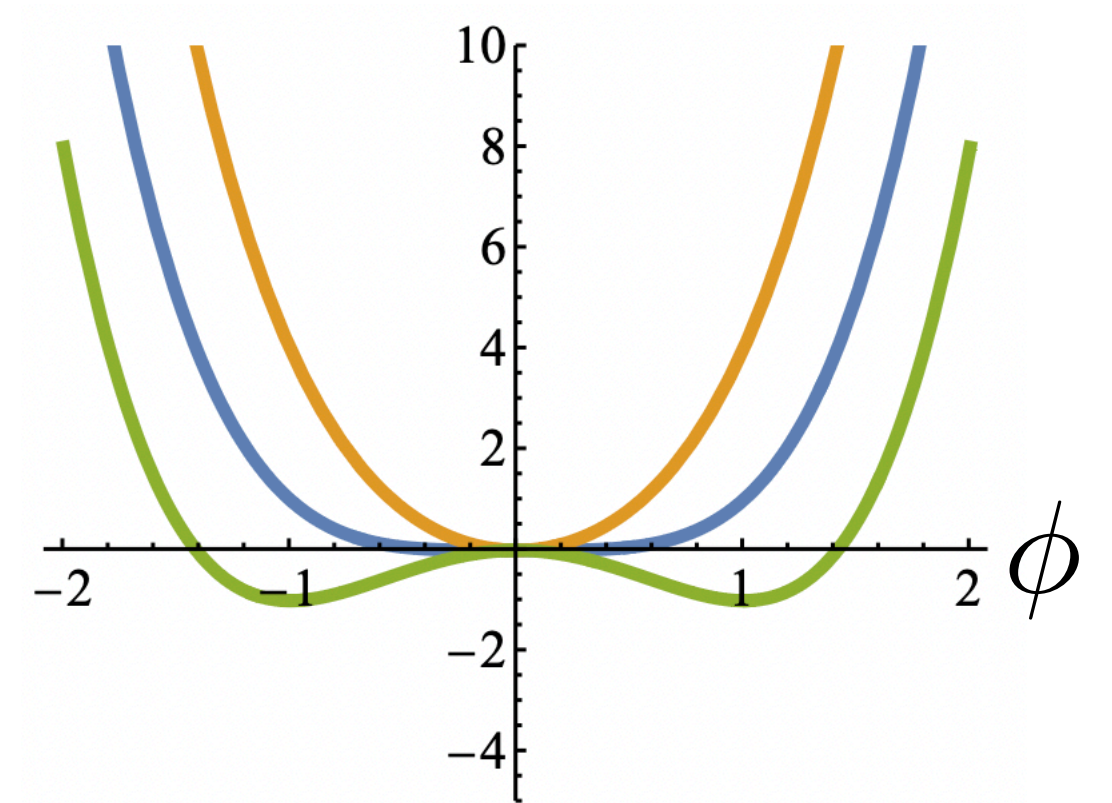
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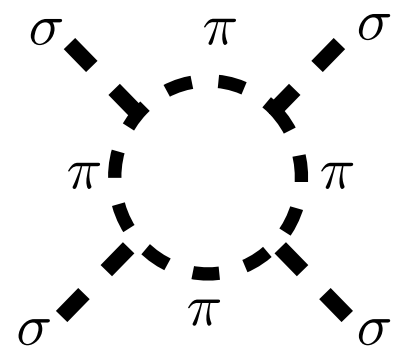
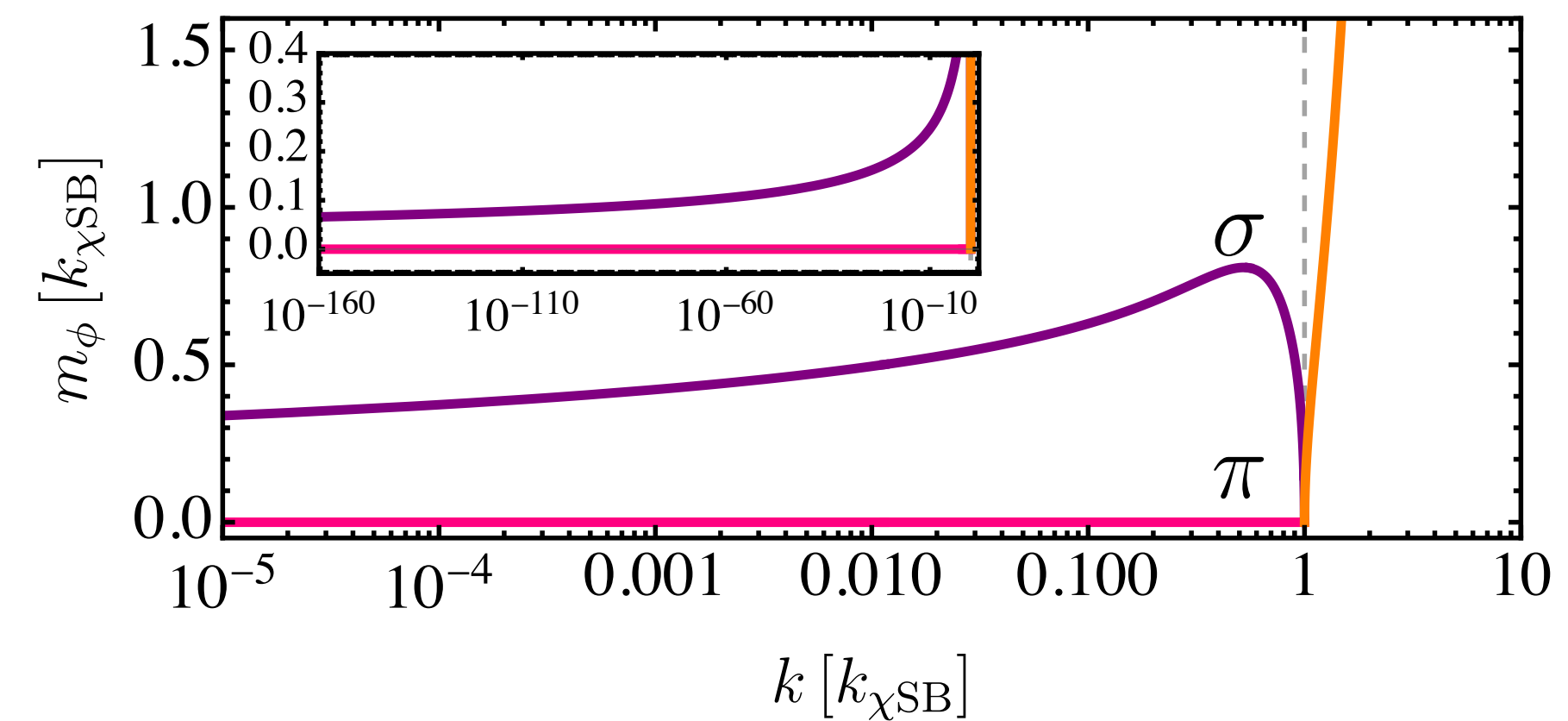
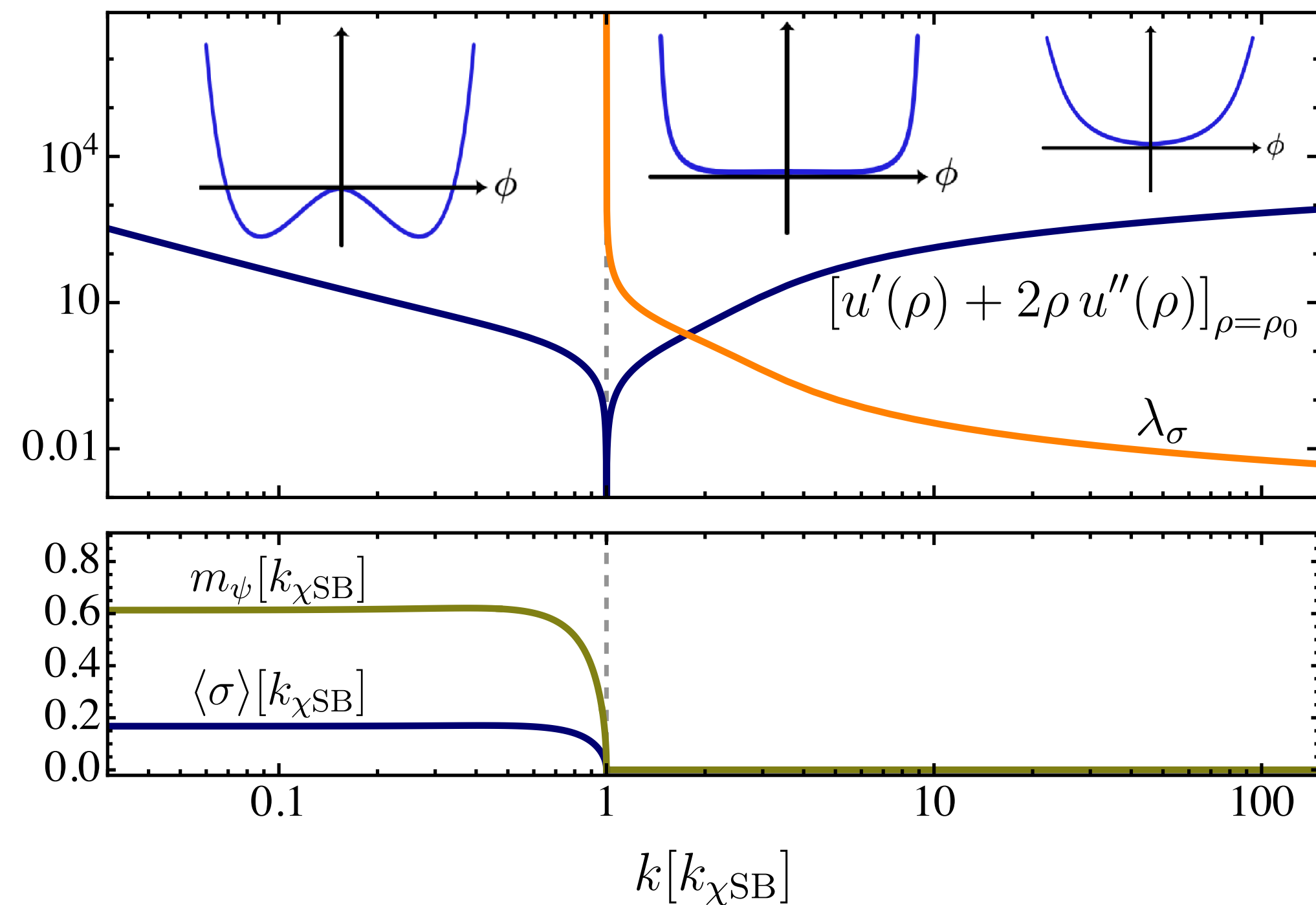
$$\phi = (\sigma, i\gamma_5 \pi^a)$$

$$+ h \bar{\psi} (T_f^0 \sigma + i\gamma_5 T_f^a \pi^a) \psi + \frac{1}{2} (\partial_\mu \phi)^2 + V(\phi^2)$$

$$V(\phi^2) = \sum_{n=1}^{N_{\max}} \frac{\lambda_n}{n!} \left(\frac{\phi^2}{2} \right)^n$$

Dynamical chiral symmetry breaking

$$N_f = 2$$



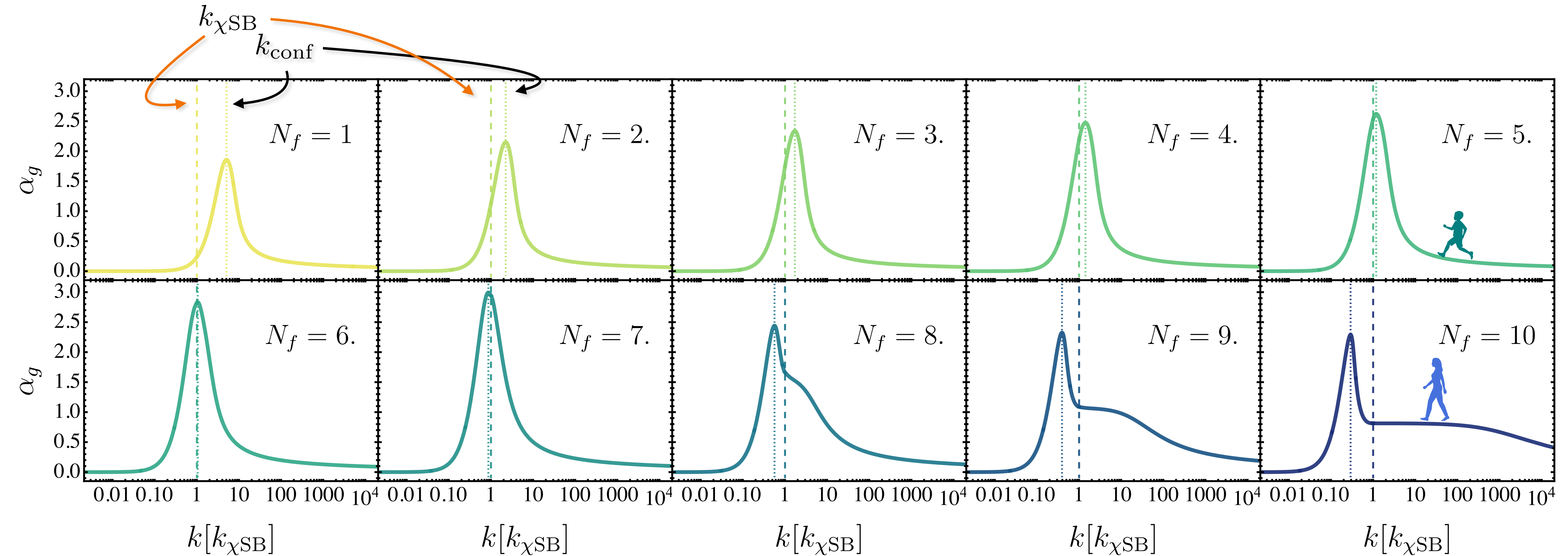
- ♦ Flows computed: $\{h, V(\phi), Z_\psi, Z_\phi, \lambda_i\}$
- ♦ **Continuous** interpolation between chirally **symmetric** and **broken** regimes
- ♦ A **clear** and **precise** way to **diagnose** χSB

♦ Obtaining **fundamental parameters**

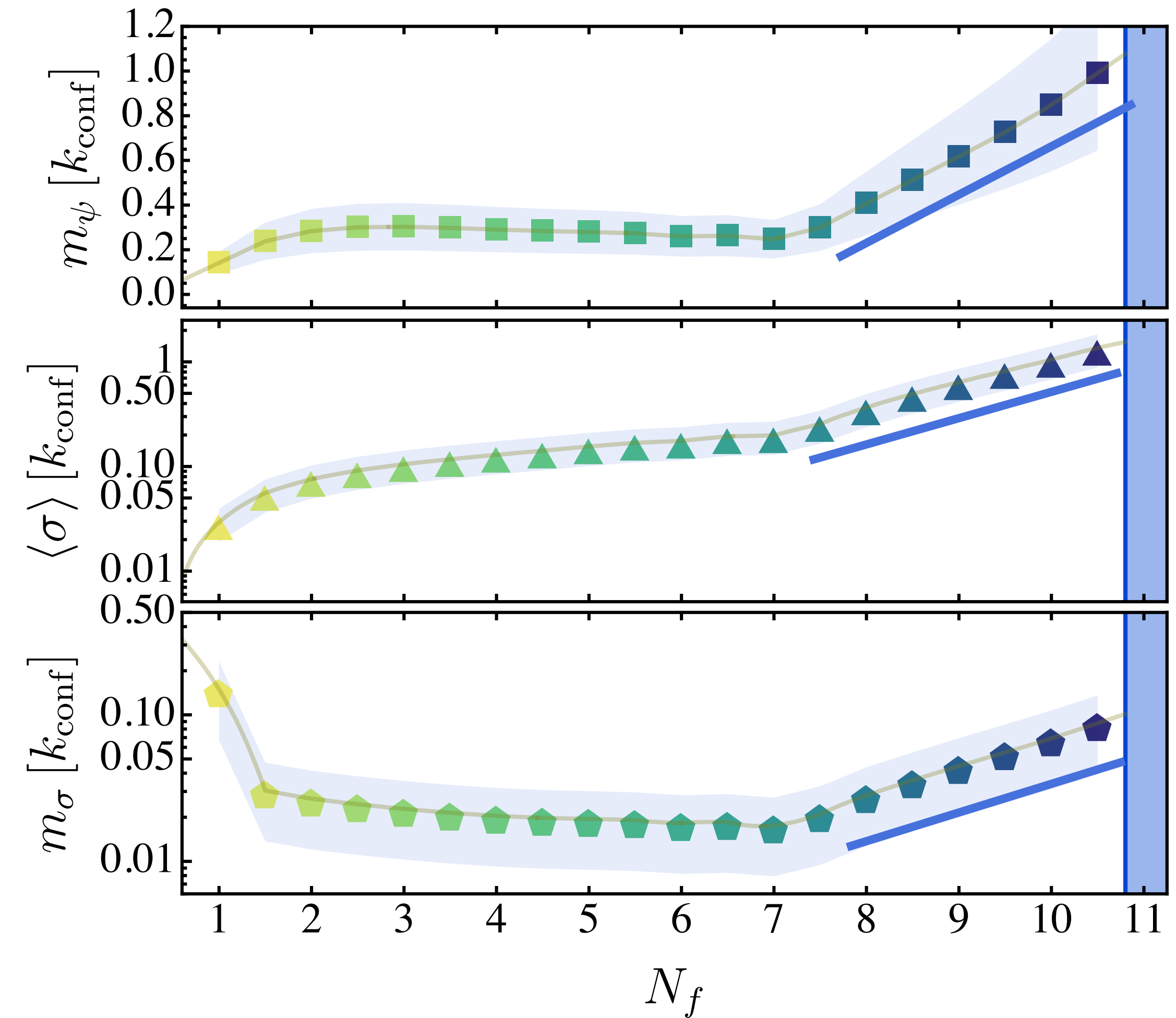
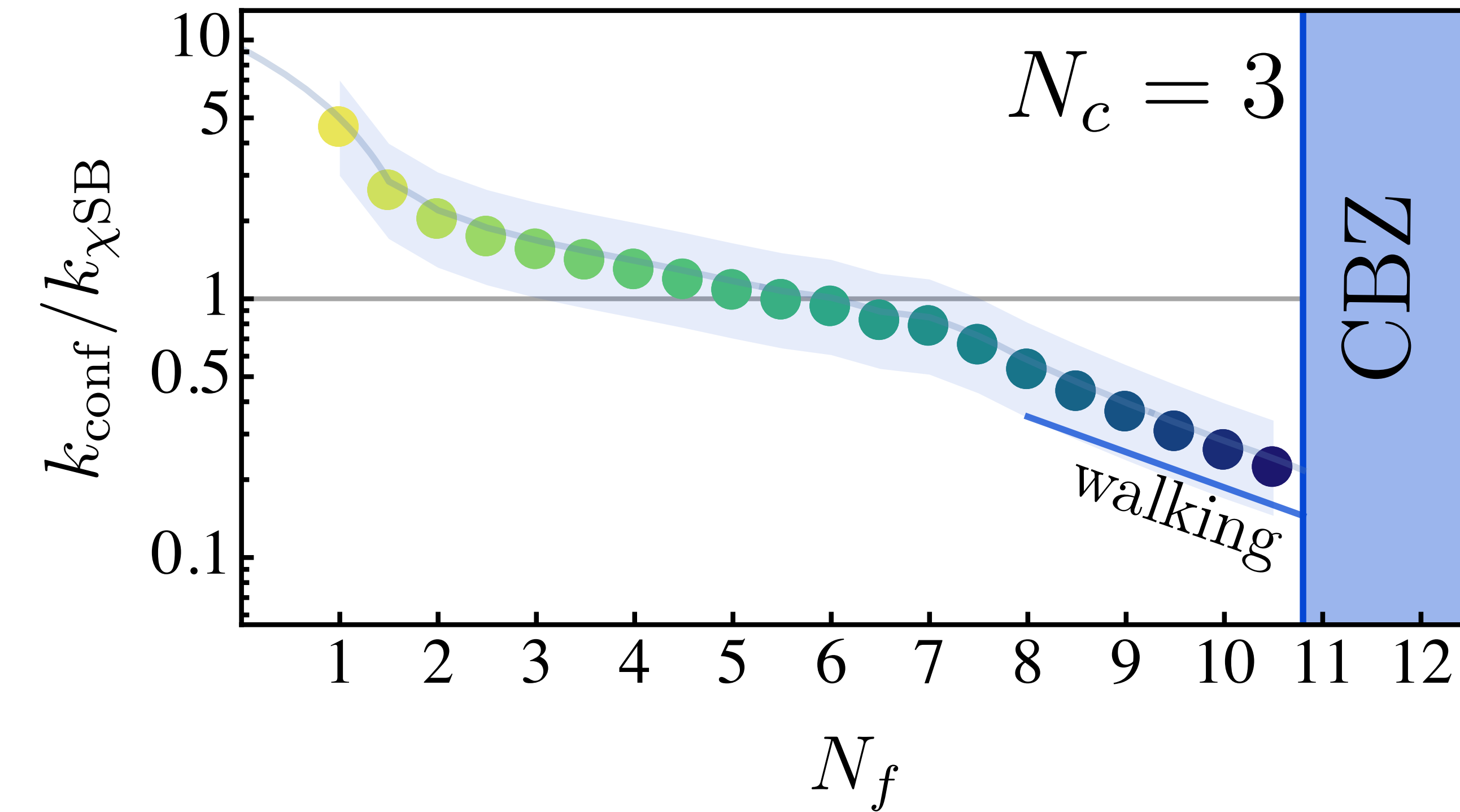
- Constituent fermion masses: m_ψ (within 10% error)
- Chiral condensate: $\langle \sigma \rangle \sim f_\pi$
- Composite masses of bosonised channels: m_σ, m_π

- ♦ Account for **higher dimensional fermionic** operators via higher-order scalar potential

