Quark pairing in heavy ion collisions

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fQCD collaboration :
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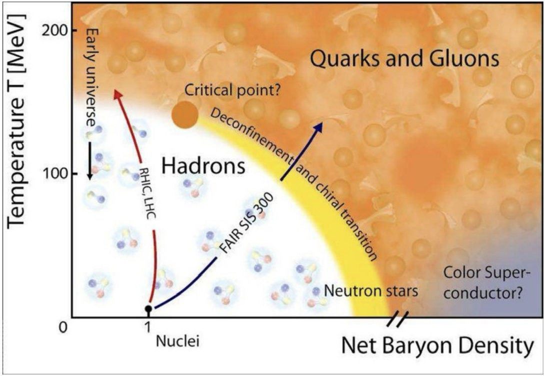
QCD in heavy ion collisions:

Roughly speaking, a transition between hadrons and asymptotic quarks/gluons.

- Transitions coming from the mass scale change of quark and gluon:
  Quark mass generation=Chiral PT
  Gluon mass generation ≈ Deconfinement
- Crossover at low density connected with a critical end point (CEP)
- Fine structures to classify phases like chiral spin symmetric phase, inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity phase



fQCD approaches:
Schwinger-Dyson equations and functional Renormalization group approach

The principle of fQCD: We don't do models, we do simplification.

QCD in vacuum:

Cyrol, Mitter, Pawlowski, Strodhoff, PRD 97 (2018) 5, 054006.

Binosi, Chang, Papavassiliou, Qin, Roberts, PLB 742, (2015) 183

Williams, Fischer, Heupel, PRD 93, (2016)034026.

Mitter, Pawlowski, Strodthoff, PRD 91, (2015)054035.

Qin, Chang, Liu, Roberts, Schmidt, PLB 722 (2013) 384

Chang, Roberts, PRL 106 (2011) 072001 ...

Yang-Mills sector:

Eichmann, Pawlowski, Silva, PRD 104 (2021) 11, 114016

Aguilar, Ferreira, Papavassiliou, PRD 105 (2022) 1, 014030

Huber, PR 879, 1 (2020)

Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Aguilar, Binosi, Papavassiliou, PRD 86 (2012) 014032 ...

Phase Structure:

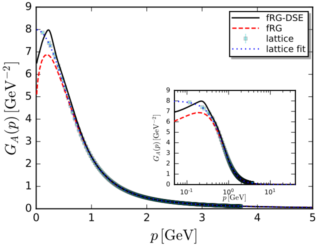
Fu, Pawlowski, Renneke, PRD 101 (2020) 5, 054032; Gao, Chen, Liu, Roberts, Schmidt, PRD 93 (2016) 9, 094019; Fischer, PPNP 105,(2019)1;Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 Isserstedt, Buballa, Fischer, PRD 100 (2019) 7, 074011; Qin, Chang, Chen, Liu, Roberts, PRL106 (2011) 172301...

The minimal requirements for a truncation scheme that describes QCD:

- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*

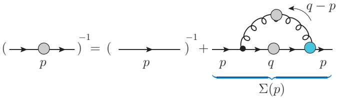
The minimal scheme

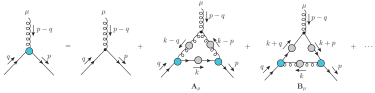
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.

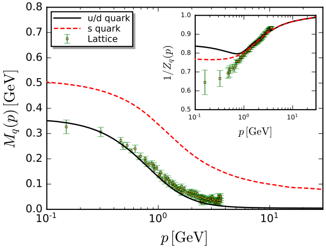


Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Solve the DSEs of quark propagator and quark gluon vertex:







lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

A further simplification on the quark gluon vertex:

Quark gluon vertex In Landau gauge:

$$\Gamma^{\mu}(q,p) = \sum_{i=1}^{8} t_i(q,p) P^{\mu\nu}(q-p) \mathcal{T}_i^{\nu}(q,p) \,,$$

The dominant structures are Dirac and Pauli term:

