# Thermodynamics of color-superconducting quark matter

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QuantFunc2024, Valencia, September 4, 2024

[based on AG, T. Gorda, and J. Braun '24]

# QCD Phase Diagram

temperature $T$

lattice

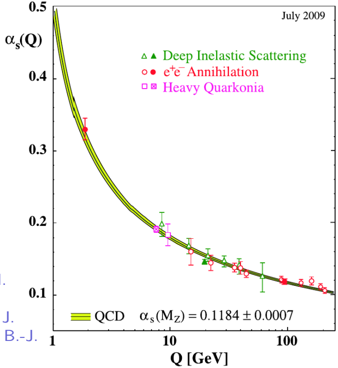
[H. T. Dings et al. (HotQCD) '19; S. Borsanyi et al. '20; F. Cuteri et al. '22; O. Philipsen '23; ...]

functional methods

[J. Bernhardt and C. S. Fischer '23; K.Fukushima, J. M. Pawlowski et al. '21; M. Drews and W. Weise '15; R.-A.Tripold, C. Jung, L. v. Smekal, J. Wambach, '21; J. Braun, F. Rennecke et al. '23; K. Otto, M. Oertel, and B.-J. Schäfer '19 and '20; W. Fu, F. Rennecke al. '23; ...]

$\chi$EFT

[K. Hebeler '20; E. Epelbaum, H.W. Hammer, and Ulf-G. Meißner '08; . . . ]



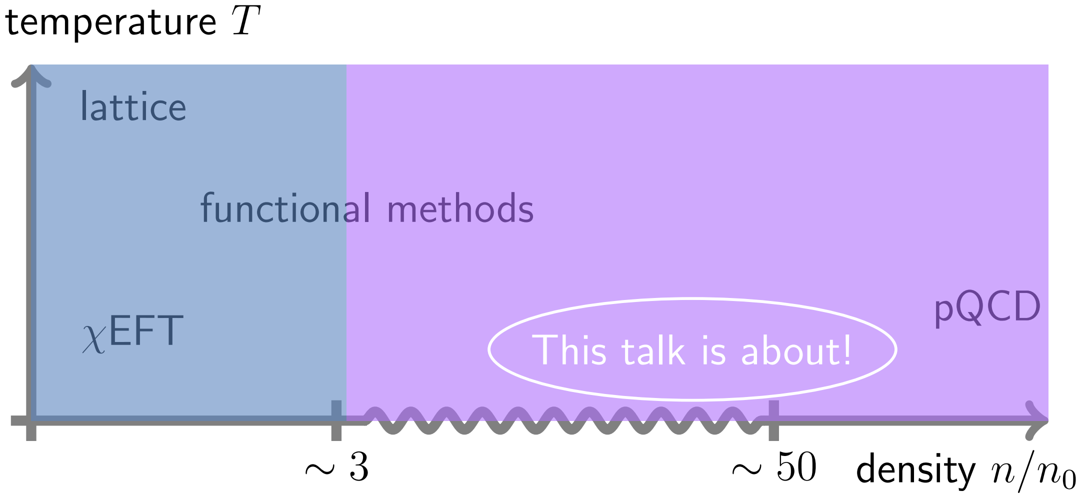

[Bethke '09]

pQCD

[A. Kurkela, P. Romatschke, and A. Vuorinen '10, A. Kurkela et al. '14; T. Gorda et al. '18; . . . ]

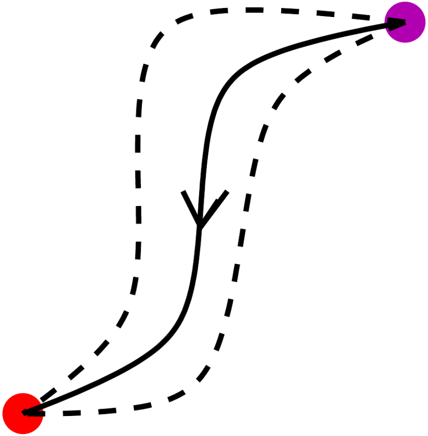
$\sim 3$ $\sim 50$ density $n/n_0$

# QCD Phase Diagram



# Degrees of Freedom

[Wetterich '93; (figure adapted from) Gies '06]



$\Gamma_{k=\Lambda} = S$

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, ...