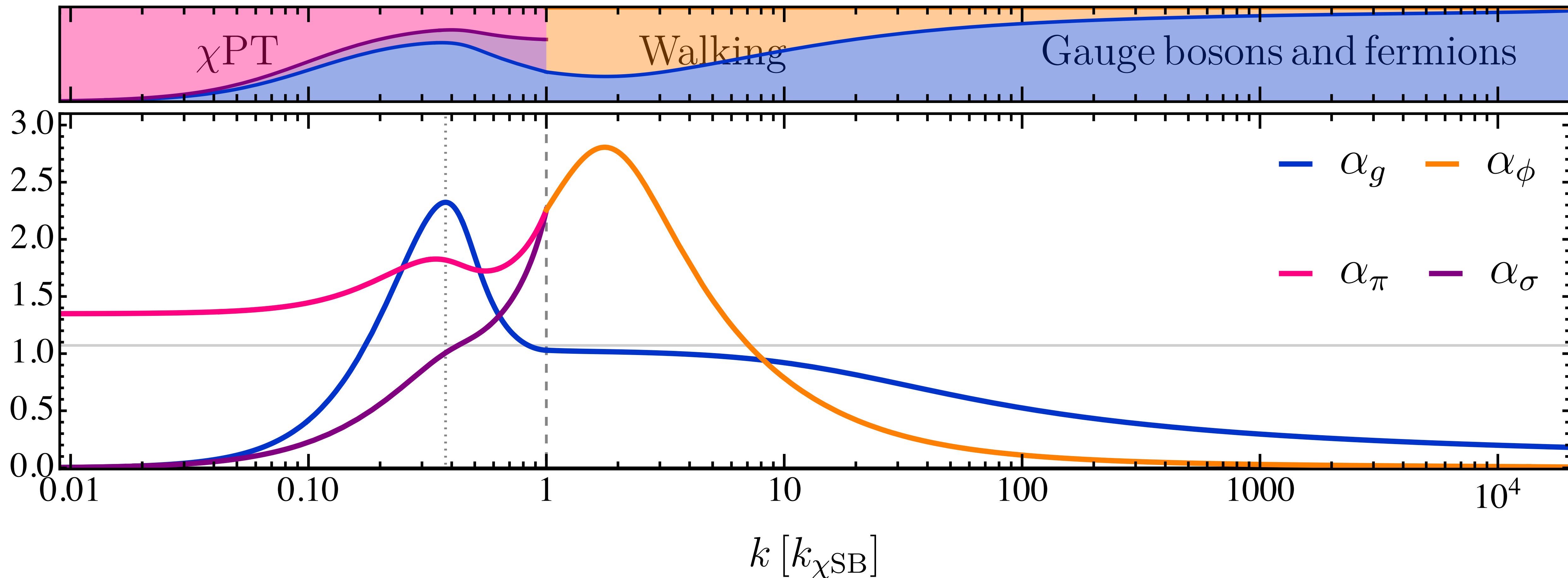
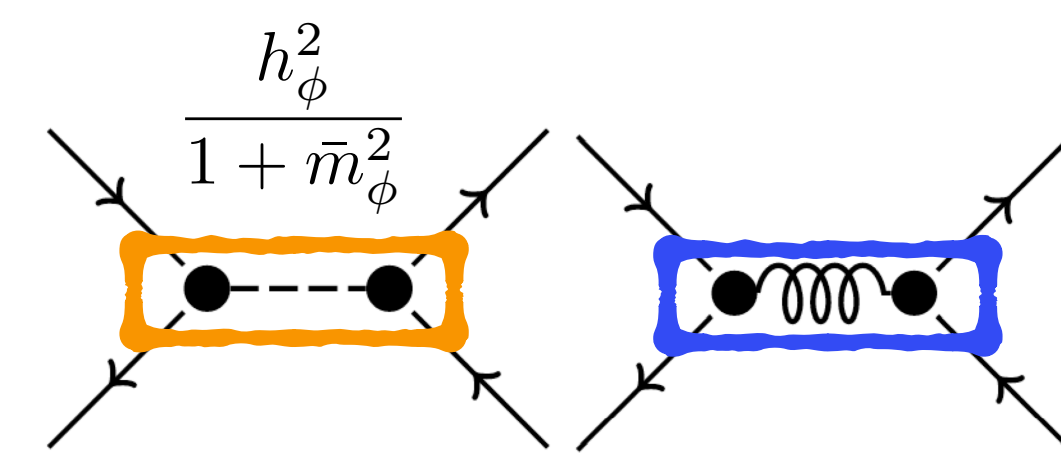
Many flavour gauge dynamics



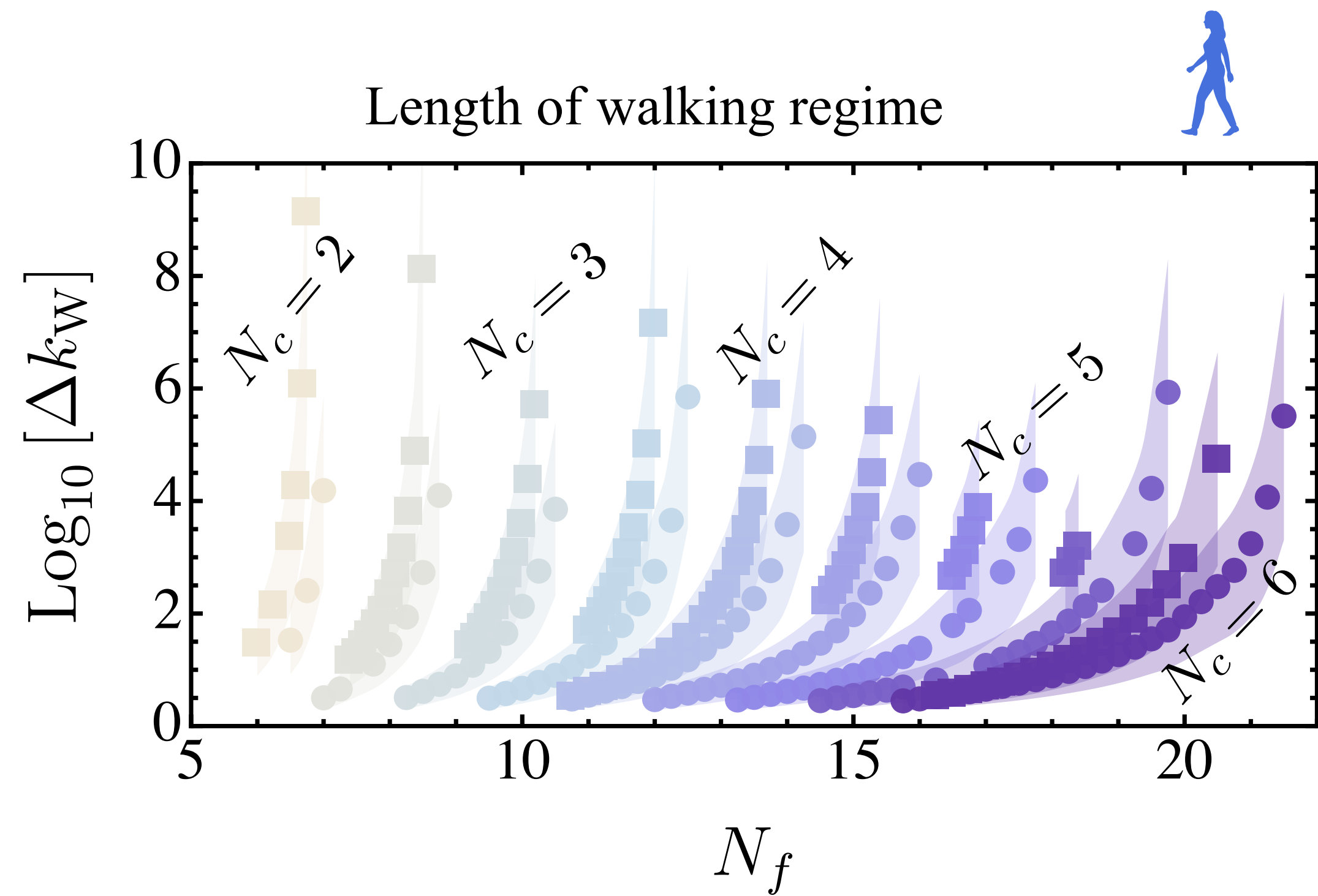
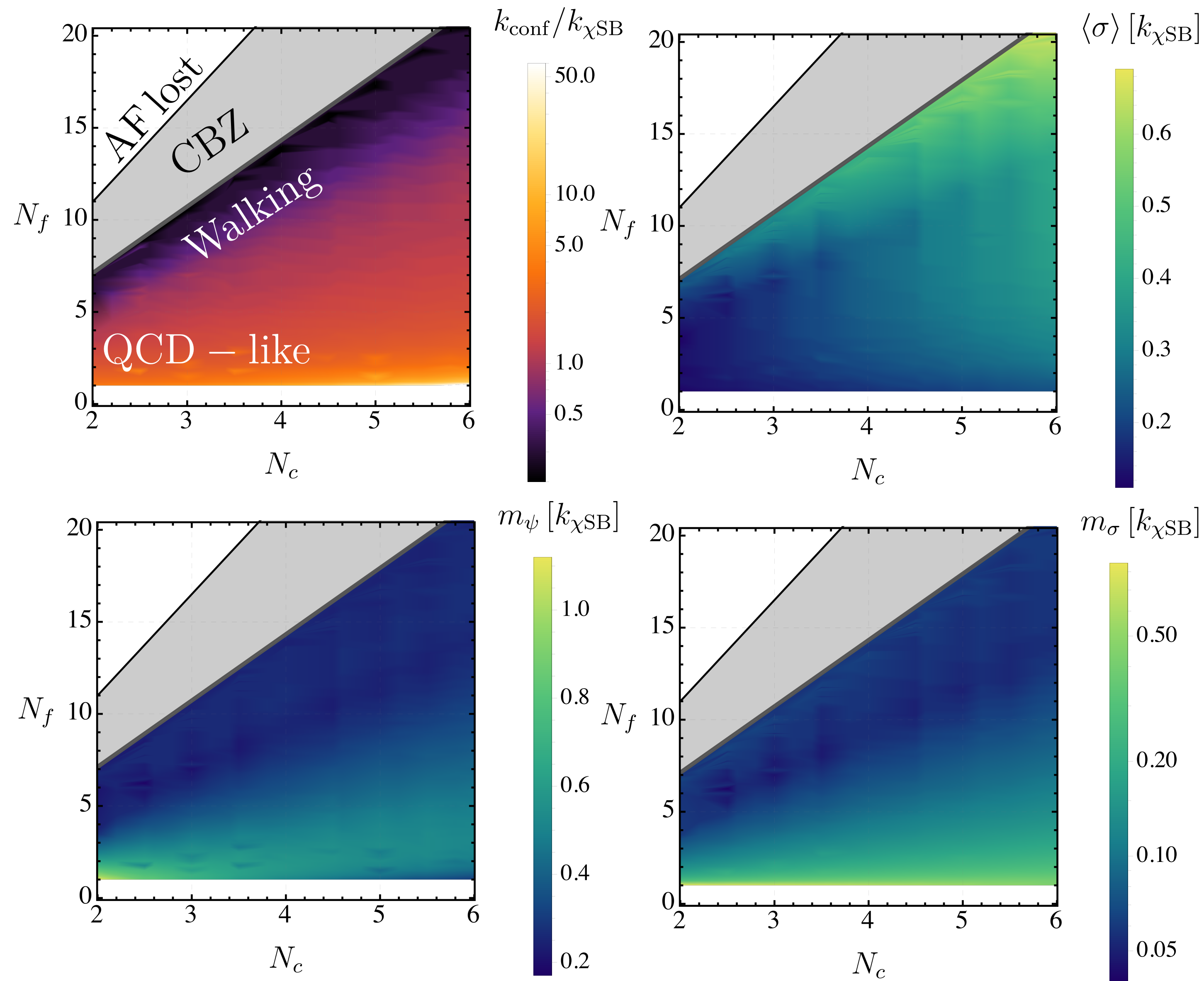
Interplay of scales and fundamental parameters



Walking dynamics and flow over EFTs



Cartography

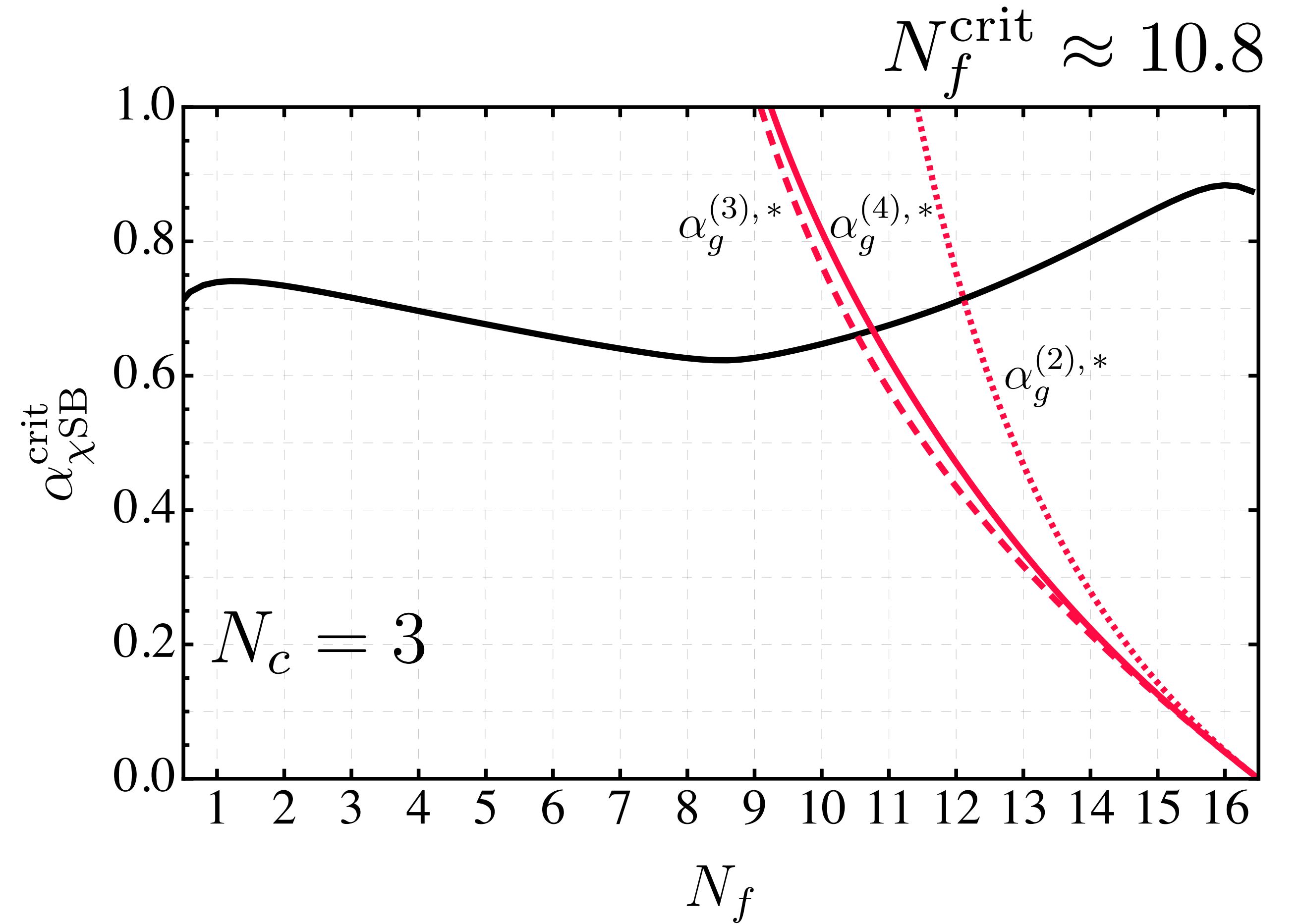
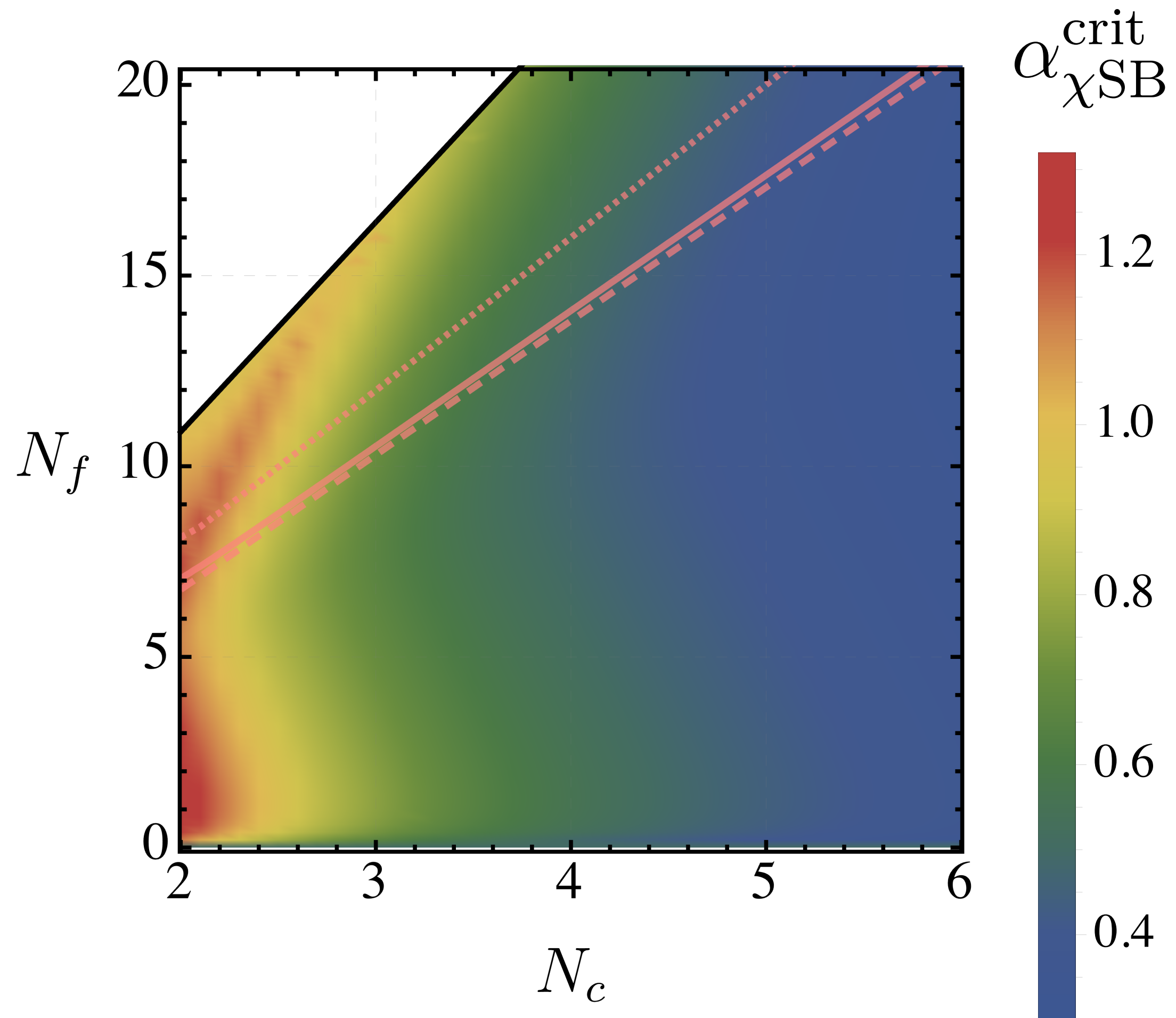


Boundary of the conformal window

$\alpha_{\chi\text{SB}}^{\text{crit}}$: Critical strength of **gauge dynamics** necessary to trigger dynamically χSB