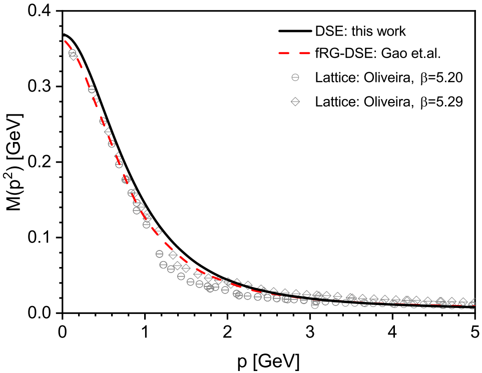
$$\mathcal{T}_1(p,q) = -i\gamma^{\mu} \,, \mathcal{T}_4^{\mu}(p,q) = \sigma_{\mu\nu}(p-q)^{\nu} \,,$$

$$t_1(p,q) = F(k^2) \frac{A(p^2) + A(q^2)}{2}$$

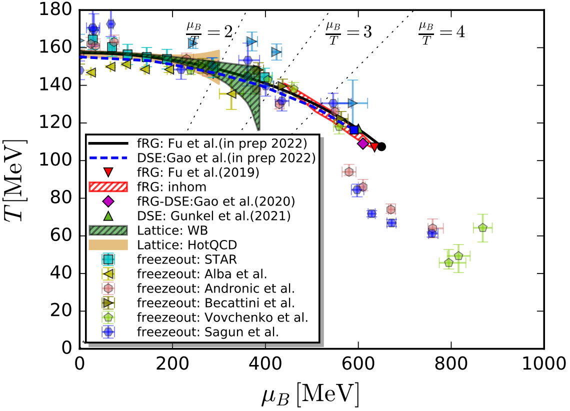
$$t_4(p,q) = \left[Z(k^2)\right]^{-1/2} \frac{B(p^2) - B(q^2)}{p^2 - q^2}$$

FG, J. Papavassiliou, J. Pawlowski, PRD 103.094013 (2021).
Y, Lu, **FG**, YX Liu, J. Pawlowski, arXiv:2310.16345.

All quantities are expressed by the running of two point functions.
The Quark Mass function:



Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

- $T_c = 155\,\text{MeV}$ and $\kappa \sim 0.016$
- Estimated range of CEP: $T \in (100, 110)$ MeV $\mu_B \in (600, 700)$ MeV
- $\sqrt{s_{\text{NN}}} \approx 3 - 5\text{GeV}$

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and J. Pawlowski, PRD 102, 034027 (2020)
FG and J. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

Deconfinement phase transition

A direct measurement of deconfinement is Polyakov loop:

- Reflects the $Z(N_c)$ center symmetry
- Stands for a nontrivial stationary point in gauge field potential.
- Related to the gluon mass scale (L Fister, J. Pawlowski, PRD 88, 045010 (2013)).

However, there might be some different scenaria for deconfined phase:

- Quasi quarks that breaks center symmetry
- The quark is confined into colored bound states (Diquark) , with no asymptotic quarks but still breaks the symmetry(Partial deconfinement).

The diquark/quark pairing in the deconfined phase is an additional characteristic property.

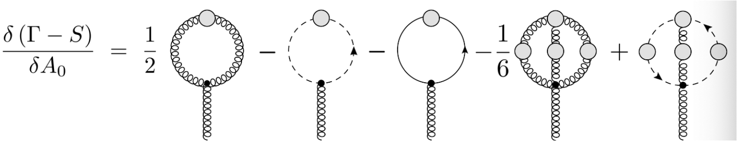
Deconfinement phase with Polyakov loop

Polyakov loop in background field approach is related to A_0^a condensate as:

$$\mathcal{L}(A_0) = \frac{1}{N_c} \mathrm{tr}\, \mathcal{P} e^{ig\int dx_0 A_0} = \frac{1}{3}\left[1 + 2\cos\left(g\beta A_0/2\right)\right]$$

