Studying the $^3P_0$ decay model from QCD in Landau gauge

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Univ. Complutense de Madrid (FJLE, ASB) & Univ. of Graz (RA)

Based on : Phys.Rev.D 109 (2024) 7, 074015

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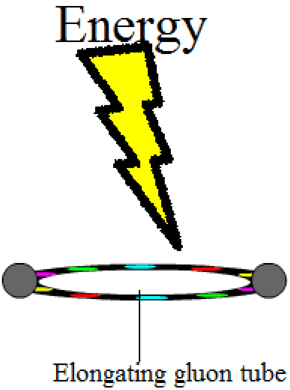
Hadron decay and String Breaking

Usual picture for hadron decay:



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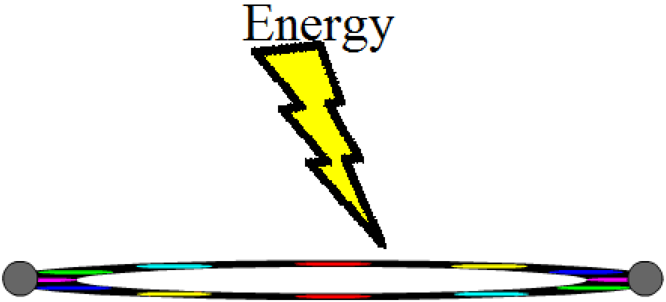

flux tube energy grows with inter-quark separation and creates a $q\bar{q}$ breaking the tube.

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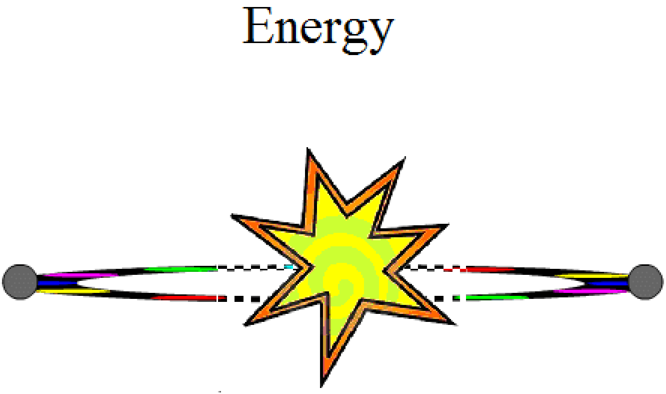

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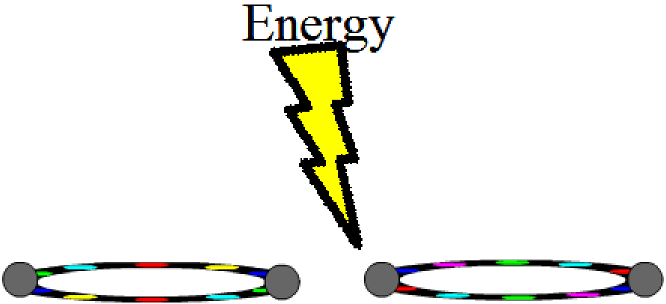

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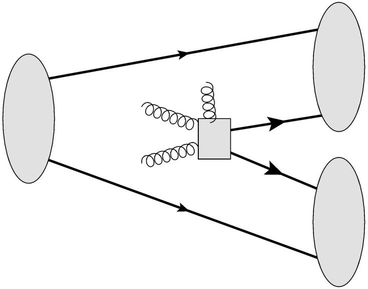
$q\bar{q}$ OZI-allowed meson decays



L. Micu, NPB 10 (1969) 521-526

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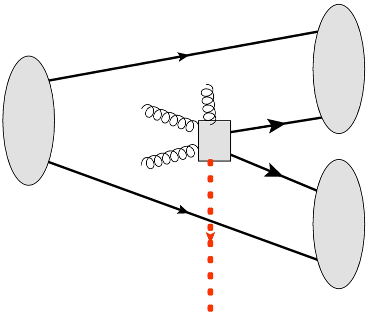

$$^1S_0, ^1P_1, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \ldots ^{2S+1}L_J$$

Possible Q# of produced pair

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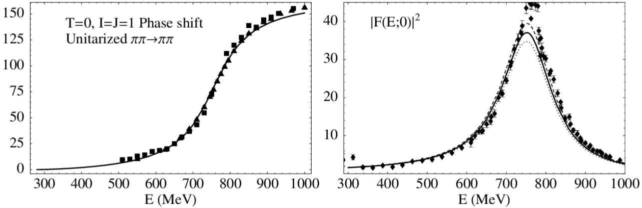

$$\cancel{{}^1S_0}, \cancel{{}^1P_1}, {}^3S_1, {}^3P_0, {}^3P_1, {}^3P_2 \ldots$$

Think of $\rho(\uparrow\uparrow) \to \pi(\uparrow\downarrow)\pi(\uparrow\downarrow)$