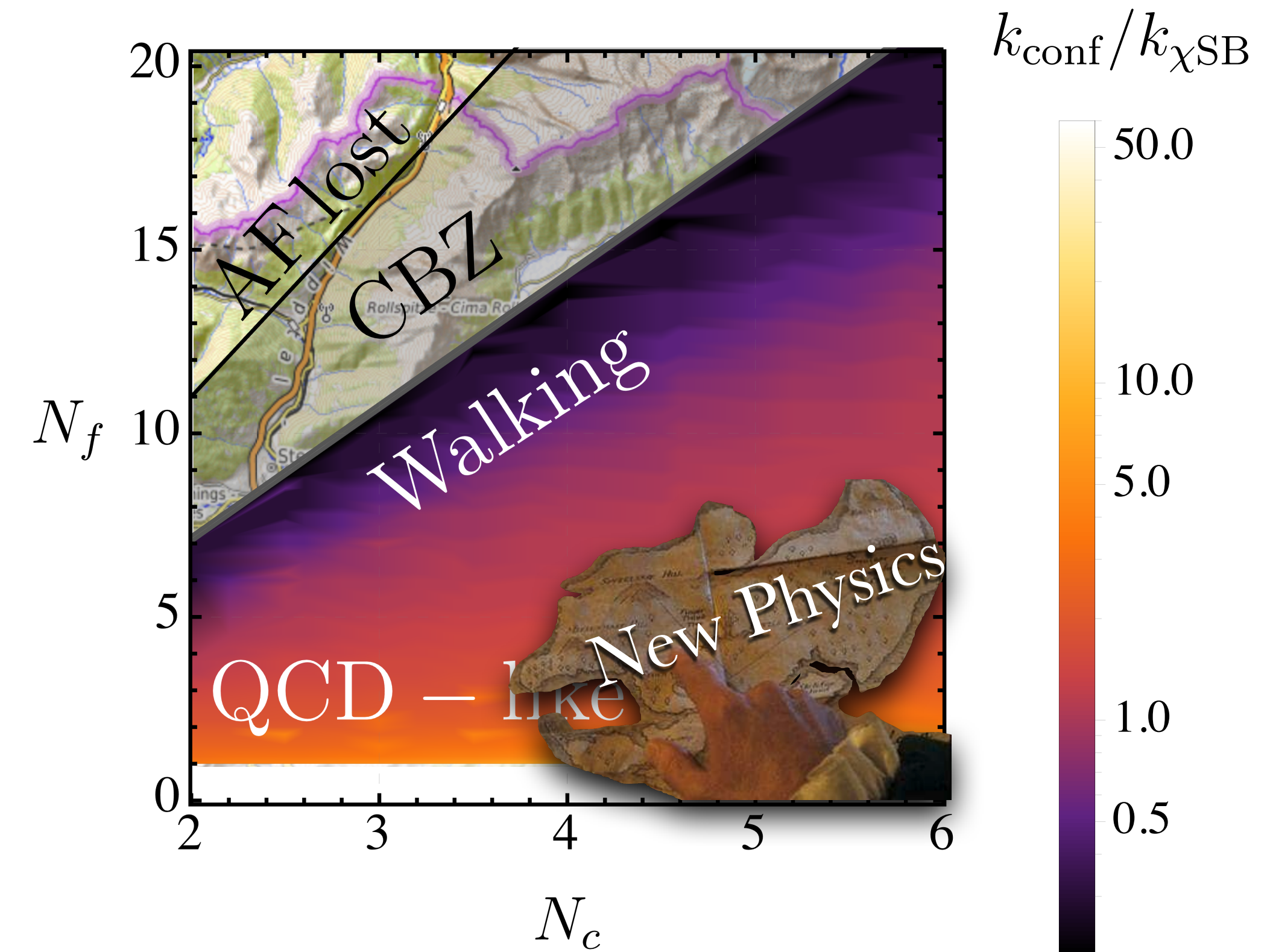
Gies, Jaeckel '05

Braun, Gies'05'06



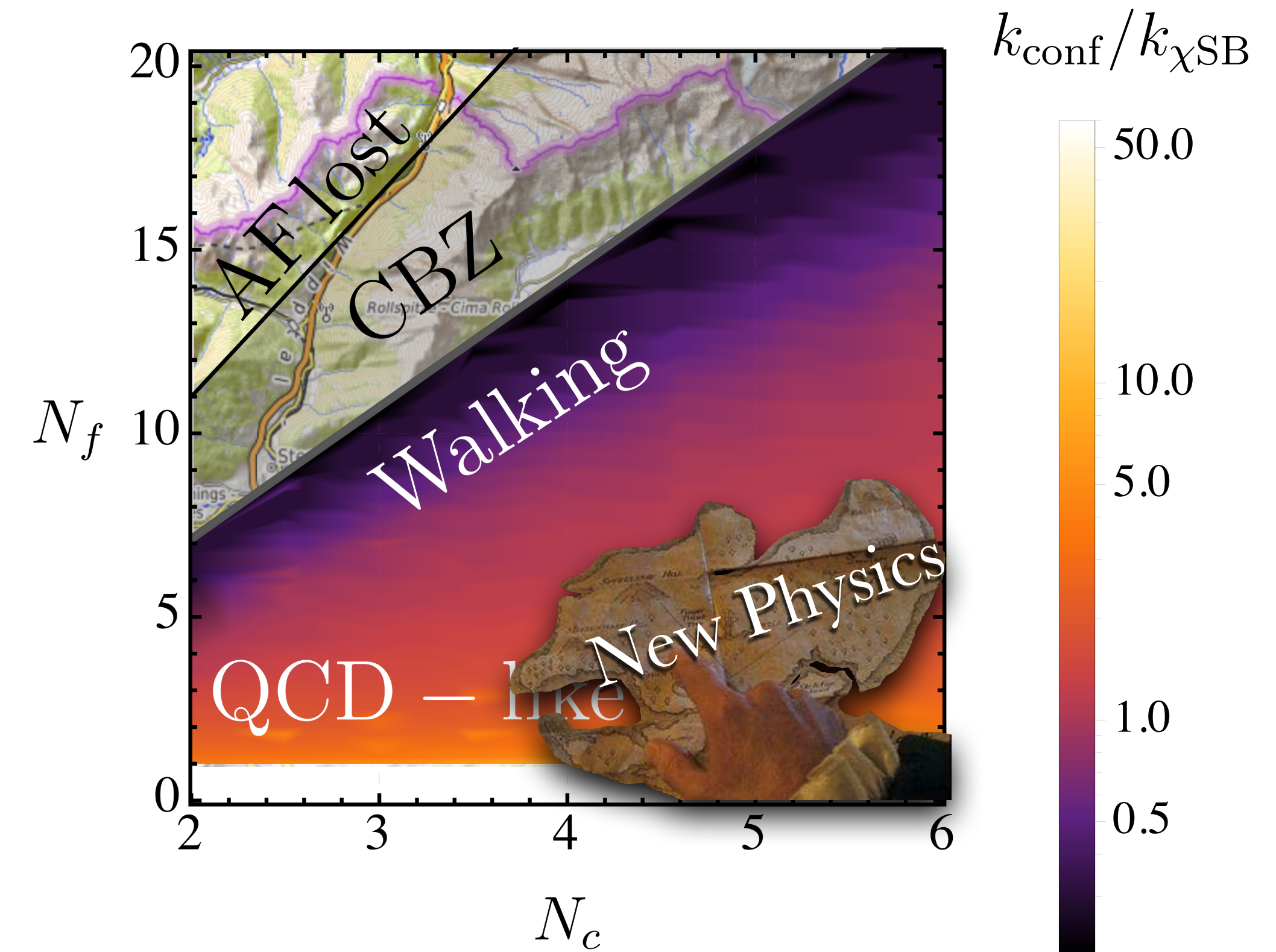
Summary and conclusions

- ♦ The fRG is a **powerful approach** to strong dynamics:
 - “*Easy*” new **treatment of confinement**
 - Charted the **landscape of gauge-fermion theories** from **first principles**
 - Determination of **fundamental parameters**:
 - ➔ m_ψ , $\langle\sigma\rangle$, m_σ , size of walking regime, ...
 - Quantitative estimate for the **lower boundary of the CBZ**
 - **conformal window**
 - Flow over EFTs
 - Input for CFTs
- } ask me!



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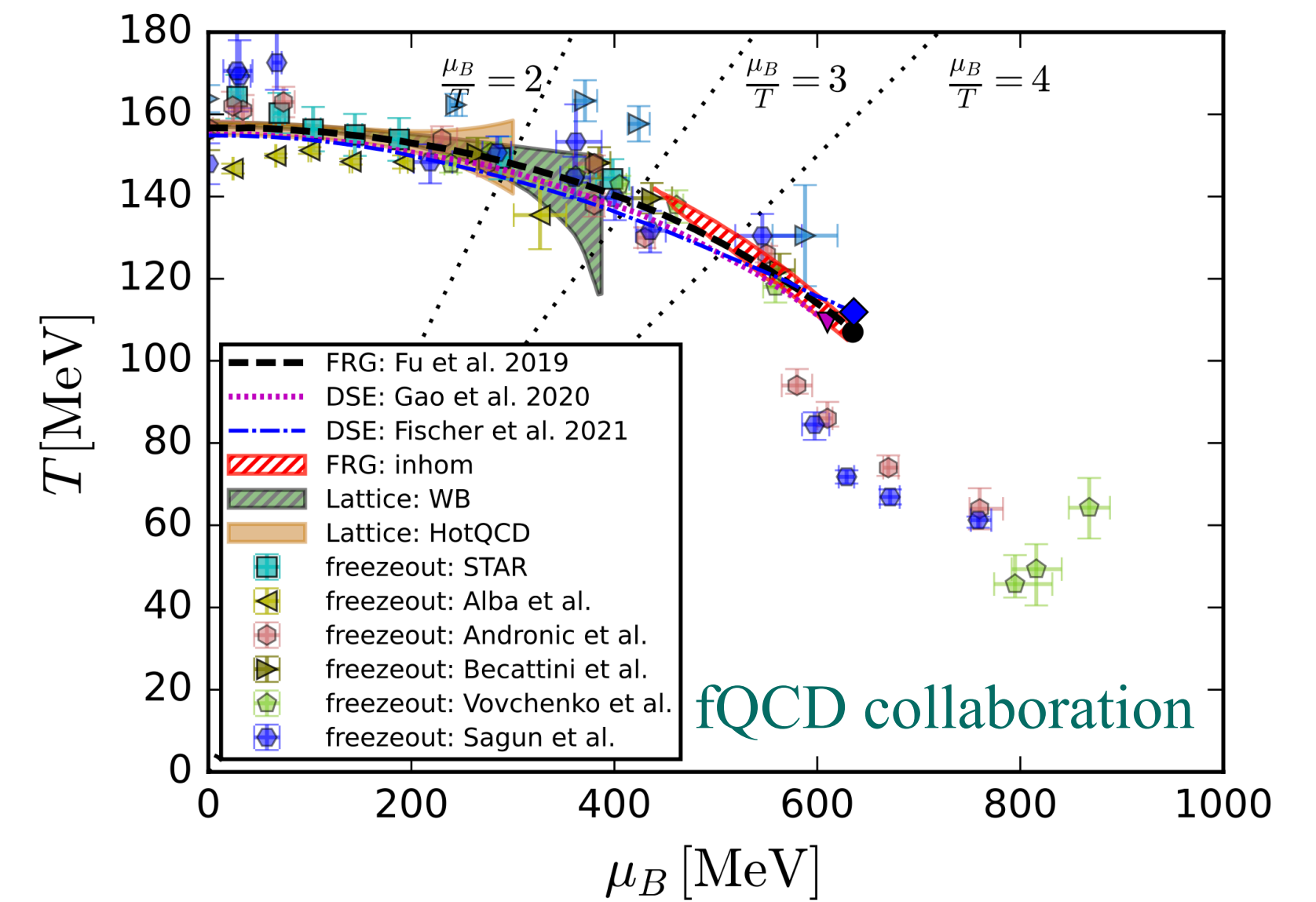
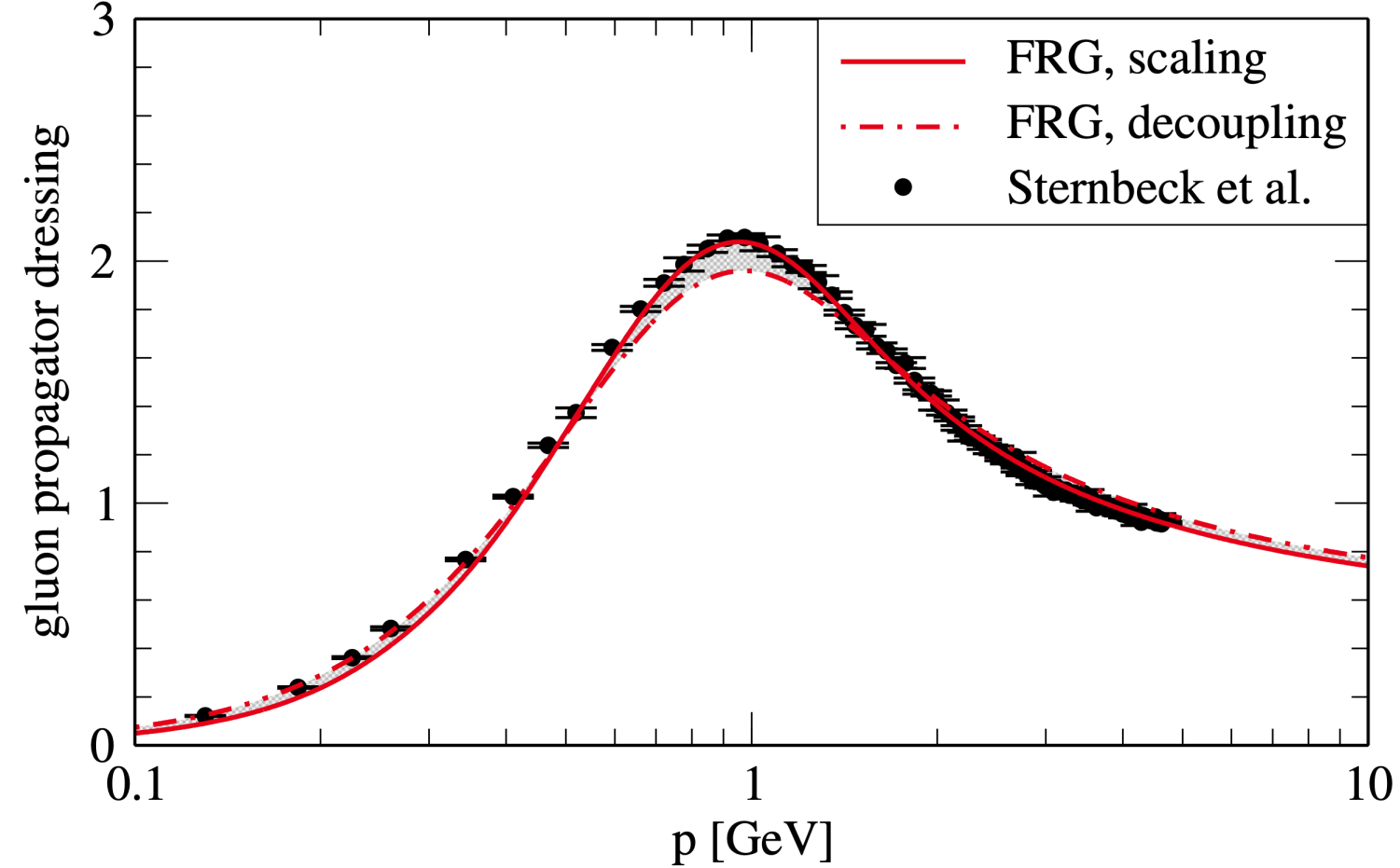


Thank you for your attention!

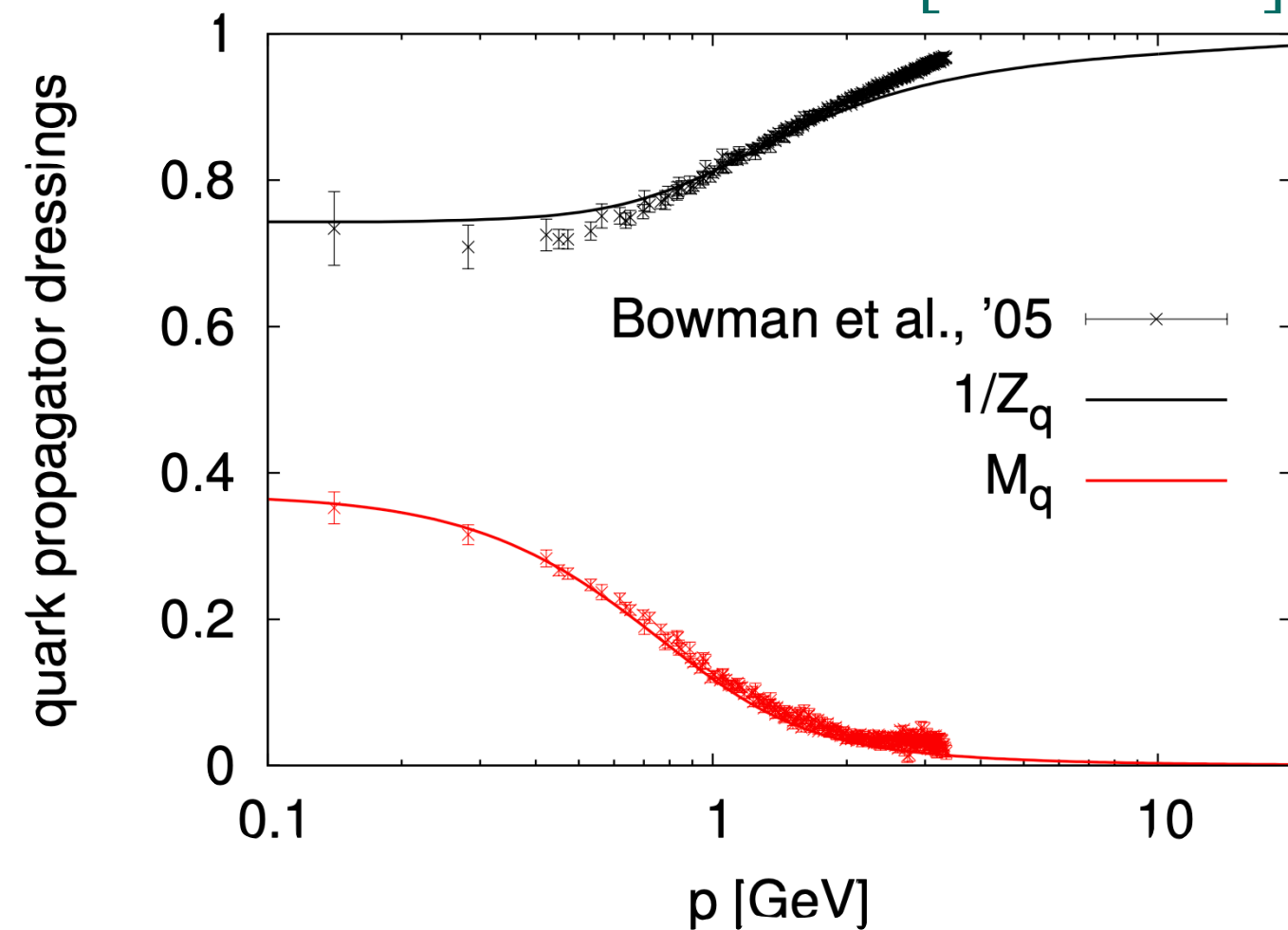
Additional slides

Functional Renormalisation Group in QCD

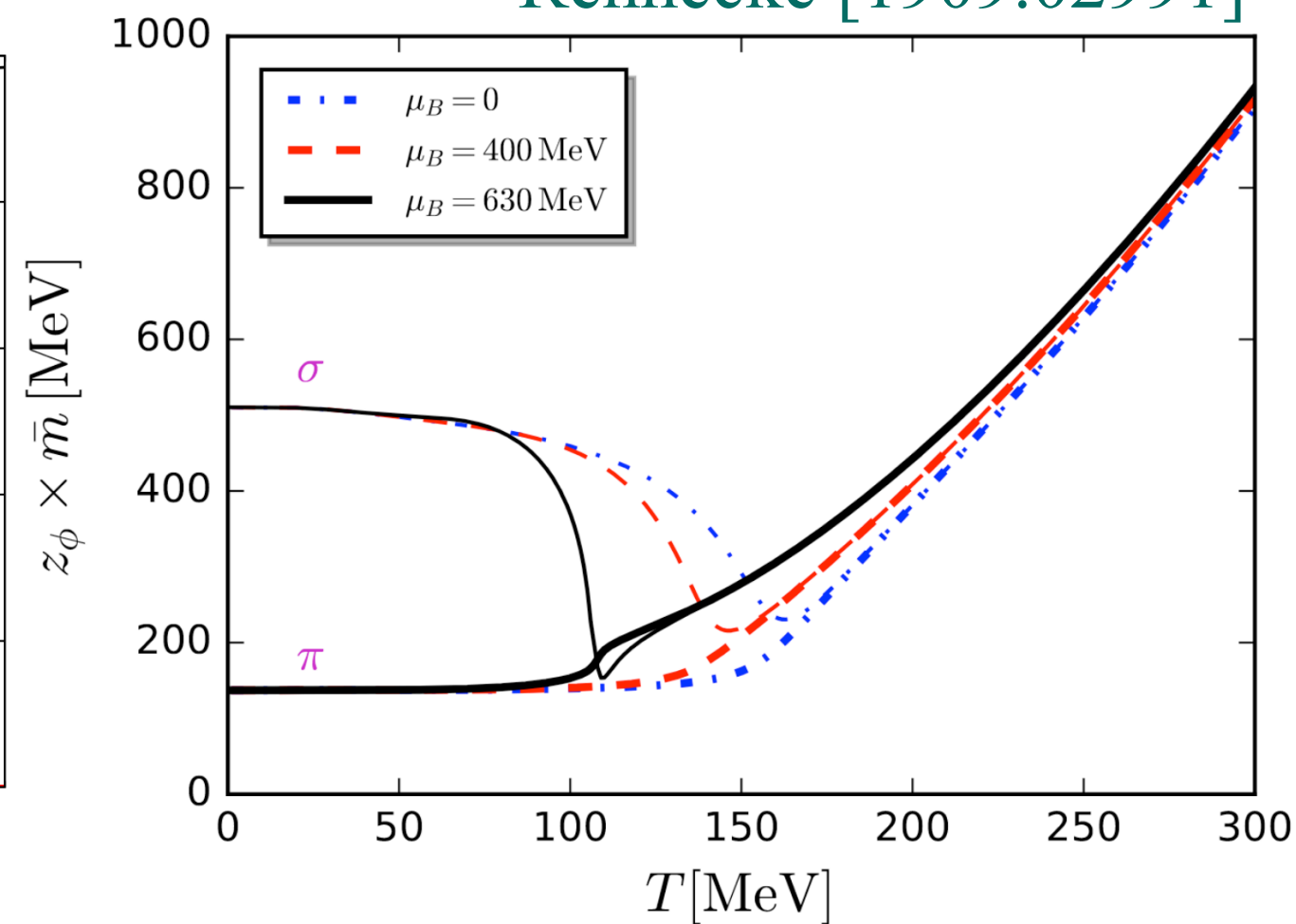
Cyrol, Fister, Mitter, Pawłowski [1605.01856]