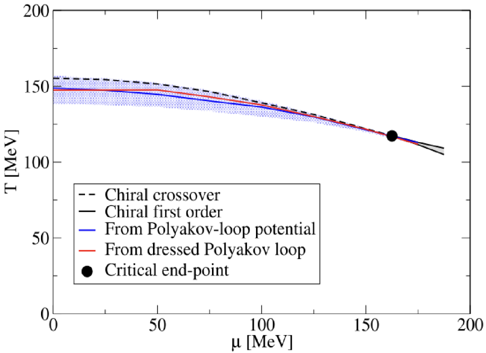
A_0^a condensate is equivalent to the colored imaginary chemical potential: Center symmetry is a symmetry for the shift in the imaginary time axis which is thus related to the pole structure of propagators.

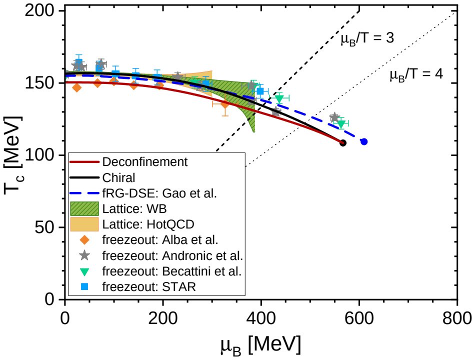
One can obtain A_0 condensate by solving the The DSE of A_0^a as $\frac{\delta(\Gamma - S_A)}{\delta A_0} = 0$.
The diagrammatic representation is:



Deconfinement phase diagram

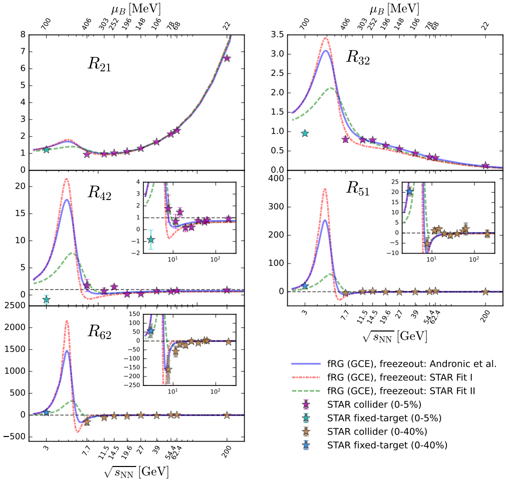
The deconfinement characterized by Polyakov loop is in agreement with Chiral PT with the same CEP location.





C. S. Fischer et al, PLB 732, 273(2014); Yi Lu, FG, J. Pawlowski, Yuxin Liu, in preparation

The thermodynamic quantities in fQCD

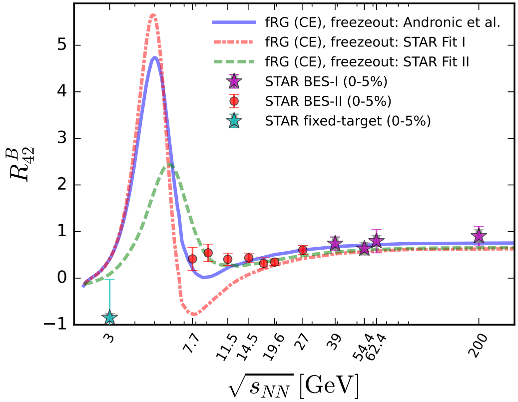


W. Fu, X. Luo, J. Pawlowski, F. Rennecke, S. Yin, arXiv:2308.15508.

$$R_{nm} = \frac{\chi_n^B}{\chi_m^B}$$

- Good agreements between R_{32} and R_{21} for $\sqrt{s_{NN}} \geq 11.5$ GeV.
- For lower col. energy, the non-equilibrium and finite volume effect on R_{32} and R_{21} becomes sizable.
- No wiggling behavior/peak structure in the BES energy range in R_{42} from fQCD since the CEP is at $\sqrt{s_{NN}} \approx 3 - 5$ GeV.

