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Noise-aware variational eigensolvers: a dissipative route for lattice gauge theories

Enrique Rico Ortega

Monday 02 September 2024

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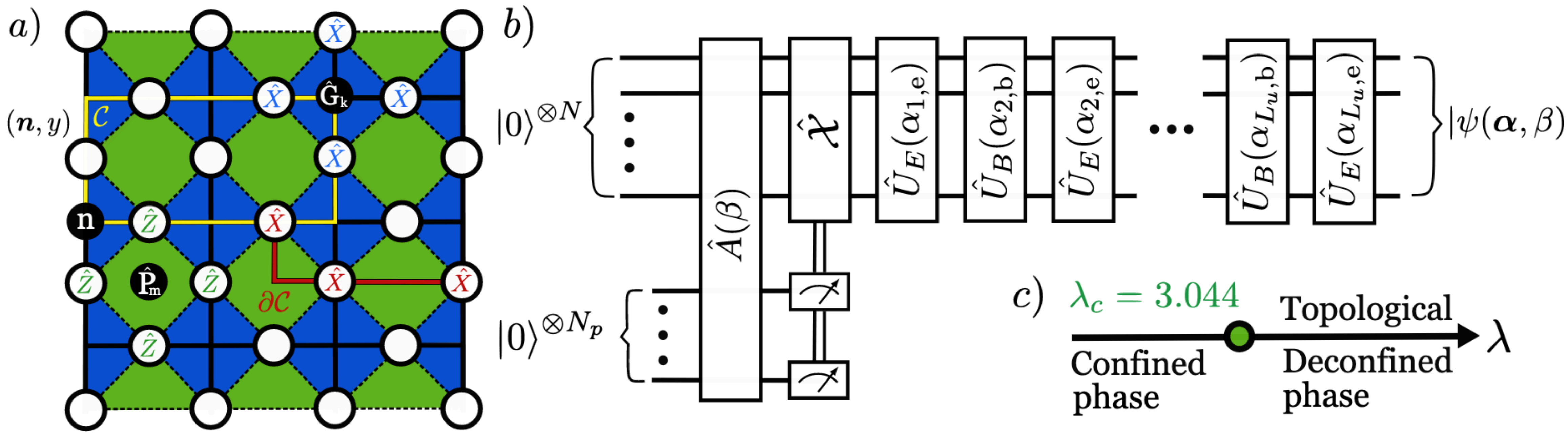
Noise-Aware Variational Eigensolvers: A Dissipative Route for Lattice Gauge Theories

Jesús Cobos, David F. Locher, Alejandro Bermudez, Markus Müller, and Enrique Rico

PRX Quantum **5**, 030340 – Published 26 August 2024

arXiv:2308.03618



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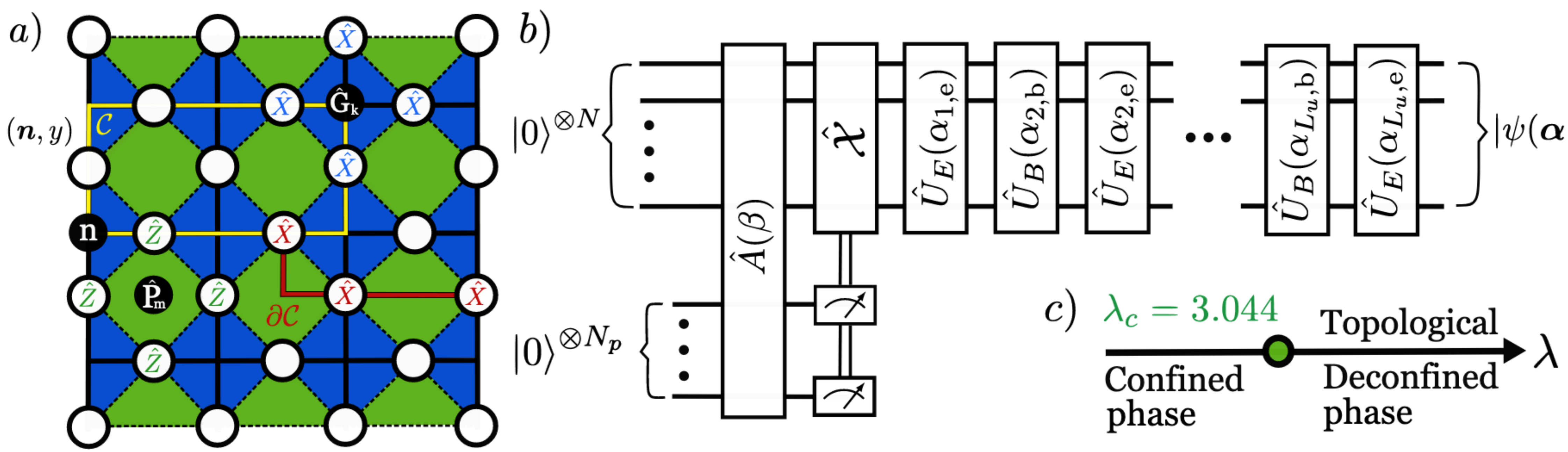
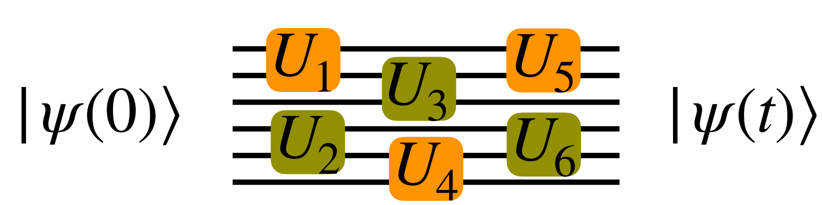


Quantum State Preparation



We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory

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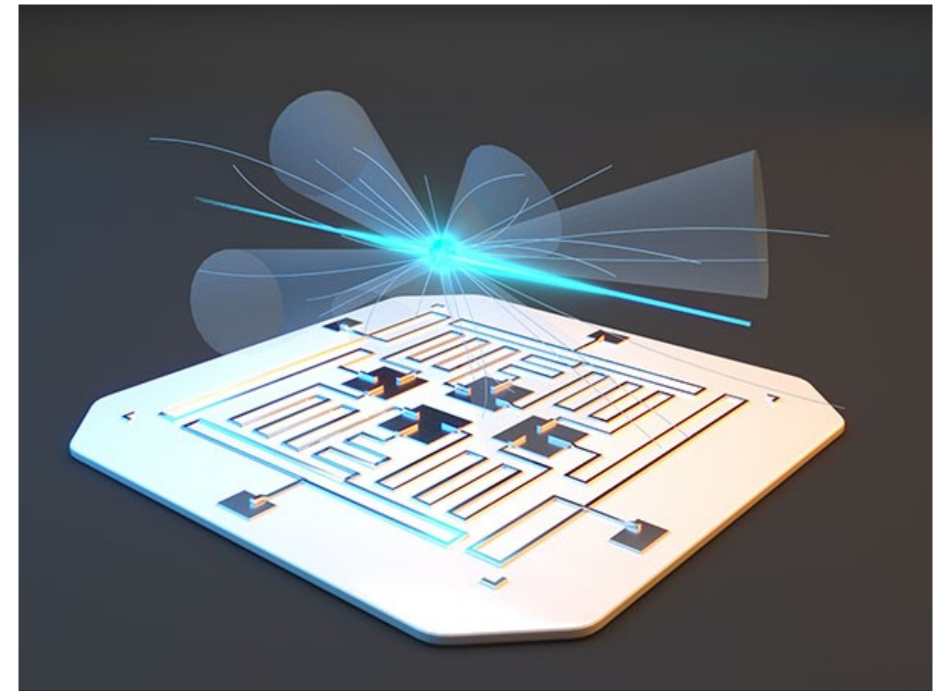
Simulating lattice gauge theories within quantum technologies



Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller…

Eur. Phys. J. D (2020) 74: 165
https://doi.org/10.1140/epjd/e2020-100571-8

THE EUROPEAN PHYSICAL JOURNAL D

Colloquium

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

Quantum Technologies for Lattice Gauge Theories

Quantum Simulation for High Energy Physics
C.W. Bauer, Z. Davoudi, A.B. Balantekin, T. Bhattacharya, M. Carena, W.A. de Jong, P. Draper, A. El-Khadra, N. Gemelke, M. Hanada, D. Kharzeev, H. Lamm, Y.-Y. Li, J. Liu, M. Lukin, Y. Meurice, C. Monroe, B. Nachman, G. Pagano, J. Preskill, E. Rinaldi, A. Roggero, D.I. Santiago, M.J. Savage, I. Siddiqi, G. Siopsis, D. Van Zanten, N. Wiebe, Y. Yamauchi, K. Yeter-Aydeniz, S. Zorzetti
arXiv:2204.03381

Lattice gauge theories simulations in the quantum information era
M. Dalmonte, S. Montangero
Contemporary Physics 57, 388 (2016)

Quantum Simulations of Lattice Gauge Theories using Ultracold Atoms in Optical Lattices
E. Zohar, J.I. Cirac, B. Reznik
Rep. Prog. Phys. 79, 014401 (2016)

Towards Quantum Simulating QCD
U.-J. Wiese
Nucl.Phys. A931, 246-256 (2014)

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A fruitful dialogue (two-way communication)

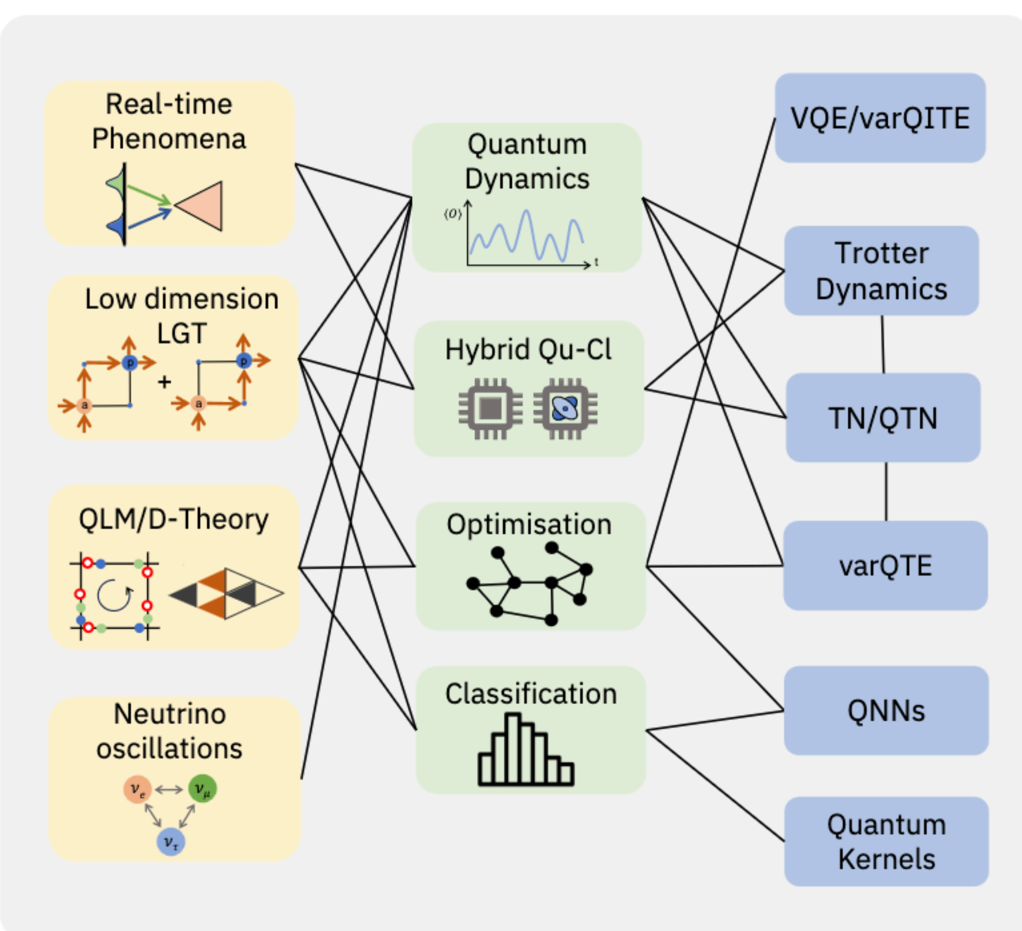
PRX QUANTUM **5,** 037001 (2024)

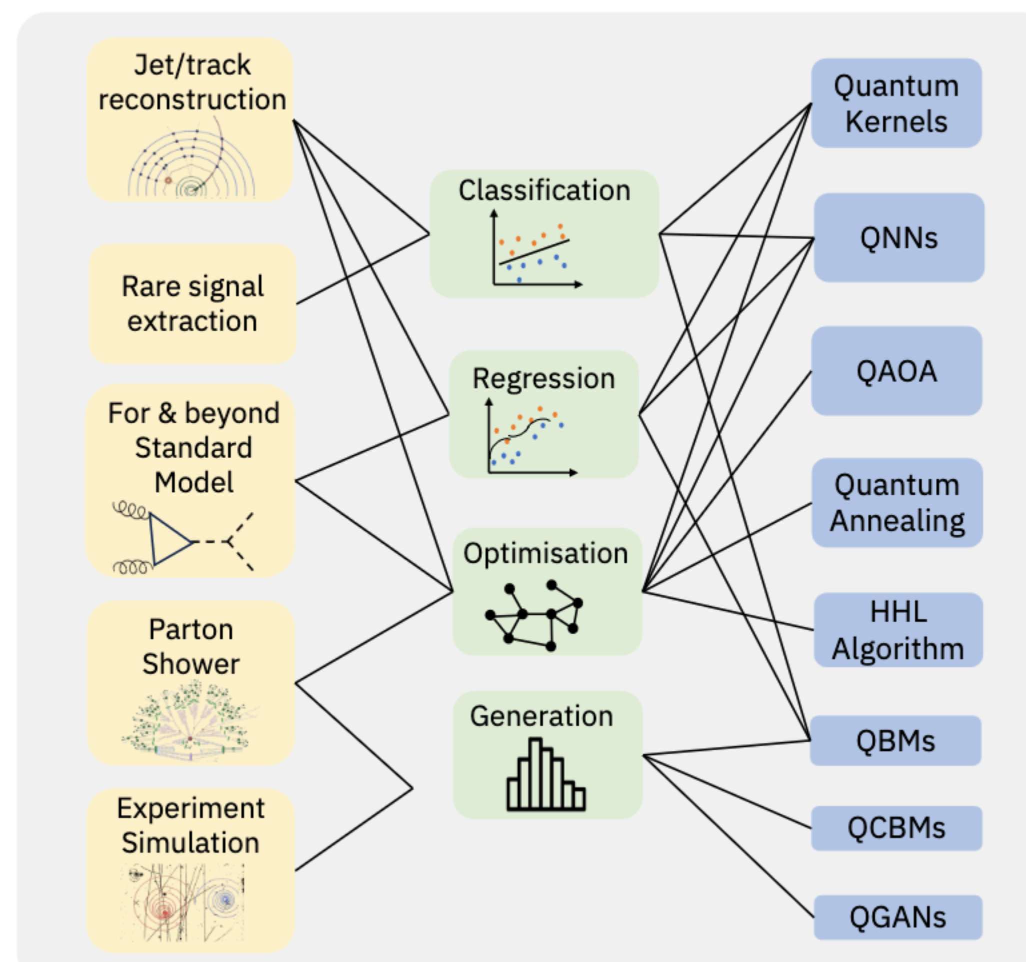
Roadmap

arXiv:2307.03236

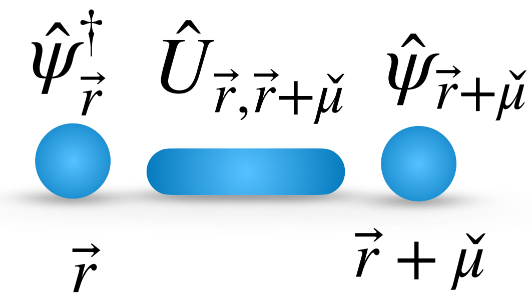
Quantum Computing for High-Energy Physics: State of the Art and Challenges