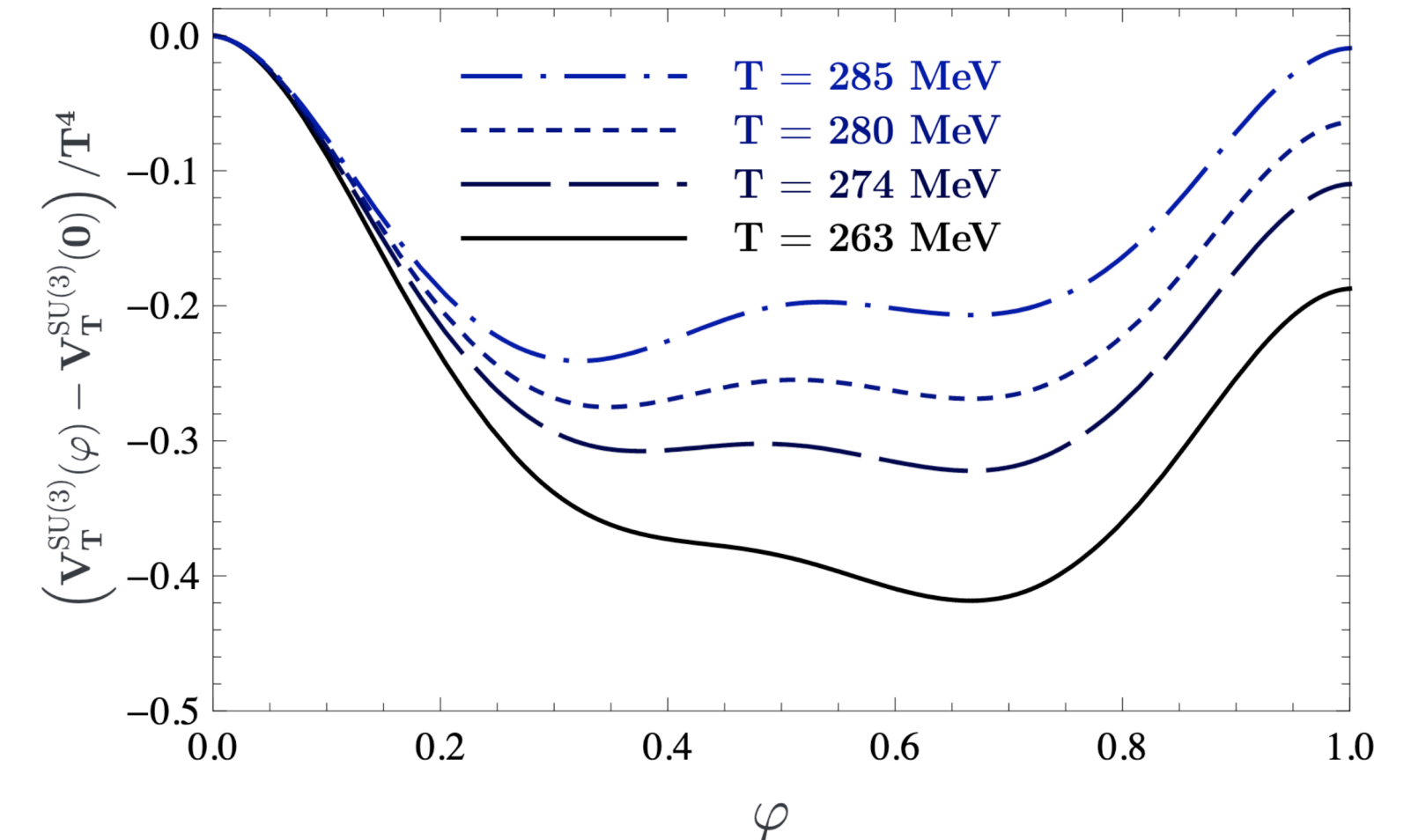
Mitter, Pawłowski, Strodthoff [1411.7978]



Fu, Pawłowski, Rennecke [1909.02991]



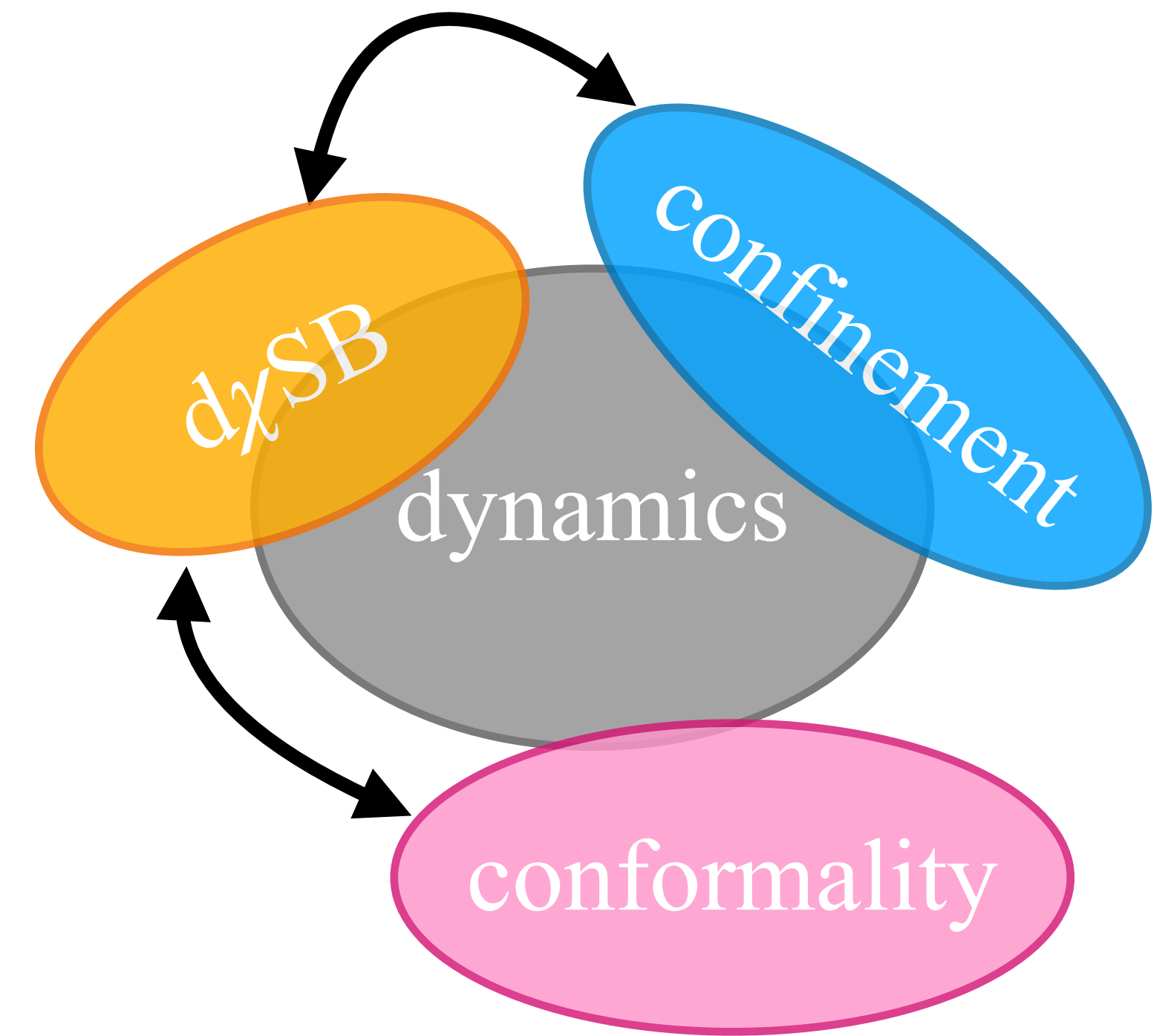
Fister, Pawłowski [1301.4163]



Towards the uncharted regime

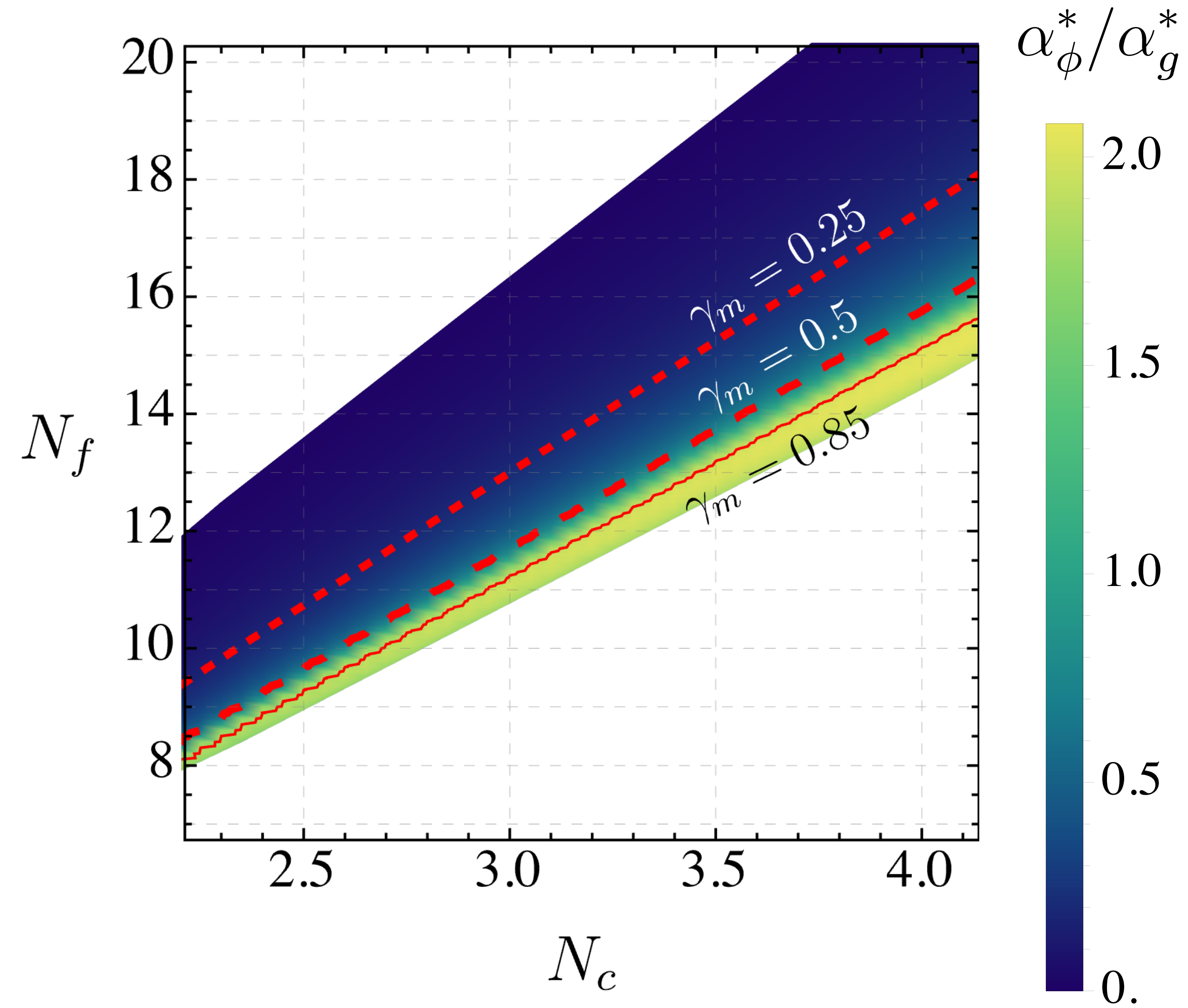
1. Chart the **landscape** of gauge-chiral theories
2. Understand **dynamics** in the many-flavour limit from **first principles**
3. Obtain **fundamental parameters** and the relation between scales
4. Determine the **boundary** between phases: conformal & non-conformal
5. Provide **UV input** to low-energy effective theories

- ◆ Functional methods in the continuum limit: the **functional Renormalisation Group**
 - No difficulties in the exact chiral limit



$$\Gamma_k[\phi]$$

In the conformal window



Deriving correlation functions

$$\Gamma_k[\phi] = \sum_n \int_p \Gamma_{k, \phi_{i_1} \dots \phi_{i_n}}^{(n)}(p_1, \dots, p_n) \phi_{i_n}(p_n) \dots \phi_{i_1}(p_1)$$

$$\Gamma_{k, \phi_{i_1} \dots \phi_{i_n}}^{(n)}(p_1, \dots, p_n) = \frac{\delta}{\delta \phi_{i_1}(p_1)} \dots \frac{\delta}{\delta \phi_{i_n}(p_n)} \Gamma_k[\phi]$$

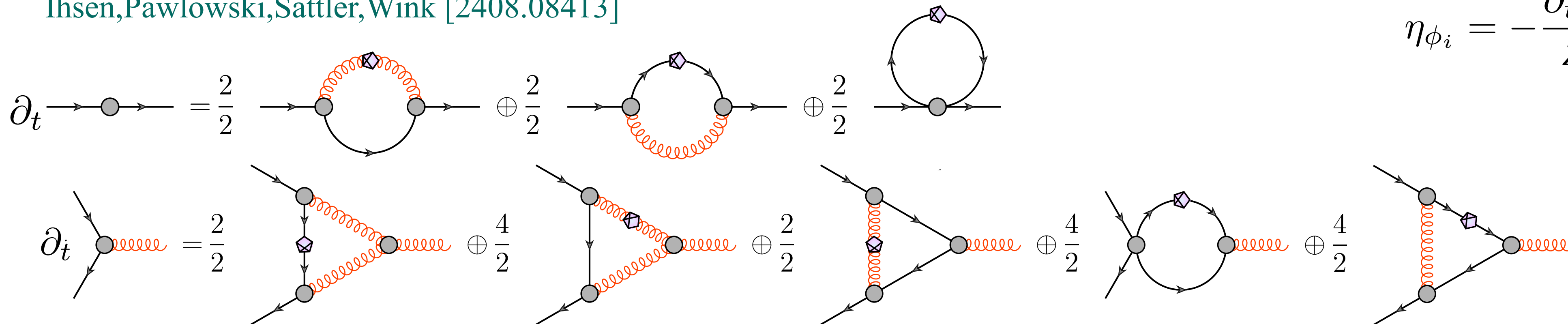
Example: quark-gluon vertex and gauge coupling

$$\partial_t \Gamma_k^{(\bar{\psi}\psi A)} = \partial_t \left(Z_A^{1/2} Z_\psi g_{\bar{\psi}\psi A} \cdot \mathcal{T}_\mu \right) \longrightarrow \partial_t g_{\bar{\psi}\psi A} = \frac{\text{Tr} \left[\mathcal{T}_\mu \partial_t \Gamma_k^{(\bar{\psi}\psi A)} \right]}{\text{Tr} \left[\mathcal{T}_\mu^2 \right] Z_A^{1/2} Z_\psi} + \left(\frac{1}{2} \eta_A + \eta_\psi \right) g_{\bar{\psi}\psi A}$$

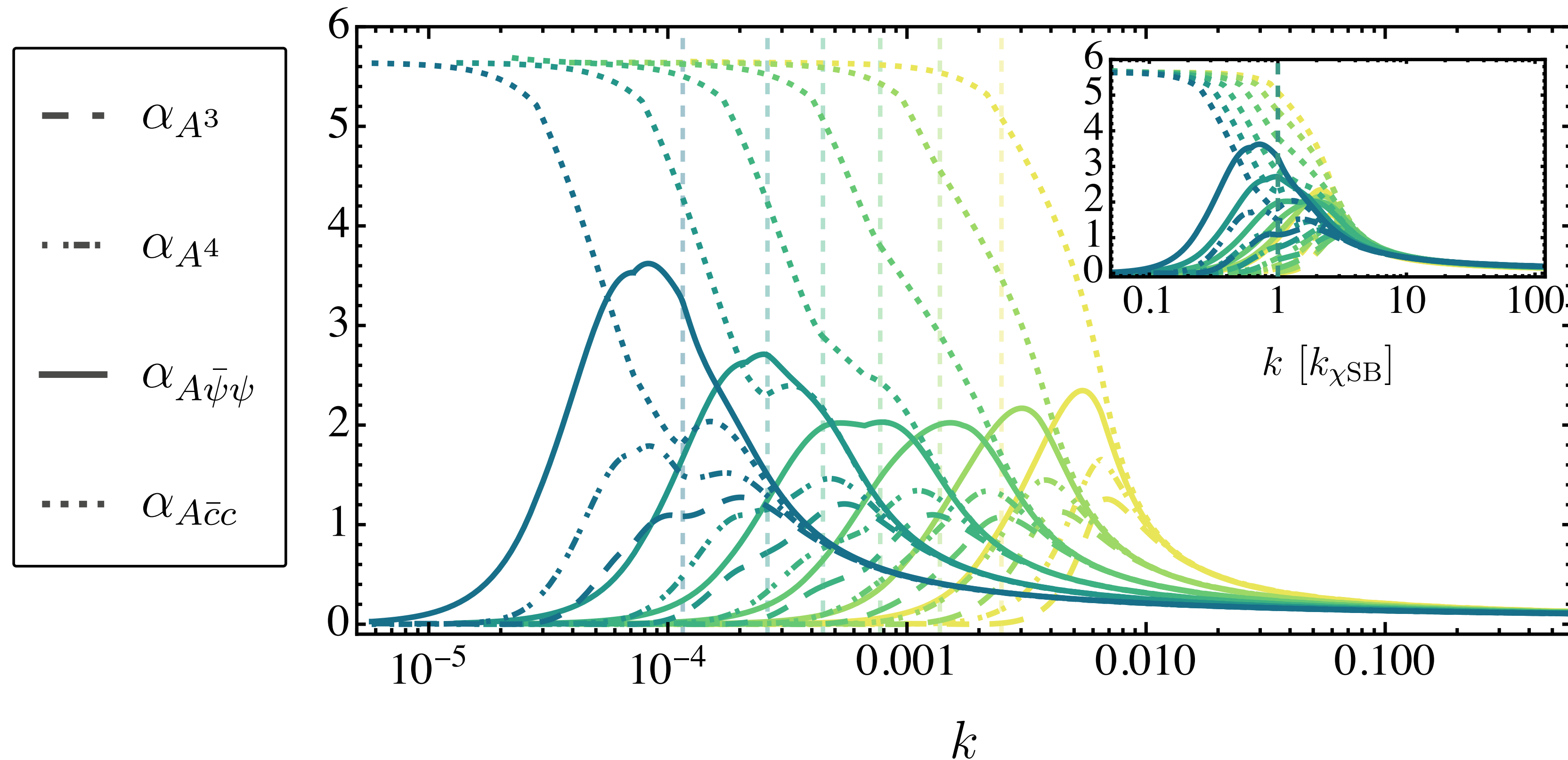
with anomalous dimensions:

$$\eta_{\phi_i} = - \frac{\partial_t Z_{\phi_i}}{Z_{\phi_i}} = - \frac{\partial_{p^2} \partial_t \Gamma_k^{(\phi_i \phi_i)}}{Z_{\phi_i}}$$

Ihsen, Pawłowski, Sattler, Wink [2408.08413]



Many flavour full truncation



♦ Observations:

- The peak of the pure-gluon vertex is rather constant and independent of the number of flavours
- The ghost-gauge coupling in the deep IR always reaches the same constant coupling given it is only colour-group dependent
- Increment of gauge-chiral coupling strength. The stronger value of this avatar causes a flattening of the chiral potential and consequently an earlier χSB .