The comparisons of the cumulants between fQCD and BESII



W. Fu, X. Luo, J. Pawlowski, F. Rennecke, S. Yin, arXiv:2308.15508.

CEP estimation from fQCD:
$\mu_B \in (600 - 700)$ *MeV*
$\sqrt{s_{NN}} \approx 3 - 5$ *GeV*

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and Pawlowski, PRD 102, 034027 (2020)
FG and Pawlowski, PLB 820, 136584(2021)
Yi Lu et al, arXiv:2310.18383 (2023)
Huiwen Zheng et al, arXiv:2312.00382 (2023).
...

Chiral PT and the measured cumulants

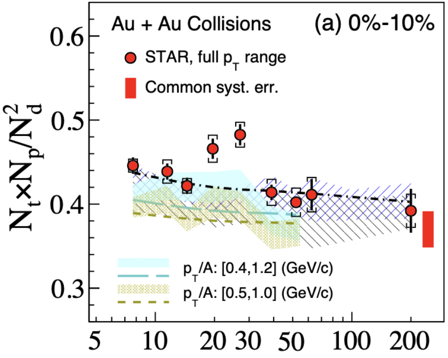
Consistences:

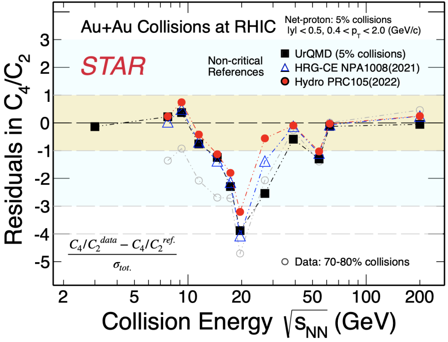
- The measured cumulants in BESII are consistent with the computation from fQCD which estimates the CEP of Chiral pT at $\sqrt{s_{\mathrm{NN}}} \approx 3 - 5$ GeV;
- The measurements are then also consistent with the fact that the CEP of Chiral PT is at $\sqrt{s_{\mathrm{NN}}} \geq 7.7$ GeV, and hence no critical behavior of Chiral PT has been observed in RHIC.

However, finer comparison reveals some inconsistencies at $\sqrt{s_{\mathrm{NN}}} \approx 20 - 30$ GeV:

- Triton yield ration $N_t \times N_p / N_d^2$ even from BESI (STAR, PRL 130, 202301(2023));
- Kurtosis in BESII deviates from UrQMD, HRG and Hydro simulations.

Finer structure in the measurement





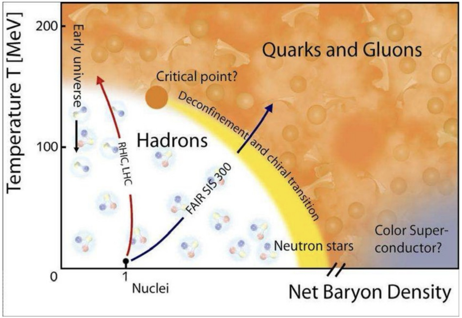
- Density Inhomogeneity from first order phase transition can enhance the Triton yield ratio (Kaijia Sun et al, arXiv: 2205.11010) and also affects the fluctuations.
- Cannot be Chiral and deconfinement PT because the col. energy is too large.

If something happens at $\sqrt{s} \sim 20 - 30$ GeV, what would it be?

A deeper look to the phases above T_c

Besides of these phase transitions, QCD has rich phases above chiral phase transition T_c:

- Strongly coupled quark gluon plasma (sQGP) at low μ
- The counterparts of sQGP at large μ: inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity (CSC) phase
- enhanced chiral spin symmetry, small viscosities, exotic behaviors of thermodynamic quantities....



How the sQGP evolves into CSC phase with increasing μ?
Is there another phase transition there?

The conventional CSC phase can only exist at high chemical potential and hence low temperature.