Alberto Di Meglio,[1,*] Karl Jansen,[2,3,†] Ivano Tavernelli,[4,‡] Constantia Alexandrou,[3,5] Srinivasan Arunachalam,[6] Christian W. Bauer,[7] Kerstin Borras,[8,9] Stefano Carrazza,[1,10] Arianna Crippa,[2,11] Vincent Croft,[12] Roland de Putter,[6] Andrea Delgado,[13] Vedran Dunjko,[12] Daniel J. Egger,[4] Elias Fernández-Combarro,[14] Elina Fuchs,[1,15,16] Lena Funcke,[17] Daniel González-Cuadra,[18,19] Michele Grossi,[1] Jad C. Halimeh,[20,21] Zoë Holmes,[22] Stefan Kühn,[2] Denis Lacroix,[23] Randy Lewis,[24] Donatella Lucchesi,[1,25] Miriam Lucio Martinez,[26,27] Federico Meloni,[8] Antonio Mezzacapo,[6] Simone Montangero,[1,25] Lento Nagano,[28] Vincent R. Pascuzzi,[6] Voica Radescu,[29] Enrique Rico Ortega,[30,31,32,33] Alessandro Roggero,[34,35] Julian Schuhmacher,[4] Joao Seixas,[36,37,38] Pietro Silvi,[1,25] Panagiotis Spentzouris,[39] Francesco Tacchino,[4] Kristan Temme,[6] Koji Terashi,[28] Jordi Tura,[12,40] Cenk Tüysüz,[2,11] Sofia Vallecorsa,[1] Uwe-Jens Wiese,[41] Shinjae Yoo,[42] and Jinglei Zhang[43,44]

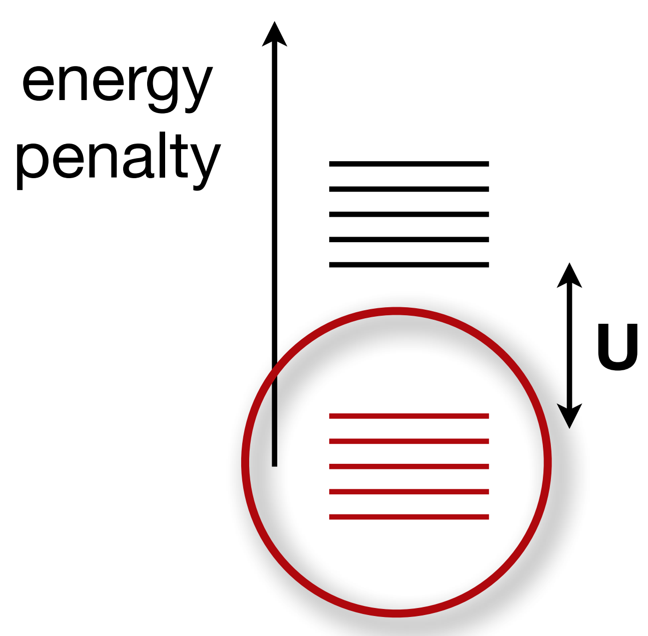


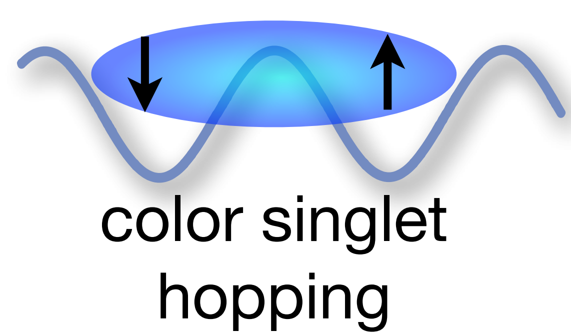


Simulating lattice gauge theories within quantum technologies

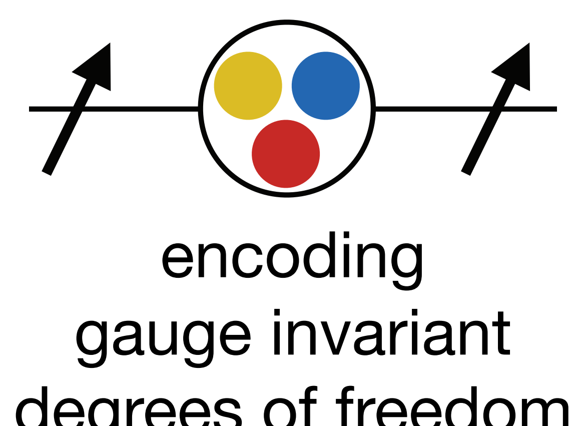


- Implementing the gauge invariant dynamics

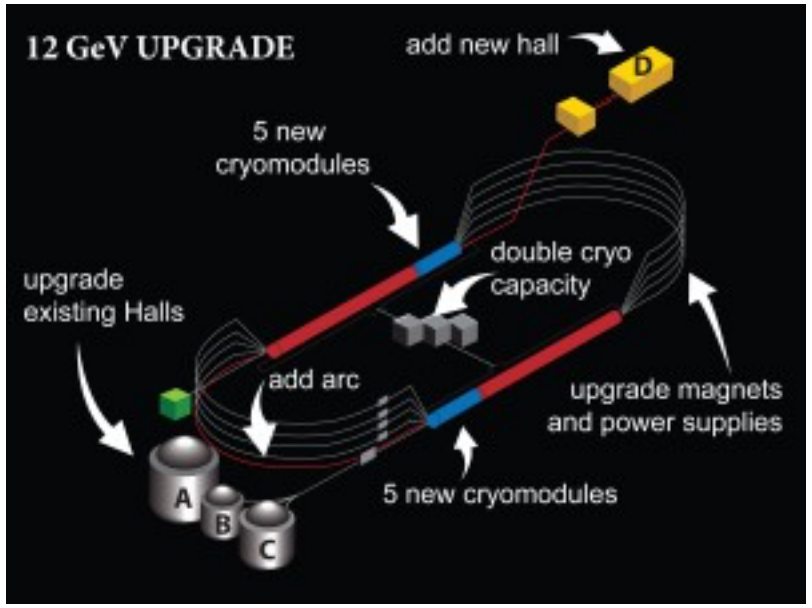


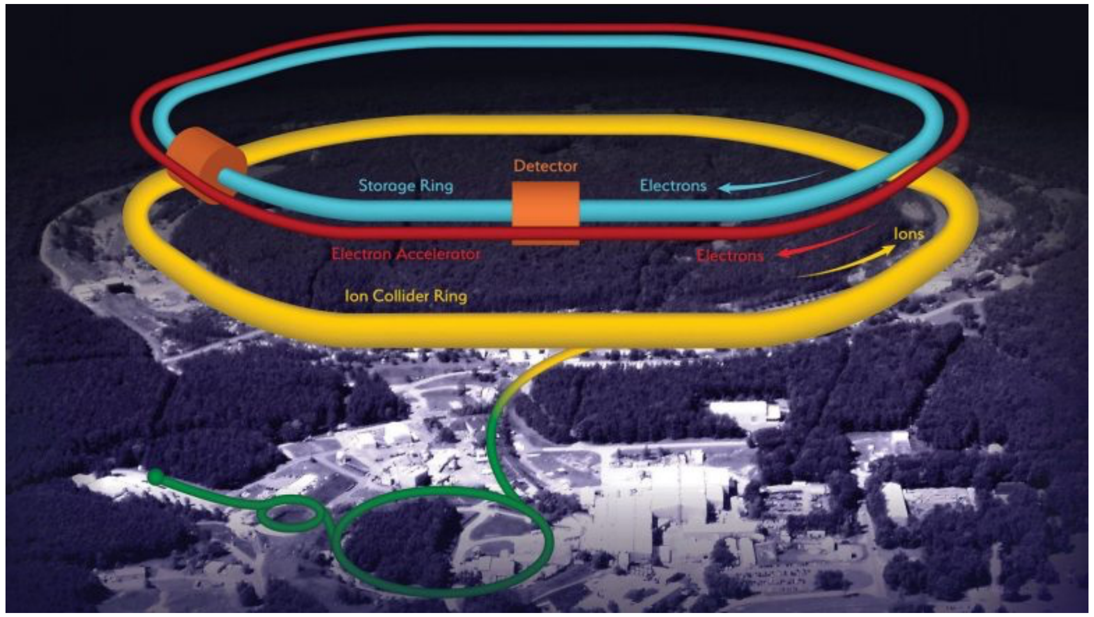


Internal symmetry

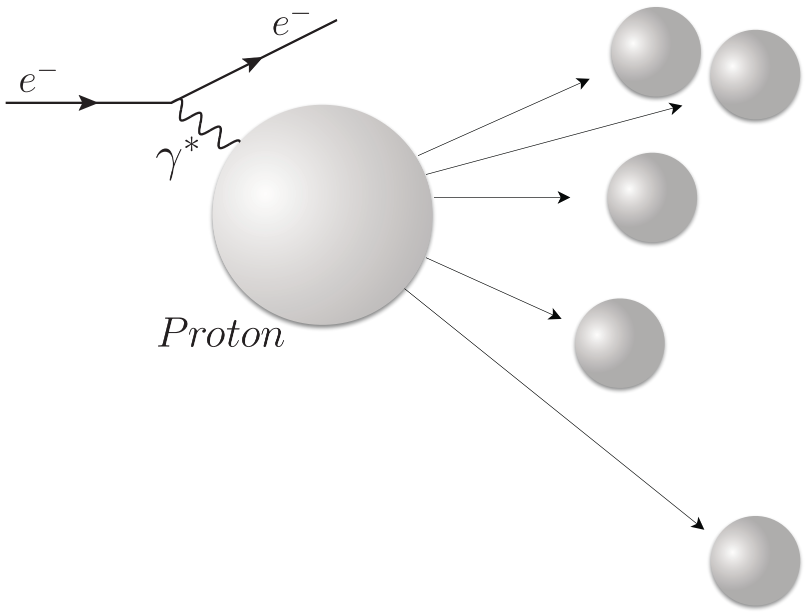


Quantum simulation of light-front parton correlators





modern microscopes



(semi-inclusive) deep-inelastic lepton scattering

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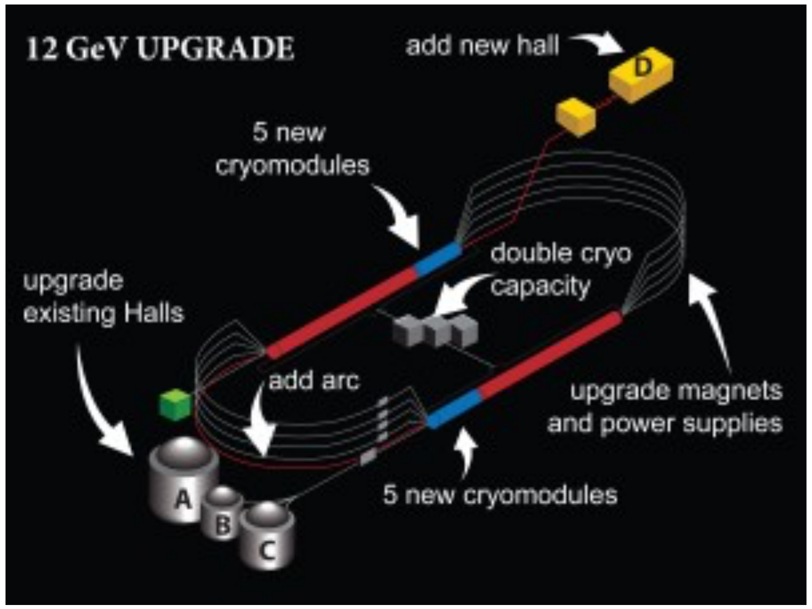
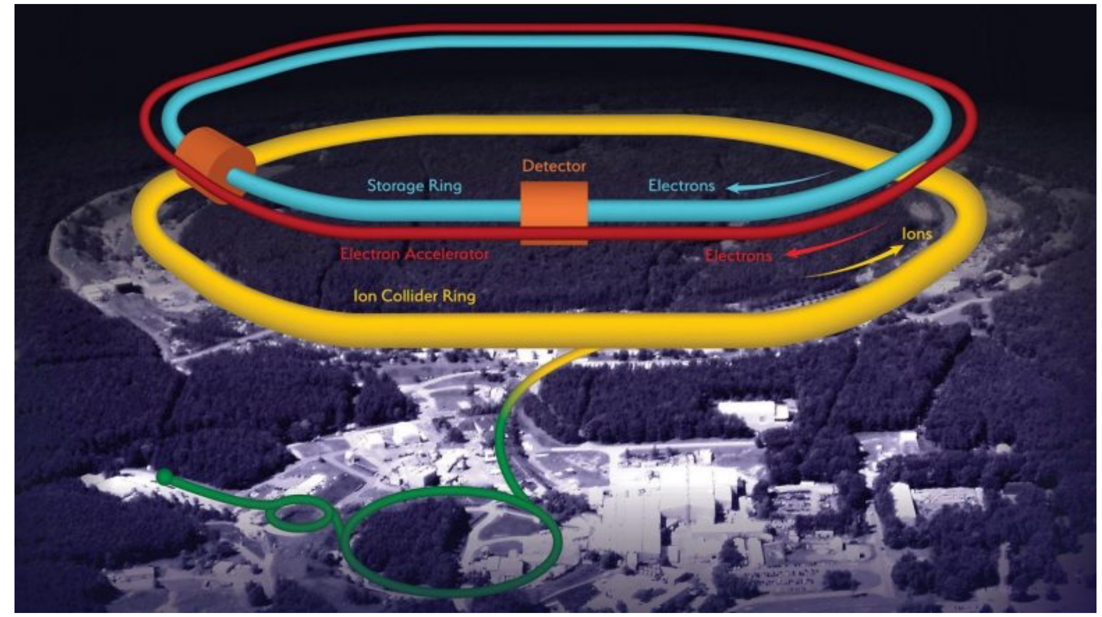
Quantum simulation of light-front parton correlators





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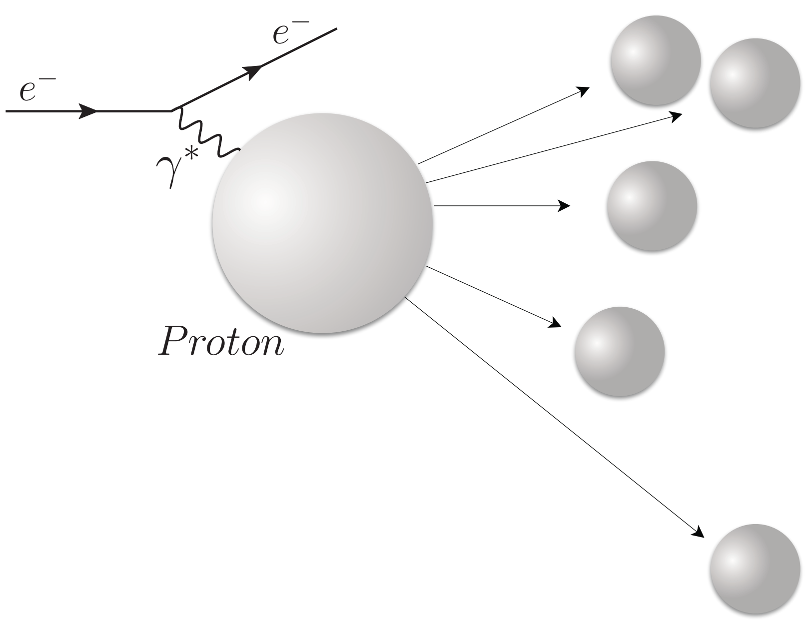