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

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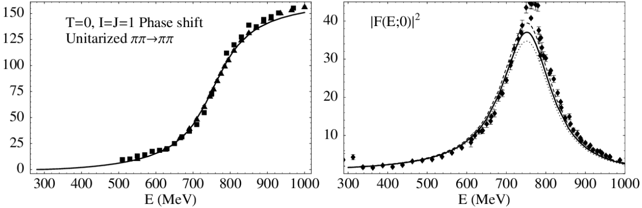
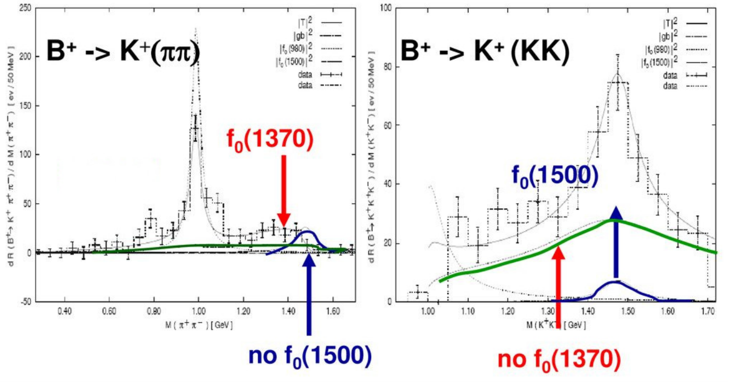


$$^1S_0, {}^1P_1, {}^3\cancel{S}_1, {}^3P_0, {}^3P_1, {}^3P_2 \ldots$$

$$\text{Think of } \rho(\text{s-wave}) \to \underbrace{\pi(\text{s-wave})\pi(\text{s-wave})}_{L=1}$$

Transition amplitude must carry a P wave.

A. Gómez-Nicola *et al.* PLB **606** 351-360 (2005)

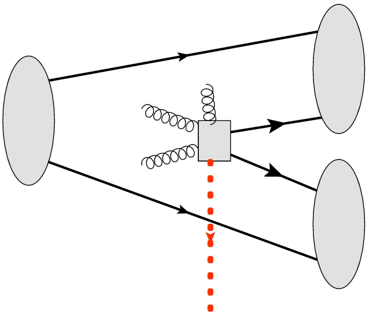


$${}^1S_0, {}^1P_1, {}^3S_1, {}^3P_0, \cancel{{}^3P_1}, \cancel{{}^3P_2} \ldots$$

Move on to $\underbrace{f_0}_{\text{mainly } {}^3P_0} \to \underbrace{\pi\pi}_{J=0}$

E. Klempt `https://slideplayer.com/slide/14648261/`

Lore: important $^3P_0$ pair production mechanism



$$^1S_0, ^1P_1, ^3S_1, ^3P_0, ^3P_1, ^3P_2 \ldots ^{2S+1}L_J$$

Possible Q# of produced pair

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Not visible in QCD (or QED)

- $\int d^3x \; \bar{\psi} \; \boldsymbol{\gamma}\cdot\mathbf{A} \; \psi$ seems ${}^3S_1$

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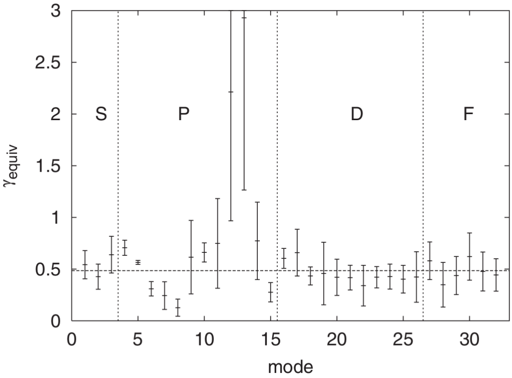
- $\int d^3x \; \bar{\psi} \; \boldsymbol{\gamma} \cdot \mathbf{A} \; \psi$ seems ${}^3S_1$
- Chiral-symmetry respecting at all orders in perturbation theory
- But ${}^3P_0$ breaks chiral symmetry

Modelling the D/D_s spectrum with $^3P_0$: 32 modes studied by Close and Swanson



F. Close and E.S.Swanson PRD**72** 094004 (2005)

Effective Hamiltonian

$$H_{^3P_0} = \sqrt{3} g_s \int d^3\boldsymbol{x} \bar{\psi}(\boldsymbol{x}) \psi(\boldsymbol{x})$$

$\gamma = \frac{g_s}{2m}$

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Chiral-symmetry breaking:

$$[Q_5, H_{^3P_0}] = \left[\int d^3\boldsymbol{x} \psi^\dagger(\boldsymbol{x}) \gamma_5 \psi(\boldsymbol{x}), H_{^3P_0} \right] \neq 0$$

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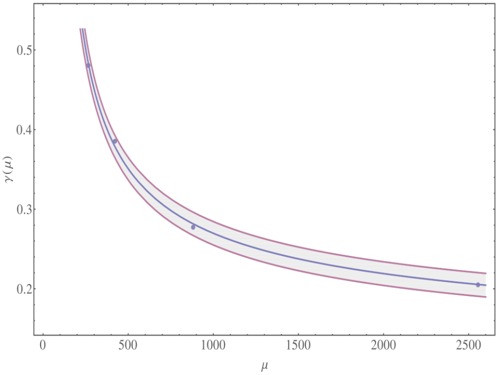
$$i(2\pi)^4\delta^{(4)}(p+q)\mathcal{M}^{ss'}_{^3P_0}(p,q) = \langle \boldsymbol{p}s, \boldsymbol{q}s' | iT_{^3P_0} | 0 \rangle =$$
$$= \left(i\,(2\pi)^4\delta^{(4)}(p+q)\right)(-\sqrt{3}g_s)\bar{u}^s(p)v^{s'}(q)$$

$$\boxed{\Rightarrow \bar{u}^s(\boldsymbol{p})v^{s'}(-\boldsymbol{p}) = 2\boldsymbol{p}\cdot\boldsymbol{\sigma}^{ss'}}$$