Yang-Mills phases and confinement

$$\Gamma_k^{(AA)}(p^2) = Z_{A,k}(p) (p^2 + m_{A,k}^2)$$

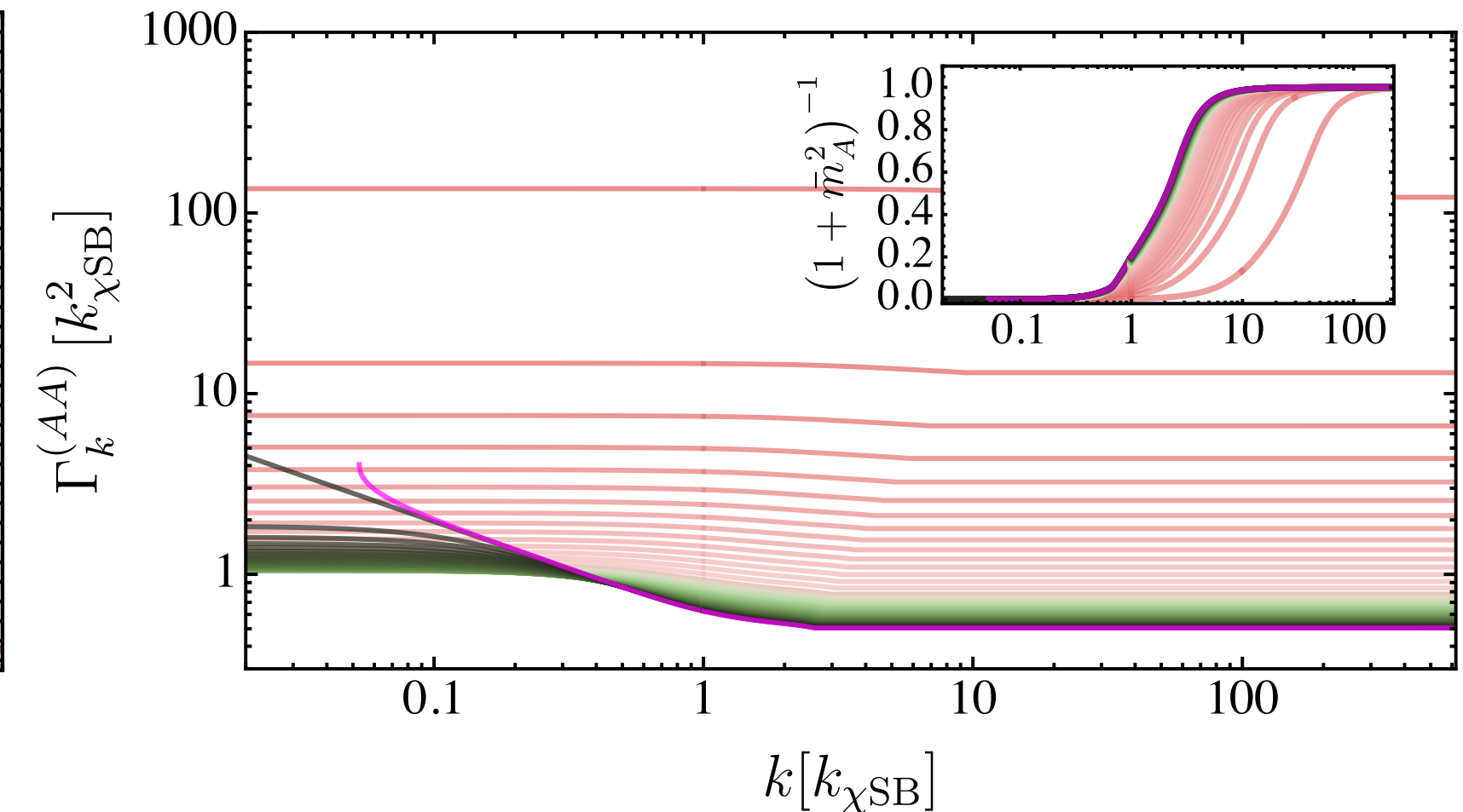
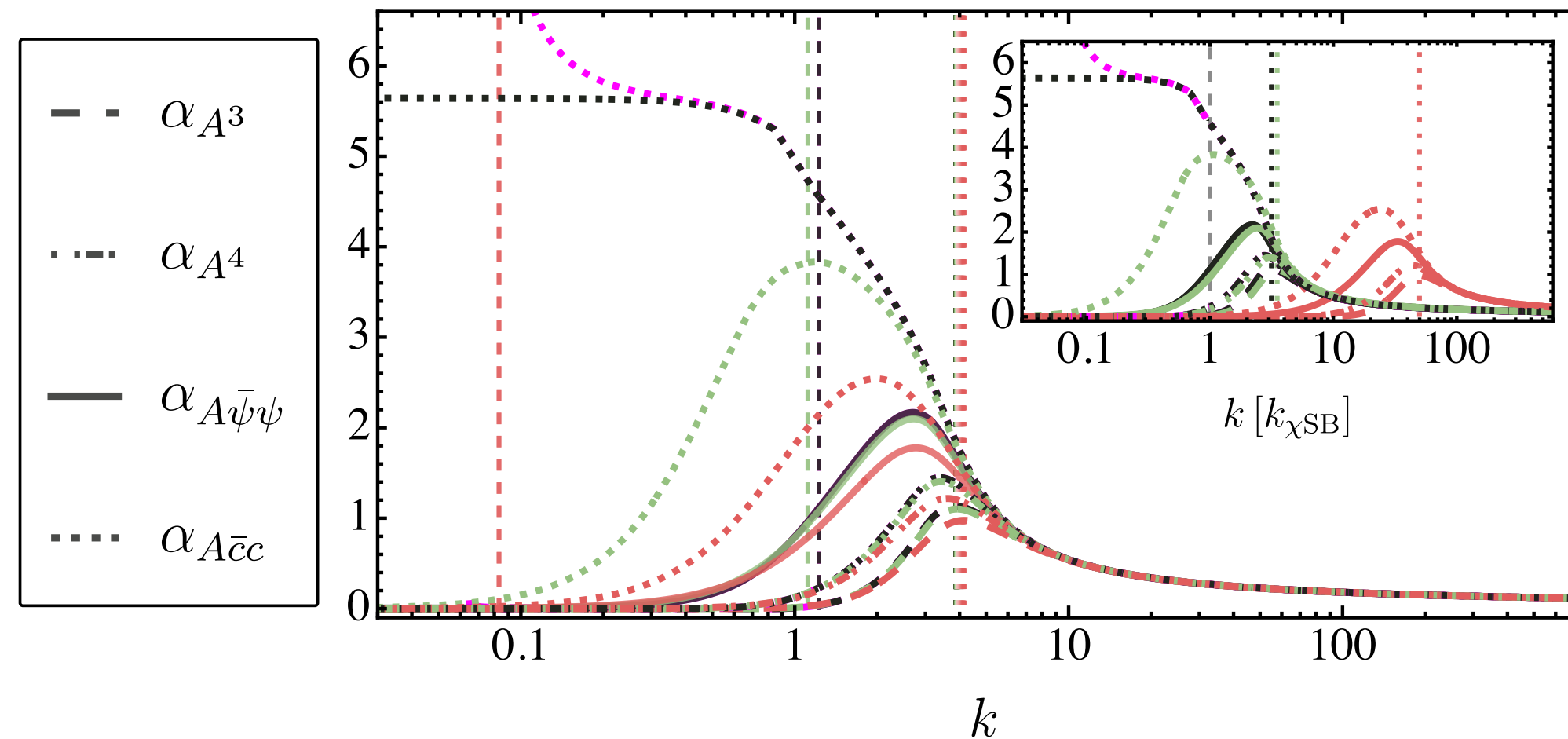
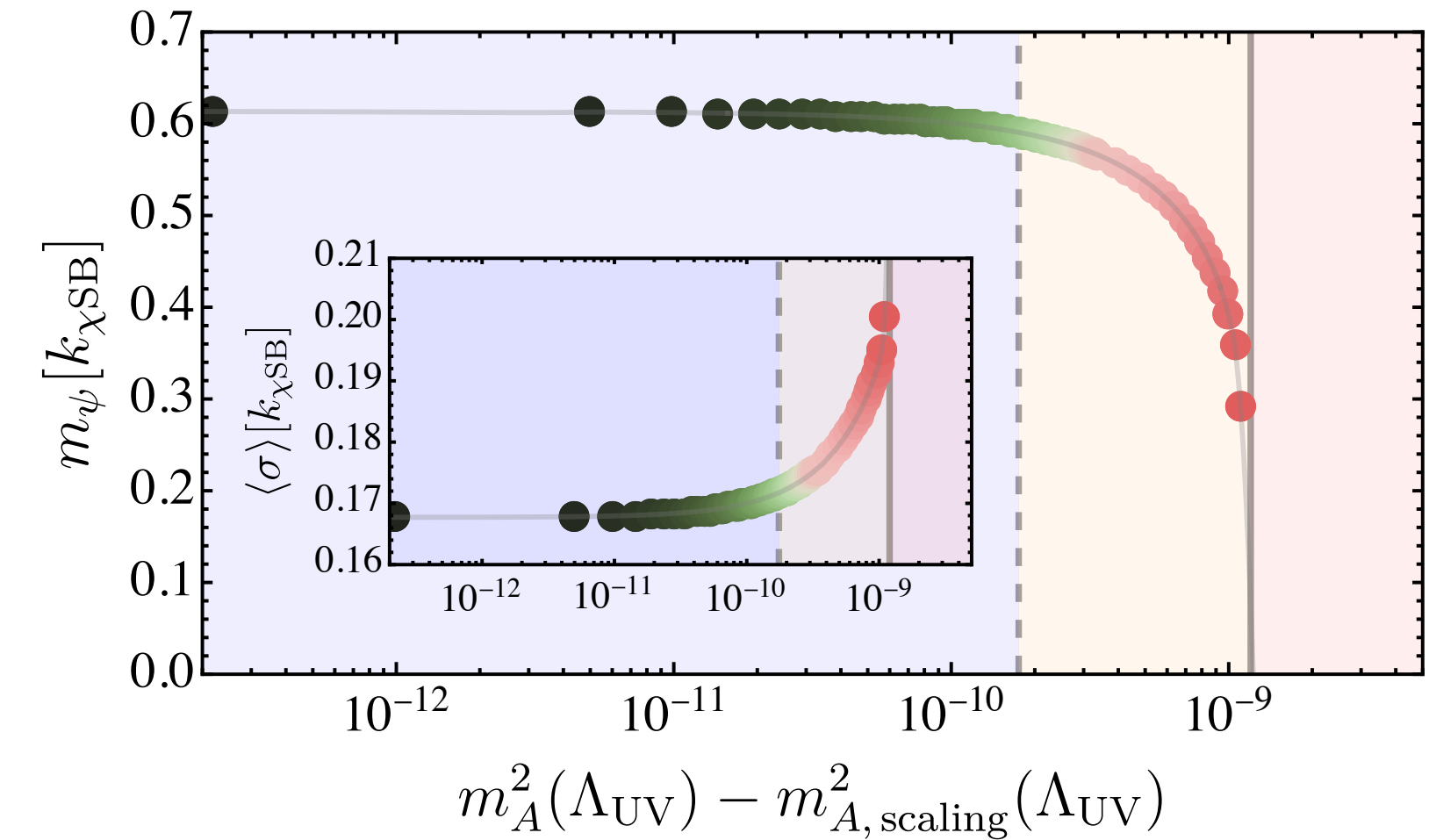
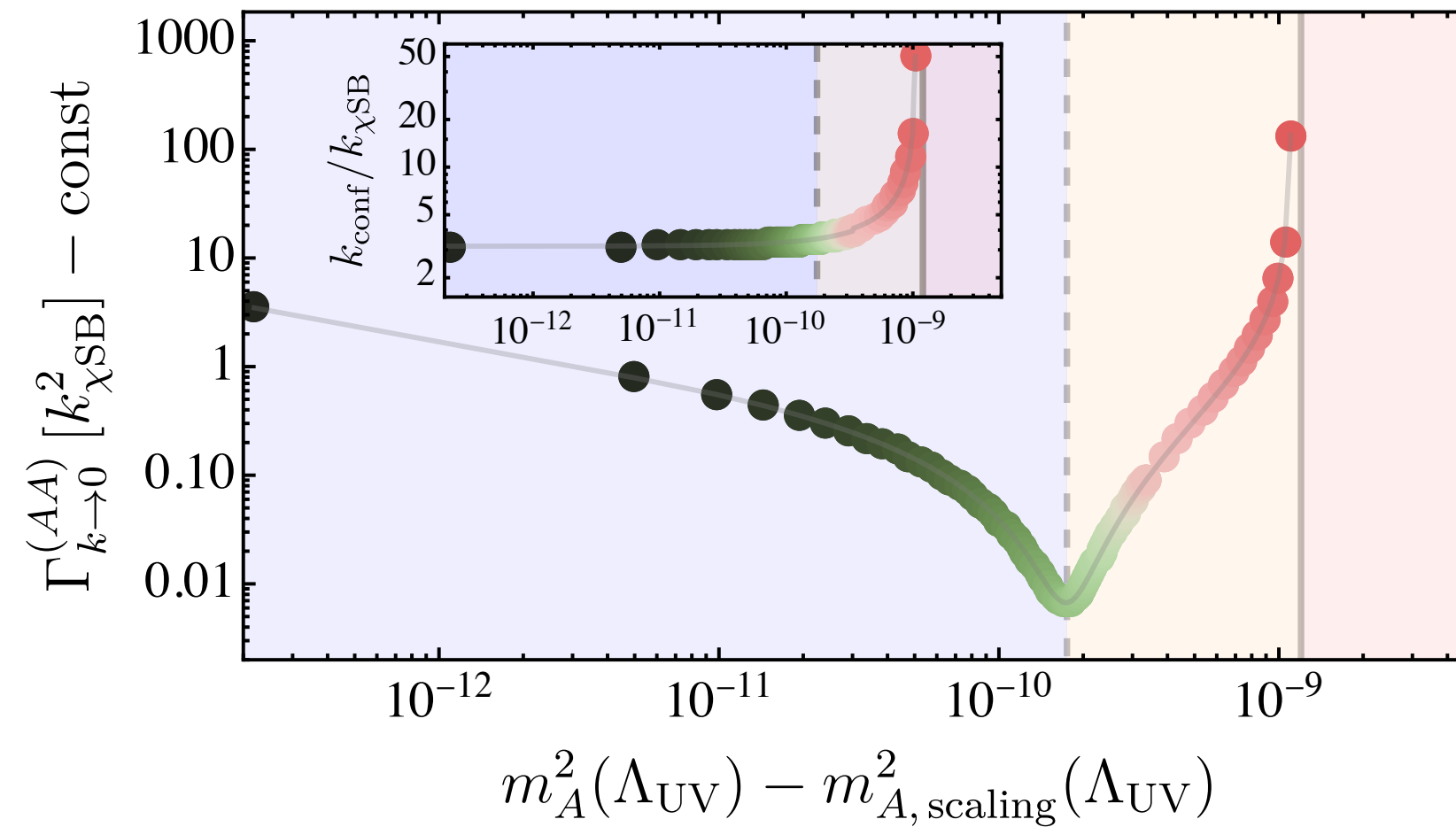
◆ Mass gap as a dialing parameter

◆ Confining solutions:

- Scaling
- Decoupling

◆ Non-confining solutions:

- Coulomb phase
- Massive Yang-Mills



Simplified truncation

♦ Purpose:

- **Practical** simplification of the pure gauge sector
- **Connection** with high-order **perturbative computations** towards the **CBZ** limit
- Implement **IR fixed-point** solutions from perturbative approaches

♦ Implementation:

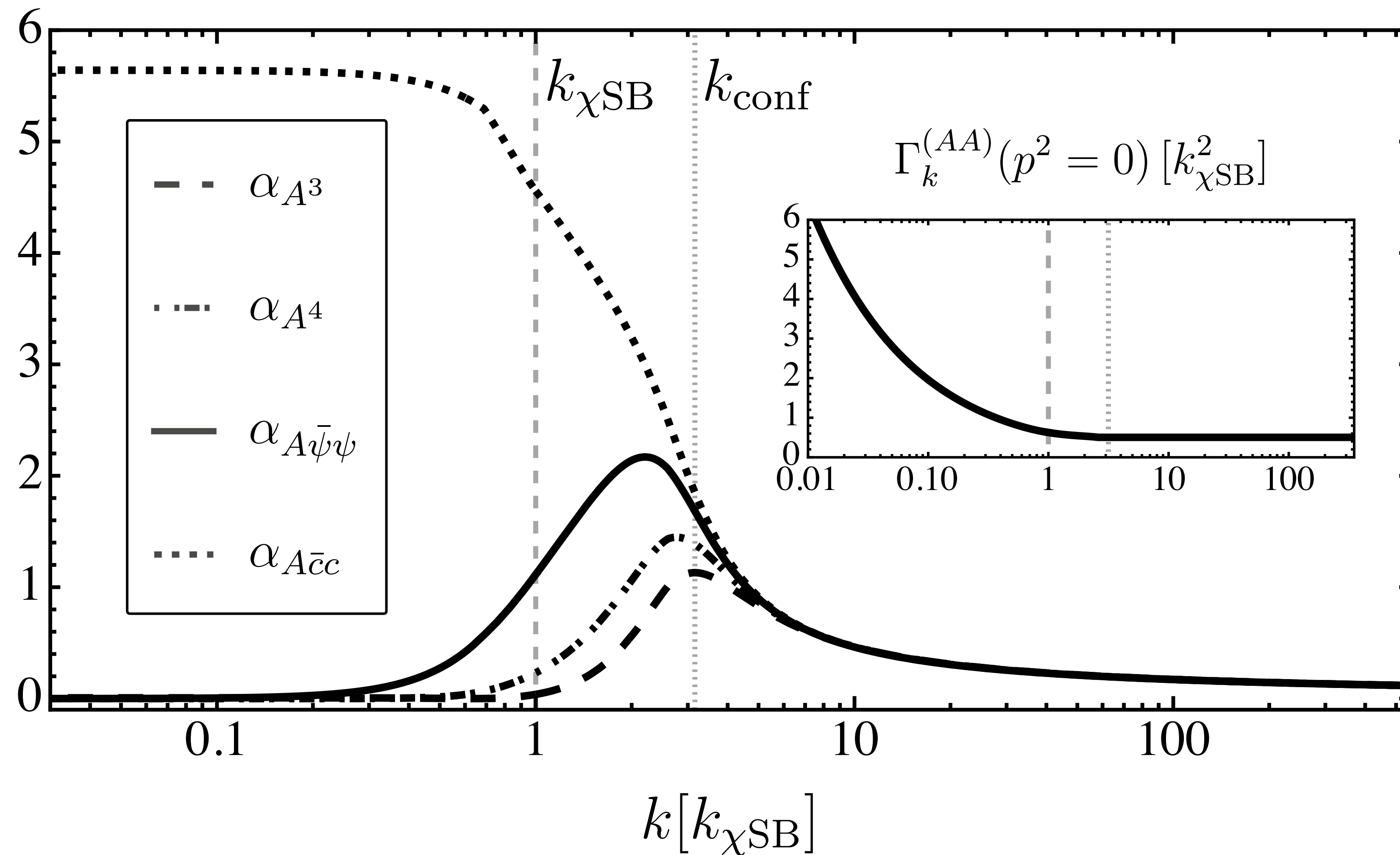
- Employ a **single adapted** gauge coupling **avatar**
- **Bosonised sector in full glory**

Smooth decoupling of dof

$$\partial_t g = \underbrace{\partial_t g_{A\bar{\psi}\psi}^{(1)}}_{\text{Non-perturbative}} + \underbrace{\frac{\partial_t g_{A\bar{\psi}\psi}^{(>1)}}{\partial_t g_{A\bar{\psi}\psi}^{(>1)} \big|_{\{\bar{m}_A, \bar{m}_\psi\}=0}}}_{\text{Non-perturbative}} \sum_{n=2}^{N_{\max}} \left\{ \sum_{i=1}^{i_{\max}} \underbrace{\frac{1}{(1+i_{\max})} \frac{1}{(1+\bar{m}_A^2)^{3n+i}}}_{\text{Smooth decoupling of dof}} \left(\underbrace{\beta_A^{(n)}}_{\text{Perturbative}} + \left[\frac{(2+\bar{m}_\psi^2)}{2(1+\bar{m}_\psi^2)^3} \right]^{N_f \text{ in loop}} \underbrace{\beta_\psi^{(n)}}_{\text{Perturbative}} \right) \right\}$$

* Fundamental parameters in good agreement with best truncations

Criticality of low Nf



Conformality and scale invariance

$$S = \int_x \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} + \bar{\psi} \not{D} \psi$$

Conformality and scale invariance

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$\text{SU}(N_f)_L \times \text{SU}(N_f)_R$

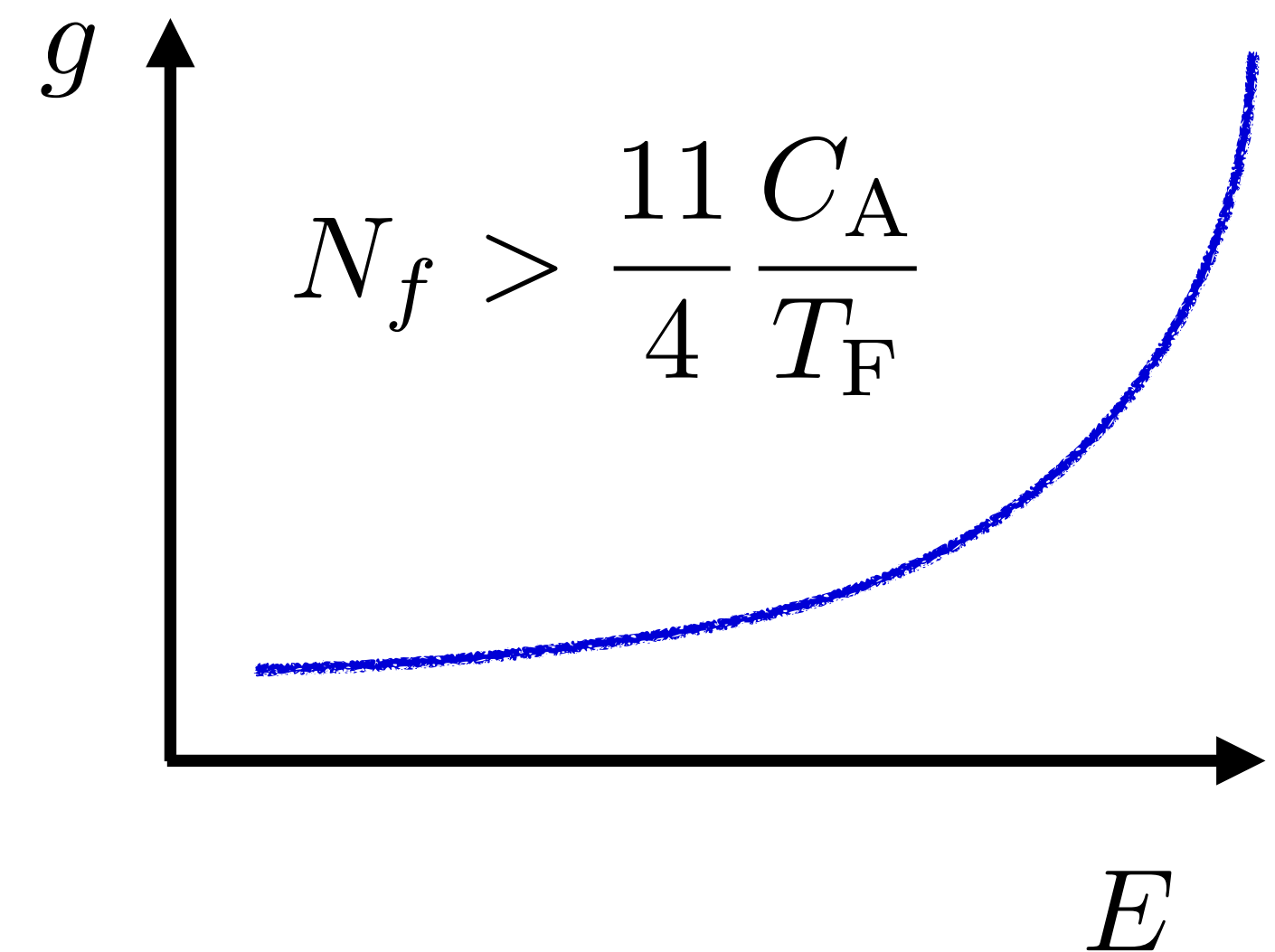
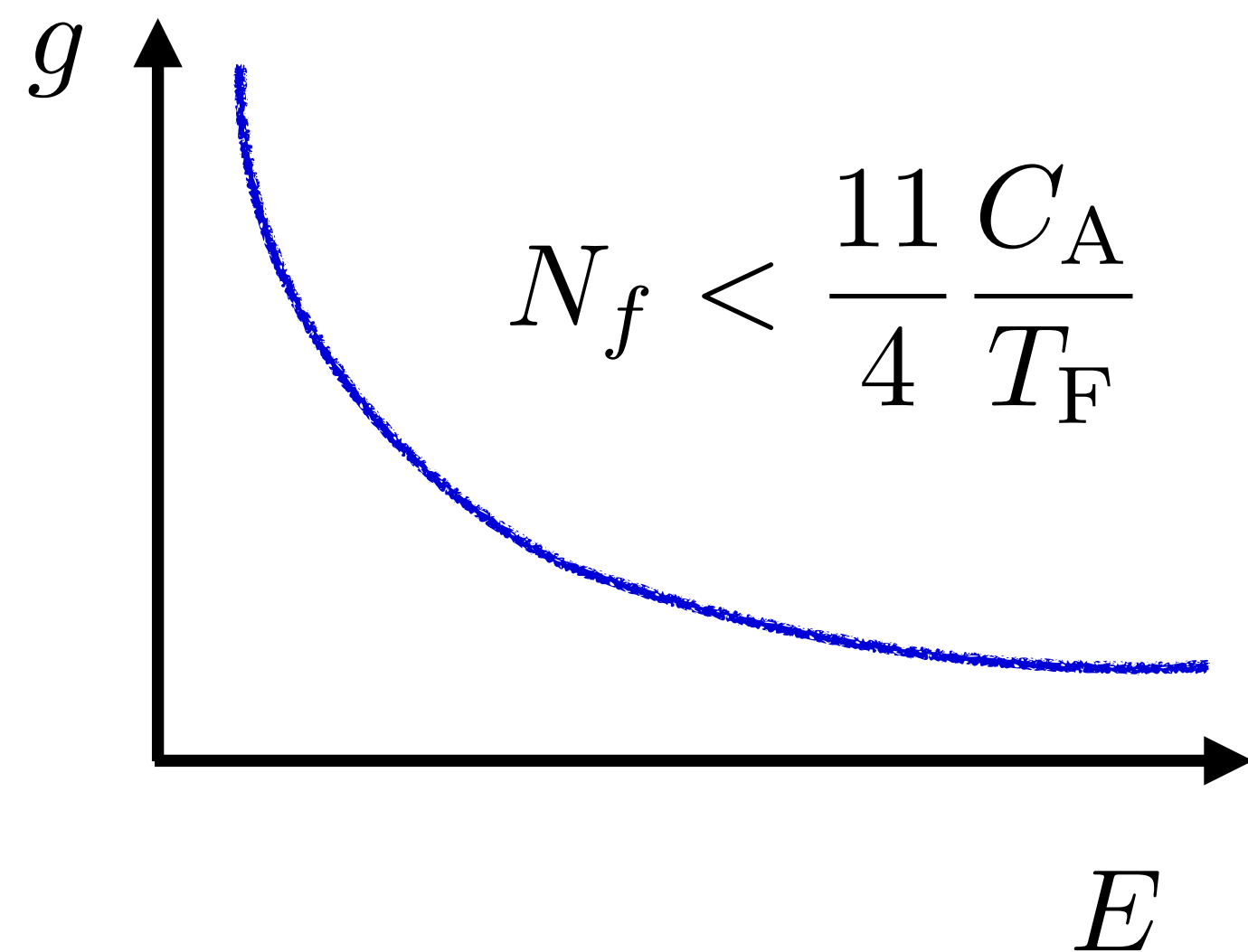
$$\rightarrow \beta_g = -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3} C_A - \frac{4}{3} T_F N_f \right)$$