The Cooper pair Δ in conventional CSC is generated through the gap equation as:

$$\Delta = g^2 T \sum_m \int d^3\vec{q} \frac{\Delta}{q^2} G(p-q)$$

This type of propagator gives a gap that is proportional to chemical potential μ as $\Delta \sim \mu\, e^{-\frac{\mathrm{const}}{g}}$ in weak coupling limit. (D. Son, PRD 59, 094019 (1999); R. Pisarski, D. Rischke, PRD 61, 074017 (2000))
The conventional CSC in QCD is based on the Abelian approximation (bare or BC type vertex)and thus obtains the same type of pairing as in QED.

M. G. Alford, et al, RMP 80, 1455 (2008); D. Nickel, et al, PRD 73, 114028 (2006)

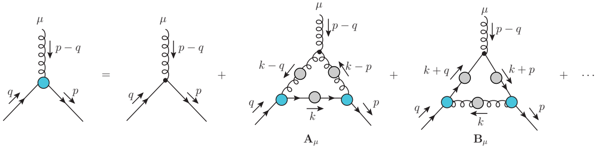
To study the quark pairing in QCD, one needs to compute the gap equation i.e. the quark propagator Schwinger-Dyson equation, in the Nambu-Gorkov basis. It is to extend the fermion field as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C),$$

$$\mathbf{S}^{-1}(p) = \mathbf{S}_0^{-1}(p) + \Sigma(p), \qquad \Sigma(p) = \int_q g^2 G^{aa'}_{\mu\nu}(q-p)[\Gamma^{(0)}_\mu]^a \mathbf{S}(q)\, \Gamma^{a'}_\nu(q,p)$$

The quark gluon vertex is essential input for the quark pairing gap equation.

In vertex DSE, diagram A is non Abelian diagram and diagram B is the Abelian diagram similar to QED.



FG, Y Lu, YX Liu, arXiv:2403.16816

The quark gluon vertex in NG basis

$$\Gamma_\nu^a = \begin{pmatrix} \Gamma_{\nu+}^a, & \Xi_{\nu-}^a \\ \Xi_{\nu+}^a, & \Gamma_{\nu-}^a \end{pmatrix},$$

$$\Gamma_{\nu,+}^a(p,q) = \frac{\lambda^a}{2} F(q-p)\gamma_\nu$$

$$\Xi_{\nu,+}^a(p,q) = \frac{K^a}{2} t_4(q-p)\sigma_{\mu\nu}(p-q)_\mu$$

Solving the quark gluon vertex and obtaining t_4.

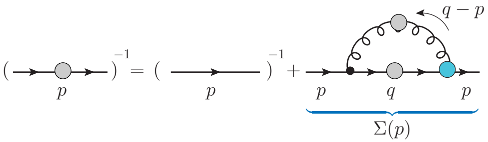
With diagram A:

- In ultraviolet region with for instance $p \to 0$, the term $Z_1\Delta$ is dominant as Z_1 is proportional to $1/p^2$ and leads to the coefficient of quark gluon vertex $t_4 \sim \Delta/p^2$.
- In the infrared limit, Z_1 and Z are finite constants. Considering Δ to be small, the two solutions become $t_4 = Z_1\Delta$ and $t_4 = \frac{1}{Z\Delta}$.

With diagram B:

- one only gets the first solution $t_4 \propto \Delta$.

The dynamics related to diagram A is very different from that from diagram B.



$$\mathbf{S} = \begin{pmatrix} S_+, & S_+\gamma_5\mathcal{M}S_-\Delta^* \\ S_-\gamma_5\mathcal{M}S_+\Delta, & S_- \end{pmatrix},$$

Putting the coefficient of vertex t_4 into off diagonal part of gap equation:

- With $t_4 \propto \frac{1}{\Delta}$, the gap equation becomes: $\Delta \propto \int \frac{Z}{k^2\Delta} G(\bar{k}^2)$

For $Z > 0$, a finite solution for Δ; For $Z < 0$, the trivial solution as $\lambda_4 = \Delta = 0$.