Dependence on the quark mass by Salamanca group



J. Segovia, D. R. Entem, F. Fernández Phys.Lett.B **715** (2012) 322-327

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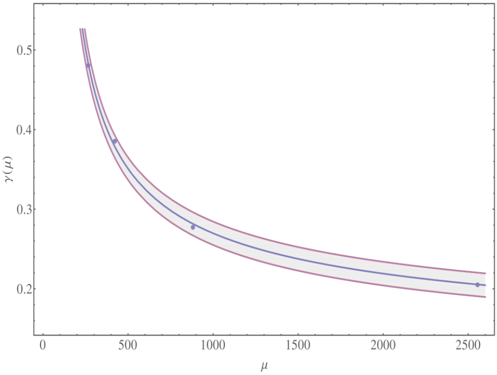

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Recent work pointing towards the direction that this sub-process may also have a measurable impact on hadron structure: 2406.05920 M. Karliner and J.L. Rosner.

Our work:
connect Quark-model pheno w. Landau gauge QCD

Can we obtain the $^3P_0$ effective Hamiltonian from *ab initio* QCD calculations?

Our work: connect Quark-model pheno w. Landau gauge QCD

Can we obtain the $^3P_0$ effective Hamiltonian from *ab initio* QCD calculations?

- N-gluon to $\bar{q}q$ kernel not known from first principles
- What to do with the information at hand?

Strategy: couple dynamical quarks to the flux tube background

$\boxed{\text{Flux tube}} \longleftrightarrow \boxed{\text{Dynamical quarks}}$

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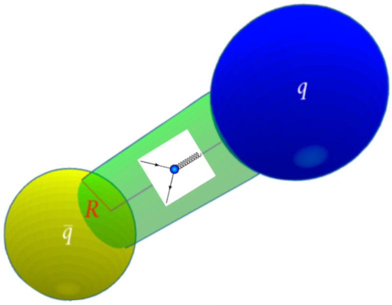
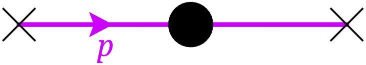
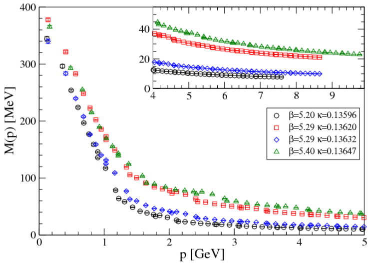
Through Landau-gauge $\boxed{\text{DSE}}$ primitive QCD Green's functions



Extensive lattice+DSE work on Landau gauge primitive Green's functions





Lattice data from O. Oliveira *et al.* Acta Phys.Polon.Supp. 9 (2016) 363-368

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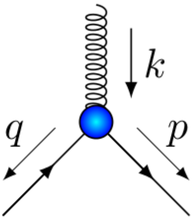

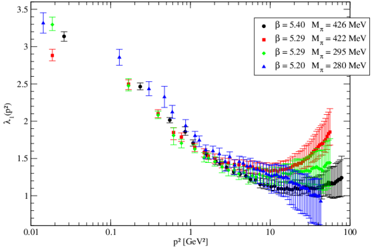

And also the pure Yang-Mills primitive Green's functions...

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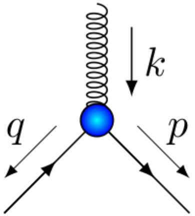
Quark-Gluon vertex spin structure



$$\Gamma^\mu_T(q_E, p_E; k_E) = \sum_{i=1}^{8} g_i(\bar{p}_E^2)\rho_i^\mu(q_E, p_E)$$

$$k^\mu = p^\mu + q^\mu$$

- It includes chiral-symmetry respecting and breaking pieces

Parametrization of transverse part of the vertex

- The tree-level vertex $\rho^{\mu}_{1,E} = (\delta^{\mu\nu} - \hat{k}^{\mu}_{E}\hat{k}^{\nu}_{E})\gamma^{\mu}_{E} \equiv \gamma^{\mu}_{T,E}$

  with $\quad g_1(x) = 1 + \frac{1.67+0.204x}{1+0.683x+0.000851x^2}$

Parametrization of transverse part of the vertex

- Chiral-symmetry breaking structures ($s_E^\mu = (\delta^{\mu\nu} - \hat{k}_E^\mu \hat{k}_E^\nu)\bar{p}_E^\nu$)

$$\rho_{2,E}^\mu = i\hat{s}_E^\mu \quad \text{and} \quad \rho_{3,E}^\mu = i\hat{\not k}_E \gamma_{T,E}^\mu$$

with $\quad g_3(x) = -1.45 g_2(x) = \frac{0.365x}{0.0187 + 0.353x + x^2}$;

Parametrization of Landau-transverse part of the vertex

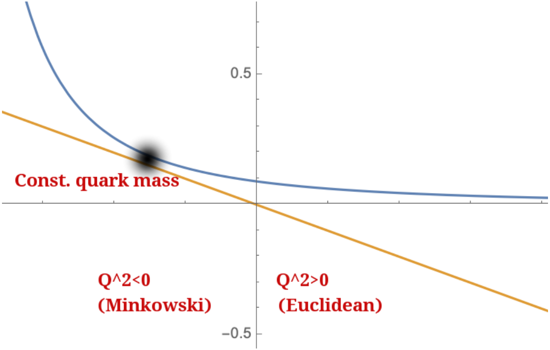
- The chirally symmetric structures

$$\rho^{\mu}_{4,E} = \hat{\not{k}}_E s^{\mu}_E \quad \text{and} \quad \rho^{\mu}_{7,E} = \hat{\not{s}}_E \hat{\not{k}}_E \gamma^{\mu}_{T,E}$$