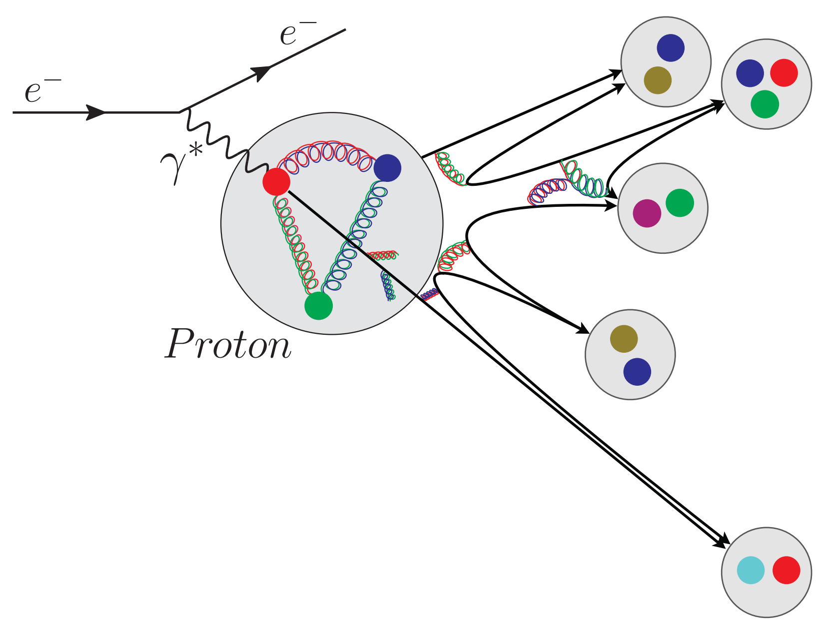

(semi-inclusive) deep-inelastic lepton scattering

highly virtual photons resolve inner (partonic) structure

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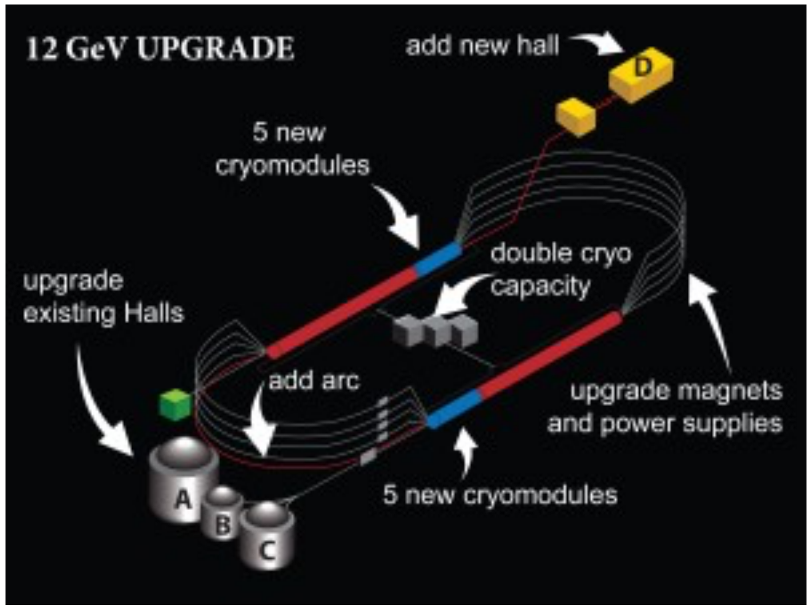
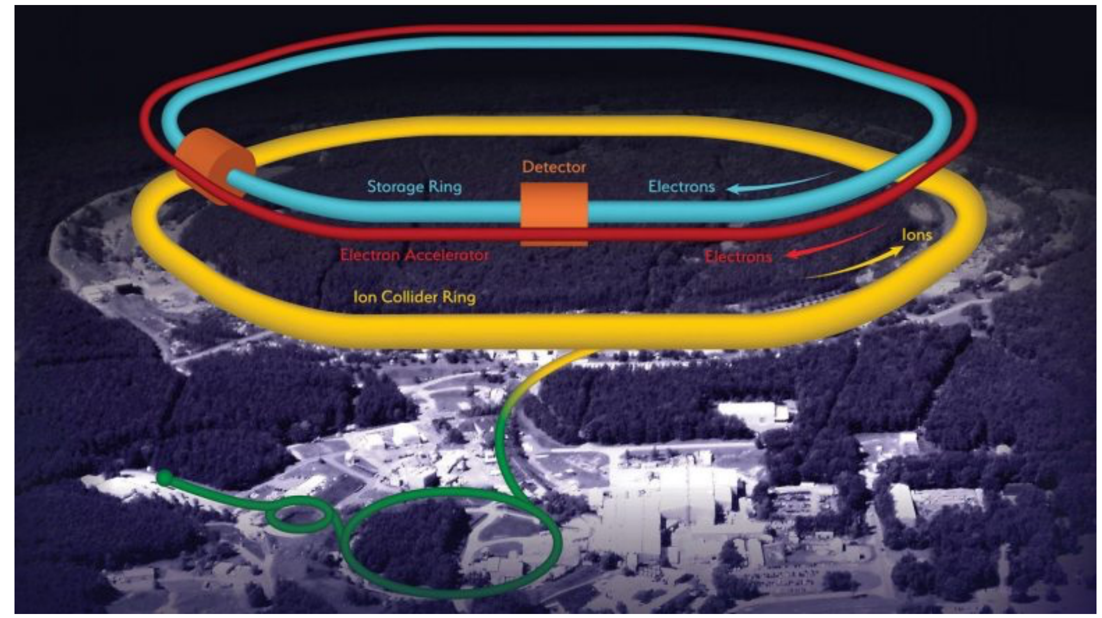
Quantum simulation of light-front parton correlators





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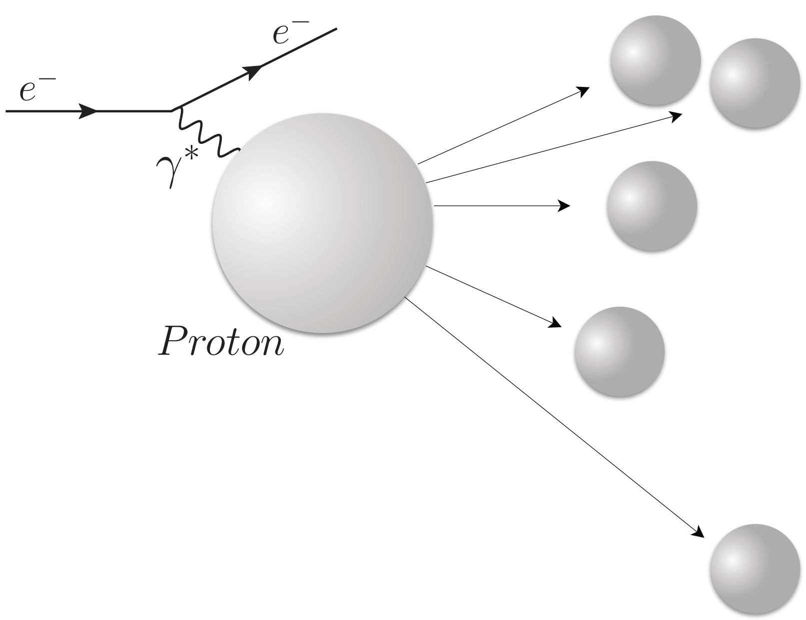

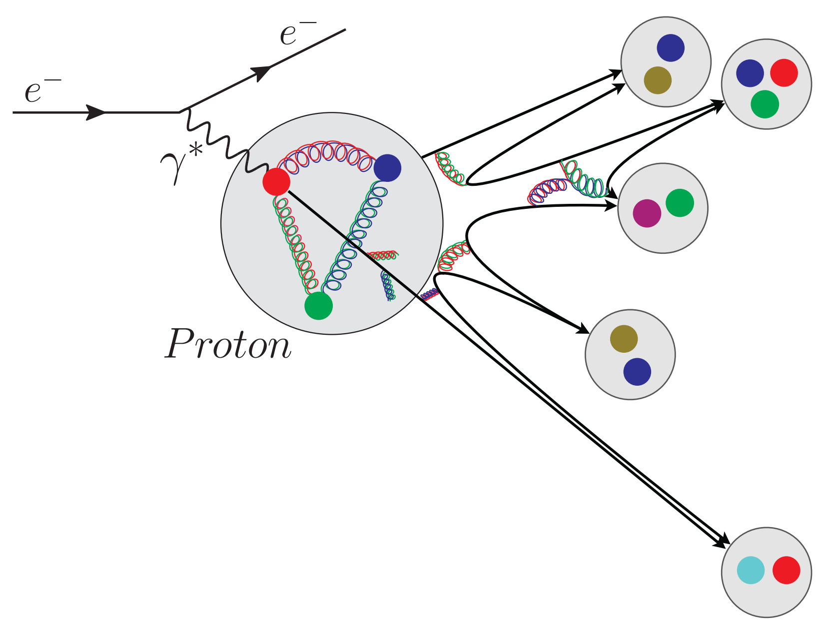

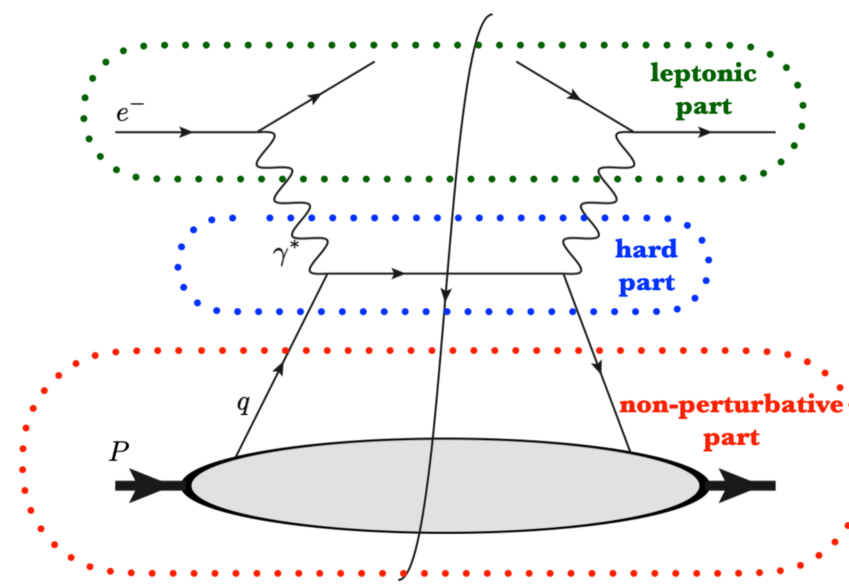

(semi-inclusive) deep-inelastic lepton scattering

highly virtual photons resolve inner (partonic) structure

factorization theorems separate non-calculable from calculable parts

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PHYSICAL REVIEW D **104,** 014512 (2021)

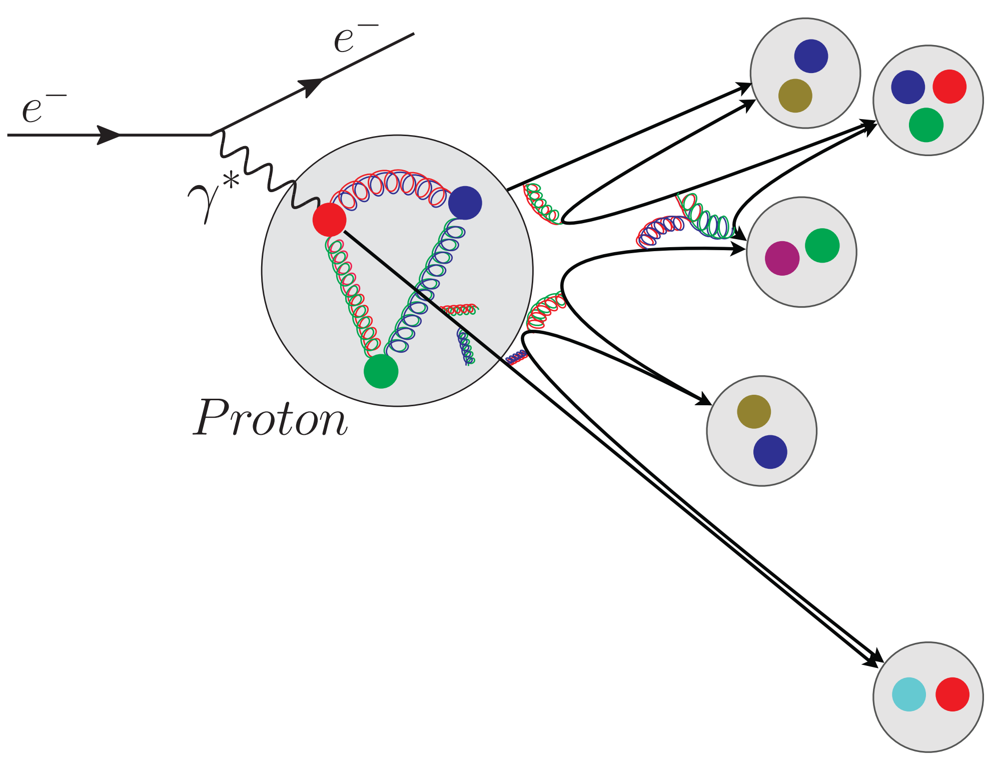
Quantum simulation of light-front parton correlators

M. G. Echevarria,[1,*] I. L. Egusquiza,[2,†] E. Rico,[3,4,‡] and G. Schnell[2,4,§]

arXiv:2011.01275

Phys. Rev. D 104, 014512 (2021)

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell



Quantum simulation of light-front parton correlators

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Quantum simulation of light-front parton correlators

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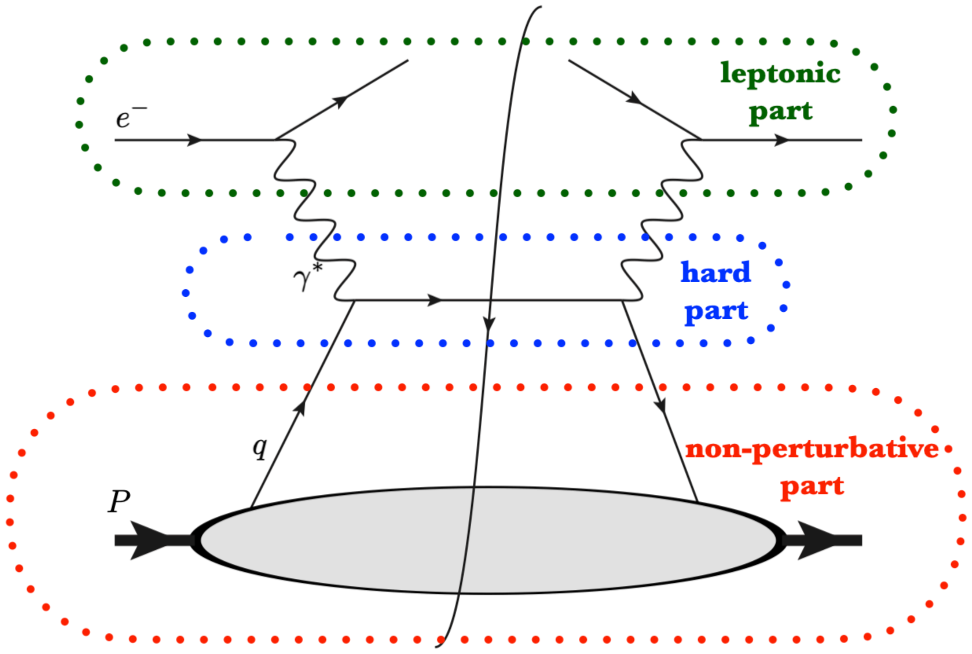
cross section:

$$\sigma(\xi, Q^2) = \sum_f \int_\xi^1 \mathrm{d}\bar{\xi}\, \hat{\sigma}(\bar{\xi}, Q^2)\, f_{f/P}(\xi/\bar{\xi}) \quad + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$