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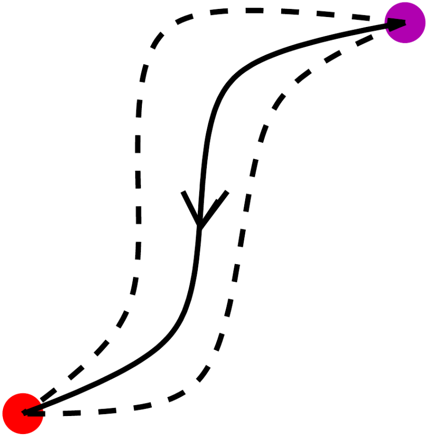

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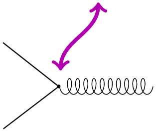
$(N_{\mathrm{f}} = 2)$



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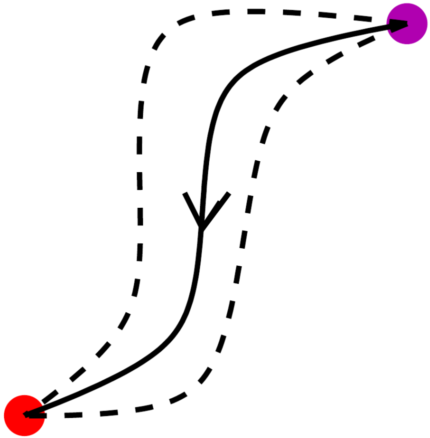

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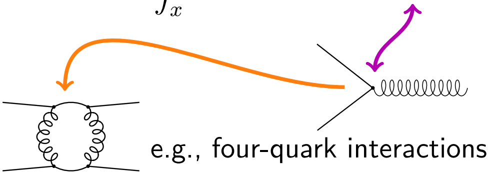


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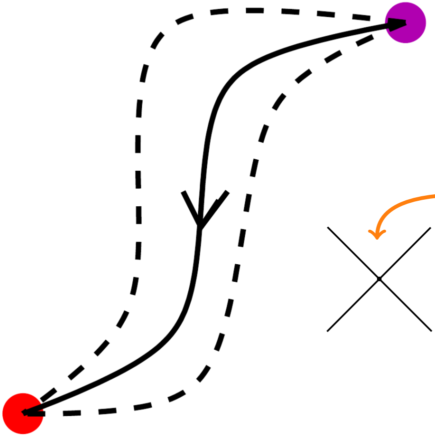

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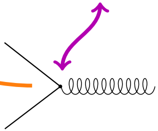

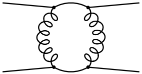

e.g., four-quark interactions

resonant four-quark interaction (at high densities)

[M. Leonhardt et al. '19; J. Braun and B. Schallmo '21, B. Schallmo, AG, J. Braun (in prep.)]

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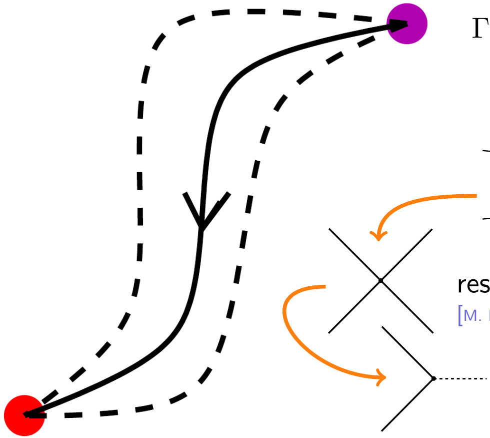

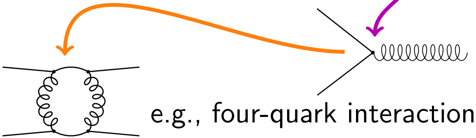

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auxiliary diquark field $\Delta_a$ (Hubbard-Stratonovich)