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Conformality and scale invariance

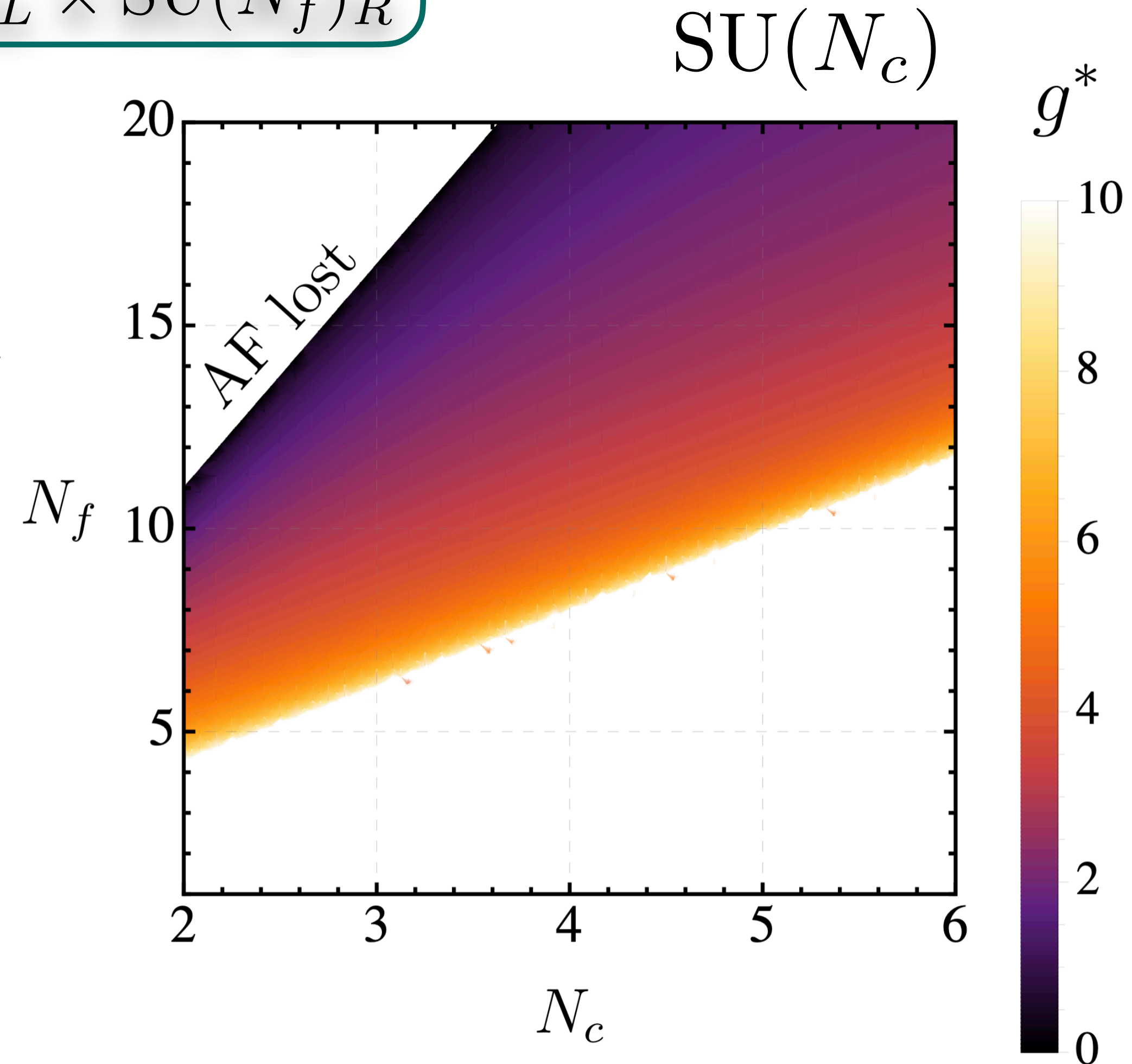
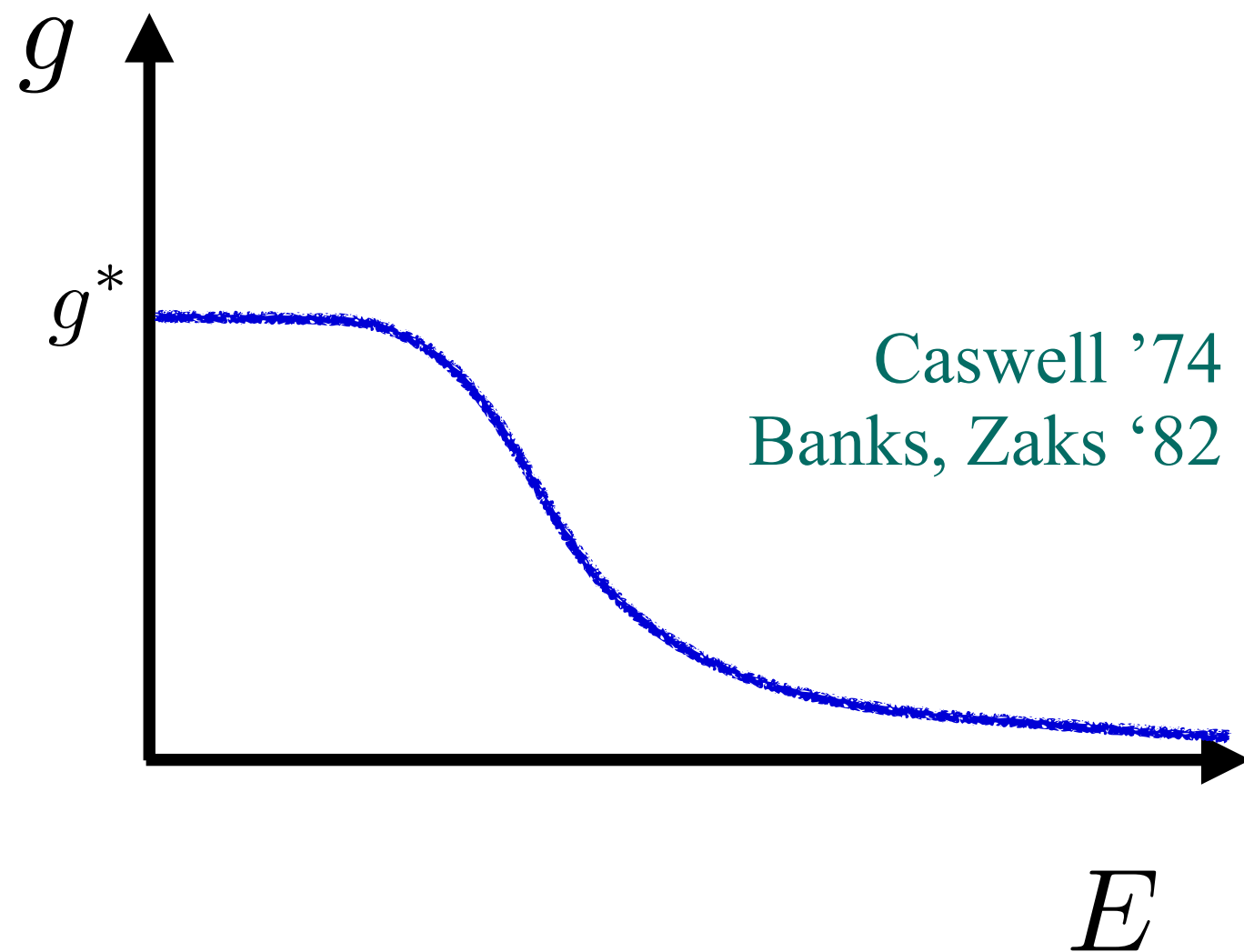
$$S = \int_x \left[\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} \right]_{\mathcal{G}_{\text{TC}}} + \bar{\psi} \not{D} \psi_{\text{SU}(N_f)_L \times \text{SU}(N_f)_R}$$

$$\begin{aligned} \rightarrow \beta_g = & -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3} C_A - \frac{4}{3} T_F N_f \right) \\ & - \frac{g^5}{(4\pi)^4} \left(\frac{34}{3} C_A - 4 C_F T_F N_f - \frac{20}{3} C_A T_F N_f \right) + \dots \end{aligned}$$

Conformality and scale invariance

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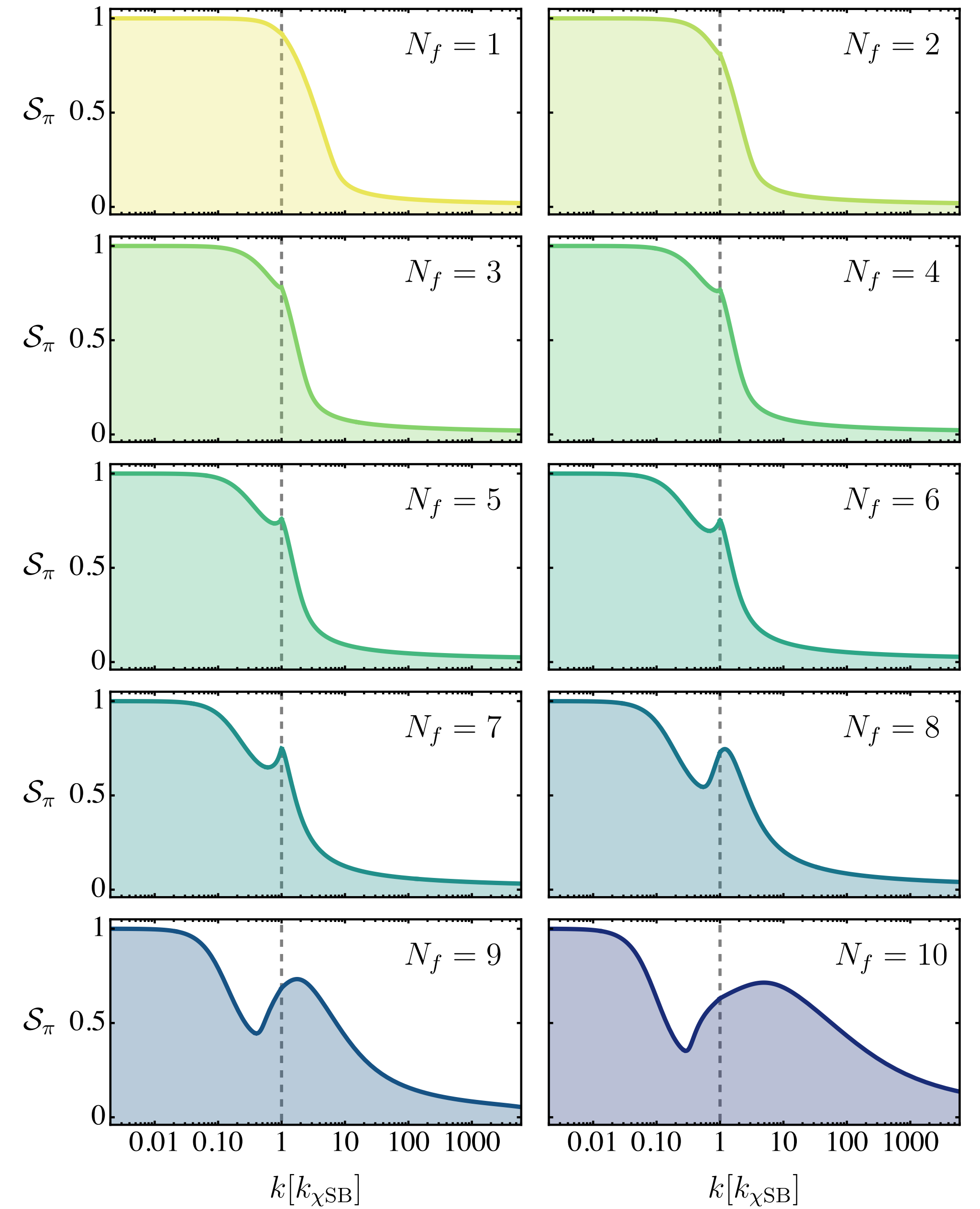


Fermion masses

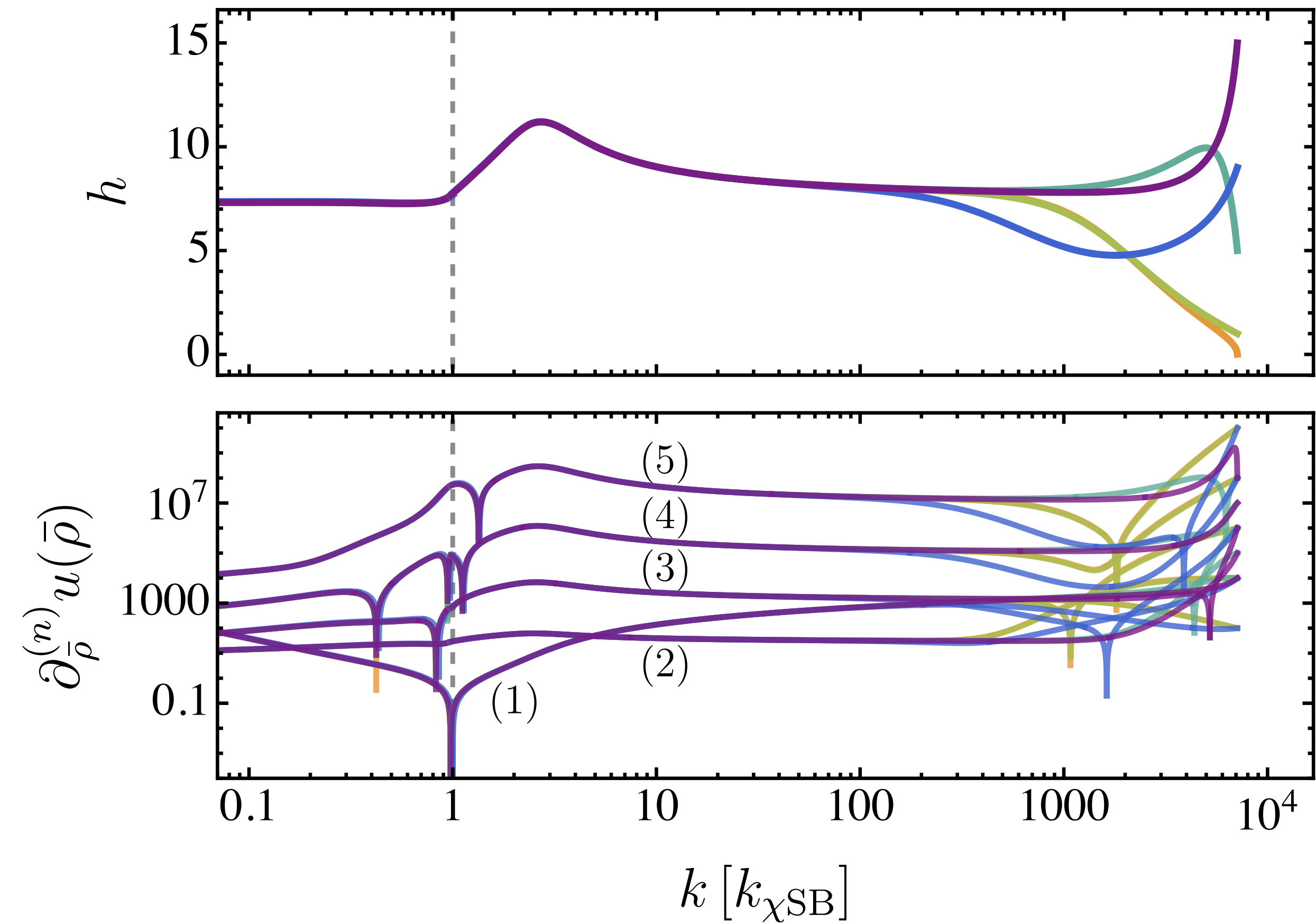
$m_\psi [k_{\chi\text{SB}}]$	$N_f = 2$	$N_f = 3$
fQCD [20]	0.80	0.74*
Enhancement	0.74	0.53
No enhancement	0.63	0.50
Simplified	0.64	0.52

TABLE I. m_ψ obtained in the different approximations put forward in [Section III](#) (No/Enhancement) and in [Section V A](#) (Simplified) as well as with existing results from functional computations in the chiral limit (fQCD) [20]. The * highlights that fQCD $N_f = 3$ results are obtained in the chiral limit for two flavours in the chiral limit and one heavy, justifying the large m_ψ value.