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Quantum simulation of light-front parton correlators

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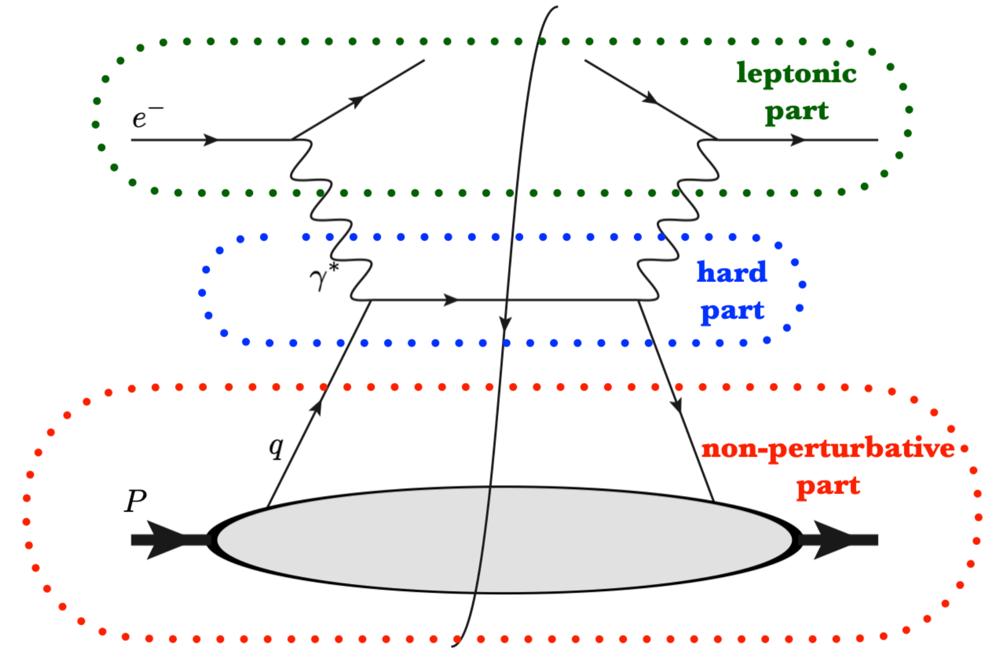
partonic cross section: calculable



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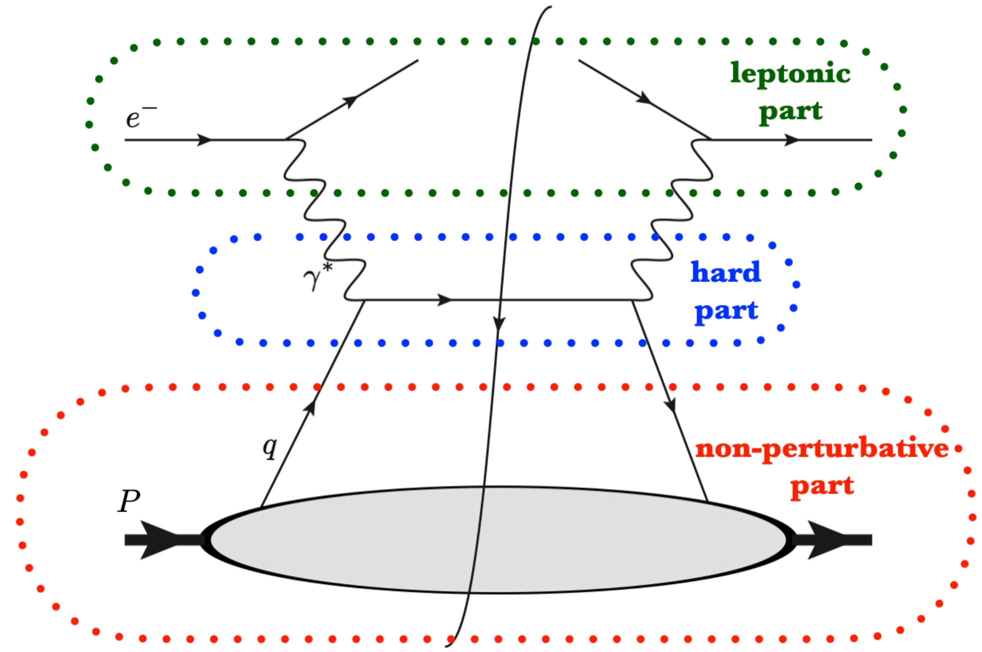

partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts

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Quantum simulation of light-front parton correlators

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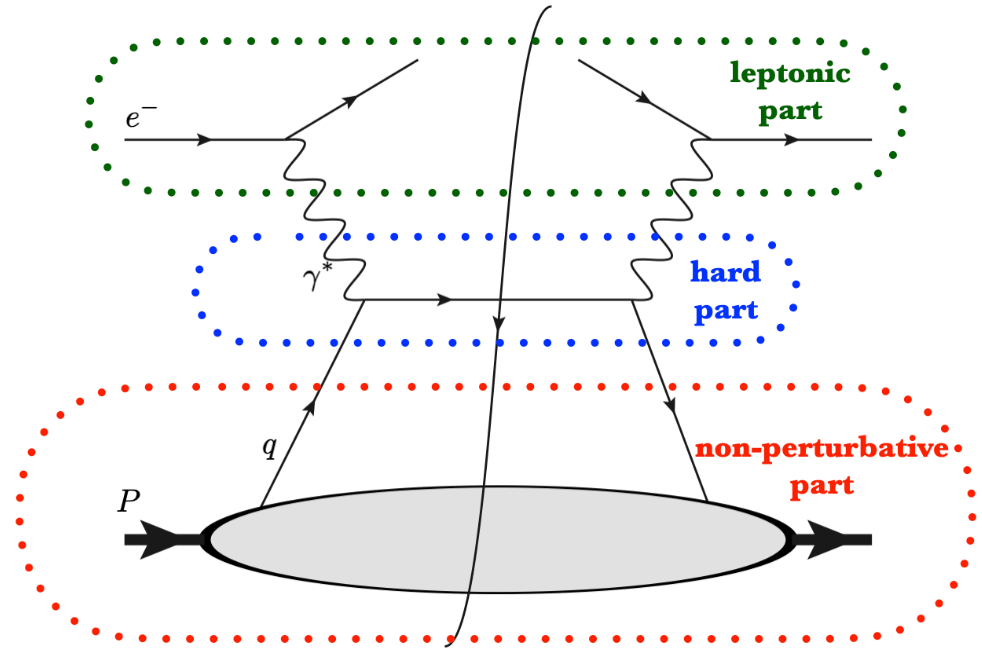

corrections

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Quantum simulation of light-front parton correlators

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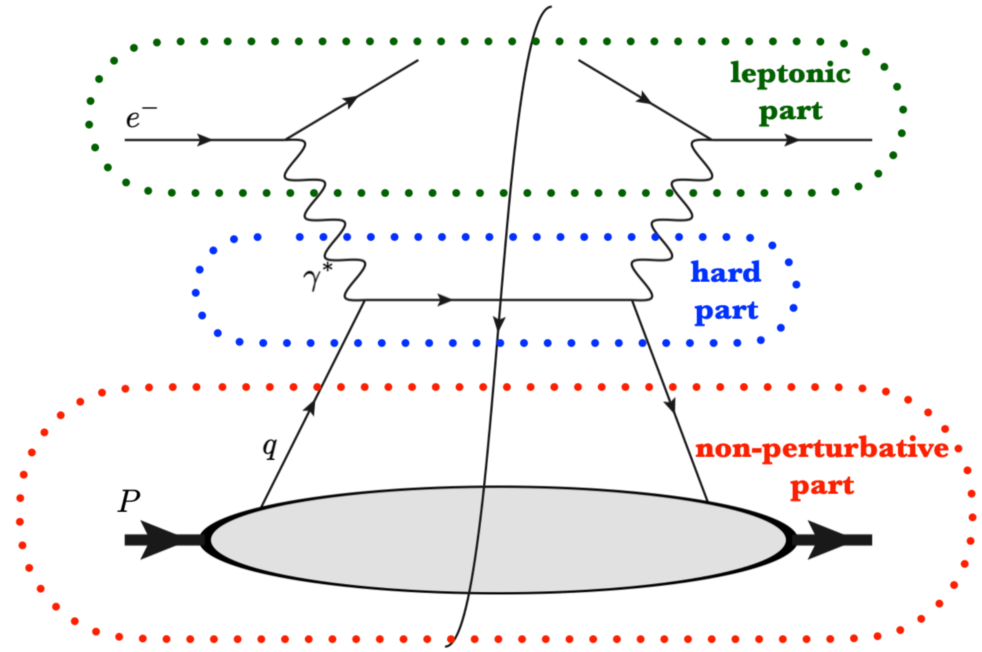

corrections

partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts

$$f_{f/P}(\xi) = \sum_S \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle PS \,|\, [\bar{\psi}\mathcal{U}](y^-) \frac{\gamma^+}{2} [\mathcal{U}^\dagger \psi](0) \,|\, PS \rangle$$

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Quantum simulation of light-front parton correlators

cross section:

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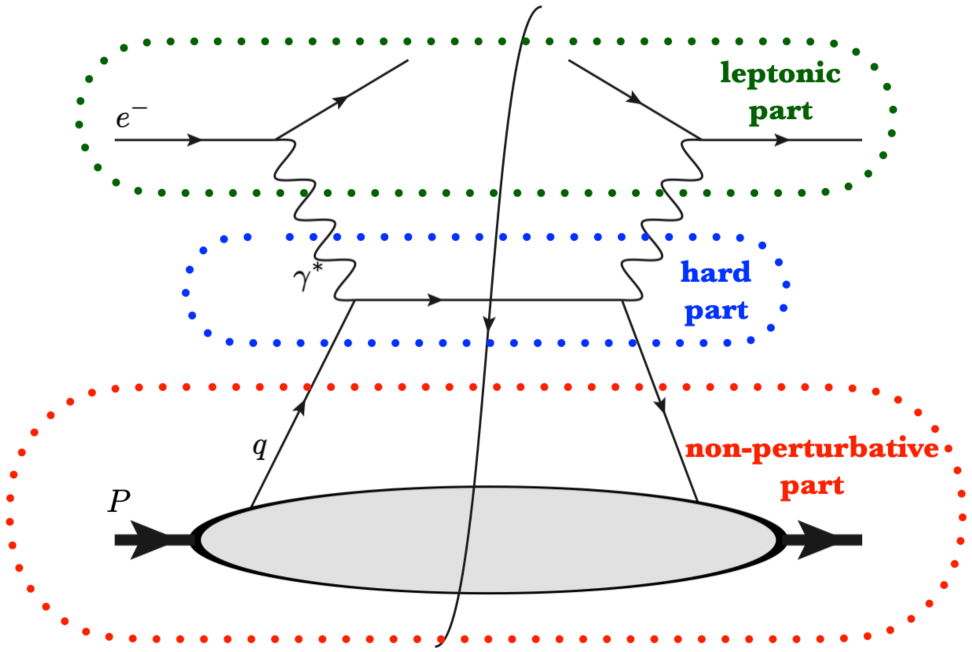
corrections



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Non-local (space-time) matrix elements require Wilson lines for gauge invariance

We study the quantum simulation of Wilson loops in space and real-time

Quantum simulation of light-front parton correlators

Non-local (space-time) matrix elements require Wilson lines for gauge invariance
We study the quantum simulation of Wilson loops in space and real-time

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Requirements for the quantum simulation of parton correlators:

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Quantum simulation of light-front parton correlators

Non-local (space-time) matrix elements require Wilson lines for gauge invariance
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$$f_{f/P}(\xi) = \sum_S \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle PS | \left[\bar{\psi}\mathscr{U}\right](y^-) \frac{\gamma^+}{2} \left[\mathscr{U}^\dagger \psi\right](0) | PS \rangle$$

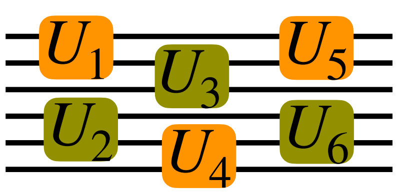
Requirements for the quantum simulation of parton correlators:

- encode in quantum degrees of freedom both matter and gauge fields
- preparation of a reference state, e.g., vacuum, proton, glue-ball
- simulate gauge-invariant quantities, e.g., minimal gauge-matter coupling
- real-time evolution, since the Wilson line is non-local in time
- carry out measurements after the evolution, i.e., quantum interferometer

Quantum simulation of light-front parton correlators

Digital simulation: Universal simulator

$|\psi(0)\rangle$



$|\psi(t)\rangle$

Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator

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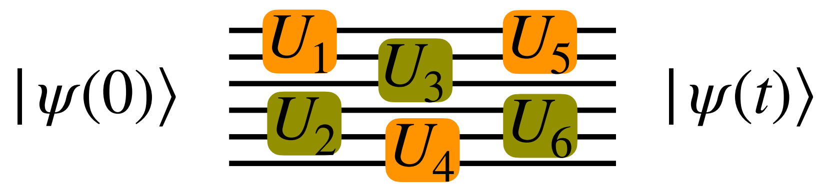
Quantum simulation of light-front parton correlators

Digital simulation: Universal simulator



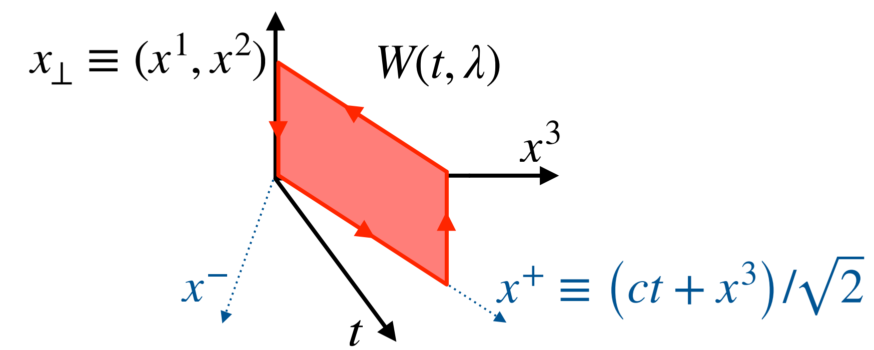
Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator