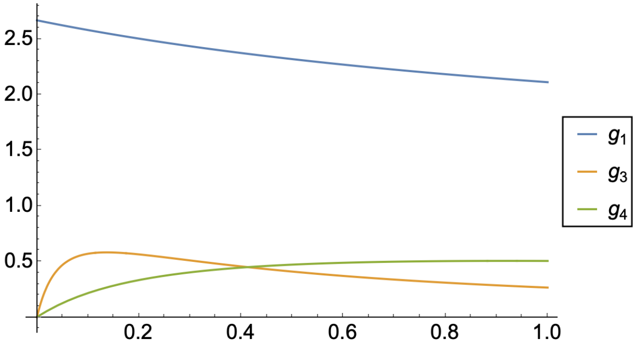
$$\text{with} \quad g_4(x) = g_7(x) = \frac{2.59x}{0.859+3.27x+x^2} \, .$$

Extension to physical Minkowski space

First, the propagator mass function:

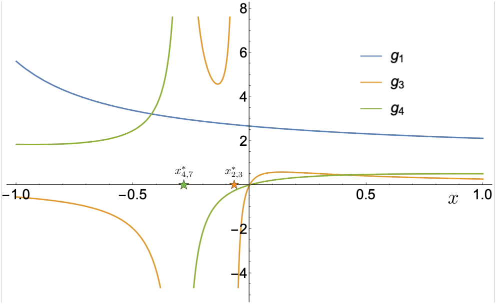


Euclidean q_E^2 functions (input from lattice, DSEs)



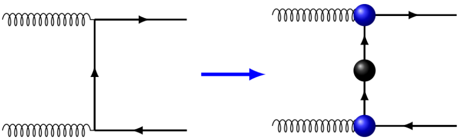
Extension to physical Minkowski space

Next, the vertex dressing form factors:



Note the $Q^2 < 0$ enhancement of the chiral symmetry breaking piece!

Breit-Wheeler process for $q\bar{q}$ creation



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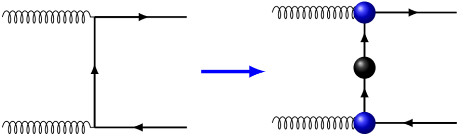
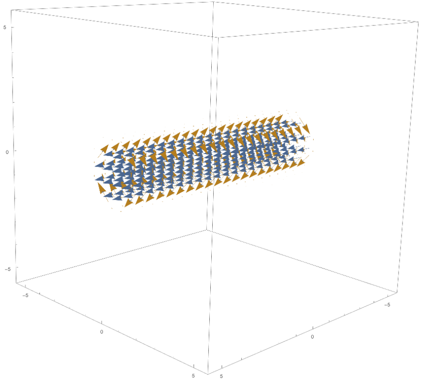


We couple flux-tube “gluons” to the quarks with the DSE functions

In a constant chromoelectric flux tube:



- Simplify to a constant chromo-E (parallel-plate capacitor)
  Background Landau-gauge field $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$

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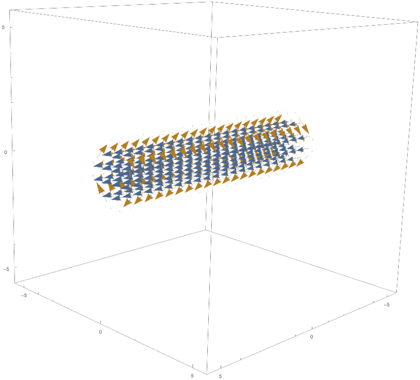

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- Think of the Schwinger pair-creation mechanism in QED.

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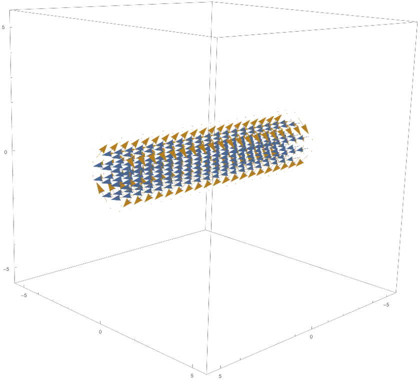

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  Background Landau-gauge field $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$
- Think of the Schwinger pair-creation mechanism in QED.
- Rotation symmetry is broken: **must use notation for diatomic molecules!**. ${}^3P_0 \rightarrow {}^3\Pi_0$.

$$\langle \boldsymbol{p}s, \boldsymbol{q}s'|iT_{\text{singlet}}|0\rangle =$$

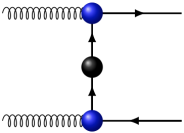
$$\langle \boldsymbol{p}s, \boldsymbol{q}s'| - \frac{g^2}{2}\int d^4x\bar{\psi}_i(x)T^a_{ij}A^a_\mu(x)\Gamma^\mu\psi_j(x)\int d^4y\bar{\psi}_i(y)T^a_{ij}A^a_\nu(y)\Gamma^\nu\psi_j(y)|0\rangle =$$

$$= -g^2\int d^4xd^4y\int\frac{d^4t}{(2\pi)^4}\tilde{A}^a_0(p-t)\tilde{A}^a_0(q+t)\mathcal{K}^{ss'}_{ab}(p,q,t)$$

where

$$\mathcal{K}^{ss'}_{ab}(p,q,t) \equiv \left[\bar{u}^s_i(p)T^a_{ij}\Gamma^0(p,-t)S(t)T^b_{jk}\Gamma^0(q,t)v^{s'}_k(q)\right]$$

and $S(t)$ is the dressed fermion propagator.