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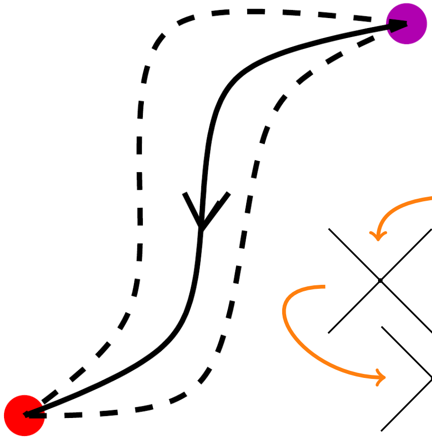

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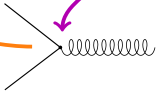

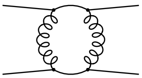

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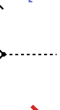

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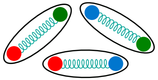


color-superconducting gap

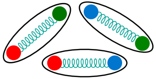
diquark condensate

$\Gamma_{k=0} = \Gamma \rightarrow$ pressure, expectation values, . . .

[R. Rapp et al. ’98; M. G. Alford et al. ’98; T. Schäfer and F. Wilczek ’99, J. Berges and K. Rajagopal ’99; M. Buballa ’05]

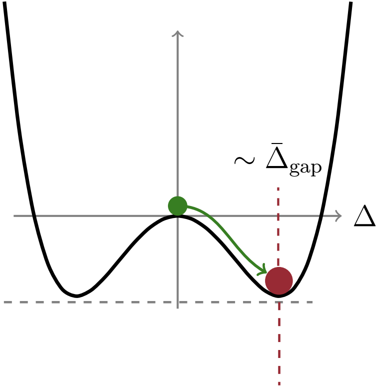
# Color Superconductivity

quark cooper pairs
$\Delta_a \sim \langle \psi^T \mathcal{C} \gamma_5 \tau_2 \epsilon_a \psi \rangle$



ground state lowered

$\mathrm{U}(1)_V$ symmetry breaking




diquark condensate

2-flavor diquark condensate is chirally symmetric

# Expansion of the Pressure

[J. Braun, AG, and B. Schallmo '22]

- effective action $\Gamma$ in the presence of a color superconducting condensate:

$$p = -\frac{\Gamma}{V_4}\bigg|_{\mu,\,\Delta=\bar{\Delta}_{\mathrm{gap}}} + \mathrm{const.}$$

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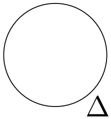
**expansion about vanishing gap**

$$p = p_{\rm free}\left(\gamma_0(g) + \gamma_1(g)\left(\frac{|\bar{\Delta}_{\rm gap}|}{\mu}\right)^2 + \dots\right)$$

- expanding the $\gamma_i$ perturbatively
- gap implicitly depends on $g$

# One Loop

- zeroth order of the $\gamma_i$'s
- the gap enters the fermionic propagator




- minimize the effective potential
- expansion of the pressure yields

$$p^{1-\text{loop}} = p_{\text{free}} \left( 1 + 2 \left( \frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 + \ldots \right)$$

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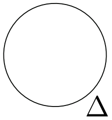



- minimize the effective potential
- expansion of the pressure yields

like in the diquark model

ground state is phase with the highest pressure

$$p^{1-\mathrm{loop}} = p_{\mathrm{free}} \left( 1 + 2 \left( \frac{|\bar{\Delta}_{\mathrm{gap}}|}{\mu} \right)^2 + \dots \right)$$

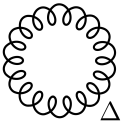
# One Loop

- What about the gluons?
- diquarks carry color charge
- Anderson-Higgs mechanism
  $\Longrightarrow$ some gluons become massive