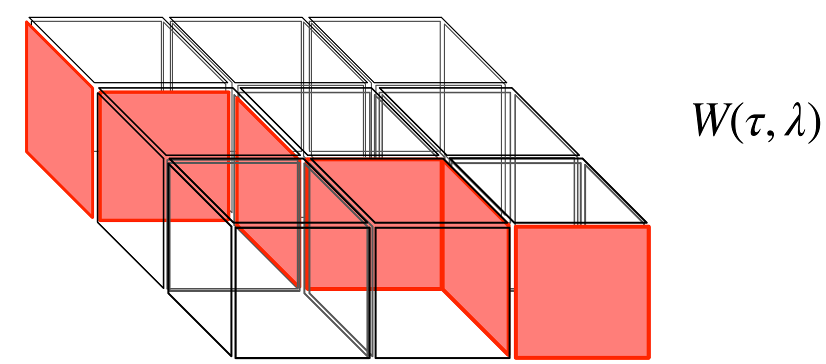
Note: in the Hamiltonian formulation the temporal gauge $A_0=0$ is chosen

Discretisation of space-time in a Hamiltonian formulation

(a)

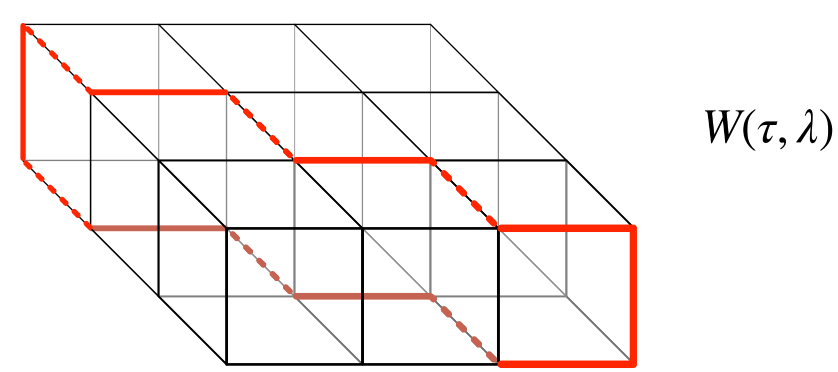


(b)



$$W(\tau,\lambda) = W_{C_1} W_{\tau_1} W_{C_2} W_{\tau_2} \cdots W_{C_k} W_{\tau_k} \cdots$$

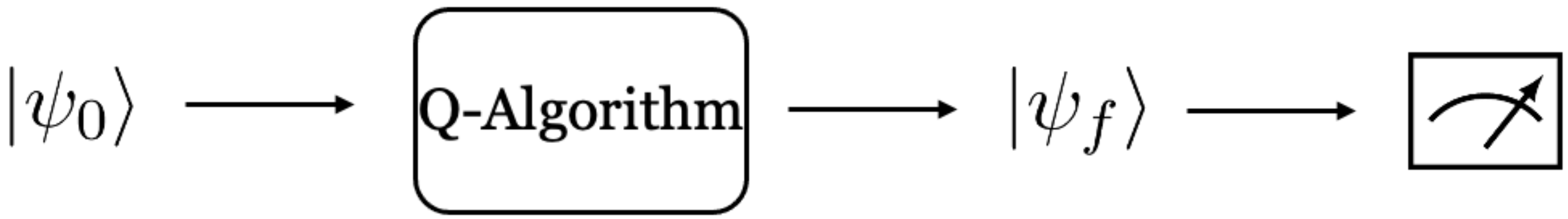
(c)



$$W(\tau,\lambda) = \mathcal{U}_1 e^{-i\tau_1 H} \mathcal{U}_2 e^{-i\tau_2 H} \cdots \mathcal{U}_k e^{-i\tau_k H} \cdots \mathcal{U}_N$$

Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states



For our purposes

$$|\psi_f\rangle \simeq |E_0\rangle$$

- We classify algorithms depending on how they manipulate quantum states.

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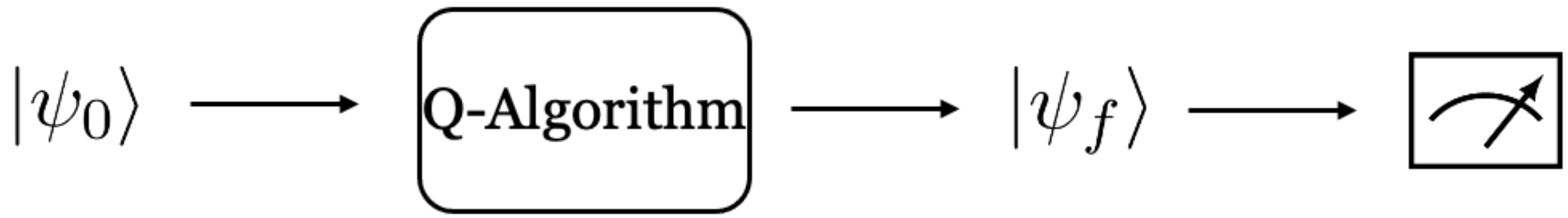

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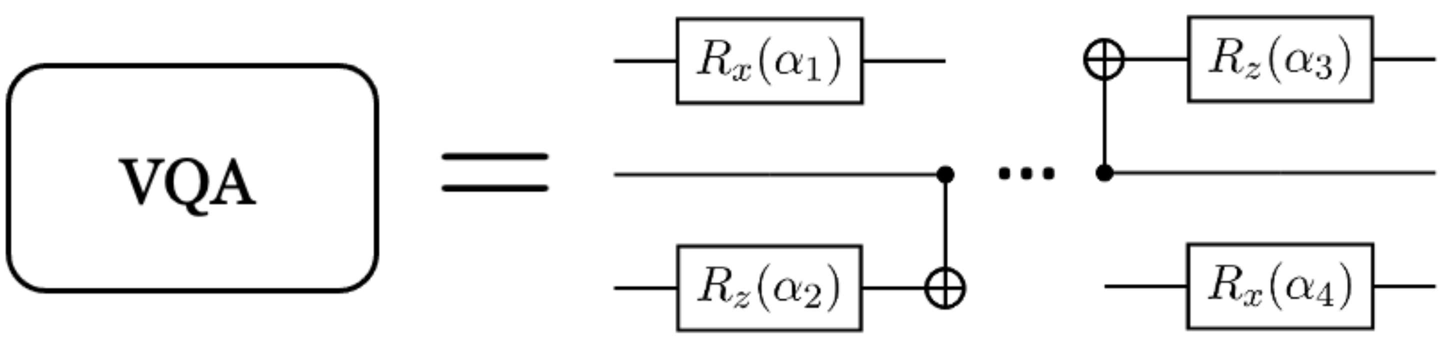
Variational quantum algorithms



Short depth

Hard Optimization

$$|\psi(\boldsymbol{\alpha})\rangle = U_k(\alpha_N)U_{k-1}(\alpha_{N-1})\ldots U_1(\alpha_1)\,|\psi_0\rangle$$

$$|\psi_f\rangle = |\psi(\boldsymbol{\alpha}^*)\rangle \qquad \boldsymbol{\alpha}^* = \underset{\boldsymbol{\alpha}}{\operatorname{argmin}} \langle\psi(\boldsymbol{\alpha})|\,\hat{H}\,|\psi(\boldsymbol{\alpha})\rangle \qquad \langle\psi|\,H\,|\psi\rangle \geq E_0\ \forall\,|\psi\rangle$$

Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states

$|\psi_0\rangle \longrightarrow$ Q-Algorithm $\longrightarrow |\psi_f\rangle \longrightarrow$ 

For our purposes
$$|\psi_f\rangle \simeq |E_0\rangle$$

- We classify algorithms depending on how they manipulate quantum states.

Adiabatic algorithms

$$\text{AA} = \mathcal{T}\left\{\int_0^T \exp\left[-\frac{it}{\hbar}H(t)\right]\right\}$$

Less sensitive to noise

$$H(t) = [1-\lambda(t)]H_0 + \lambda(t)H_f$$

$$\lambda(0) = 0 \qquad \lambda(T) = 1$$

$$T \sim \mathcal{O}\left(\frac{1}{\Delta}\right) \qquad \Delta \longrightarrow \text{Min. Gap}$$

Hamiltonian engineering

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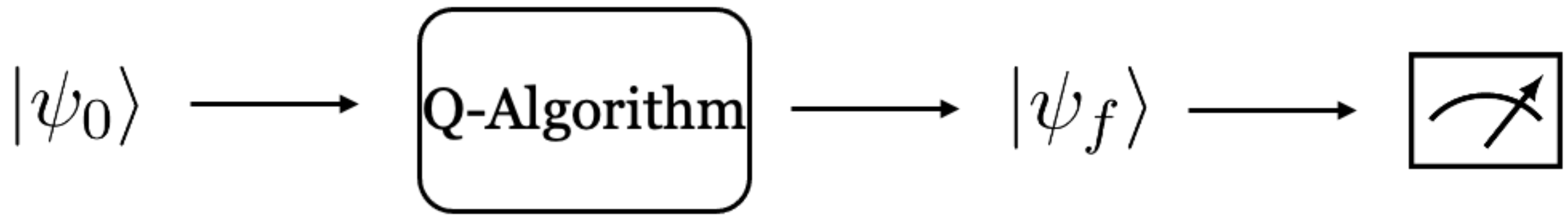
Quantum algorithms for quantum state preparation

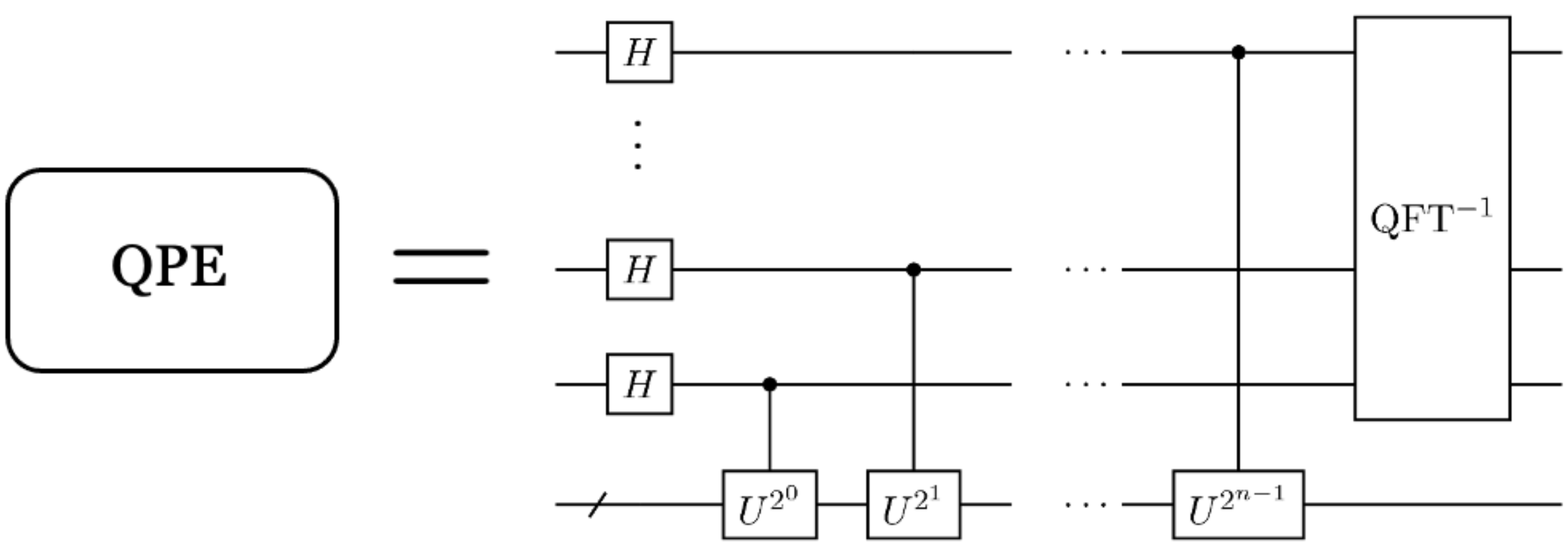
- Quantum algorithms are recipes that manipulate quantum states



For our purposes
$|\psi_f\rangle \simeq |E_0\rangle$

- We classify algorithms depending on how they manipulate quantum states.

<u>Provable algorithms</u>



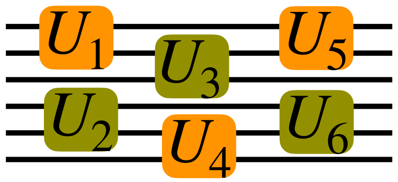
Guarantee of success

Long depths

Noise-Aware Variational Eigensolvers: A Dissipative Route for Lattice Gauge Theories

Jesús Cobos, David F. Locher, Alejandro Bermudez, Markus Müller, and Enrique Rico
PRX Quantum **5**, 030340 – Published 26 August 2024
arXiv:2308.03618

Quantum State Preparation

$|\psi(0)\rangle$  $|\psi(t)\rangle$

In the ***general case***, it is know to be a QMA problem (analogue of NP problem)

With ***unitary circuits***, it is know that the depth scales with the system size (topological order)

Bravyi, Hastings, Verstraete (2006)

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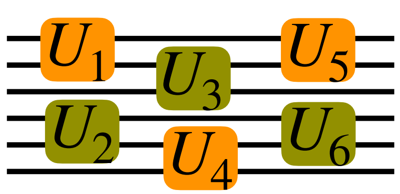
$|\psi(0)\rangle$  $|\psi(t)\rangle$

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Bravyi, Hastings, Verstraete (2006)

We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory

$$\hat{H}_{\mathbb{Z}_2} = -\sum_{\text{link}} \hat{\sigma}_l^x - \lambda \sum_{\text{plaq}} (\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z)_{\text{plaq}}$$

$\lambda_c = 3.04438$

confined | deconfined topological order | λ

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We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

The $\mathbb{Z}_2$ lattice gauge theory

- Hamiltonian

$$\hat{H} = -\underbrace{\sum_{n,i} \hat{\sigma}^x_{(n,i)}}_{\text{Electric term}} - \lambda \underbrace{\sum_{n} \hat{P}_n}_{\text{Magnetic term}} \qquad \lambda \in [0, \infty)$$

- Gauge invariance

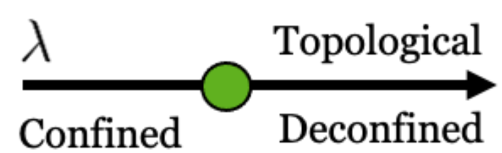
$$\left[\hat{G}_k, \hat{H}\right] = 0 \qquad \forall\, k = 0, 1 \ldots N_g$$

- Gauss' Law

$$(\nabla \cdot \mathbf{E})(k) \equiv 0 \implies \hat{G}_k \left|\psi\right\rangle = \left|\psi\right\rangle$$