A relation between the gluon to quark kernel $\mathcal{K}$ and the pair production amplitude

$$\boxed{\langle \boldsymbol{p}s, \boldsymbol{q}s' | iT_{\text{singlet}} | 0 \rangle = -(2\pi)^4 \delta^{(4)}(p+q)(gE)^2 \left[\frac{\partial}{\partial p^3} \frac{\partial}{\partial q^3} \mathcal{K}_{ab}^{ss'}(p,q,t) \right] \Big|_{t=-q}}$$

- With the primitive Green's functions construct this skeleton kernel ✓
- Project it over ${}^{2S+1}L_J$ and numerically compare
  (But you can see that the chiral symmetry breaking part will be important, perhaps even dominant)

Angular Momentum projections

$$\mathcal{A}^{^1S_0}(|\boldsymbol{p}|) = \sum_{s} \int d\Omega\, \mathcal{A}^{ss}(\boldsymbol{p})$$

$$\mathcal{A}_i^{^3S_1}(|\boldsymbol{p}|) = \sum_{s,t} \int d\Omega\, \boldsymbol{\sigma}_i^{st} \mathcal{A}^{ts}(\boldsymbol{p})$$

$$\mathcal{A}^{^3P_0}(|\boldsymbol{p}|) = \sum_{s,t} \int d\Omega\, \hat{\boldsymbol{p}} \cdot \boldsymbol{\sigma}^{st} \mathcal{A}^{ts}(\boldsymbol{p})$$

$$\mathcal{A}_i^{^1P_1}(|\boldsymbol{p}|) = \sum_{s} \int d\Omega\, \hat{\boldsymbol{p}}_i\, \mathcal{A}^{ss}(\boldsymbol{p})\ .$$

Remember spherical symmetry is reduced to cylindrical symmetry:

$$\mathcal{A} = A\sigma_\perp \cdot \mathbf{p}_\perp + B\sigma_z p_z + C\sigma_\perp + D\sigma_z$$

$$\mathcal{A}^{^1S_0}(|\boldsymbol{p}|) = \sum_s \int d\Omega\, \mathcal{A}^{ss}(\boldsymbol{p})$$

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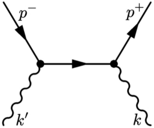
Use ${}^{2S+1}\Lambda_{j_z}$ quantum numbers.

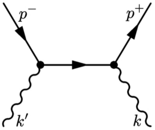
QED computation



For a $(A_\rho, A_\theta, A_z, A_0) = (0, 0, 0, -Ez)$

$$\frac{\partial}{\partial p_-^3}\frac{\partial}{\partial p_+^3}\left[\bar{u}^s(\boldsymbol{p}_+)\left(\gamma^0 - \frac{(\not{p}_+ - \not{t})}{p_+^0}\right)\frac{\not{t}+m}{t^2-m^2}\left(\gamma^0 - \frac{(\not{p}_- + \not{t})}{p_-^0}\right)v^{s'}(\boldsymbol{p}_-)\right]\Bigg|_{\boldsymbol{t}=}$$

QED computation



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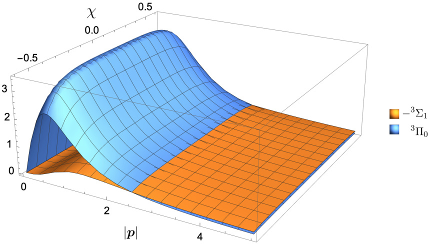
This has a $^3P_0$ contribution:

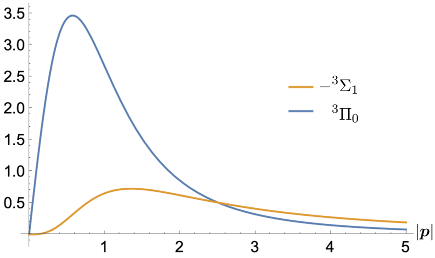
$$\mathcal{A}_{\text{QED}}^{^3\Sigma_1}(|\boldsymbol{p}|) \propto -2\pi|\boldsymbol{p}|\left(\frac{E_{\boldsymbol{p}} - m}{E_{\boldsymbol{p}}^4}\right)$$

$$\mathcal{A}_{\text{QED}}^{^3\Sigma_0}(|\boldsymbol{p}|) = 0$$

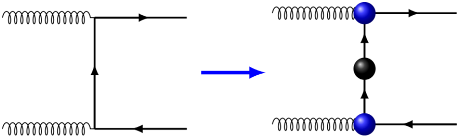
$$\mathcal{A}_{\text{QED}}^{^3\Pi_0}(|\boldsymbol{p}|) \propto \frac{32m|\boldsymbol{p}|}{3E_{\boldsymbol{p}}^4} \ . \quad \Rightarrow \frac{\mathcal{A}_{\text{QED}}^{^3\Pi_0}}{\mathcal{A}_{\text{QED}}^{^3\Sigma_1}} = O\left(\frac{m^2}{|\mathbf{p}|^2}\right)$$

QED comparison of two field insertions





In QCD: two field insertions for getting the singlet



Skeleton-expanded kernel.