Covariant derivative gives

$$\sim A^a_\mu T^a_{cd} \bar{\Delta}_d A^b_\mu T^b_{ce} \bar{\Delta}^*_e$$

mass term for the gluon



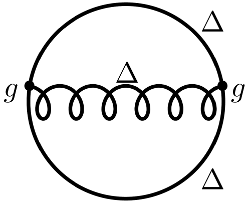

$$\Longrightarrow \ p \sim |\bar{\Delta}_{\text{gap}}|^4$$

- higher order in $|\bar{\Delta}_{\text{gap}}|$

# Two Loops

- next order in loop expansion $\Longrightarrow$ sunset diagram

- gap enters all propagators



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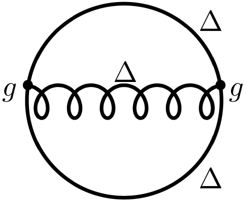


$$\Longrightarrow \; p^{2-\text{loop}} = p_{\text{free}} \left( -\frac{g^2}{2\pi^2} + 1.09(4)\, g^2 \left( \frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 + \ldots \right)$$

# Expansion of the Pressure

[AG, T. Gorda, and J. Braun '24]

**double expansion**

$$p \approx p_{\text{free}} \left( 1 - \frac{g^2}{2\pi^2} + \left(2 + 1.09(4)\, g^2\right) \left( \frac{|\bar{\Delta}_{\text{gap}}|}{\mu} \right)^2 \right)$$

free pressure

pert. contribution

gapped quark loop

gapped 2-loop contribution

$\bar{\Delta}_{\text{gap}}(g)$

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$N_\mathrm{f} = 2$

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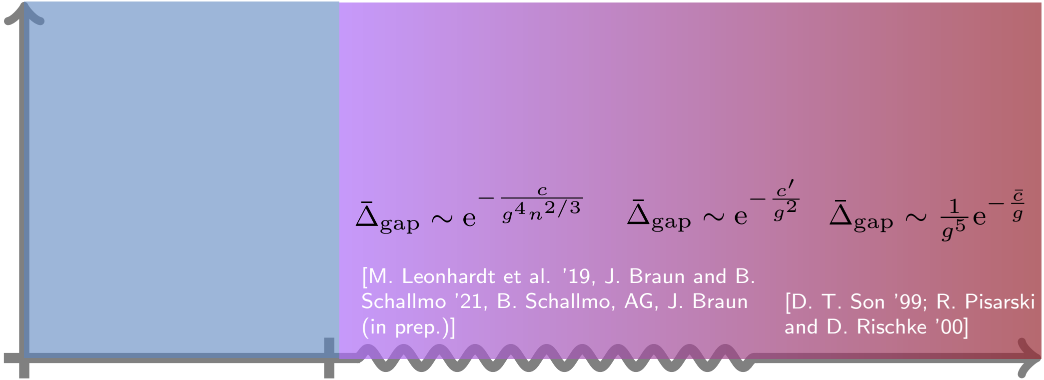
# Diquark Gap