$\left|\psi\right\rangle \longrightarrow$ *Physical* states

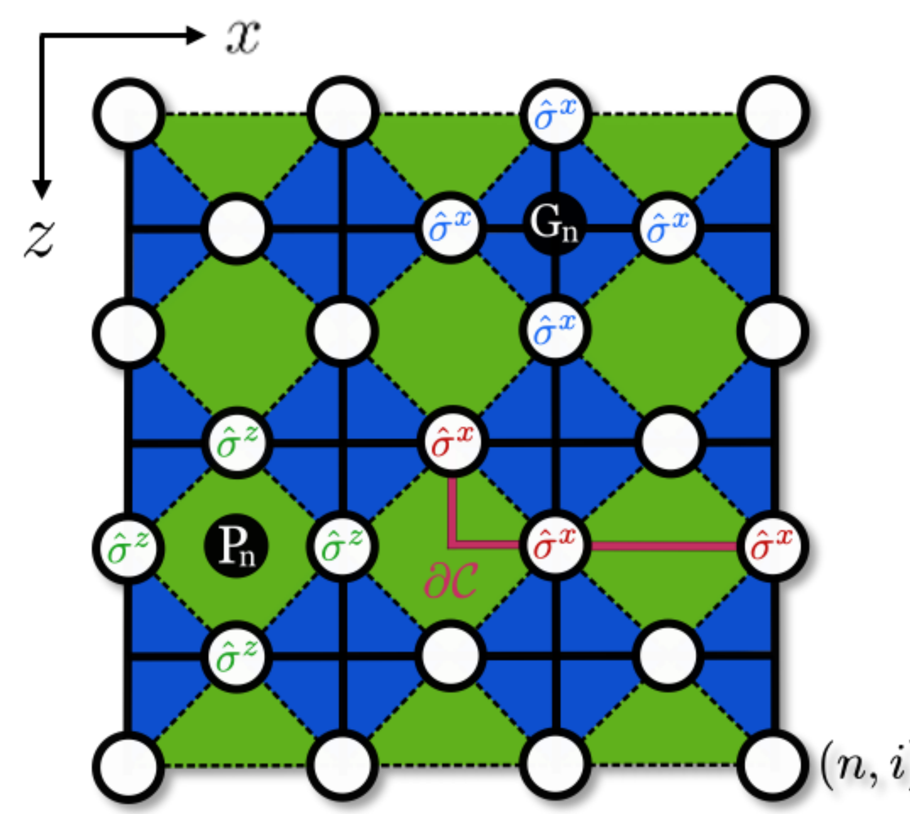
- Phase diagram



$$\lambda_c = 3.04438$$

- Dual magnetization

$$\hat{M}_n = \prod_{(n,i)\in\partial\mathcal{C}_n} \hat{\sigma}^x_{(n,i)}$$



$d = 4$

Variational gauge invariant state

$$|\psi\rangle = \frac{e^{\beta \sum_{\text{plaq}} (\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z)_{\text{plaq}}}}{Z} \otimes_{\text{link}} |+\rangle_l$$

$$\begin{cases} |\psi\rangle = \otimes_{\text{link}} |+\rangle_l & \lambda = 0 \\ |\psi\rangle = \otimes_{\text{plaq}} \frac{\mathbb{I} + (\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z)_{\text{plaq}}}{2} \otimes_{\text{link}} |+\rangle_l & \lambda \gg 1 \end{cases}$$

$$\hat{G}_{\text{vertex}} |\psi\rangle = \left(\hat{\sigma}_x\hat{\sigma}_x\hat{\sigma}_x\hat{\sigma}_x\right)_{\text{vertex}} |\psi\rangle = |\psi\rangle$$

Cardy, Hamber (1980)

Variational gauge invariant state

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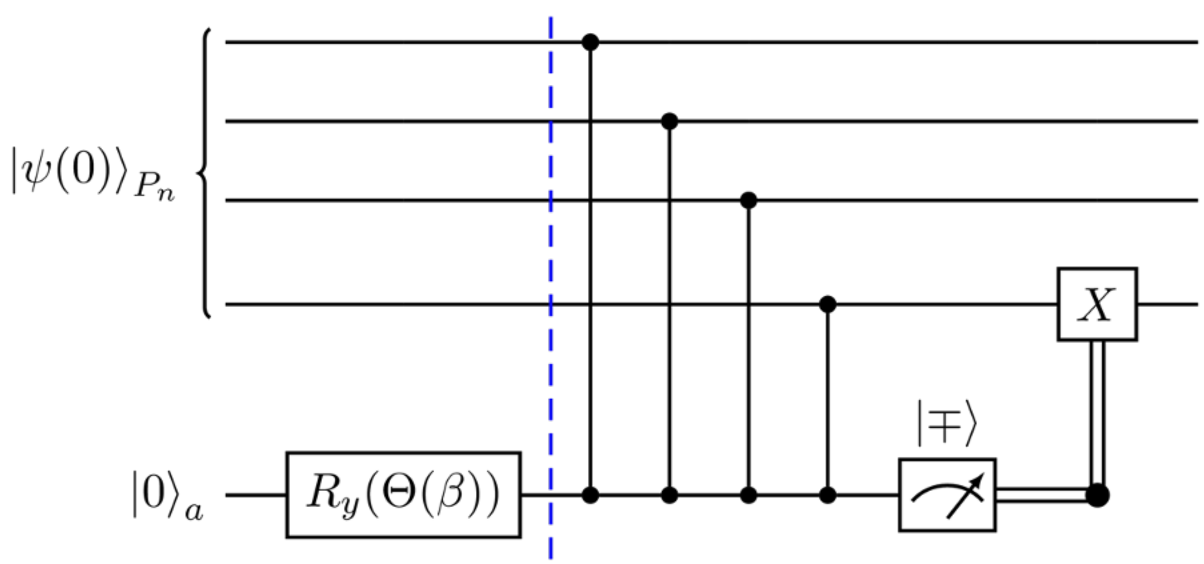

$$\Theta(\beta) = \tan^{-1}(\tanh\beta)$$

$$|\psi(0)\rangle_{P_n} |0\rangle_a \xrightarrow{R_y(\Theta(\beta))} \left[\frac{|0\rangle_a + \tanh\beta |1\rangle_a}{\sqrt{1+\tanh^2\beta}}\right] |\psi(0)\rangle_{P_n}$$

Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)

Variational gauge invariant state

$$|\psi\rangle = \frac{e^{\beta \sum_{\text{plaq}} (\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z\hat{\sigma}^z)_{\text{plaq}}}}{Z} \otimes_{\text{link}} |+\rangle_l$$

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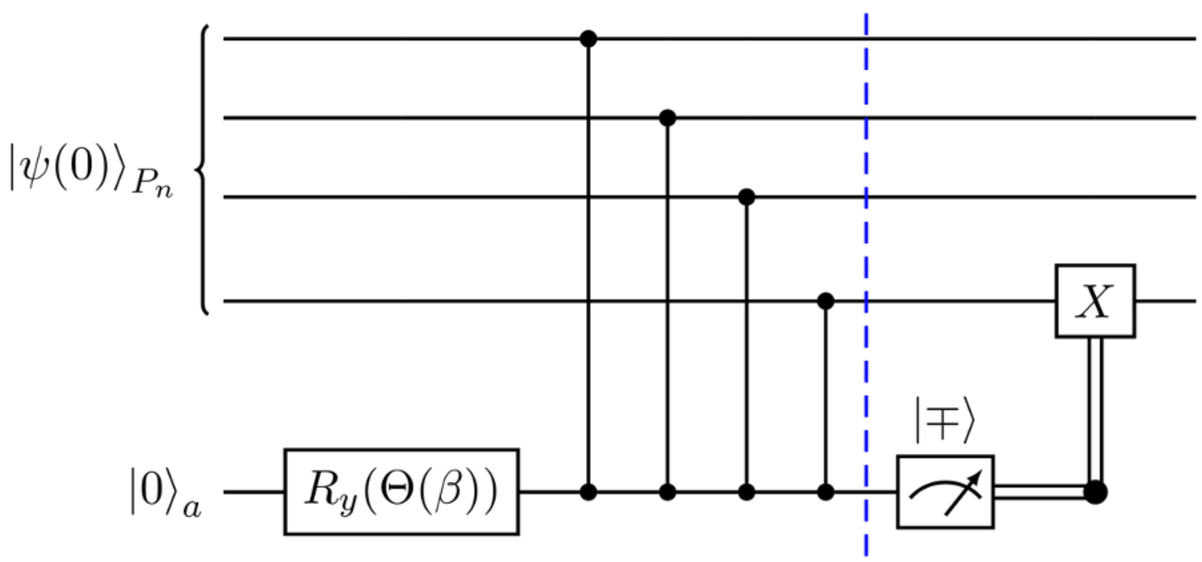


$$\hat{\sigma}^x_{m\in P_n}\, e^{-\hat{P}_n} = e^{\hat{P}_n}\, \hat{\sigma}^x_{m\in P_n}$$

$$\hat{\sigma}^x_m\, |\psi(0)\rangle = |\psi(0)\rangle \;\; \forall\, m$$

$$\left[\frac{|0\rangle_a + \tanh\beta|1\rangle_a}{\sqrt{1+\tanh^2\beta}}\right] |\psi(0)\rangle_{P_n} \xrightarrow{CZ} \frac{1}{\sqrt{2}} \left\{ \left[\frac{e^{\beta\hat{P}_n}}{(\cosh 2\beta)^{N_p/2}}\right] |+\rangle_a + \left[\frac{e^{-\beta\hat{P}_n}}{(\cosh 2\beta)^{N_p/2}}\right] |-\rangle_a \right\} |\psi(0)\rangle_{P_n}$$

Variational gauge invariant state

We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

Variational ansatz

$$|\psi(\boldsymbol{\alpha},\boldsymbol{\beta})\rangle = \left[\prod_{k=2}^{\ell} e^{i\alpha_k H_E} e^{i\beta_k H_B}\right] e^{i\alpha_1 H_E} \frac{e^{\beta_1 H_B}}{(\cosh 2\beta_1)^{N_p/2}} |\Omega_E\rangle$$

Unitary Dissipative

$$H_E = \sum_{n,i} \sigma^x_{(n,i)} \quad H_B = \sum_n P_n \quad |\Omega_E\rangle = \bigotimes_{n,i} |+\rangle_{(n,i)}$$

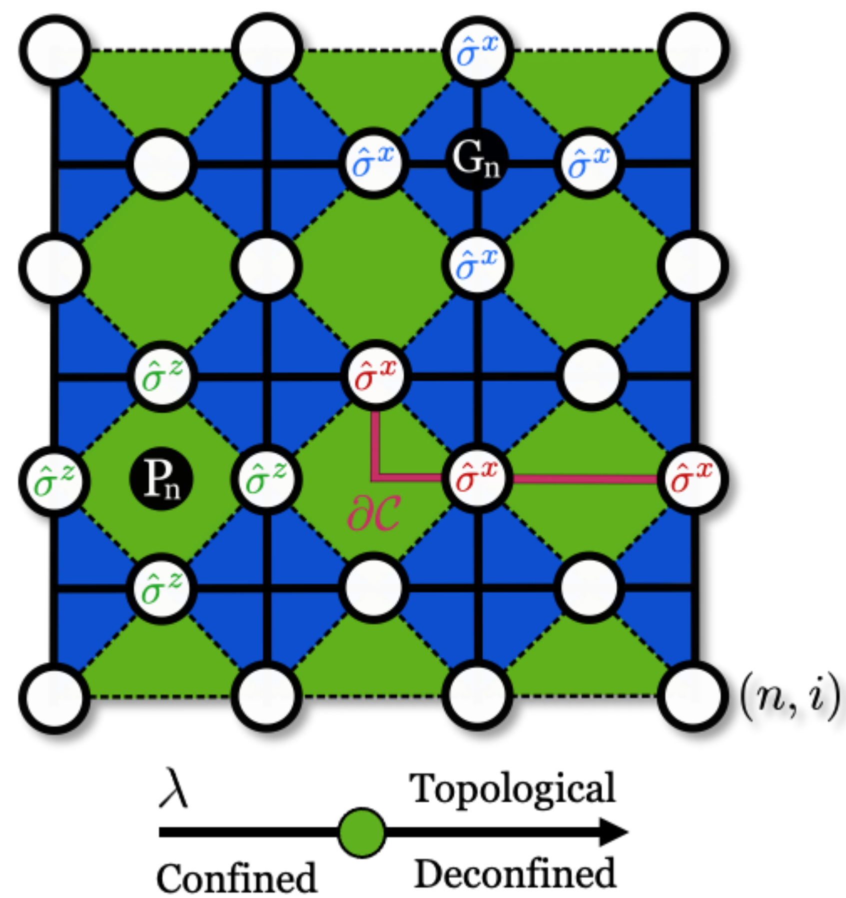
$$|\psi(\mathbf{0},\beta_1=\infty)\rangle = \prod_n \left[\frac{1+P_n}{\sqrt{2}}\right] |\Omega_E\rangle \longrightarrow$$

The ansatz captures the ground states of H_E, H_B

$$\lim_{\tau\to\infty} e^{-\tau\hat{H}} |\psi\rangle \longrightarrow |\text{g.s.}\rangle$$

$$e^{-\tau\hat{H}} |\psi\rangle = e^{-\tau E_0} |\text{g.s.}\rangle + e^{-\tau E_1} |E_1\rangle + e^{-\tau E_2} |E_2\rangle + \dots$$

$$H = -H_E - \lambda H_B$$



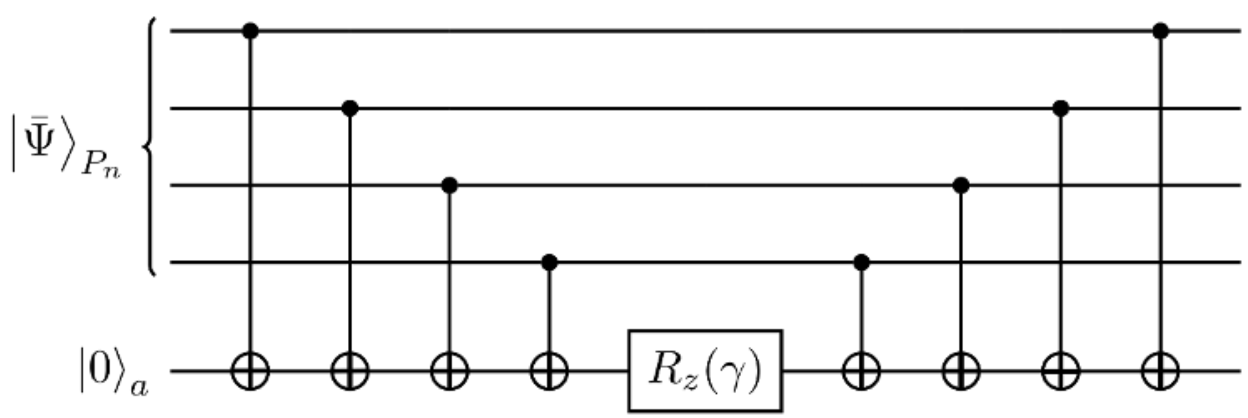
$\lambda_c = 3.04438$

Variational gauge invariant state

We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

Unitary part implementation

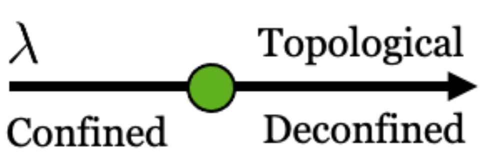
- $e^{i\alpha\hat{\sigma}^x}$ Single qubit rotations
- Circuit implementation of $e^{i\gamma_k \hat{P}_n}$



Hamiltonian variational ansatz

$$|\phi_{\mathrm{u,e}}(\boldsymbol{\alpha},\boldsymbol{\beta})\rangle = \left[\prod_{k=2}^{\ell} e^{i\alpha_k H_E} e^{i\beta_k H_B}\right] |\Omega_E\rangle$$