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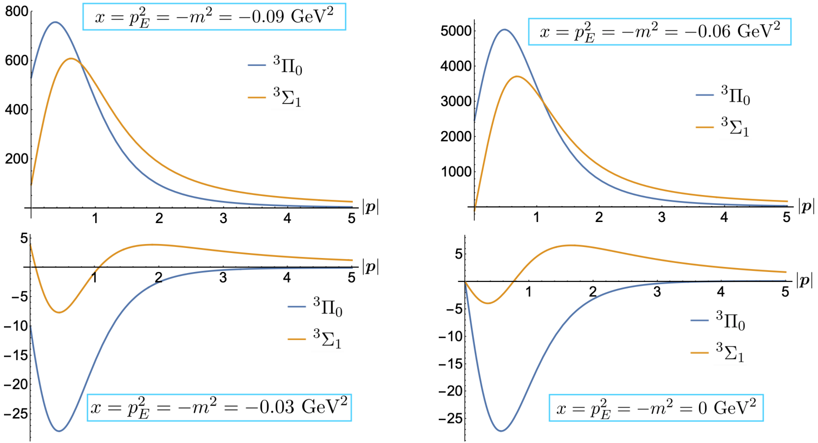
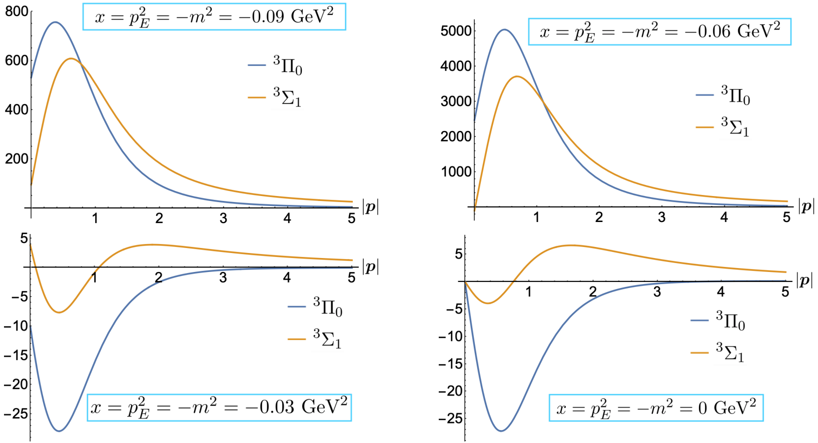


In QCD: two field insertions for getting the singlet



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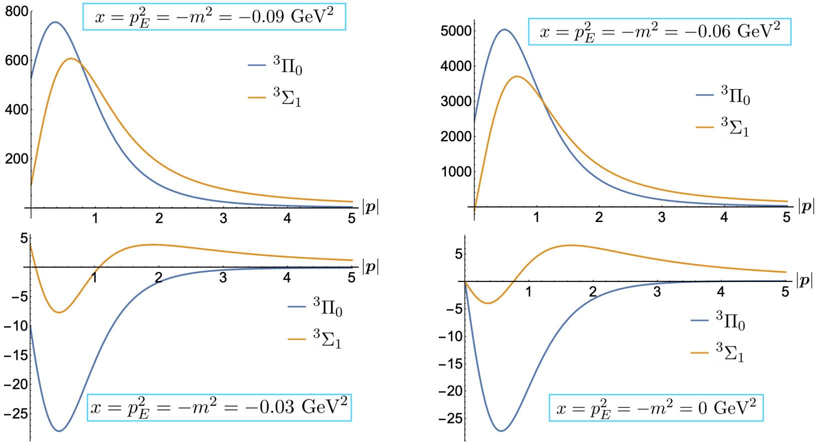
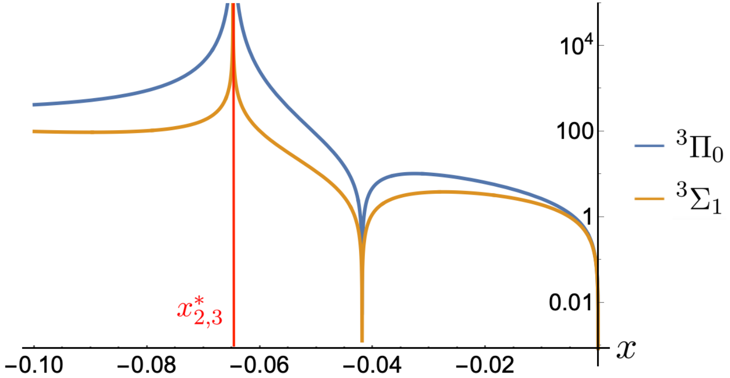


$^3\Sigma_1$ dominates at high momenta: spontaneous chiral symmetry breaking is less important. $^3P_0$ dominates at low momenta!