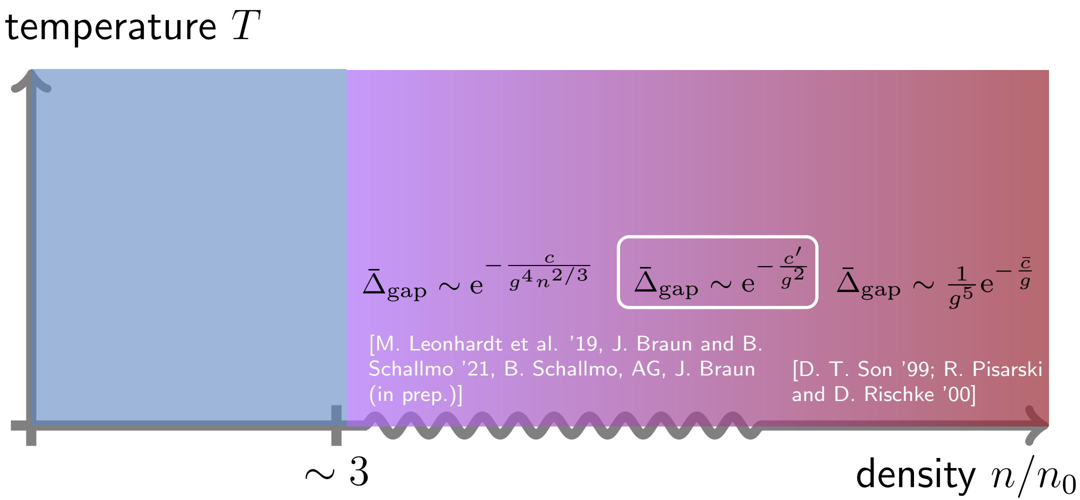
temperature $T$




$\sim 3$

density $n/n_0$

# Diquark Gap


temperature $T$
$\bar{\Delta}_{\mathrm{gap}} \sim \mathrm{e}^{-\frac{c}{g^4 n^{2/3}}}$
$\bar{\Delta}_{\mathrm{gap}} \sim \mathrm{e}^{-\frac{c'}{g^2}}$
$\bar{\Delta}_{\mathrm{gap}} \sim \frac{1}{g^5}\mathrm{e}^{-\frac{\bar{c}}{g}}$
[M. Leonhardt et al. '19, J. Braun and B. Schallmo '21, B. Schallmo, AG, J. Braun (in prep.)]
[D. T. Son '99; R. Pisarski and D. Rischke '00]
$\sim 3$
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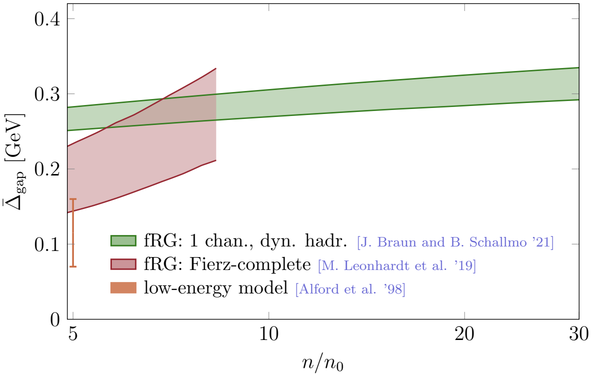
# Diquark Gap

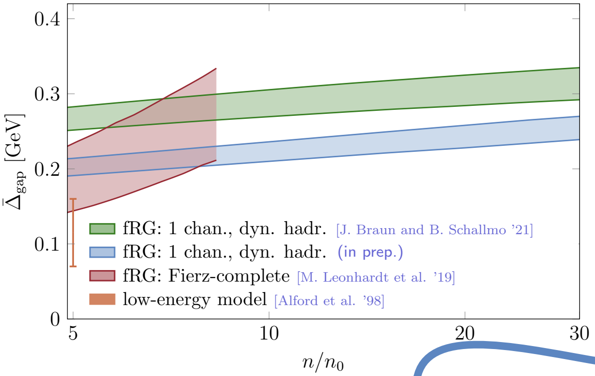
[M. Leonhardt et al. ’19; J. Braun, AG, and B. Schallmo (in prep.)]



# Diquark Gap

[B. Schallmo, AG, and J. Braun (in prep.)]

no Taylor expansion about $g = 0$




**ext. truncation**

higher order fluctuation effects

very high densities (weak-coupling)

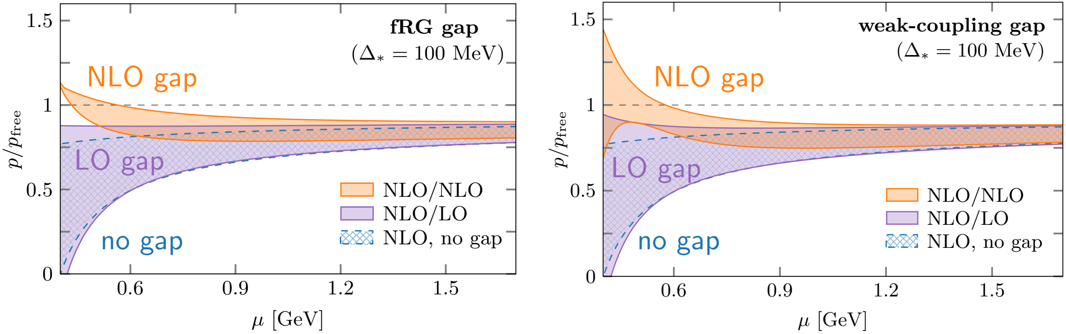
$$\bar{\Delta}_{\text{gap}} \sim \Lambda_{\text{QCD}} \exp\left(-\frac{c'}{g^2}\right), \quad c' > 0$$