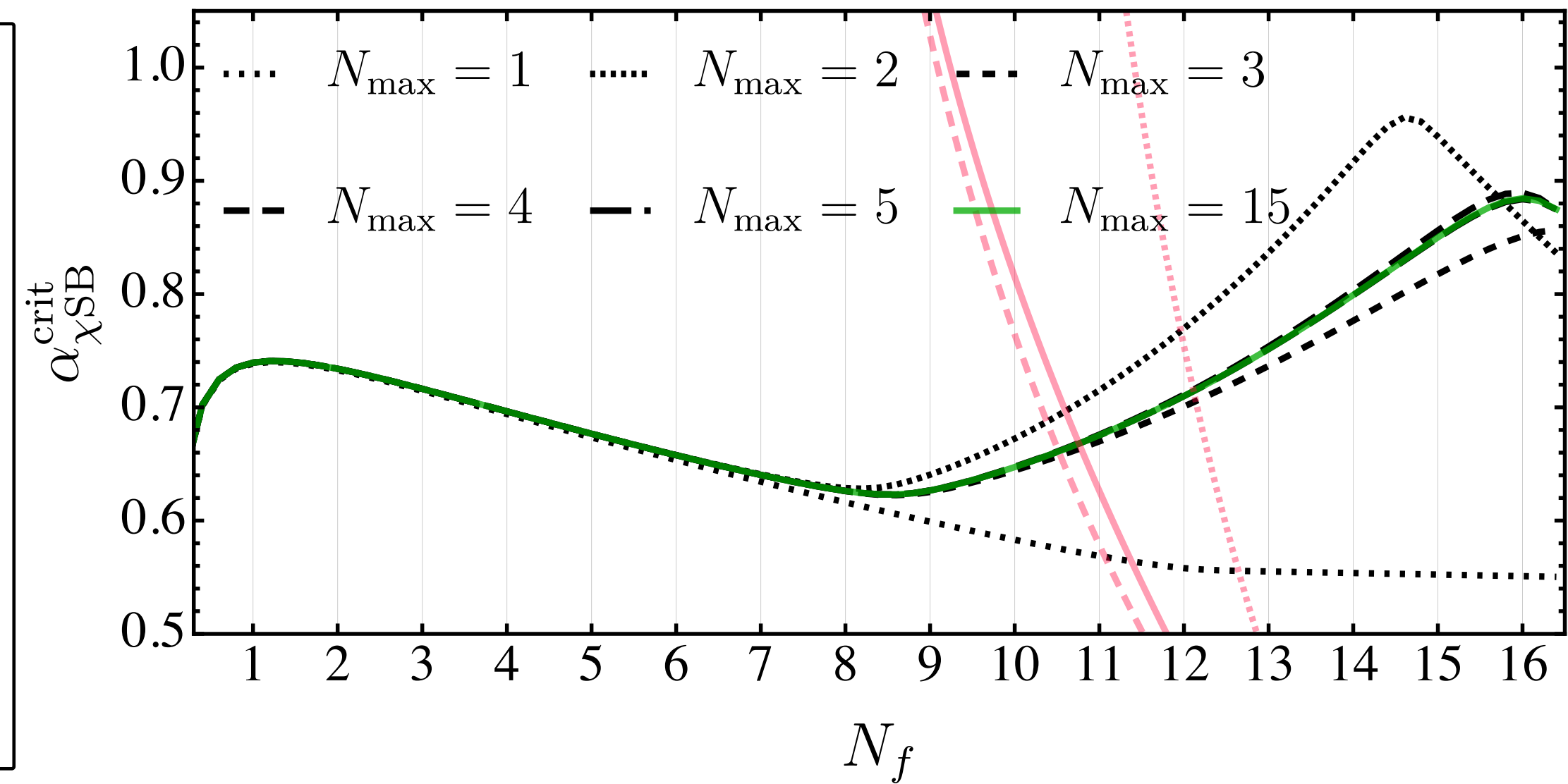
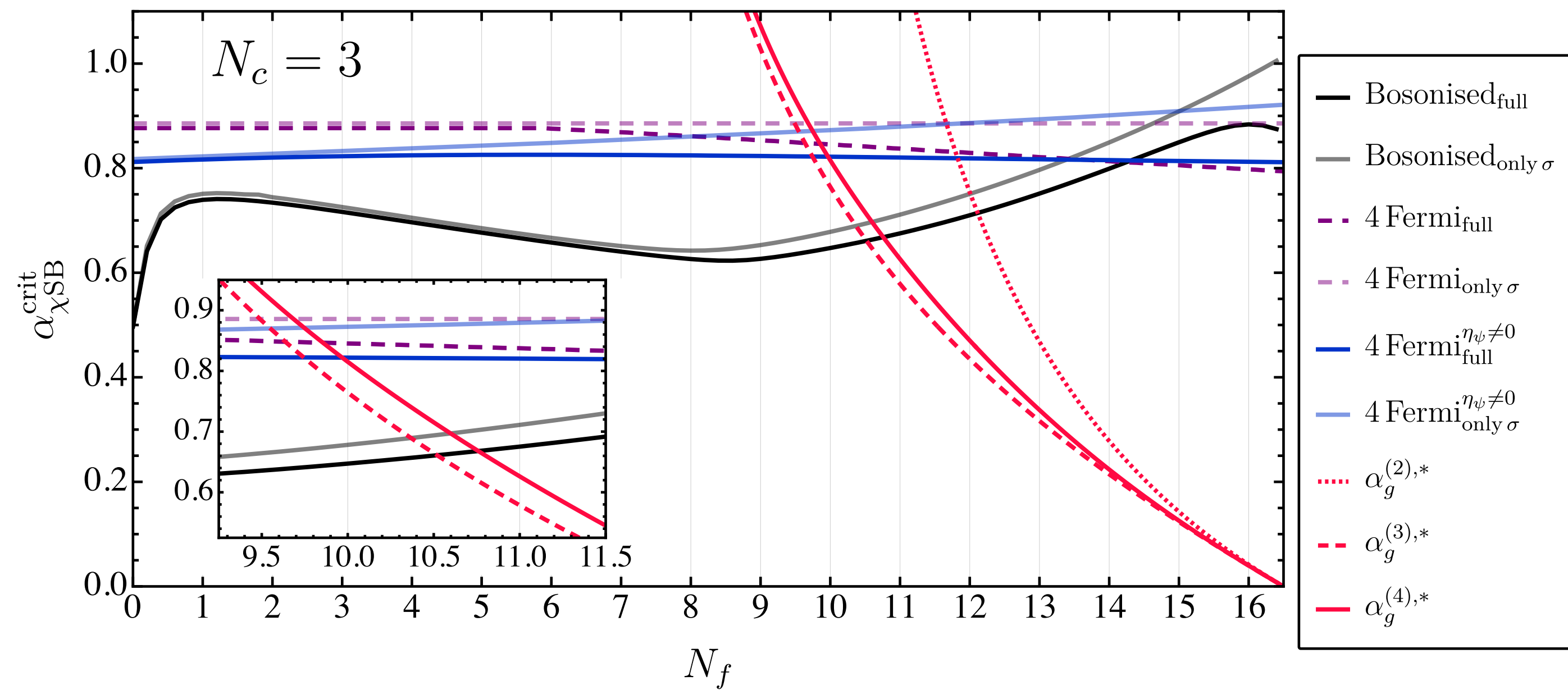
Validity of LEFTS



Dynamical bosonisation



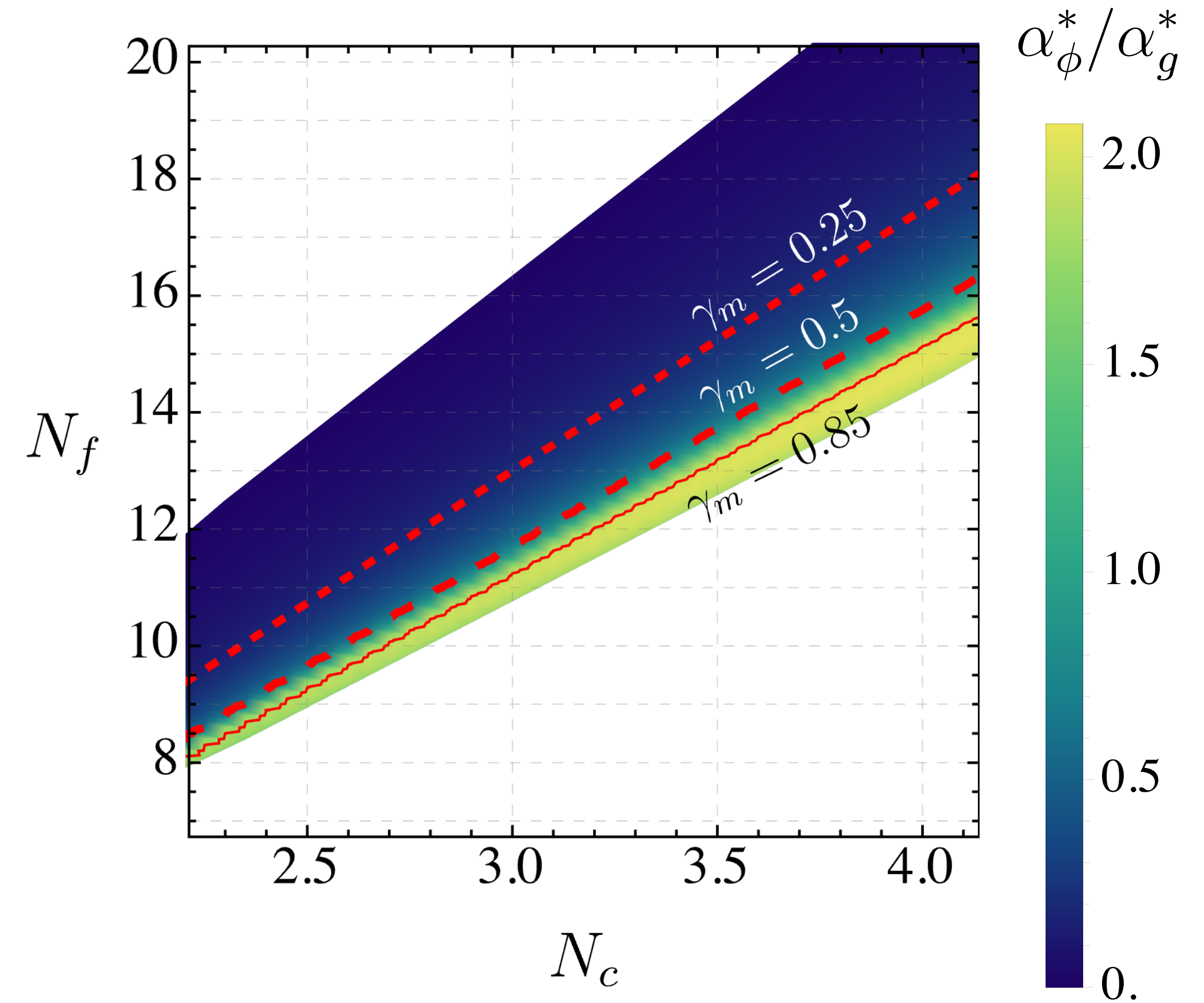
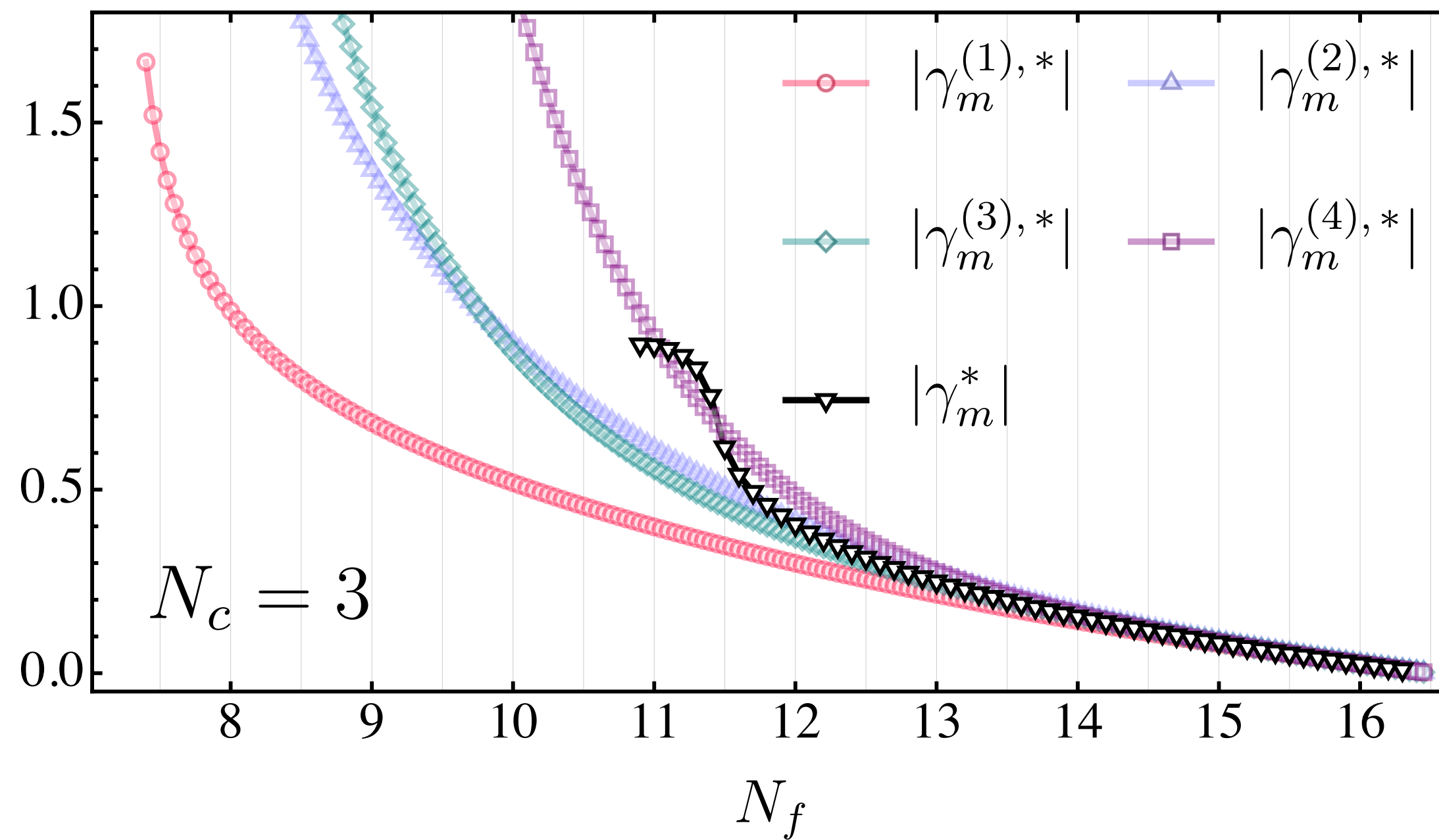
Boundary of the conformal window: systematics



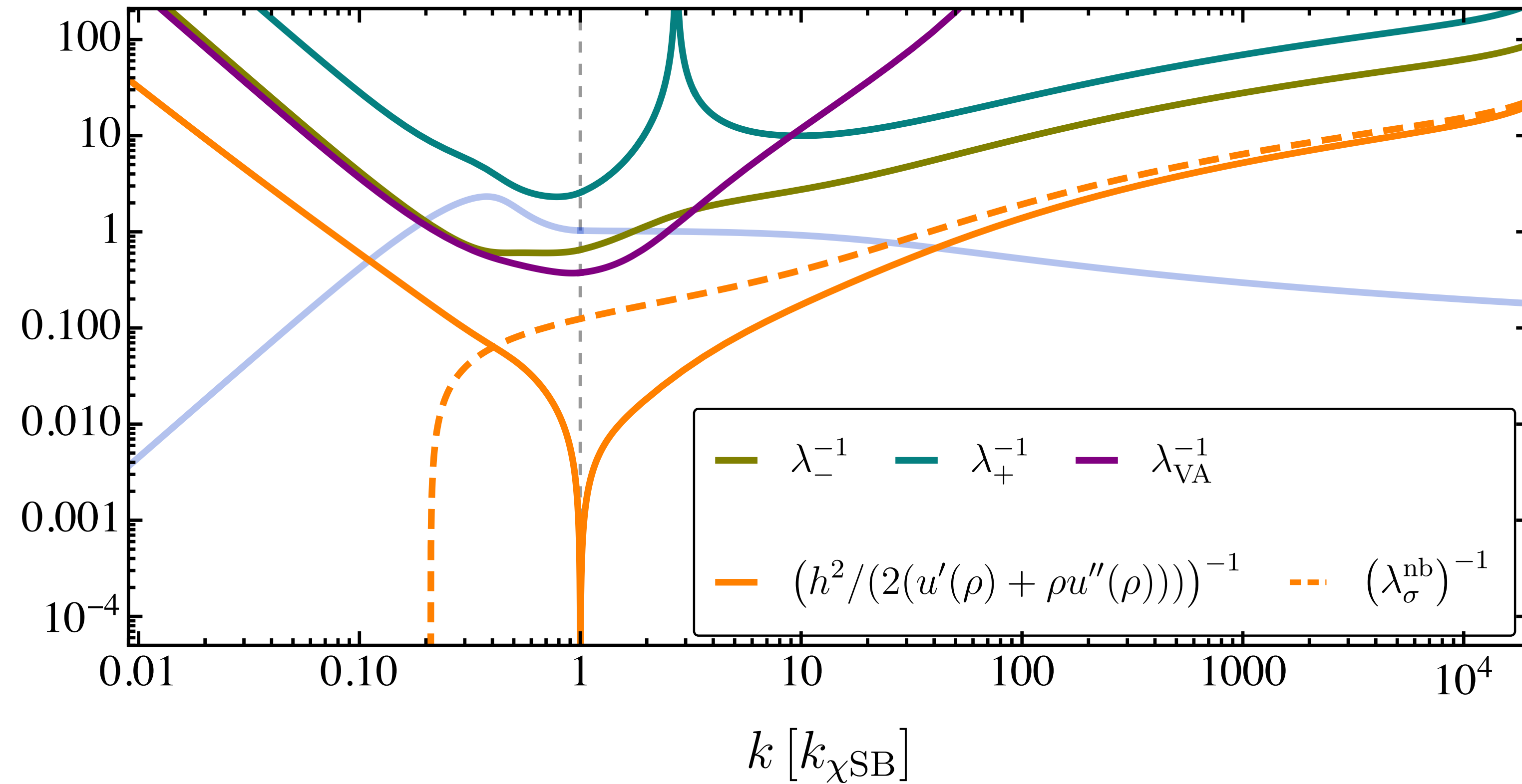
Boundary of the conformal window: γ_m

$$\gamma_m = \frac{\partial_t \bar{m}_\psi}{\bar{m}_\psi} = \left[\frac{\partial_t \left(T_f^0 \Gamma_k^{(\bar{\psi}\psi)}(p^2 = 0) \right)}{Z_\psi m_\psi} + \eta_\psi \right]$$

$$= \gamma_m^{(1)} + \gamma_m^{(>1)} = \frac{3 \alpha_g C_F}{2\pi} + \dots$$



Different tensor structures



Dynamical hadronisation and flow over EFTs

✓ Well defined implementation with the fRG

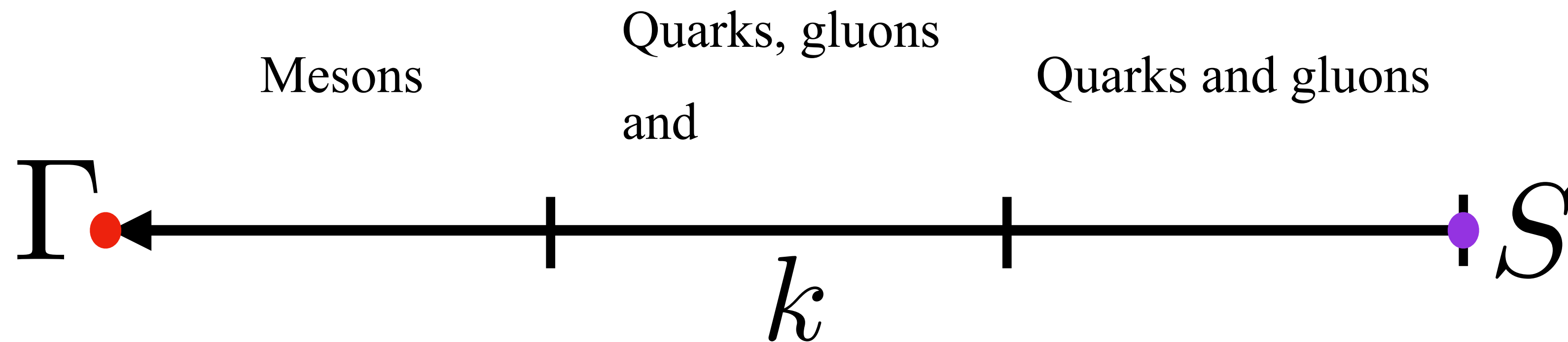
$$\begin{aligned} & \left(\partial_t + \int \dot{\phi}_c \frac{\delta}{\delta \phi_c} \right) \Gamma_k[\phi] \\ &= \frac{1}{2} \text{Tr} \left[\left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)_{ij} \left(\partial_t \delta^{jn} + 2 \frac{\delta \dot{\phi}_n}{\delta \phi_j} \right) R_k^{mi} \right] \end{aligned}$$

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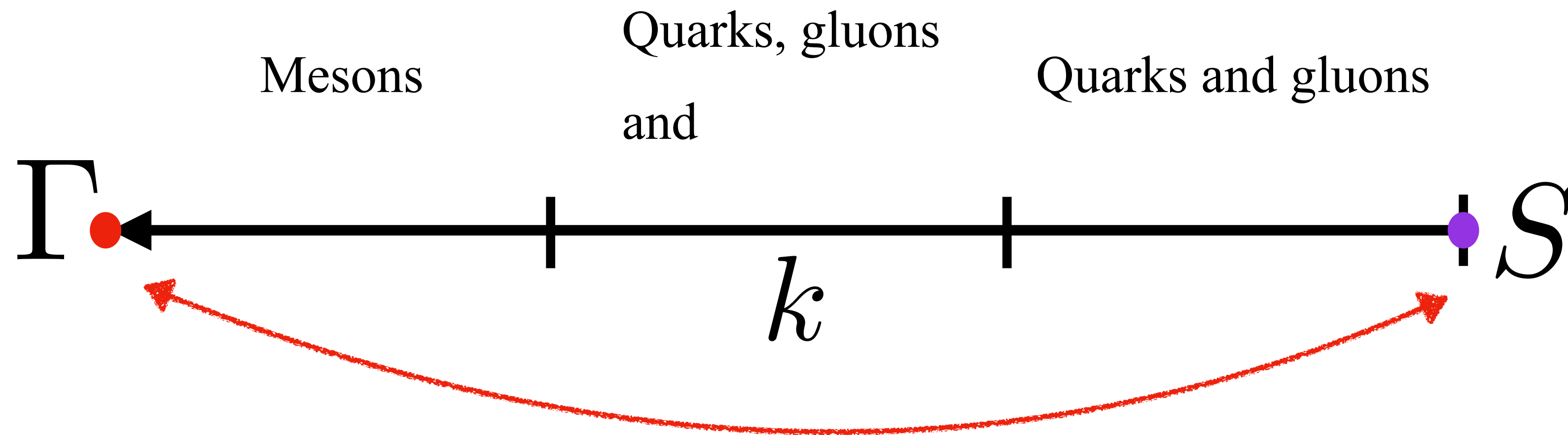


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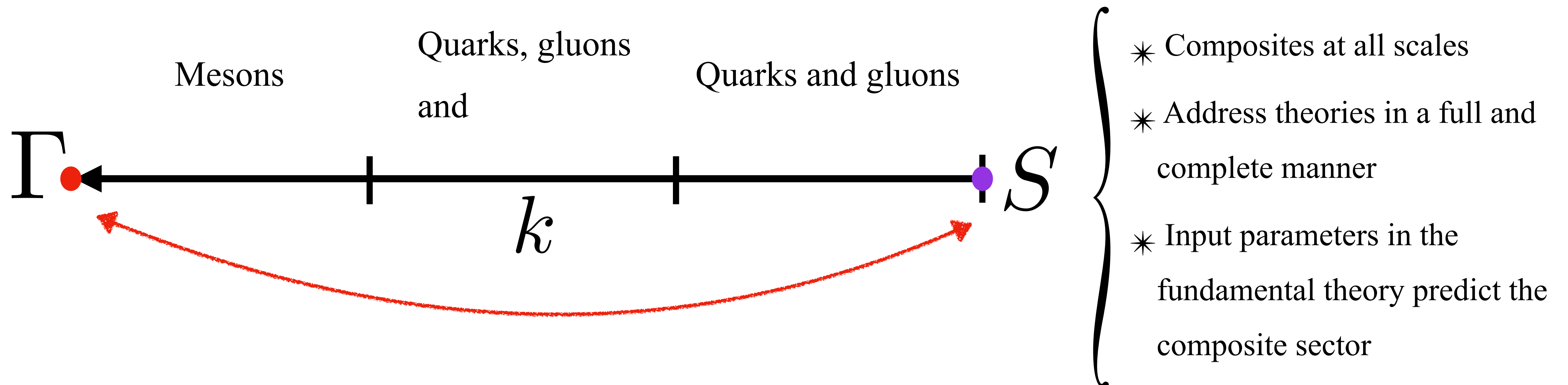