Threshold pair production comparison in Minkowski region



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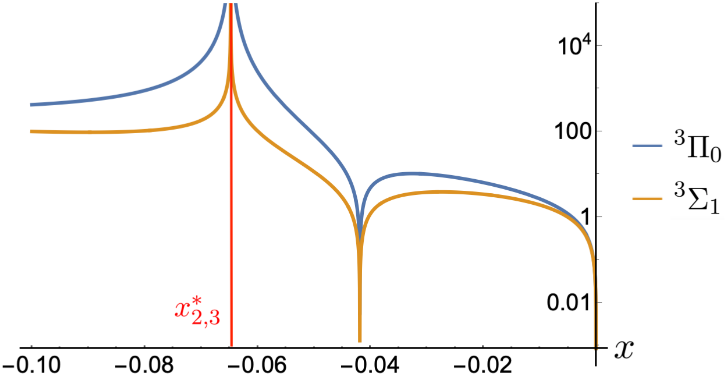

$^3\Pi_0$ **Dominance at low momenta!**

${}^3P_0$ vs. ${}^3\Pi_0$

${}^3\Pi_0$ requires $m_L = 1 \implies L \geq 1$, without restricting total J, the smallest angular momenta is ${}^3P_0$. To distinguish ${}^3P_0$ and ${}^3\Pi_0$ does not seem possible with basic meson decays, interpreted as $(q\overline{q}) \to (q\overline{q})(q\overline{q})$.

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For the parent meson, spin is $s_i = 0, 1$, internal orbital angular momentum l, and parity $P_i = (-1)^{l+1}$. The two daughter mesons in the final state have $s_f = 0, 1, 2$, internal and relative orbital l_1, l_2, L and parity (one antiparticle has been produced) $P_f = (-1)^{l_1+l_2+L+2}$, $\Rightarrow$ orbital angular momentum has to change by an odd number of units to preserve parity.

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Total angular momentum conservation implies that $\Delta J = 0$, $\Rightarrow |\Delta S| = |\Delta L|$. Because $\Delta S = 0, \pm(1, 2)$, these are the values that ΔL can take. Among them, $\Delta L = 1$ is common to both $^3P_0$ and $^3\Pi_0$, $\Delta L = 0$ to none, and only $\Delta L = 2$ could distinguish the two mechanisms.
Not allowed by parity conservation!

Recap

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- Distinguishing ${}^3P_0$ from ${}^3\Pi_0$ may not be straightforward.
- Currently extending to open color decays of hybrid mesons (qqg).

Acknowledgments

Work of FJLE done as part of the Exotic Hadrons (ExoHad) Topical Coll. This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093; grants MICINN: PID2019-108655GB-I00, PID2019-106080GB-C21 (Spain); UCM research group 910309 and the IPARCOS institute. Funded by research grant PID2022-137003NB-I00 from spanish MCIN/AEI/10.13039/501100011033/ and EU FEDER.

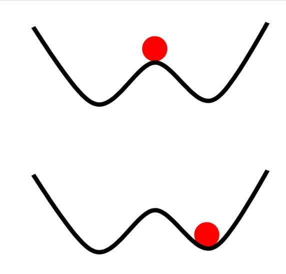
$$
\begin{aligned}
\mathcal{A}^{^3\Sigma_1}_{\rm QCD}(|\boldsymbol{p}|) \propto & \frac{-\pi}{E_{\boldsymbol{p}}^3 \left(2E_B^2|\boldsymbol{p}|^4 + \Lambda_B^4 E_{\boldsymbol{p}}^2 - |\boldsymbol{p}|^6\right)} \frac{E_B^2}{E_A^2} \Bigg(\\
& g_1^2 \; 2|\boldsymbol{p}|E_{\boldsymbol{p}} \left[E_A^2 m(Z_0-2)\Lambda_B^2 + E_{AZ}^2\left(E_B^2 E_{\boldsymbol{p}} + m|\boldsymbol{p}|^2\right) - m|\boldsymbol{p}|^2(Z_0-1)\Lambda_B^2\right] \\
& + g_2^2 m|\boldsymbol{p}|E_{\boldsymbol{p}} \left[E_A^2\left(E_B^2(Z_0+1) - |\boldsymbol{p}|^2\right) - E_B^2|\boldsymbol{p}|^2(Z_0-1)\right] \\
& + g_3^2 \; 2|\boldsymbol{p}|E_{\boldsymbol{p}} \left(2E_A^2\Lambda_B^2 m + E_{AZ}^2 E_B^2 m + E_{AZ}^2 E_B^2 E_{\boldsymbol{p}}\right) \\
& + g_4^2 m|\boldsymbol{p}|E_{\boldsymbol{p}} \left(E_A^2\Lambda_B^2 - E_{AZ}^2 E_B^2\right) \\
& + g_7^2 \; 2m|\boldsymbol{p}|E_{\boldsymbol{p}} \left(-E_A^2\Lambda_B^2 - 2E_{AZ}^2 E_B^2\right) \\
& + g_1 g_2 \; 2E_{\boldsymbol{p}} [\Lambda_A^2 E_B^2|\boldsymbol{p}|^2 Z_0 + mE_A^2\Lambda_B^2(E_{\boldsymbol{p}} - m) + E_B^2|\boldsymbol{p}|^4] \\
& + g_1 g_3 \; 4|\boldsymbol{p}|E_{\boldsymbol{p}} E_{AZ}^2 E_B^2 \left(m + E_{\boldsymbol{p}}\right) \\
& + g_1 g_7 \; 2E_{\boldsymbol{p}} \left[E_A^2\left(m|\boldsymbol{p}|^2\left(m+3E_{\boldsymbol{p}}\right) - E_B^2\left(\left(m^2 - |\boldsymbol{p}|^2 Z_0\right) + 3mE_{\boldsymbol{p}}\right)\right) - E_B^2|\boldsymbol{p}|^4(Z_0-1)\right] \\
& + g_3 g_7 \; 2E_{\boldsymbol{p}} \left(3E_A^2 m|\boldsymbol{p}|^2(E_{\boldsymbol{p}}+m) + 3E_{AZ}^2 E_B^2|\boldsymbol{p}|^2 - 3E_A^2 E_B^2 m^2 - mE_A^2 E_B^2 E_{\boldsymbol{p}}\right) \\
& + g_4 g_7 \; 2m|\boldsymbol{p}|E_{\boldsymbol{p}} \left(E_{AZ}^2 E_B^2 - E_A^2 E_B^2 + E_A^2|\boldsymbol{p}|^2\right) \\
& + g_2 g_3 \; 2\left(-E_A^2\Lambda_B^2 m(E_{\boldsymbol{p}} - m) - \Lambda_A^2 E_B^2|\boldsymbol{p}|^2 Z_0 - E_B^2|\boldsymbol{p}|^4\right) \\
& + 2g_2 g_7 m|\boldsymbol{p}|E_{\boldsymbol{p}} \left(E_A^2\left(E_B^2(Z_0+1) - |\boldsymbol{p}|^2\right) - E_B^2|\boldsymbol{p}|^2(Z_0-1)\right) \Bigg)
\end{aligned} \tag{1}
$$