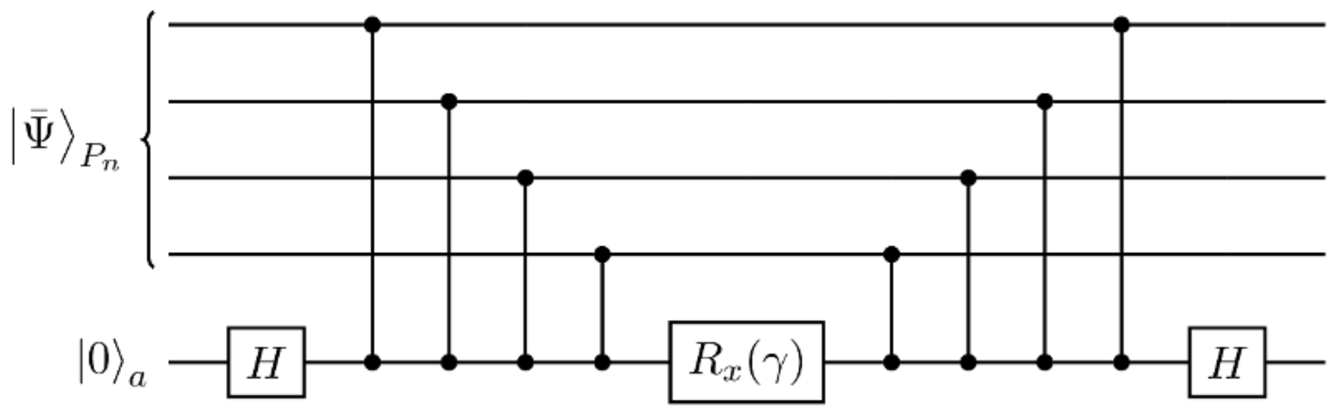
$$|\phi_{\mathrm{u,m}}(\boldsymbol{\alpha},\boldsymbol{\beta})\rangle = \left[\prod_{k=2}^{\ell} e^{i\beta_k H_B} e^{i\alpha_k H_E}\right] \left[\prod_{n} \frac{1+P_n}{\sqrt{2}}\right] |\Omega_E\rangle$$



$$\lambda_c = 3.04438$$

Modified variational gauge invariant state

$$|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\rangle = \left[\prod_{k=2}^{L} e^{i\theta_k \hat{H}'_E}\, e^{i\gamma_k \hat{H}'_B}\right]\, e^{i\alpha \hat{H}'_E}\, \frac{e^{\beta \hat{H}'_B}}{(\cosh 2\beta)^{N_p/2}} \left(\bigotimes_{n=0}^{N} |+\rangle\right)$$



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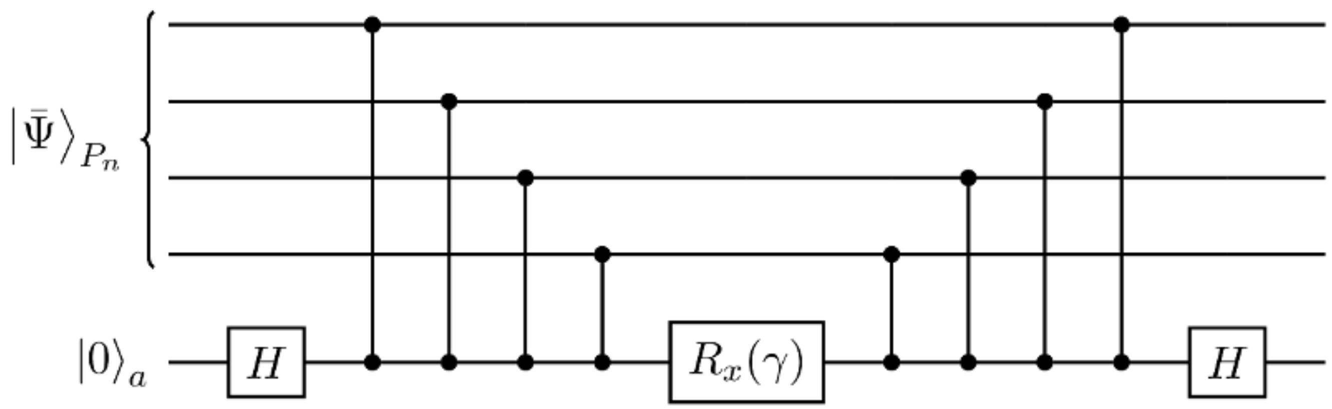
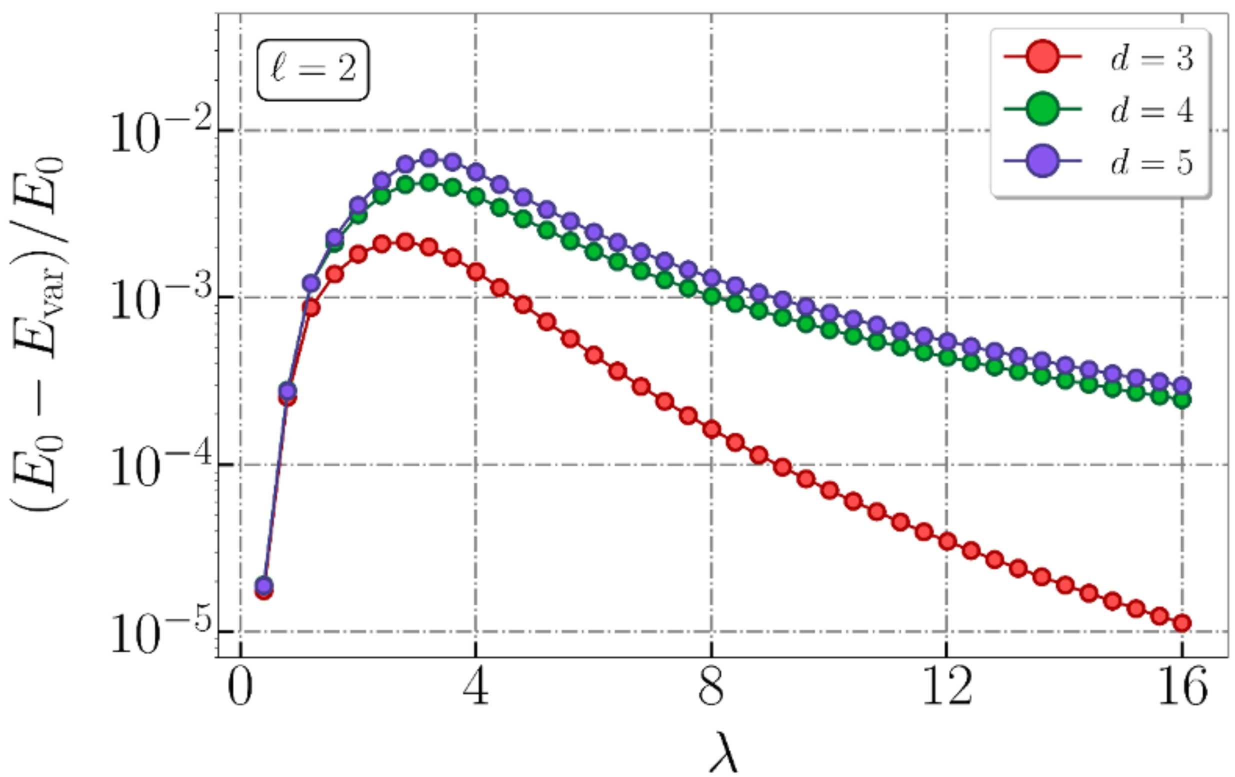
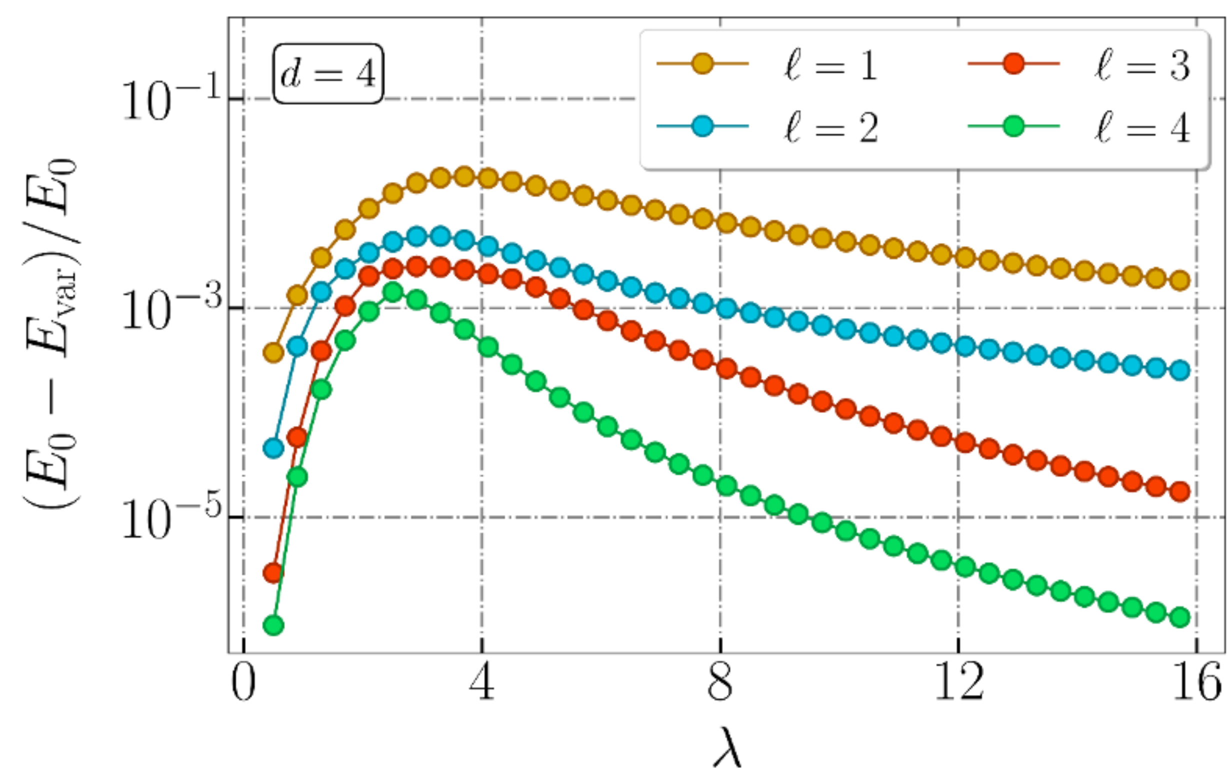


Energy deviation

- Energy difference with the exact ground state





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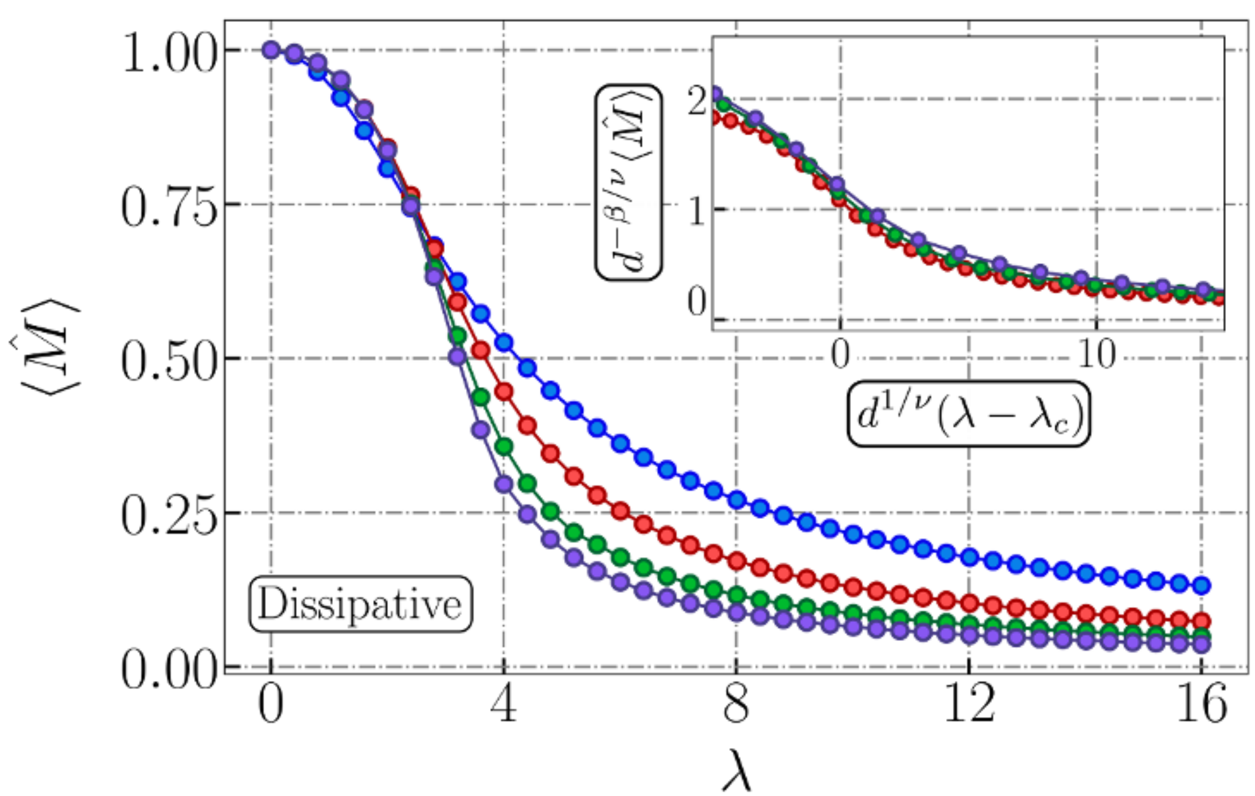
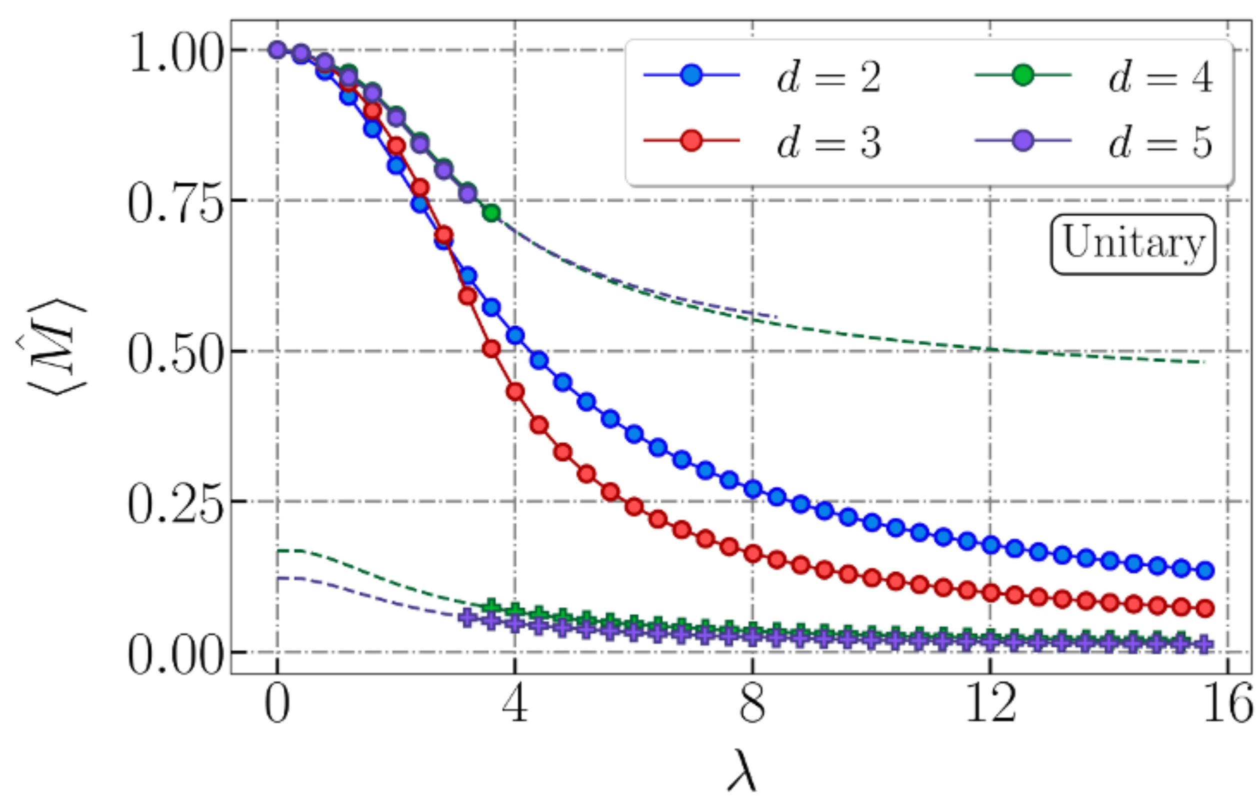
Dual magnetization