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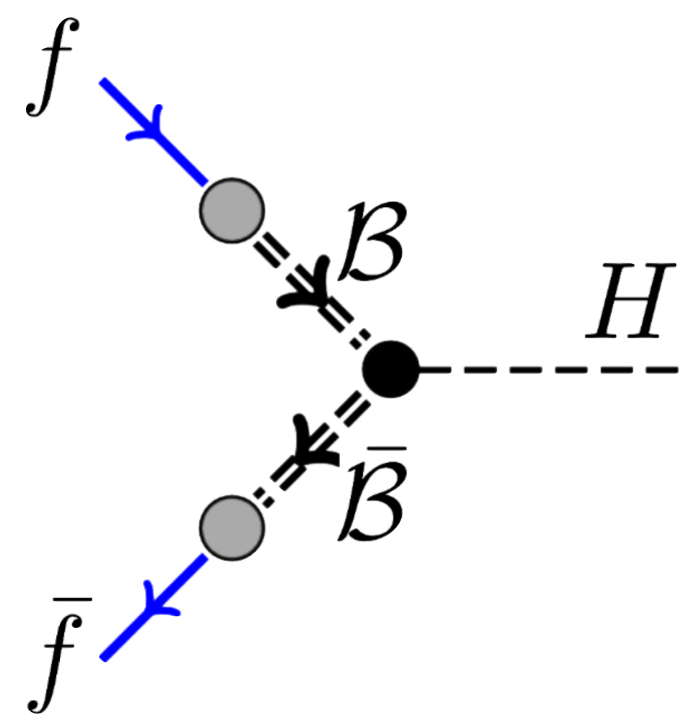


Flavour hierarchies from Fundamental Partial Compositeness

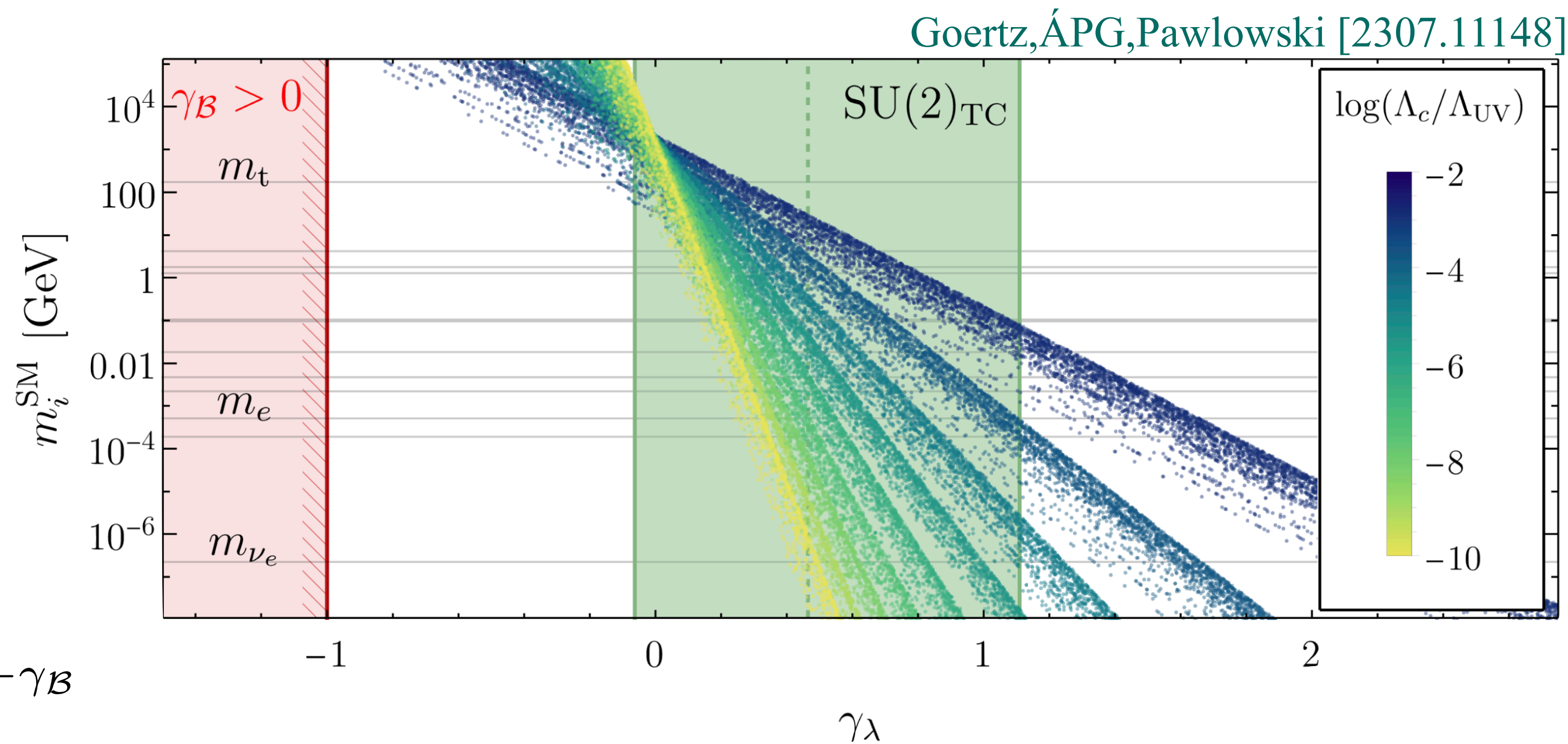
Sannino, Strumia, Tesi, Vigiani [1607.01659]
Cacciapaglia, Gertov, Sannino, Thomsen [1704.07845]

$$\bar{\Gamma}_{\text{mix}} = \int_x \left\{ \lambda_t^L \bar{q}_L \mathcal{B}_R^q + \lambda_t^R \bar{t}_R \mathcal{B}_L^t + \text{h.c.} \right\}$$

$$\mathcal{B} \sim \mathcal{F} \mathcal{S} \quad \underline{\text{MFPC}} \begin{cases} \text{Sp}(24)_{\mathcal{S}} \\ \text{SU}(4)_{\mathcal{F}} \end{cases}$$



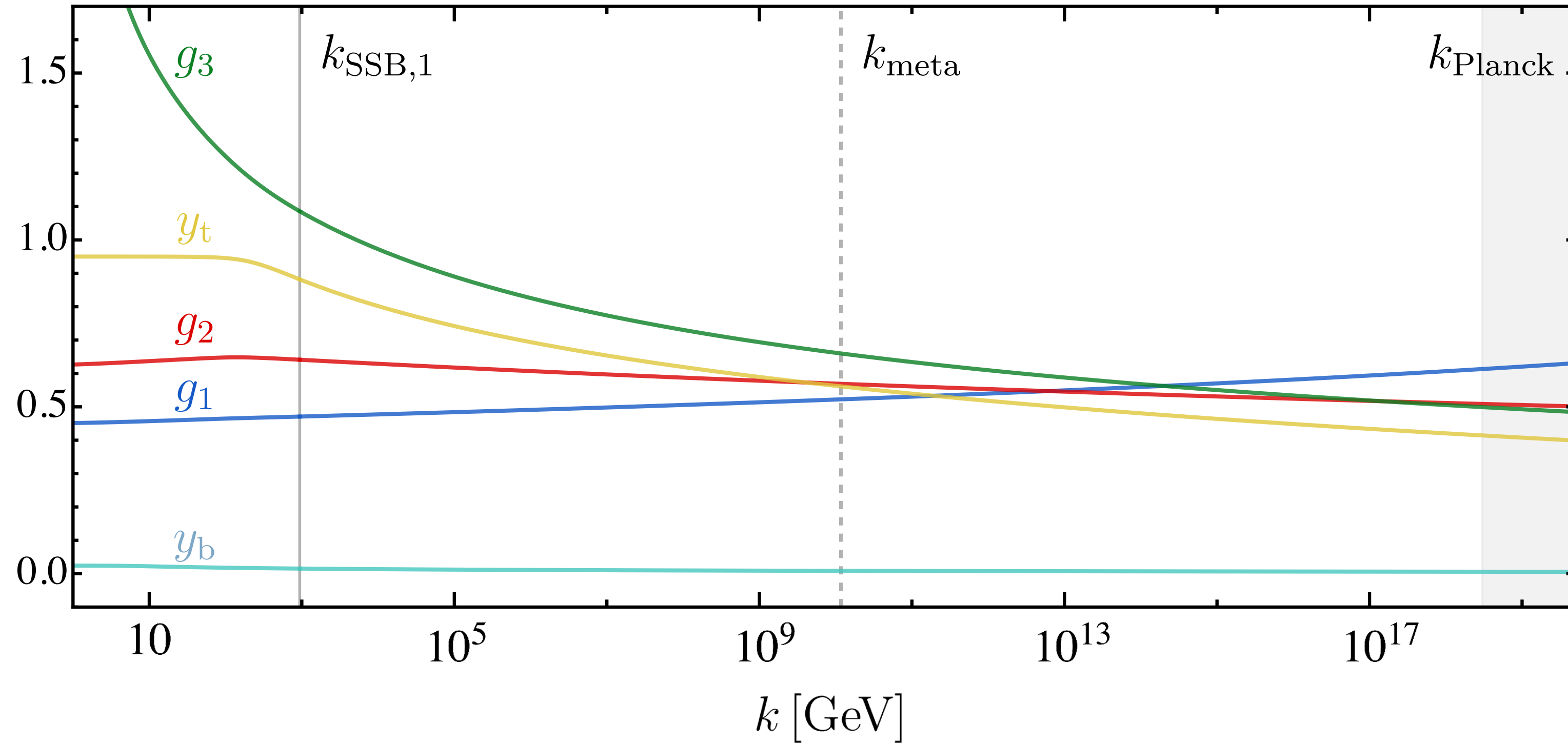
$$m_f \sim \frac{\mathcal{Y}_f v}{\sqrt{2}} \left(\frac{\Lambda_c}{\Lambda_{\text{UV}}} \right)^{-2-\gamma_{\mathcal{B}}}$$



The SM from the fRG

ÁPG,Pawlowski,Reichert [2207.09817]

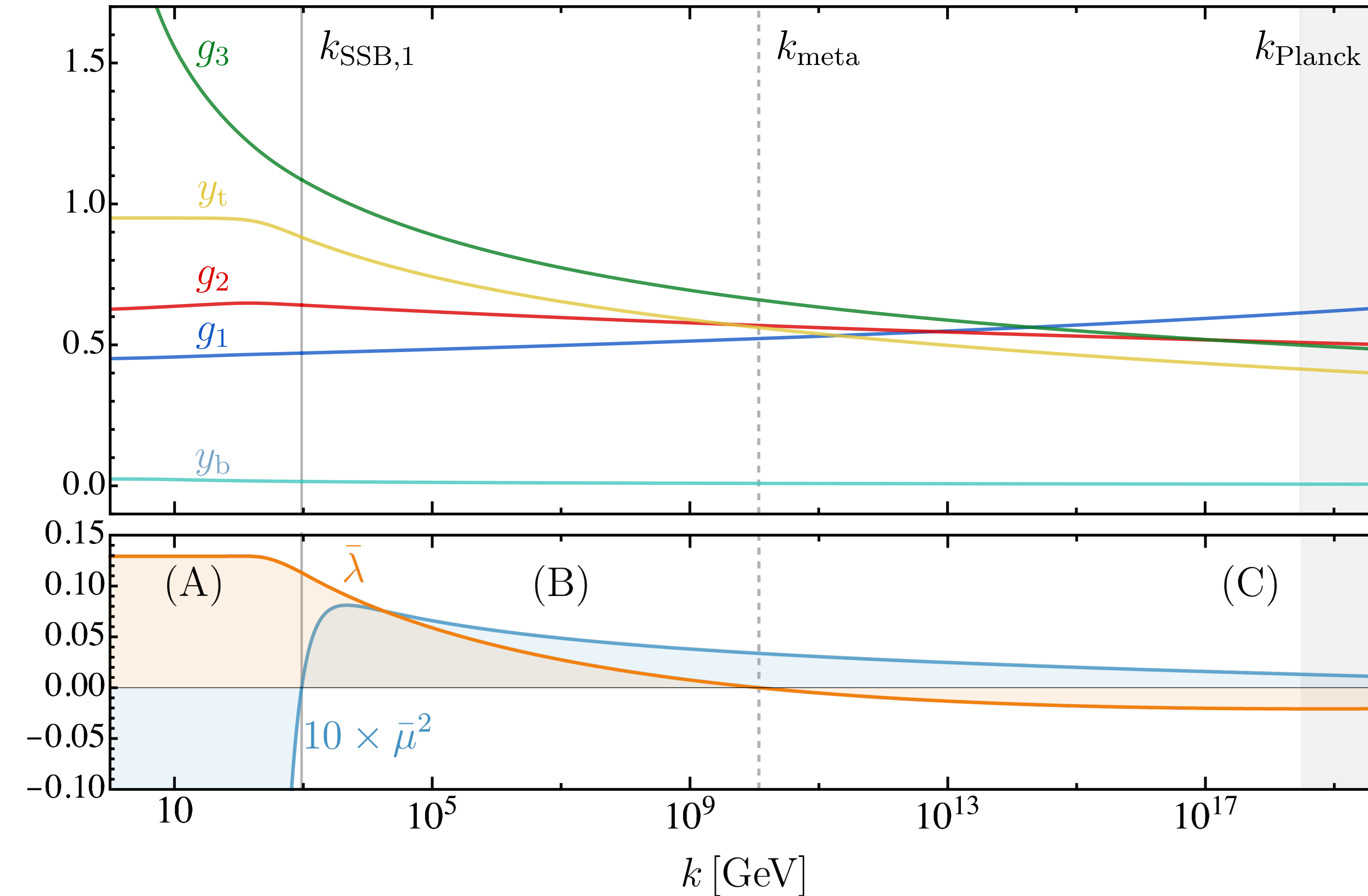
Goertz,ÁPG [2308.13594]



The SM from the fRG

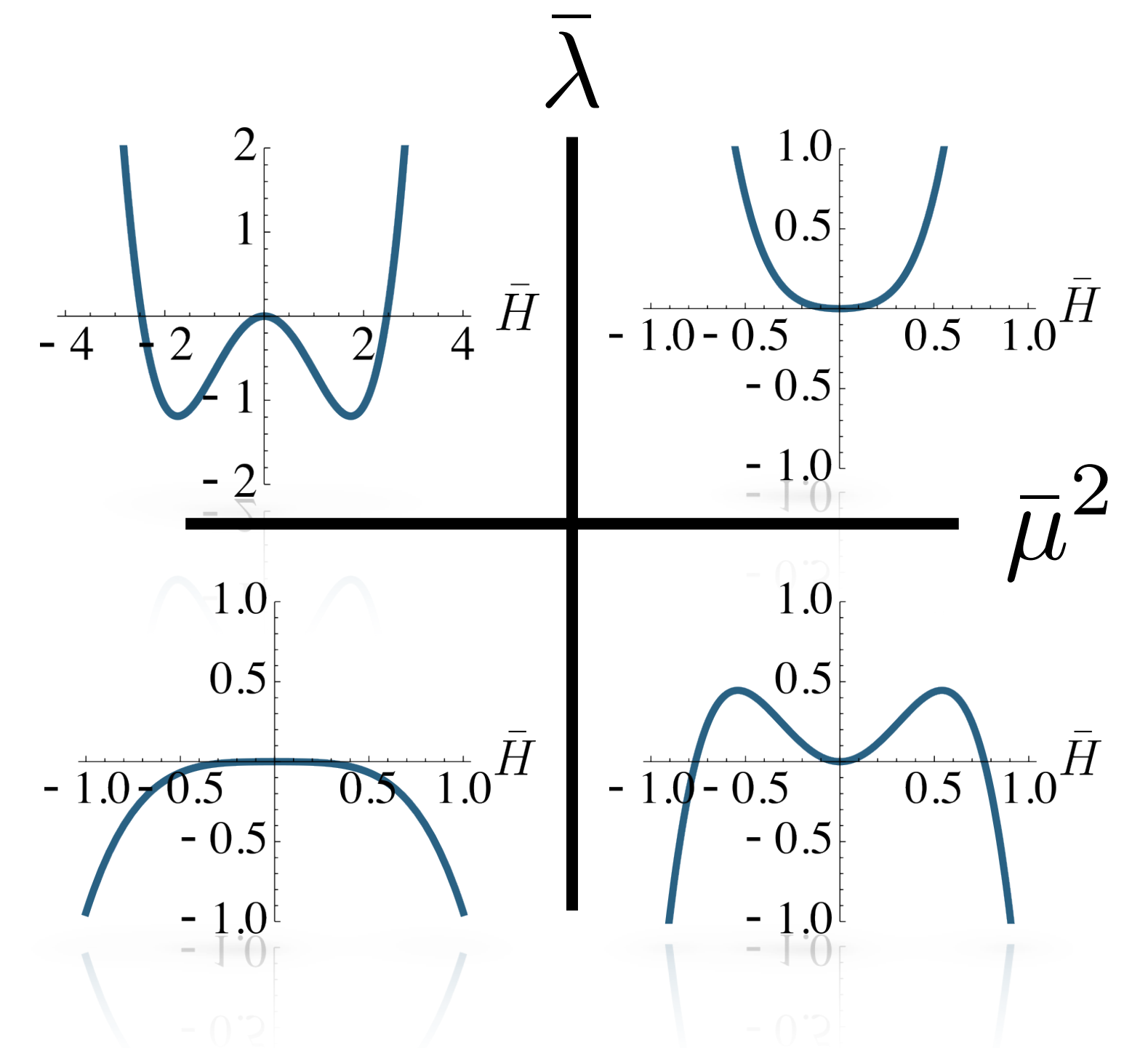
ÁPG,Pawlowski,Reichert [2207.09817]

Goertz,ÁPG [2308.13594]



$$u(\bar{\rho}) = V_{\text{eff}}(\rho) / k^4 = \bar{\mu}^2 \bar{\rho} + \bar{\lambda} \bar{\rho}^2$$

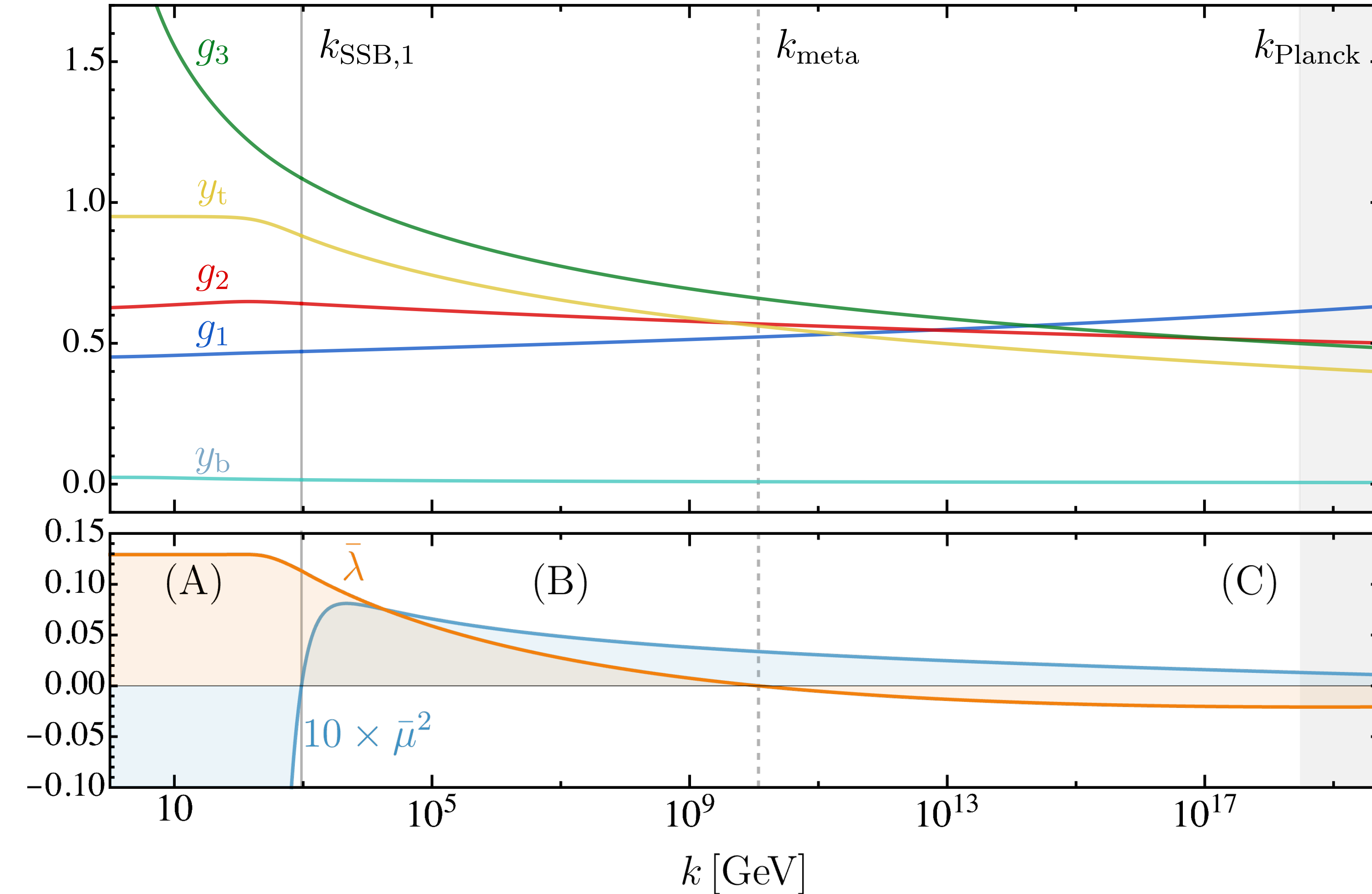
$$\bar{\rho} = Z_{\Phi} \frac{\text{tr } \Phi^{\dagger} \Phi}{k^2} \quad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathcal{G}_1 + i\mathcal{G}_2 \\ H + i\mathcal{G}_3 \end{pmatrix}$$



The SM from the fRG

ÁPG,Pawlowski,Reichert [2207.09817]

Goertz,ÁPG [2308.13594]



ÁPG,Pawlowski,Reichert [2207.09817]