Amplitudes 2

$$(2)$$

$$
\begin{aligned}
\mathcal{A}^{^3\Pi_0}_{\mathrm{QCD}}(|\boldsymbol{p}|) \propto & \frac{-16/3}{E_{\boldsymbol{p}}^2\left(-2E_B^2|\boldsymbol{p}|^4 - \Lambda_B^4 E_{\boldsymbol{p}}^2 + |\boldsymbol{p}|^6\right)} \frac{E_B^2}{E_A^2}\Bigg(\\
& g_1^2\ m|\boldsymbol{p}|\left(E_A^2\left(E_B^2(Z_0-3)+3|\boldsymbol{p}|^2\right) - E_B^2|\boldsymbol{p}|^2(Z_0-1)\right) \\
& + g_2^2\ m|\boldsymbol{p}|\left(E_A^2\left(E_B^2(Z_0+1)-|\boldsymbol{p}|^2\right) - E_B^2|\boldsymbol{p}|^2(Z_0-1)\right) \\
& + g_3^2\ m|\boldsymbol{p}|\left(E_A^2\left(E_B^2(Z_0+3)-3|\boldsymbol{p}|^2\right) - E_B^2|\boldsymbol{p}|^2(Z_0-1)\right) \\
& + g_4^2\ m|\boldsymbol{p}|\left(E_B^2|\boldsymbol{p}|^2(Z_0-1) - E_A^2\left(E_B^2(Z_0-1)+|\boldsymbol{p}|^2\right)\right) \\
& - g_7^2\ m|\boldsymbol{p}|\left(E_A^2\left(3Z_0E_B^2+\Lambda_B^2\right) + 3|\boldsymbol{p}|^2E_B^2(1-Z_0)\right) \\
& + g_1g_2\ 2\left(-m^2E_A^2\Lambda_B^2 + |\boldsymbol{p}|^2\Lambda_A^2E_B^2Z_0 + |\boldsymbol{p}|^4E_B^2\right) \\
& + g_1g_3\ 2|\boldsymbol{p}|E_{\boldsymbol{p}}E_{AZ}^2E_B^2 \\
& - g_1g_7\ 4mE_{\boldsymbol{p}}E_A^2\Lambda_B^2 \\
& - g_2g_3\ 2mE_{\boldsymbol{p}}E_A^2\Lambda_B^2 \\
& + g_3g_7\ 4\left(-m^2E_A^2\Lambda_B^2 + |\boldsymbol{p}|^2\Lambda_A^2E_B^2Z_0 + |\boldsymbol{p}|^4E_B^2)\right) \\
& + g_4g_7 2m|\boldsymbol{p}|\left(\Lambda_A^2E_B^2(Z_0-1) + |\boldsymbol{p}|^2E_A^2\right)\Bigg)\,. \qquad (3)
\end{aligned}
$$

Chiral symmetry breaking



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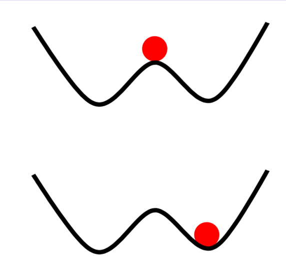

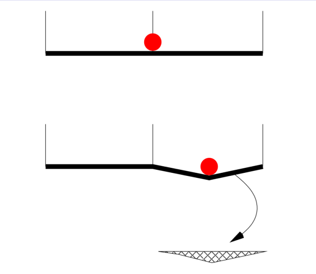

R. Alkofer *et al.* Annals Phys. **324** (2009) 106-172

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List of $S=0$ quantum numbers:

L	$J^{P(C)}$	
0	$0^{-(+)}$	π, η, $K\dots$
1	$1^{+(-)}$	h_1, b_1, $\dots$
2	$2^{-(+)}$	π_2, η_2 $\dots$
3	$3^{+(-)}$	h_3, b_3, $\dots$
$\dots$		