$$\bar{\Delta}_{\text{gap}} \sim \frac{\mu}{g^5} \exp\left(-\frac{\bar{c}}{g}\right), \quad \bar{c} > 0$$

[D. T. Son '99; R. Pisarski and D. Rischke '00]

# Pressure

[AG, T. Gorda, and J. Braun '24]




- gap effects become more important for lower densities
- NLO/NLO correction of the same order as NLO/LO contribution
- specific scaling of the gap less relevant for the pressure

# Another way of characterizing dense matter

- Speed of sound

$$c_s^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{\mu}\left(\frac{\partial p}{\partial \mu}\right) \Bigg/ \left(\frac{\partial^2 p}{\partial \mu \partial \mu}\right)$$

- Causality

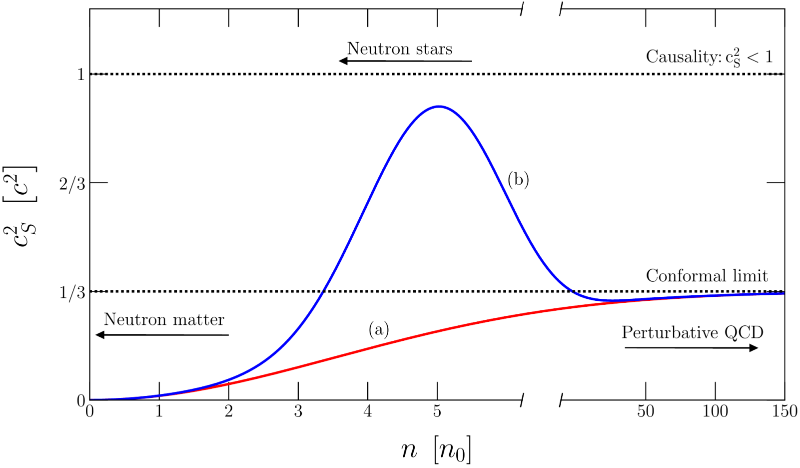
$$c_s^2 \leq 1$$

- Thermodynamic stability

$$c_s^2 \geq 0$$

- Speed of sound is a measure for the **stiffness** of the equation of state. **Stiffness** is needed to prevent a neutron star from collapsing to a black hole.

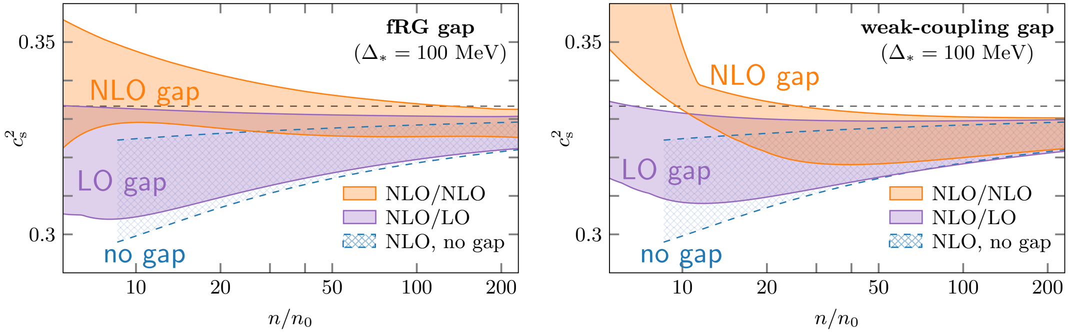
# Speed of Sound




[I. Tews et al. '18]

# Speed of Sound

[AG, T. Gorda, and J. Braun '24]