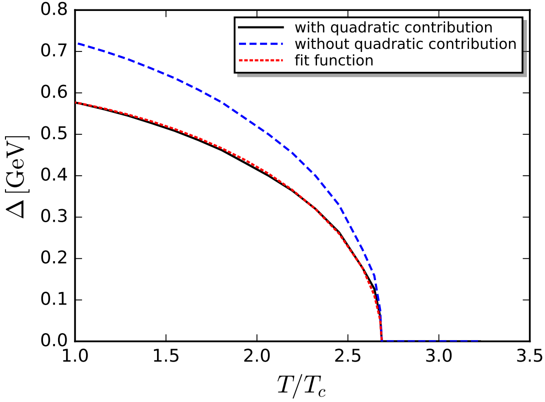
- With $t_4 \propto \Delta$, the gap equation becomes: $\Delta \propto \int_k \frac{\Delta}{k^2} G(\bar{k}^2)$

which gives the conventional CSC gap and proportional to chemical potential μ.

A new type of pairing at zero chemical potential

The pairing can be expanded as:

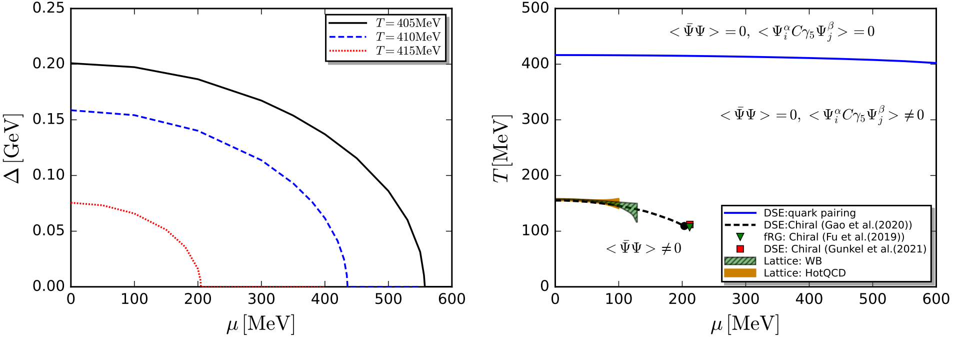
$$\Delta \propto \frac{3}{2}\langle g^2 A^2 \rangle - \frac{3}{2}\langle g^2 \frac{k_4^+ p_4^+}{k_+^2}(G_L(\bar{k}^2) + 2G_T(\bar{k}^2))\rangle,$$



- The quark pairing gap is related to the dimensional 2 gluon condensate and thus dominant by the glue dynamics.
- A second order phase transition at temperature T_Δ, as one has $\Delta = 0$ above T_Δ, and near below T_Δ:

$$\Delta^2 \propto 1 - (T/T_\Delta)^a,$$

  with the best fit as $a = 2.16$.
- The relation then yields a mean field critical exponent as $\beta = 1/2$.



The pairing phase in $T-\mu$ plane:

- Represents a color deconfined phase above the chiral phase transition;
- Quarks are confined into colored bound states as a partial deconfined phase;
- Temperature range $T \in [T_c, T_\Delta \approx 2-3\,T_c]$, overlaping with Chiral Spin Symmetric phase and the other conjectured strongly coupled states in sQGP.

Combination of fQCD computations and BES measurements

The conclusions and discussions:

- The chiral PT and deconfinement is with a CEP at $\mu_B \approx 600 - 700$ MeV/ $\sqrt{s_{\rm NN}} \approx 3 - 5$ GeV, which is consistent with the current measurements in BESII.
- The deviation of the measurements and the transport model/hydro-simulations at $\sqrt{s_{\rm NN}} \approx 20 - 30$ GeV, cannot be explained through the critical behavior of chiral phase transition and deconfinement.
- The phase structures near above T_c are rich and can be possibly unified by a quark pairing gap (No first order phase transition though).

Is there something new in heavy ion collisions at $\sqrt{s_{\rm NN}} \approx 20 - 30$ GeV?

The answer is possibly in the phases in the temperature region $T \in [T_c, 3T_c]$.

Thank you!