	Dissipative	Monte Carlo	Exact diag.	Unitary $\lvert\phi_{\mathrm{u,e}}\rangle$	Unitary $\lvert\phi_{\mathrm{u,m}}\rangle$
λ_c	3.24	3.04	3.06	2.56	2.09
β	0.35	0.33	0.36	0.04	-1.27
ν	0.59	0.63	0.64	-0.20	-0.40





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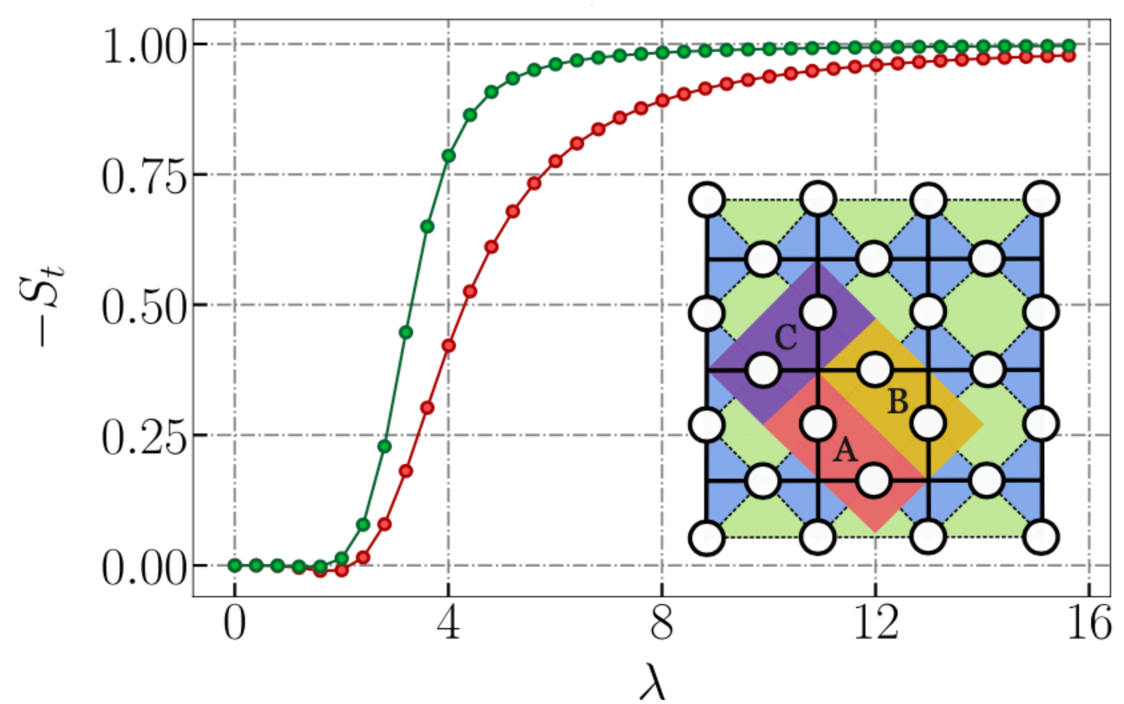
Topological entanglement entropy



$$S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

Modified variational gauge invariant state

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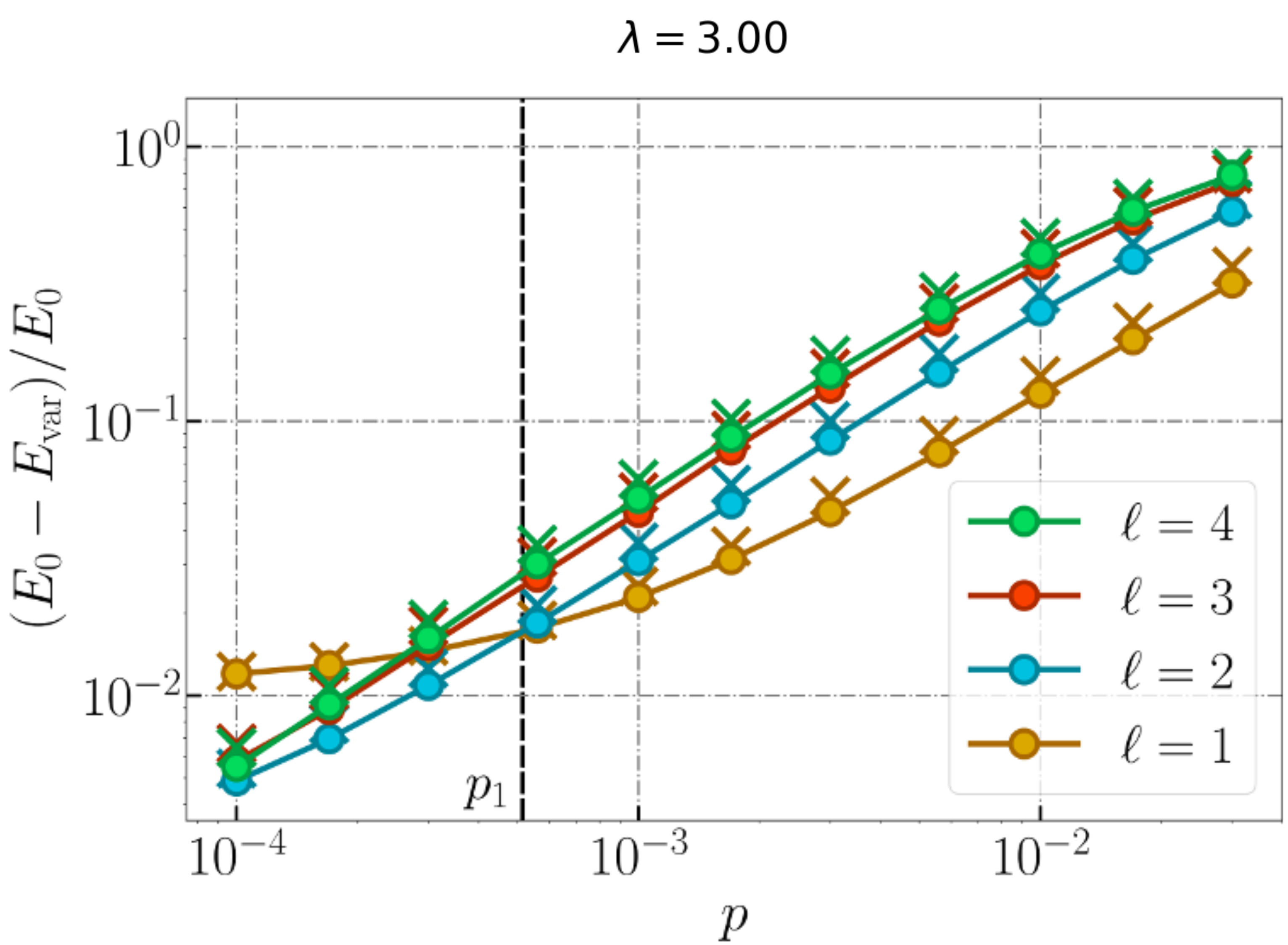
State preparation with noisy gates



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Noise-Aware Variational Eigensolvers: A Dissipative Route for Lattice Gauge Theories

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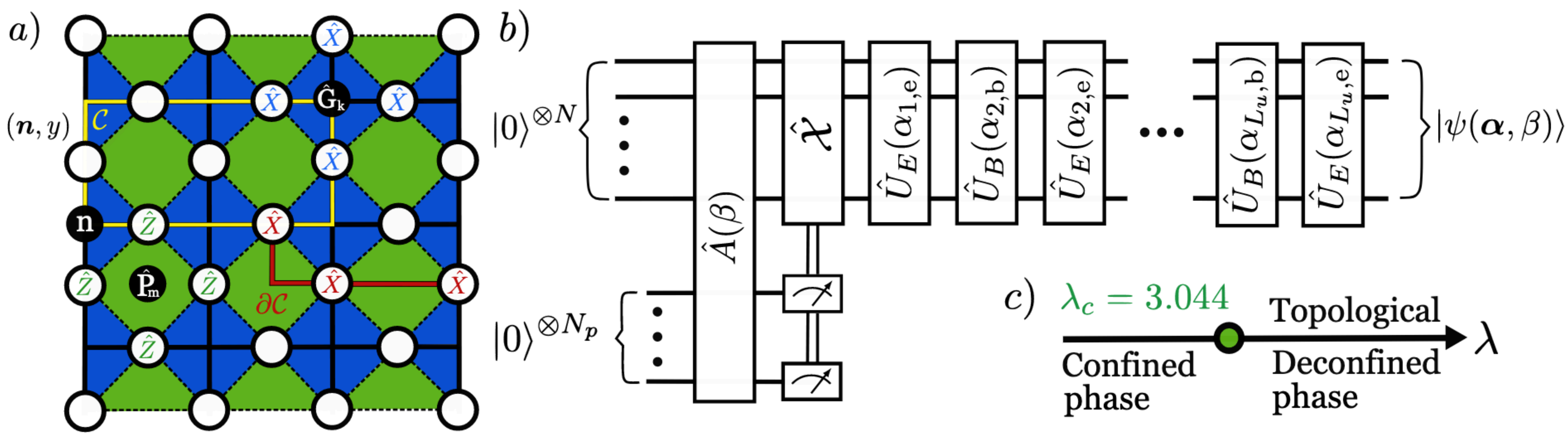
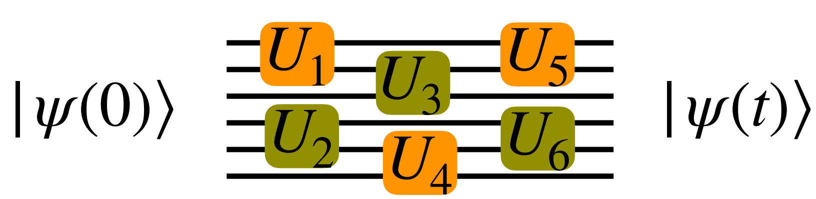


Quantum State Preparation



We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory

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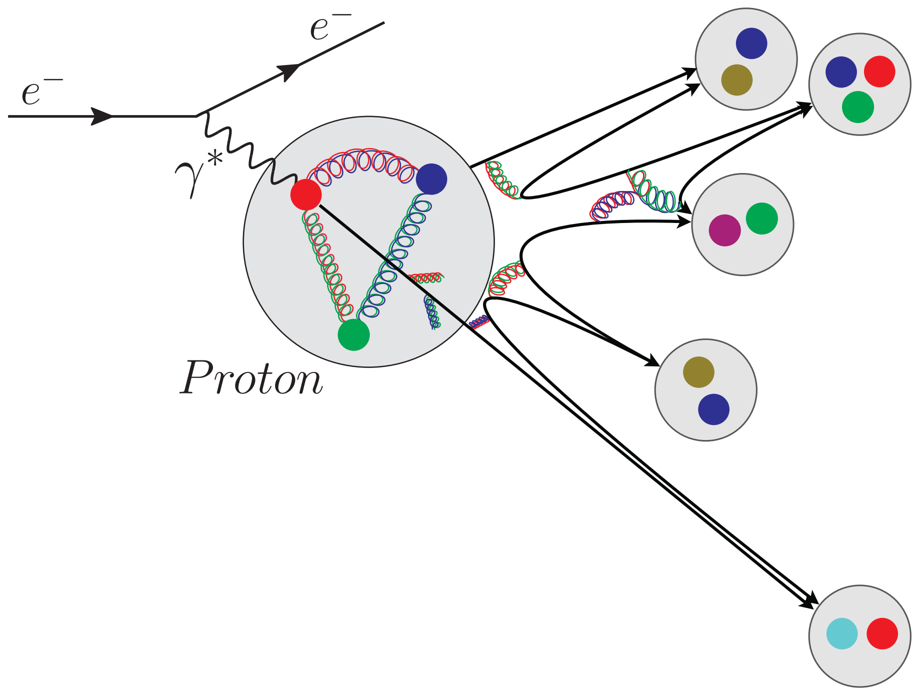
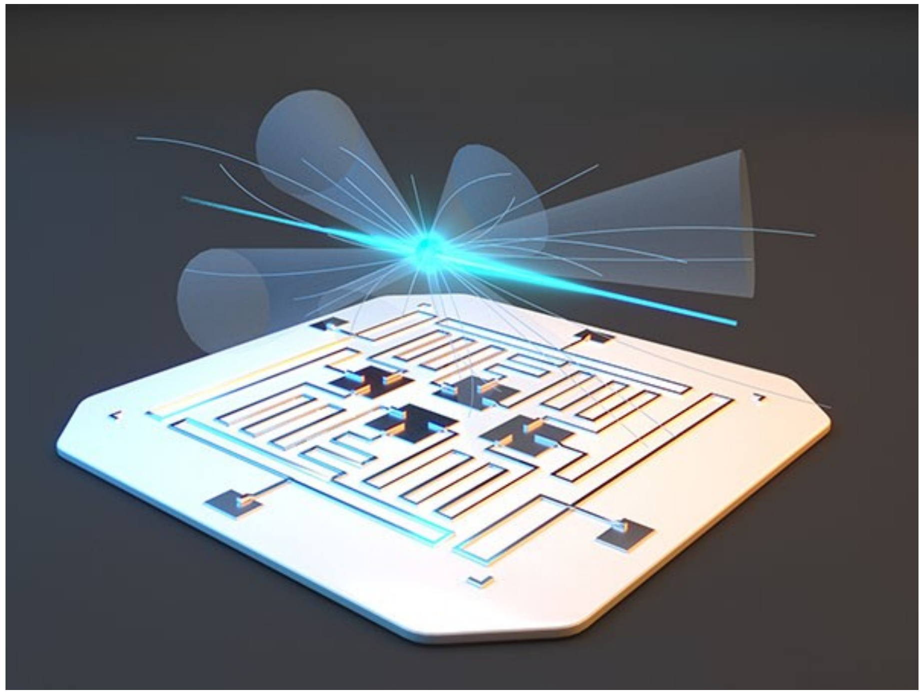
A fruitful dialogue (two-way communication)





High-Energy and Nuclear Physics

Quantum Information Science and Technology

- The first successful implementations of gauge-field theory dynamics on quantum simulators have emerged for small systems.
- Efficient Hamiltonian formulations for (non-Abelian) gauge theories along with best approaches to state preparation and measurement will continue to develop.
- Abelian and non-Abelian lattice gauge theories in higher than 1+1 dimensions present significant challenge but progress is being made.
- Theory-experiment collaborations will be highly beneficial.
- New results in the frontier between HEP and Quant-Ph