- gap effects become more important for lower densities
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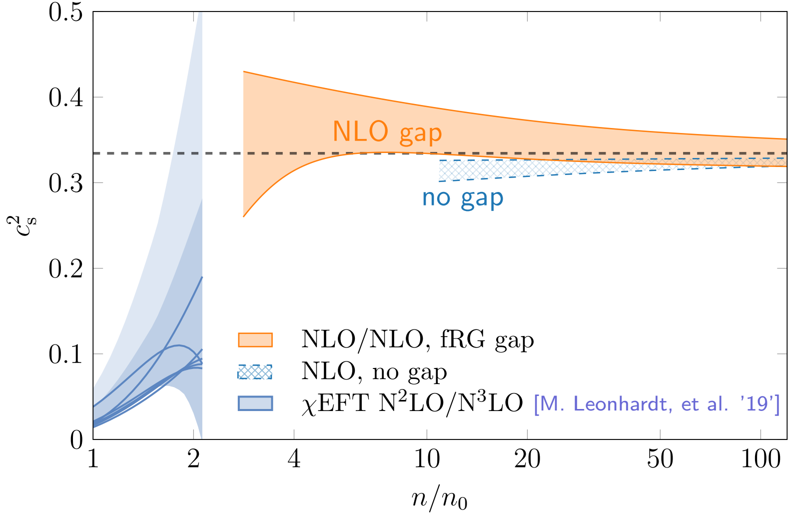
## Speed of Sound

[AG, T. Gorda, and J. Braun '24]



# Expansion of the Pressure (CFL)

[M. Alford, K. Rajagopal, and F. Wilczek '98] [AG, T. Gorda, and J. Braun (in prep.)]

- color flavor locking (CFL) assumed to be the true ground state

$N_{\mathrm{f}} = 3$

**double expansion**

$$p \approx p_{\mathrm{free}} \left( 1 - \frac{g^2}{2\pi^2} + (4 + 3.29(3) g^2) \left( \frac{|\bar{\Delta}^{\mathrm{CFL}}_{\mathrm{gap}}|}{\mu} \right)^2 \right)$$

preliminary!

free pressure

pert. contribution

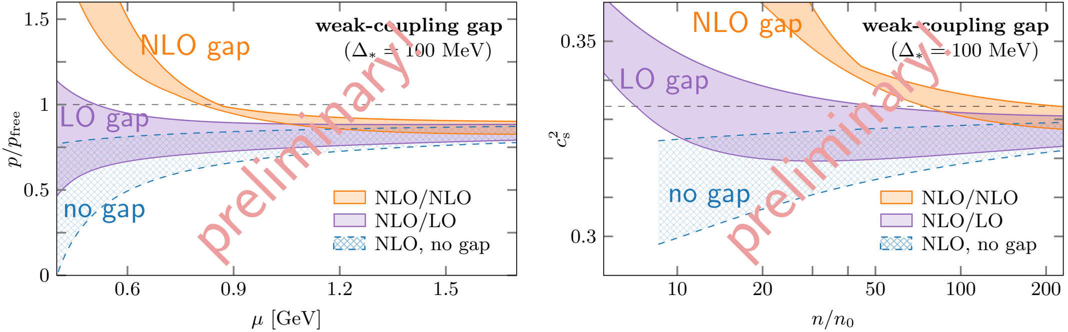
gapped quark loop

gapped 2-loop contribution

$\bar{\Delta}^{\mathrm{CFL}}_{\mathrm{gap}}(g)$

# Pressure and Speed of Sound (CFL)

[AG, T. Gorda, and J. Braun (in prep.)]




- enhancement compared to the two-flavor case
- gap effects become more important for lower densities
- NLO/NLO correction even larger than NLO/LO contribution

# Take Home Message

A color-superconducting **gap** suggests a **maximum** in the speed of sound at supranuclear densities.

At even higher densities, the speed of sound again **crosses** the conformal limit and approaches it from **below**.

What's next?

Bounds on the CFL gap.

Include further channel into fRG gap calculations.

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# Thank you for your attention!

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